Revisiting Bohr's principle of complementarity with a quantum device

Jian-Shun Tang, Yu-Long Li, Chuan-Feng Li,* and Guang-Can Guo
Key Laboratory of Quantum Information, University of Science and Technology of China, CAS, Hefei 230026, China
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Bohr's principle of complementarity (BPC) is the cornerstone of quantum mechanics. According to this principle, the total wavelike and particlelike information of a particle is limited by the Englert-Greenberger (EG) duality relation. Here, by introducing a quantum detecting device into the experiment, we find that the limit of the EG duality relation is exceeded because of the interference between the wave and particle properties of the photon. A generalized EG duality relation is further developed. Our work provides a generalization of BPC and gives new insights into quantum mechanics.

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Introduction. Bohr's principle of complementarity (BPC) has been the cornerstone of quantum theory since it was proposed in 1928 [1,2]. This principle states that some physical objects have multiple properties, but the exhibition of these properties depends on what type of exclusive detecting devices are used. One well-known example is the wave-particle duality considered for a single particle in a two-way interferometer [3]. One can choose to observe the wavelike or particlelike behaviors of the particle through different detection arrangements. Interference fringes have been observed for massive particles, such as neutrons [4], electrons [5], atoms [6,7], and molecules [8]; these species were all previously thought to only be particlelike. These observations show the unfamiliar wavelike side of these particles. In the case of light, both the antibunching effect and interference fringes, which are associated with the particlelike and wavelike properties, respectively, have been previously demonstrated [9–11].

In addition to these all-or-nothing situations, some intermediate stages actually exist [12–16], in which the which-path information corresponding to the particlelike property is partially detected. This detection then results in a reduced interference visibility. This issue was first discussed by Wootters and Zurek in 1979 [12]. Later, Greenberger and Yasin found this phenomenon in an analysis of some unbalanced neutron interferometry experiments [17]. Consistent conclusions were then derived by Jaeger *et al.* in 1995 [18] and Englert in 1996 [19] independently. They derived the inequality

$$V^2 + D^2 \leqslant 1, \tag{1}$$

where V is the visibility of the interference fringes and D is the path distinguishability of the particle, which stands for the available which-path information of the system. This inequality is also known as the Englert-Greenberger (EG) duality relation. Many experiments have demonstrated this inequality with atoms [20], nuclear magnetic resonance [21,22], a faint laser [23], and single photons in a delayed-choice scheme [24]. Recently, this duality relation has been extended to the more general case of an asymmetric interferometer in which only a single output port is considered, and this inequality still holds [25].

One of the most efficient quantum systems for tests of the BPC is a single photon in a Mach-Zehnder interferometer (MZI) [24]. For example, if the second beam splitter (BS) in the MZI is replaced by a series of unbalanced BSs, the visibility and path distinguishability will always satisfy the EG duality relation. Here, we notice that all beam splitters of this type are classical devices. What will happen when this BS is replaced by a quantum BS (q-BS) [26,27]? The q-BS was proposed by Ionicioiu *et al.* and was implemented in our previous work [28]. In the language of q-BS, the unbalanced BSs used in the previous experiment can be described by a set of eigenstates. The same results will be found if the q-BS is selected to collapse onto these eigenstates.

In our experiment, the q-BS is in a quantum superposition state of two eigenstates, which are denoted as $|a\rangle$ ($R = 0$) and $|p\rangle$ ($R = 0.5$) and correspond to the absence and presence of a balanced BS, respectively. We place this q-BS in the MZI, and not only the eigenstates but also the quantum superposition states of the q-BS are selected as the bases for collapse during detection. We find that the EG duality relation is exceeded if a certain detection basis for the q-BS is chosen. We also develop a method to generalize the EG relation in this Brief Report.

Theoretical framework. The experimental setup is sketched in Fig. 1(a). The single photons are split by a 50 : 50 BS into two paths, followed by a phase shift of φ, and are then recombined with a q-BS. The use of the q-BS is the primary difference between this setup and a regular MZI.

As discussed in Ref. [25], we need to derive the photon state after the q-BS and know the probabilities of each path taken by the photon, in order to calculate the visibility. The state of the q-BS is $|\text{q-BS}\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |p\rangle)$; hence, we derive the photon state (before the q-BS state is detected) as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\text{particle}\rangle|a\rangle + \frac{1}{\sqrt{2}}|\text{wave}\rangle|p\rangle \tag{2}$$

according to Ref. [26], where $|\text{particle}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\varphi}|2\rangle)$ corresponds to the particle state, and $|\text{wave}\rangle = e^{i\frac{\varphi}{2}}(\cos\frac{\varphi}{2}|1\rangle e^{i\delta_1} - i\sin\frac{\varphi}{2}|2\rangle e^{i\delta_2})$ corresponds to the wave state. δ_1 and δ_2 are two additional constant phases, which can be adjusted in the experiment. In this experiment, δ_1 and δ_2 are both set to be 0. The q-BS state is then collapsed onto an arbitrary basis $|b\rangle = \sin\beta|a\rangle + \cos\beta|p\rangle$; therefore the photon state becomes

$$\rho = \tilde{\rho}/\mathrm{Tr}(\tilde{\rho}), \tag{3}$$

*cfli@ustc.edu.cn

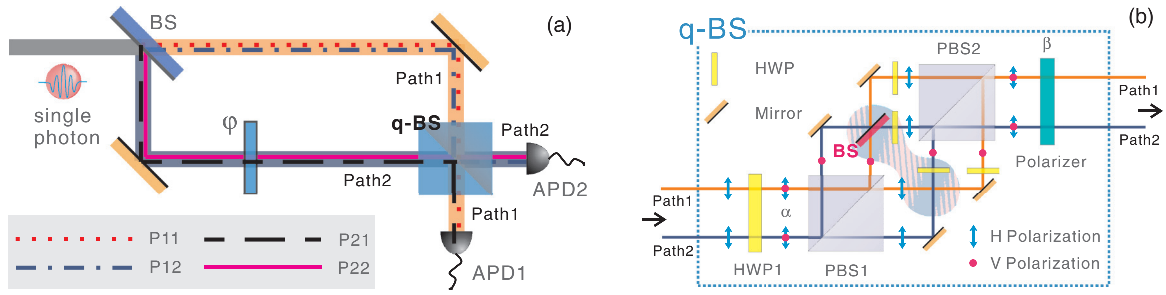

FIG. 1. (Color online) (a) The MZI with a q-BS. The primary difference between this setup and a regular MZI is that the second BS is replaced with a q-BS. P_{ij} $(i,j = 1,2)$ are the four possible subpaths for the single photon used to define the distinguishability D. (b) The simplified setup of the q-BS. Path 1 and Path 2 are each divided into two components, which are in quantum superposition states. Each component corresponds to an eigenstate of the photon polarization. One component constructs the closed MZI (a BS is present), and the other constructs the open MZI (no BS). PBS2 then recombines these two components and forms a quantum superposition state of the closed and open MZIs. The direction of the photon polarization before PBS1, α, controls the states of q-BS. The polarizer with a β-oriented axis selects the detection basis for the q-BS.

where $\tilde{\rho} = \mathrm{Tr}_{\text{q-BS}}(P_b|\psi\rangle\langle\psi|)$ with $P_b = |b\rangle\langle b|$ as the projection operator. Here, we derive the probability that the photon takes Path 2 as

$$p_2(\varphi) = \mathrm{Tr}(|2\rangle\langle 2|\rho). \tag{4}$$

From this probability, we have the visibility of Path 2,

$$V = \frac{p_{\max} - p_{\min}}{p_{\max} + p_{\min}}. \tag{5}$$

As shown in Fig. 1(a), each photon has four possible subpaths from the first BS to the single-photon avalanche photodiodes (APDs) (P11: the photon passes through both the first BS and the q-BS; P12: the photon passes through the first BS and then is reflected by the q-BS; P21: the photon is reflected by both the first BS and the q-BS; P22: the photon is reflected by the first BS and then passes through the q-BS). When a photon is finally found to appear in Path 2, it can come from the first BS to APD2 through either subpath P12 or P22. Assuming that this photon has the probabilities w_{12} and w_{22} for P12 and P22, respectively, the distinguishability of Path 2 can be written as

$$D = |w_{12} - w_{22}|. \tag{6}$$

If the photons definitely come from P12 or P22, then $D = 1$; if the chance that the photons come from either of the two paths is equal, $D = 0$. The same definitions of V and D are also found in Ref. [25], in which the inequality (1) is shown to be correct for a general situation with classical unbalanced beam splitters.

Experimental demonstration and results. The single photons are generated from a single InAs/GaAs quantum dot, and $g^{(2)}(0)$ is measured to be 0.167 ± 0.062 here [28,29]. The q-BS in our experiment is realized by using the polarization state of the photon as an ancilla to control the absence or presence of the BS. A simplified setup for the q-BS is shown in Fig. 1(b). The photon polarization for either path is first rotated by HWP1 (half-wave plate) in the direction of α, which corresponds to the q-BS state of $|\text{q-BS}\rangle = \sin\alpha|a\rangle + \cos\alpha|p\rangle$. For this experiment, we fix this angle at $\alpha = 45^\circ$. The photons are then split by PBS1 (polarizing beam splitter) into two components. In one direction, the photons pass through a closed MZI with a 50:50 BS; in the other direction, the photons pass through an open MZI with no BS. The two components are then recombined by PBS2, and the photon state at that point is exactly described by Eq. (2) with $|a\rangle \leftrightarrow |V\rangle$ and $|p\rangle \leftrightarrow |H\rangle$. Note that $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarization states of the photons, respectively. The polarizer set at the angle β chooses the detection basis for the q-BS.

To measure the visibility, we leave both paths in Fig. 1(a) unblocked, count the number of photons detected by the APDs, and then calculate the probability that the photon takes Path 2, i.e., $p_2(\varphi)$. The results are shown in Fig. 2. The solid lines are the theoretical fits corresponding to each set of experimental data. Figure 2(a) is the $\beta = 0$ case, in which the q-BS state is detected in the basis of $|b\rangle = |p\rangle$, which is the eigenstate associated with the closed MZI. Therefore, the photons behave as a wave, and the visibility [shown in Fig. 3(a)] of the interference fringe reaches 0.961 ± 0.004. This result coincides with the result for the classical-BS experiment found in Ref. [24]. Figure 2(c) corresponds to the case $\beta = \frac{\pi}{2}$. Similarly, the q-BS state is detected in the other eigenstate $|b\rangle = |a\rangle$, which is associated with the open MZI. Thus, the photons behave as particles. The result is also the same as that shown in the classical BS case.

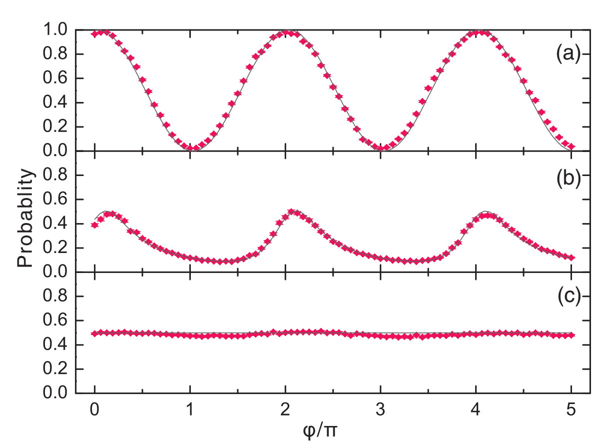

FIG. 2. (Color online) Probability that the photon takes Path 2. (a)–(c) correspond to the cases with $\beta = 0$, $\frac{3\pi}{16}$, and $\frac{\pi}{2}$, respectively. The solid lines are the corresponding theoretical fits for each case.

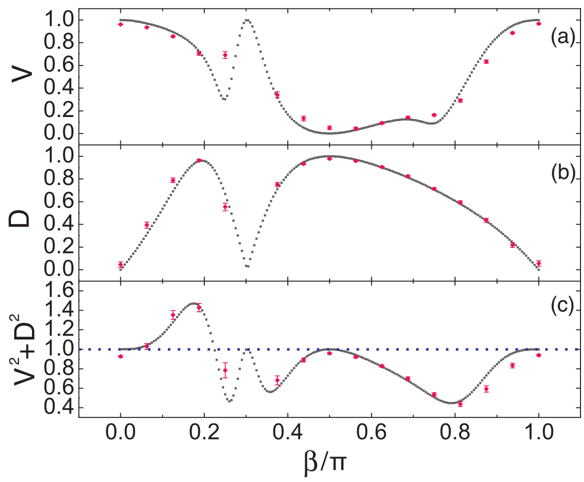

FIG. 3. (Color online) (a) The visibility V, (b) the path distinguishability D, and (c) $V^2 + D^2$. The dashed line in (c) is the limit of the EG duality relation (1), and is exceeded in this situation.

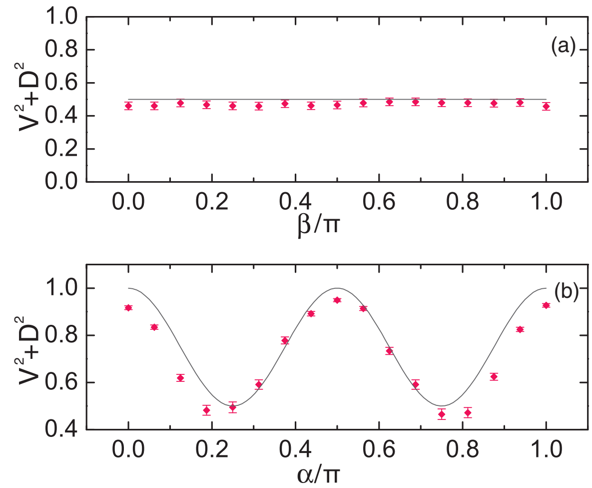

FIG. 4. (Color online) $V_g^2 + D_g^2$ after combination of the photon numbers of two orthogonal-base cases with (a) varying β and fixed $\alpha = \frac{\pi}{4}$ and (b) varying α and arbitrary β. The generalized EG duality relation holds for these results.

However, $\beta = \frac{3\pi}{16}$ for Fig. 2(b), so the detection basis here is a quantum superposition state, which is related to the MZI in both a closed and an opened state. The visibility in this case is 0.707 ± 0.017. The photon behaves as a quantum superposition of a wave and a particle; this behavior is well illustrated by the expression of the photon's state ρ; i.e., $C_1(\sin\beta|\text{particle}\rangle + \cos\beta|\text{wave}\rangle)$ (where C_1 is a coefficient). This phenomenon does not have a counterpart in the classical BS experiment. The differences between the experimental and theoretical values are caused by the counting statistics, imperfections in the optical glasses, the dark and background counts, and tiny instabilities in the MZIs. In particular, we want to note that although δ_1 and δ_2 are set to 0, they will vary slightly in the experiment. In Fig. 2(b), these two phases are fitted to be $\delta_1 = (0.070 \pm 0.040)\pi$ and $\delta_2 = (-0.050 \pm 0.007)\pi$. This difference causes the interference fringes to deviate slightly from the $\delta_1 = \delta_2 = 0$ case, but this difference does not influence the conclusions. In Fig. 3, the gray dotted lines are simulated from these fitted values.

Then, we measure the distinguishability D. We first block Path 1 after the first BS in Fig. 1(a) and count the number of photons detected by APD2. This subpath is P22, and the number is denoted N_{22}. Then, we block Path 2 after the first BS and count the number of photons detected by APD2. This subpath is P12, and the photon number is denoted N_{12}. Thus, we know the detected photons came from P12 and P22 with the probabilities of $\frac{N_{12}}{N_{12}+N_{22}}$ and $\frac{N_{22}}{N_{12}+N_{22}}$, respectively. Hence, the distinguishability of Path 2 can be calculated from $D = \frac{|N_{12}-N_{22}|}{N_{12}+N_{22}}$ according to Eq. (6). The result is shown by the larger dots in Fig. 3(b), and the line of smaller dots is the theoretical simulation. If $\beta = 0$ (the closed MZI), then $D = 0.045 \pm 0.024$ and no which-path information is available. However, if $\beta = \frac{\pi}{2}$ (the open MZI), then $D = 0.97751 \pm 0.0038$ and full which-path information is detected. This result is in agreement with the wavelike and particlelike behaviors of the photons as previously discussed. For these all-or-nothing cases, the q-BS collapses onto the eigenstates; therefore, these situations give the same results as the classical BS experiment: The inequality (1) holds, and the upper bound is reached [see in Fig. 3(c)]. In contrast, in the quantum intermediate case $\beta = \frac{3\pi}{16}$, the value of $V^2 + D^2$ is beyond the limit of the EG duality relation [1, the blue dashed line in Fig. 3(c)] by 10 deviations and is 1.428 ± 0.043. This result coincides with the results from the theoretical simulation.

This exceeding of the EG duality relation is caused by the quantum superposition of the wave and particle states of the photons, i.e., the interference between these two states. This interference is introduced by the q-BS and a quantum intermediate detection basis. To illustrate this point and derive a generalized EG duality relation, we combine the corresponding photon counts of the two orthogonal bases related to β and $\beta + \frac{\pi}{2}$ and then calculate $V_g^2 + D_g^2$ in the same way. The forms of V_g and D_g are the same as V and D, respectively. However, the photon counts and the meanings are different. V_g and D_g correspond to the sum of the counts of two orthogonal bases and describe the behavior of photons in these two cases as a whole; the wave-particle interference becomes an internal effect here. In contrast, V and D describe the behavior of photons in a single detection basis. We find that the generalized inequality ($V_g^2 + D_g^2 \leqslant 1$) holds for our results, as shown in Fig. 4(a). The solid line is the theoretical simulation. To further analyze this combination process, we calculate the final state of the photon after the combination. The final state is found to be $C_2(\sin^2\alpha|\text{particle}\rangle\langle\text{particle}| + \cos^2\alpha|\text{wave}\rangle\langle\text{wave}|)$, where C_2 is a coefficient. This state is a classical mixture of the wave and particle properties and is independent of the chosen orthogonal basis pair (defined by β). However, the state is related to the parameter α, which determines the state of the q-BS and the probabilities of the photon passing through the closed or open MZIs. $V_g^2 + D_g^2$ is calculated to be $\sin^4\alpha + \cos^4\alpha$, which is no larger than 1; when $\alpha = \frac{\pi}{4}$, then $V_g^2 + D_g^2 = 0.5$. We have also measured $V_g^2 + D_g^2$ for various values of α, and the results shown in Fig. 4(b) further prove our previous discussions. There is a systematic error in Fig. 4 that may be caused by the dark and background counts, decoherence processes, imperfections in the optical glasses, and a lack of precision in the experimental parameters.

Discussions. Actually, violation of the BPC, and specifically, violation of the EG duality relation, has been declared by Afshar *et al*. [30], who believe that quantum mechanics is not correct; however, others disagree with this interpretation [31–34], and the debate continues. Here, we must note that our experiment is completely unrelated to the Afshar experiment.

Although our results exceed the EG duality relation, our experiment as a whole is in agreement with quantum theory. Our experiment does not demonstrate that the EG duality relation is wrong. The EG duality relation is definitely correct within the range of classical detecting devices. However, in our experiment, a q-BS is used instead of a classical BS. This BS can remain in the quantum superposition of the two originally exclusive states of a classical BS (e.g., the absence and presence). The q-BS allows the wave and particle properties of the photon to be quantum superimposed (or to interfere); this feature of q-BS demonstrates the advantages of the q-BS over the classical BS and breaks the limitation on information extraction from the wave and particle properties. When we consider both of the orthogonal detection bases, the interference between the wave and particle properties becomes an internal effect. Then a generalized EG duality relation holds.

We emphasize that although we use a q-BS in the experiment, the q-BS is also composed of a series of classical devices since all experimental devices can be only available as classical devices. If one considers every part of our q-BS separately as a classical device and calculates the EG duality relation, he finds that this relation always holds. This approach is certainly reasonable; however, in this approach, our q-BS is not still a "quantum BS," but is a stack of "classical devices." Then, the physical meanings are changed, and our purpose of "exploring the new phenomena with quantum devices" is lost.

Conclusions. In conclusion, we introduced a polarization-controlled q-BS into an MZI, and selected some quantum-superposition states of the q-BS as the detection bases. We find that the limit of the EG duality relation is exceeded. We conclude that this result is caused by the interference between the wave and particle properties of the photons. When we combine the corresponding photon numbers of two mutually orthogonal detection bases of the q-BS, the wave-particle interference becomes an internal effect, and the generalized EG duality relation holds. This work is entirely within standard quantum theory but opens a new way for people to understand the quantum world through the replacement of classical devices with quantum ones.

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