## **Mode competition in a dual-mode quantum-dot semiconductor microlaser**

Laurent Chusseau\*

*Institut d'Électronique du Sud, UMR CNRS 5214, Université Montpellier 2, F-34095 Montpellier, France* 

Fabrice Philippe

*LIRMM, UMR CNRS 5506, 161 Rue Ada, F-34392 Montpellier, France*

Pierre Viktorovitch and Xavier Letartre

*Institut des Nanotechnologies de Lyon, UMR CNRS 5270, Ecole Centrale de Lyon, 36 Avenue Guy de Collongue, F-69134 Ecully, France ´* (Received 1 February 2013; published 18 July 2013)

This paper describes the modeling of quantum-dot lasers with the aim of assessing the conditions for stable cw dual-mode operation when the mode separation lies in the THz range. Several possible models suited for InAs quantum dots in InP barriers are analytically evaluated, in particular quantum dots electrically coupled through a direct exchange of excitation by the wetting layer or quantum dots optically coupled through the homogeneous broadening of their optical gain. A stable dual-mode regime is shown possible in all cases when quantum dots are used as active layer whereas a gain medium of quantum well or bulk type inevitably leads to bistable behavior. The choice of a quantum-dot gain medium perfectly matched the production of dual-mode lasers devoted to THz generation by photomixing.

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CW THz and millimeter wave generation using the beating frequency of a dual-mode laser is a topic that has attracted many efforts recently because there is currently a lack of versatile and easy to use sources at these frequencies [\[1\]](#page-4-0). To fill this gap, THz photomixing has many strengths, including the use of photonic technologies as the manufacture of laser diodes and complex optical systems or the ability to carry the beat on optical fiber.

Regarding the optical beat source, several alternatives were considered. The easiest way is to combine two independent lasers that are stabilized in frequency and amplitude. This solution is now commercial but it is expensive because of the use of extended cavity lasers or high power DFB requiring sophisticated control electronics [\[2\]](#page-4-0). In addition, the use of two independent lasers implies that the drift and noise are added in the beat noise which directly blames the linewidth of the THz signal generated.

The use of a single laser operating in the dual-mode regime seems the most simple and should allow an improvement of compactness. Several techniques have been proposed. First with the advent of manufacturing technologies, semiconductor lasers can now include a complex structure with multiple DFB sections operating independently [\[3\]](#page-4-0). The fact that the two modes inject each other can lead to instabilities and may seriously decrease the tuning range. Modal competition is thus at the heart of the stable dual-mode operation of a single semiconductor laser. Since the seminal work of Lamb [\[4\]](#page-4-0), it is well known that even if the laser cavity allows two simultaneous modes with similar quality factors and gains, the gain medium nonlinearity sometimes forbids their simultaneous emission. A stable dual-mode operation is only allowed when the coupling factor between the modes is not strong enough. Otherwise the laser operates in a bistable regime where one mode dominates.

Although unwanted, this case unfortunately happens with gain media such as bulk semiconductor or quantum wells (QWs). The physical origin is the short conduction-band intraband relaxation time that couples two adjacent modes sharing the same carrier population. Following Agrawal [\[5\]](#page-4-0) and taking an intraband relaxation time below 100 fs which is consistent with InGaAsP active layers [\[6\]](#page-4-0) or InGaAs QW [\[7\]](#page-4-0), it is not possible to have a stable dual-mode operation if the two modes are separated by less than 1.6 THz. This result was also obtained from rate equations [\[8,9\]](#page-4-0). Such an estimation moreover neglects the spatial hole burning which is also involved in the coupling of longitudinal modes [\[10\]](#page-4-0). This leaves no chance for a semiconductor laser emitting at 1*.*55*μ*m and incorporating a bulk or QW gain medium to operate dual mode with a beating of  $\approx$ 1 THz.

Our dual-mode laser design involving a slow optical Bloch mode in a vertical cavity structure is supposed to operate dual mode [\[11\]](#page-4-0). It is thus mandatory to incorporate a gain medium that will not preclude the laser to operate in the desired regime because of a too strong coupling between modes. As shown by the above analysis, this coupling is mainly due to the sharing of the same carrier populations in the conduction band by the two adjacent modes. The proposed solution to circumvent this strong coupling is to use InAs quantum dots (QDs). In fact dual-mode lasing has already been reported in InAs QD lasers owing to the simultaneous emission of the fundamental and excited QD states [\[12,13\]](#page-4-0), or using a QD mode-locked laser with a selectively filtered external cavity [\[14\]](#page-4-0), and recently owing to optical injection in a DFB [\[15\]](#page-4-0). It has however not yet been reported for a cw monolithic dual-mode QD laser. Our purpose is then to verify analytically that either optical or electrical coupling between InAs dot populations will remain at a sufficiently low level to maintain a possible cw dual-mode operation.

A semiconductor laser with a bulk or QW gain medium is generally described on the basis of population rate evolutions. Ahmed and Yamada [\[16\]](#page-4-0) gave a fairly complete version of

<sup>\*</sup>chusseau@univ-montp2.fr

it from which one has extracted the equations describing a dual-mode laser,

$$
\frac{dN}{dt} = \kappa - \frac{N}{\tau_n} - A_1 N S_1 - A_2 N S_2, \tag{1a}
$$

$$
\frac{dS_1}{dt} = -\frac{S_1}{\tau_{p_1}} + A_1 N S_1 (1 - \varepsilon_1 S_1 - \vartheta_1 S_2), \quad (1b)
$$

$$
\frac{dS_2}{dt} = -\frac{S_2}{\tau_{p_2}} + A_2 N S_2 (1 - \varepsilon_2 S_2 - \vartheta_2 S_1), \qquad (1c)
$$

where *N* is the carrier density beyond transparency,  $\kappa =$ *I/*(eV) is the pumping parameter which depends directly on the injection current *I* and volume of the active region *V*,  $\tau_n$  is the carrier recombination time, the  $A_i$  are the differential gains and  $S_i$  the corresponding photon densities,  $\tau_{pi}$  are the photon lifetime within the cavity, and  $\varepsilon_i$  and  $\vartheta_i$  the self-saturation and cross-saturation coefficients of optical gains. Indices *i* refer of course to the various modes allowed by the cold cavity here reduced to 2. Considering a laser running well above threshold, it is possible to neglect the nonradiative recombination of carriers  $N/\tau_n$  whose contribution can be seen as implicitly counted by a slight modification of the *κ* values which reflects the pumping threshold. In  $(1)$  the coupling terms between the two modes appear explicitly with  $\vartheta_i$ . As their physical origin is similar to the self-saturation gain terms  $\varepsilon_i$ , these must also be taken into account. The omission of the correction factor  $(1 - \varepsilon_i S_i - \vartheta_i S_j)$  in the carrier equation however provides a great simplification of the calculations at the cost of a very weak approximation.

Among the stationary solutions of  $(1)$ , only the one with simultaneous nonzero optical intensities  $S_1$  and  $S_2$  corresponds to a dual-mode laser, others accounting instead of bistable operations. In the general case these solutions are quite complicated but in the simplistic case where the two modes are close to the gain maximum with a spacing small enough that their parameters are very similar, then  $\varepsilon_i = \varepsilon$ ,  $\vartheta_i = \vartheta$ ,  $\tau_{p_i} =$  $\tau_p$ , and  $A_i = A$  yielding  $S_1^{\text{ss}} = S_2^{\text{ss}} = \frac{\kappa}{2u}$  and  $N^{\text{ss}} = \frac{\mu}{A}$ , with  $u = \frac{1}{2}(\varepsilon + \vartheta)\kappa + \frac{1}{\tau_p}$ .

The stability is further demonstrated by introducing perturbations to the stationary values,  $N = N^{ss} + n$ ,  $S_i = S_i^{ss} + s_i$ , and keeping only the first order. One obtains a matrix system of three coupled first-order differential equations. The dual-mode stationary solution is then stable if the variations diminish over time, which is true if and only if the real parts of the eigenvalues of the corresponding matrix  $\Theta$  are negative. Here we have

$$
\Theta = \begin{pmatrix} -\frac{A\kappa}{u} & -u & -u \\ \frac{A\kappa}{2u^2\tau_p} & -\frac{\varepsilon\kappa}{2} & -\frac{\vartheta\kappa}{2} \\ \frac{A\kappa}{2u^2\tau_p} & -\frac{\vartheta\kappa}{2} & -\frac{\varepsilon\kappa}{2} \end{pmatrix},
$$
(2)

whose characteristic polynomial is

$$
\mathcal{P}(\lambda) = -\left(\lambda - \frac{\kappa}{2}(\vartheta - \varepsilon)\right) \times \left[\lambda^2 + \lambda \left(\frac{A\kappa}{u} + \frac{\kappa}{2}(\vartheta + \varepsilon)\right) + A\kappa\right].
$$
 (3)

The eigenvalues of  $\Theta$  are the roots of  $\mathcal{P}$ , that is,  $\frac{\kappa}{2}(\vartheta - \varepsilon)$  and the two roots of the polynomial of degree two on the right side. With our choice of parameters, all coefficients of this polynomial are positive and then the two roots always exhibit negative real parts. Consequently, the dual-mode stationary solution proposed is stable if and only if  $\vartheta < \varepsilon$ .

The self-saturation and cross-saturation coefficients can be calculated using a quantum model of the gain medium and the result shows typically  $\vartheta \approx 4\varepsilon/3$  [\[5,16,17\]](#page-4-0). In such conditions, natural dual-mode behavior is not expected from a semiconductor laser, unless strong dispersive effects are included in the cavity to remove the condition assumed here:  $\varepsilon_i = \varepsilon$ ,  $\vartheta_i = \vartheta$ ,  $\tau_{p_i} = \tau_p$ , and  $A_i = A$ . This was recently confirmed by numerical simulations of the dynamics of semiconductor lasers made with typical parameters from GaAs or InGaAsP technologies. Bistable or highly multimode behaviors with a total power usually spread over a large number of modes is always obtained, in perfect agreement with experiments [\[9,16\]](#page-4-0). This result was also confirmed with a Maxwellian modeling of laser [\[18\]](#page-4-0) which only shows possibilities of either a single-mode behavior (actually bistable), or highly multimode. Both our analysis and these works indicate that dual-mode semiconductor lasers are of course totally unexpected!

A similar extensive stability analysis was proposed in [\[19\]](#page-4-0) to account for the degeneracy of the two polarization modes of VCSELs. This question is mathematically analogous to our problem invoking two longitudinal modes except that there is no theoretical estimations of the coupling factor in that case. A fairly comprehensive system behavior was reported, including the polarization switching of a VCSEL based on the gain saturation in a manner consistent with what is observed experimentally.

QD lasers are significantly different. Once created QDs have little interaction with each other and substantially the same ability to capture a free carrier from the barrier. Moreover the total number of active QDs in resonance with a given mode is determined by the construction, and even if this number is high, it remains much smaller than the possible number of states allowed in the conduction band of a bulk or QW gain medium.

Excited QDs are weakly interconnected. At first glance, they cannot directly exchange carriers either within the QD population of the same mode, or between QD populations addressing different modes. Such an exchange of carriers between two QDs must involve first thermionic emission to the barrier and the capture by another QD. This indirect process considerably weakened the coupling as compared to bulk or QW lasers. Given the values of the energy levels at stake in a system with InAs QDs in the InP barrier, we neglect this coupling here.

According to the QDs manufacturing method, a monoatomic InAs wetting layer appearing at the growth interface is often common to all dots. Owing to this layer, the QDs are likely to have a kind of direct electrical coupling that can transfer the excitation of a dot family to another family addressing another mode. The coupling effectiveness obviously depends on the average distance between dots, and on the nature and thickness of this wetting layer which is intimately dependent of the growth technology.

The homogeneous optical linewidth of a single QD is estimated between 3 and 12 meV at room temperature [\[20–24\]](#page-4-0). A direct optical coupling thus necessarily occurs between

Starting from multimode rate equations for semiconductor lasers [\[9\]](#page-4-0) and QD lasers [\[25\]](#page-4-0) we have used the following set of rate equations to encompass all these specific properties of an above-threshold QD dual-mode laser,

$$
\frac{dN_1}{dt} = \alpha_1 \kappa + k_1 N_2 P_1 - k_2 N_1 P_2 - \frac{N_1}{\tau_{n_1}}
$$
  
- A<sub>1</sub>(N<sub>1</sub> - P<sub>1</sub>)(S<sub>1</sub> + \epsilon S<sub>2</sub>), (4a)

$$
\frac{dN_2}{dt} = \alpha_2 \kappa + k_2 N_1 P_2 - k_1 N_2 P_1 - \frac{N_2}{\tau_{n_2}}
$$
  
- A<sub>2</sub>(N<sub>2</sub> - P<sub>2</sub>)(S<sub>2</sub> + \epsilon S<sub>1</sub>), (4b)

$$
\frac{dS_1}{dt} = -\frac{S_1}{\tau_{p_1}} + (A_1(N_1 - P_1) + \epsilon A_2(N_2 - P_2))S_1, \quad (4c)
$$

$$
\frac{dS_2}{dt} = -\frac{S_2}{\tau_{p_2}} + (A_2(N_2 - P_2) + \epsilon A_1(N_1 - P_1))S_2, \quad (4d)
$$

with  $B_i = N_i + P_i$  the total number of QD addressing mode  $i = 1, 2, N_i$  the number of excited dots, and  $P_i$  the number of unexcited dots; *κ* is the total pumping and  $0 \le \alpha_i \le 1$  with  $\alpha_1 + \alpha_2 = 1$  two parameters that reflect the distribution of the pump between the two QD families.  $A_i$ ,  $\tau_{ni}$ , and  $\tau_p$ , account for the modal gains, the carrier lifetime in the excited state and the photon lifetime within the cavity for each of the two modes.  $k_i$  are constants reflecting the exchange rate of the excited states between the two QD families via the InAs wetting layer and  $\epsilon = 1/(1 + \Delta^2)$  accounts for the direct optical coupling between dots families via  $\Delta$  the ratio between the mode energy separation to half the QD homogeneous broadening energy.

*a. Uncoupled quantum dots.* An ideal dual-mode QD laser with two uncoupled dot families perfectly centered on the two optical modes is obtained when  $k_i = \epsilon = 0$ . Corresponding stationary populations are straightforwardly obtained:

$$
N_i^{\rm ss} = \frac{B_i}{2} + \frac{1}{2A_i \tau_{p_i}},\tag{5a}
$$

$$
S_i^{\text{ss}} = \alpha_i \kappa \tau_{p_i} - \frac{1}{2} \left( \frac{B_i \tau_{p_i}}{\tau_{n_i}} + \frac{1}{A_i \tau_{n_i}} \right), \tag{5b}
$$

giving an estimate of threshold pumping  $\kappa_{\text{th}i} = \frac{1}{2\alpha_i \tau_{ni}} (B_i +$  $\frac{1}{A_i \tau_{p_i}}$ ) when  $S_i^{ss} = 0$ . This is in fact the behavior of two completely independent lasers with quantum efficiencies and thresholds evidencing no modal competition. It is easy to verify when inserting  $(5)$  in  $(4)$  that this solution is always stable and that it can be generalized to any number of *n* juxtaposed modes.

*b. Quantum dots electrically coupled through the wetting layer.* Let us now imagine that the two QD populations can interact with each other directly. Presumably an excited dot of the first family can transfer its excitation to an unexcited dot of the second family, the difference in photon energy being supplied or absorbed by the states involved in the wetting layer. This is accounted for by nonzero  $k_i$  constants in  $(4)$  that avoids to introduce a new carrier equation accounting for the population shared via the wetting layer. At the limit where  $k_i \rightarrow \infty$ , the rate of population exchanges is governed by the laws of mass action and a chemical-like equilibrium law is obtained:

In between infinitely coupled QDs and uncoupled QDs the dual-mode stationary solution for carriers and photons can be calculated. If the former is again given by  $(5a)$ , the latter is much more complicated in general. Two special cases are however interesting:

(a) when the coupling induced by the wetting layer are identical  $k_i = k$ .

$$
S_i^{\text{ss}} = \alpha_i \kappa \tau_{p_i} - \frac{1}{2} \left( \frac{B_i \tau_{p_i}}{\tau_{n_i}} + \frac{1}{A_i \tau_{n_i}} + k \tau_{p_i} \left[ \frac{B_j}{A_i \tau_{p_i}} - \frac{B_i}{A_j \tau_{p_j}} \right] \right),\tag{7}
$$

(b) when the material and cavity parameters are identical for the two modes, i.e.,  $A_i = A$ ,  $B_i = B$ ,  $\tau_{p_i} = \tau_p$ ,  $\tau_{n_i} = \tau_n$ .

$$
S_i^{\text{ss}} = \alpha_i \kappa \tau_p - \frac{1}{2} \left( \frac{B \tau_p}{\tau_n} + \frac{1}{A \tau_n} + (k_i - k_j) \tau_p \left[ \frac{1}{2A^2 \tau_p^2} - \frac{B^2}{2} \right] \right),\tag{8}
$$

where  $(i, j) = (1, 2)$  or  $(2, 1)$ . If (a) and (b) occur simultaneously, the situation is similar to that of uncoupled QDs because the  $k_i$  terms cancel out. Except this particular case, the coupling by the wetting layer induces a differentiation between the thresholds of the two laser modes obtained here by setting  $S_i = 0$  in (7) and (8). The mode that had the lowest threshold before coupling feeds its population of excited QDs by direct transfer from the one who had the highest threshold, the result is an amplification of the threshold difference. In the extreme case where the constants  $k_i$  are large, one population of excited QDs will reach first its stimulated emission threshold and will therefore clamp. In turn, the other population of excited QDs saturates and the second mode will never reach its own threshold. Strong coupling between QD populations thus leads to the destruction of the two-mode regime.

When the coupling is moderate, the dual-mode laser regime is preserved and the stability analysis is conducted. Reporting the stationary values plus a fluctuation in (4) yields the evolution matrix of these fluctuations:

$$
\mathbf{\Theta} = \begin{pmatrix} -\sigma_1 & \zeta_2 & -\delta_1 & 0 \\ \zeta_1 & -\sigma_2 & 0 & -\delta_2 \\ \beta_1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \end{pmatrix}, \tag{9}
$$

with

$$
\beta_1 = 2A_1 S_1^{ss} \quad \beta_2 = 2A_2 S_2^{ss}, \n\zeta_1 = k_1 N_2^{ss} + k_2 P_2^{ss} \quad \zeta_2 = k_2 N_1^{ss} + k_1 P_1^{ss}, \n\delta_1 = A_1 (N_1^{ss} - P_1^{ss}) \quad \delta_2 = A_2 (N_2^{ss} - P_2^{ss}), \n\sigma_1 = \beta_1 + \zeta_1 + 1/\tau_{n_1} \quad \sigma_2 = \beta_2 + \zeta_2 + 1/\tau_{n_2},
$$

which are all positive terms since the population inversion above the threshold requires  $N_i^{ss} > P_i^{ss}$ .

Fluctuations may return to equilibrium if and only if the eigenvalues  $\lambda$  of  $\Theta$  have negative real part. These eigenvalues are the roots of the characteristic polynomial,

$$
\mathcal{P}(\lambda) = \lambda^4 + (\sigma_1 + \sigma_2)\lambda^3 + (\pi_1 + \pi_2 + \sigma_1\sigma_2 - \zeta_1\zeta_2)\lambda^2 + (\pi_1\sigma_2 + \pi_2\sigma_1)\lambda + \pi_1\pi_2,
$$
 (10)

with  $\pi_i = \beta_i \delta_i$ .

According to the Routh-Hurwitz criterion, all roots of the fourth-order polynomial  $\mathcal{P}(\lambda) = \sum_{i=0}^{4} a_i \lambda^i$  have their real part negative if and only if  $a_i > 0$   $\overline{\forall i}$  and  $r = a_2 a_3 - a_1 a_4$  $a_3^2 a_0/a_1 > 0$ . The first condition is ensured because  $\sigma_i > \zeta_i$ and the second yields *r* as

$$
r = (\sigma_1 + \sigma_2)(\sigma_1 \sigma_2 - \zeta_1 \zeta_2) + \sigma_1 \sigma_2 \frac{(\pi_1 - \pi_2)^2}{\pi_1 \sigma_2 + \pi_2 \sigma_1}.
$$
 (11)

Noting that  $\sigma_i > \zeta_i$  this difference is always positive. It proves that the coupling of carriers by a chemical-like equilibrium law between the two QD populations does not prohibit a stable dual-mode behavior in the sense of modal competition, even if it is likely to push the threshold up to an unacceptable value for one of the lasing mode if too strong.

*c. Quantum dots optically coupled through the homogeneous linewidth.*We now consider the two QD families coupled only through the homogeneous gain width of each dot. The  $\epsilon$ value is calculated from the homogeneous gain width and thus ranges from  $\approx 10\%$  to 60% for a 1 THz frequency separation, depending on whether one considers a broadening of 3 meV or 10 meV.

In the general case the expression of stationary solutions is complex, but in the particular case of two populations having the same QD parameters  $\tau_{ni} = \tau_n$ ,  $\tau_{pi} = \tau_p$ ,  $A_i = A$ ,  $B_i = B$ , and identical pumps  $\alpha_i = 1/2$ , they are

$$
N_i^{\text{ss}} = \frac{B}{2} + \frac{1}{2A(1+\epsilon)\tau_p},\tag{12a}
$$

$$
S_i^{\rm ss} = \frac{\kappa \tau_p}{2} - \frac{B \tau_p}{2\tau_n} - \frac{1}{2A(1+\epsilon)\tau_n}.
$$
 (12b)

Like previously, the stability analysis of the dual-mode regime has been conducted in the general case. Again this stability is governed by the sign of the real parts of the eigenvalues of the evolution matrix,

$$
\Theta = \begin{pmatrix}\n-\sigma_1 & 0 & -\delta_1 & -\epsilon \delta_1 \\
0 & -\sigma_2 & -\epsilon \delta_2 & -\delta_2 \\
\beta_1 & \xi_2 & 0 & 0 \\
\xi_1 & \beta_2 & 0 & 0\n\end{pmatrix}.
$$
\n(13)

All the terms of this matrix were chosen positive,

$$
\beta_1 = 2A_1 S_1^{ss} \quad \beta_2 = 2A_2 S_2^{ss},
$$
  
\n
$$
\xi_1 = 2\epsilon A_1 S_2^{ss} \quad \xi_2 = 2\epsilon A_2 S_1^{ss},
$$
  
\n
$$
\delta_1 = A_1 (N_1^{ss} - P_1^{ss}) \quad \delta_2 = A_2 (N_2^{ss} - P_2^{ss}),
$$
  
\n
$$
\sigma_1 = \beta_1 + \xi_1 + 1/\tau_{n1} \quad \sigma_2 = \beta_2 + \xi_2 + 1/\tau_{n2},
$$

and the eigenvalues are the roots of the following characteristic polynomial [assuming  $\pi_i = \delta_i(\epsilon \xi_i + \beta_i)$  for further simplification],

$$
\mathcal{P}(\lambda) = \lambda^4 + (\sigma_1 + \sigma_2)\lambda^3 + (\sigma_1\sigma_2 + \pi_1 + \pi_2)\lambda^2 \n+ (\pi_1\sigma_2 + \pi_2\sigma_1)\lambda + (1 - \epsilon^2)\delta_1\delta_2(\beta_1\beta_2 - \xi_1\xi_2).
$$
 (14)

As  $\xi_1 \xi_2 = \epsilon^2 \beta_1 \beta_2$ , the constant term can be written more simply as  $c = (1 - \epsilon^2)^2 \delta_1 \delta_2 \beta_1 \beta_2$ , and all the coefficients of  $P(\lambda)$  are then positive since  $0 < \epsilon < 1$ . According to the Routh-Hurwitz criterion,  $P(\lambda)$  roots exhibit negative real parts if and only if  $r > 0$ ,

$$
r = (\sigma_1 + \sigma_2)\sigma_1\sigma_2 + (\pi_1\sigma_1 + \pi_2\sigma_2) - \frac{(\sigma_1 + \sigma_2)^2c}{\pi_1\sigma_2 + \pi_2\sigma_1}.
$$
 (15)

Notice that *r* is an increasing function of  $\epsilon$  on [0,1] as a sum of functions of such type. The required proof is thus obtained by just checking if the value at  $\epsilon = 0$  is positive. At  $\epsilon = 0$ ,  $c = \pi_1 \pi_2$  thus allows one to write as in (11)

$$
(\pi_1\sigma_1 + \pi_2\sigma_2) - \frac{(\sigma_1 + \sigma_2)^2 c}{\pi_1\sigma_2 + \pi_2\sigma_1} = \sigma_1\sigma_2 \frac{(\pi_1 - \pi_2)^2}{\pi_1\sigma_2 + \pi_2\sigma_1}.
$$

This proves that *r >* 0 and concludes again to the stability of the dual-mode regime in that case. It compares favorably with experiments involving edge-emitting QD lasers which have shown narrower spectra at ambient temperature than at liquid nitrogen temperature but always multimode [\[25\]](#page-4-0).

If  $\epsilon \to 1$  we have  $c \to 0$  and one eigenvalue becomes null while the real part of all others remains strictly negative: The two-mode regime is thus marginally stable. This is a limit case that is far from the scenario we have chosen for THz radiation by photomixing with an energy gap of 4.1 meV between modes and QD homogeneous linewidths between 3 and 10 meV.

We studied different semiconductor laser types devoted to operate continuously in stable dual-mode emission. The proposed analysis is based on the rate equations and Lamb's theory. After considering the case of laser with bulk or QW gain mediums for which it is demonstrated that the natural behavior is bistable and not dual mode, we evaluated analytically what should be expected from QD semiconductor lasers. We have considered uncoupled as well as coupled QDs either electrically through a direct exchange of excitation by the wetting layer, or optically through the homogeneous broadening of the gain. In all cases we have shown analytically that a stable dual-mode emission is possible. If the objective of building a dual-mode semiconductor laser producing a stable beating frequency in the THz range is highlighted, the selection of a QD as gain medium is the most suitable as compared to bulk or QWs that inevitably lead to a bistable behavior. The incorporation of QDs in the active membranes of future photonic crystal optoelectronic components for the 2.5D THz radiation by photomixing is on the way [\[11\]](#page-4-0).

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- <span id="page-4-0"></span>[1] M. Tonouchi, [Nature Photonics](http://dx.doi.org/10.1038/nphoton.2007.3) **1**, 97 (2007).
- [2] M. Wingender, E. A. Michael, B. Vowinkel, and R. Schieder, [Opt. Commun.](http://dx.doi.org/10.1016/S0030-4018(03)01174-X) **217**, 369 (2003).
- [3] N. Kim, J. Shin, E. Sim, C. W. Lee, D.-S. Yee, M. Y. Jeon, Y. Jang, and K. H. Park, Opt. Express **17**[, 13851 \(2009\).](http://dx.doi.org/10.1364/OE.17.013851)
- [4] M. Sargent III, M. O. Scully, and W. Lamb, *Laser Physics* (Addison-Wesley, Reading, 1974) .
- [5] G. P. Agrawal, [IEEE J. Quantum Electron.](http://dx.doi.org/10.1109/JQE.1987.1073406) **23**, 860 (1987).
- [6] M. Yamada, [IEEE J. Quantum Electron.](http://dx.doi.org/10.1109/JQE.1983.1072059) **19**, 1365 (1983).
- [7] G. Debaiseux, M. Guemmouri, S. Chelles, A. Ougazzaden, G. Hervé-Gruyer, M. Filoche, and J.-Y. Marzin, [IEEE Photonics](http://dx.doi.org/10.1109/68.634713) Technol. Lett. **9**[, 1475 \(1997\).](http://dx.doi.org/10.1109/68.634713)
- [8] L. Chusseau, J. Arnaud, and F. Philippe, [Opt. Spectroscopy](http://dx.doi.org/10.1134/1.1576846) **94**, [746 \(2003\).](http://dx.doi.org/10.1134/1.1576846)
- [9] M. Ahmed, Physica D **176**[, 212 \(2003\).](http://dx.doi.org/10.1016/S0167-2789(02)00746-7)
- [10] J. F. Lepage and N. McCarthy, [Appl. Opt.](http://dx.doi.org/10.1364/AO.41.004347) **41**, 4347 [\(2002\).](http://dx.doi.org/10.1364/AO.41.004347)
- [11] K. Kusiaku, O. Le Daif, J.-L. Leclercq, P. Rojo-Romeo, C. Seassal, P. Viktorovitch, T. Benyattou, and X. Letartre, [Opt.](http://dx.doi.org/10.1364/OE.19.015255) Express **19**[, 15255 \(2011\).](http://dx.doi.org/10.1364/OE.19.015255)
- [12] A. Markus, J. X. Chen, C. Paranthoën, A. Fiore, C. Platz, and O. Gauthier-Lafaye, [Appl. Phys. Lett.](http://dx.doi.org/10.1063/1.1563742) **82**, 1818 (2003).
- [13] F. Grillot, N. A. Naderi, J. B. Wright, R. Raghunathan, M. T. Crowley, and L. F. Lester, [Appl. Phys. Lett.](http://dx.doi.org/10.1063/1.3667193) **99**, 231110 [\(2011\).](http://dx.doi.org/10.1063/1.3667193)
- [14] Z. Jiao, J. Liu, Z. Lu, X. Zhang, P. J. Poole, P. J. Barrios, D. Poitras, and J. Caballero, [IEEE Photonics Technol. Lett.](http://dx.doi.org/10.1109/LPT.2011.2182642) **24**, [518 \(2012\).](http://dx.doi.org/10.1109/LPT.2011.2182642)
- [15] A. Hurtado, I. D. Henning, M. J. Adams, and L. F. Lester, [Appl.](http://dx.doi.org/10.1063/1.4807759) Phys. Lett. **102**[, 201117 \(2013\).](http://dx.doi.org/10.1063/1.4807759)
- [16] M. Ahmed and M. Yamada, [IEEE J. Quantum Electron.](http://dx.doi.org/10.1109/JQE.2002.1005419) **38**, 682 [\(2002\).](http://dx.doi.org/10.1109/JQE.2002.1005419)
- [17] M. Yamada, [IEEE J. Quantum Electron.](http://dx.doi.org/10.1109/JQE.1986.1073087) **22**, 1052 (1986).
- [18] C. Serrat and C. Masoller, Phys. Rev. A **73**[, 043812 \(2006\).](http://dx.doi.org/10.1103/PhysRevA.73.043812)
- [19] J. Albert, G. Van der Sande, B. Nagler, K. Panajotov, I. Veretennicoff, J. Danckaert, and T. Erneux, [Opt. Commun.](http://dx.doi.org/10.1016/j.optcom.2004.12.024) **248**[, 527 \(2005\).](http://dx.doi.org/10.1016/j.optcom.2004.12.024)
- [20] D. Gammon, E. S. Snow, B. V. Shanabrook, D. S. Katzer, and D. Park, Science **273**[, 87 \(1996\).](http://dx.doi.org/10.1126/science.273.5271.87)
- [21] P. Borri, W. Langbein, J. Mørk, J. M. Hvam, F. Heinrichsdorff, M.-H. Mao, and D. Bimberg, Phys. Rev. B **60**[, 7784 \(1999\).](http://dx.doi.org/10.1103/PhysRevB.60.7784)
- [22] K. Matsuda, K. Ikeda, T. Saiki, H. Tsuchiya, H. Saito, and K. Nishi, Phys. Rev. B **63**[, 121304\(R\) \(2001\).](http://dx.doi.org/10.1103/PhysRevB.63.121304)
- [23] C. Kammerer, G. Cassabois, C. Voisin, M. Perrin, C. Delalande, P. Roussignol, and J. M. Gérard, [Appl. Phys. Lett.](http://dx.doi.org/10.1063/1.1510158) 81, 2737 [\(2002\).](http://dx.doi.org/10.1063/1.1510158)
- [24] M. K. Bafna, P. Sen, and P. K. Sen, [J. Appl. Phys.](http://dx.doi.org/10.1063/1.2387054) **100**, 103515 [\(2006\).](http://dx.doi.org/10.1063/1.2387054)
- [25] M. Sugawara, K. Mukai, Y. Nakata, H. Ishikawa, and A. Sakamoto, Phys. Rev. B **61**[, 7595 \(2000\).](http://dx.doi.org/10.1103/PhysRevB.61.7595)