Controlling single-photon transport in waveguides with finite cross section

Jin-Feng Huang (黄金风),[1,2] Tao Shi (石弢),[3] C. P. Sun (孙昌璞),[1,4] and Franco Nori (野理)[1,5]

[1]*Advanced Science Institute, RIKEN, Wako-shi, Saitama 351-0198, Japan*

[2]*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, and University of the Chinese Academy of Sciences, Beijing 100190, China*

[3]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, Germany*

[4]*Beijing Computational Science Research Center, Beijing 100084, China*

[5]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA*

(Received 18 March 2013; published 23 July 2013)

We study the transverse-size effect of a quasi-one-dimensional rectangular waveguide on the single-photon scattering on a two-level system. We calculate the transmission and reflection coefficients for single incident photons using the scattering formalism based on the Lippmann-Schwinger equation. When the transverse size of the waveguide is larger than a critical size, we find that the transverse mode will be involved in the single-photon scattering. Including the coupling to a higher traverse mode, we find that the photon in the lowest channel will be lost into the other channel, corresponding to the other transverse modes, when the input energy is larger than the maximum bound-state energy. Three kinds of resonance phenomena are predicted: single-photon resonance, photonic Feshbach resonance, and cutoff (minimum) frequency resonance. At these resonances, the input photon is completely reflected.

DOI: 10.1103/PhysRevA.88.013836 PACS number(s): 42.50.Ct, 42.50.Pq, 03.65.Nk

I. INTRODUCTION

Current optical communications use electronic switching and thus are limited to electronic speeds of a few gigahertz. To reach much higher speeds, various proposals have been made including optical networks [1], as well as using all-optical routers [2] and switches [3–8]. Also, quantum optical networks were motivated by quantum information (communication), using elements with quantum coherence (such as superposition and entanglement) of photons. Thus the elemental device can be implemented as a generalized cavity QED system: a photon confined to a one-dimensional (1D) waveguide, and controlled by a quantum switch, made of a two (or more) energy-level systems [3–23].

There have been numerous theoretical [3–8,24] and experimental [25,26] studies for such a quantum switch, which could be realized in various physical systems, e.g., a transmission line [6,27–30] coupled to a charge qubit [31–35] and a defect cavity waveguide coupled to a quantum dot [36–38]. Most theoretical studies on these systems are excessively idealized, because the experimental system is never one dimensional.

In order to consider more realistic systems, here we study the finite cross-section effect of the waveguide on the single-photon transport controlled by a two-level system (TLS). We consider the waveguide as a quasi-1D system with a rectangular cross-section. It is well known that if a photon could be perfectly transported in a quasi-1D waveguide, its frequency must be larger than the cutoff frequency of a certain transverse mode. Moreover, to avoid the loss of the photon incident in the lowest transverse mode due to scattering into other modes, people need to make the cross section of the waveguide as small as possible. However, the cross section of realistic waveguides cannot be infinitely small, and a waveguide with a finite cross section would allow the photon transit from one transverse mode to another. Furthermore, if the incident photon frequency is far from the cutoff frequency, such as x ray [39,40], then the different transverse modes would be so close that the incident photon would be inevitably coupled to higher transverse modes. This consideration motivates us to study the incident photon transport in one mode while coupled to another (higher) mode.

We solve the Lippmann-Schwinger equation for calculating the reflection and transmission coefficients of a single photon scattered by a TLS. Since the exact dispersion relation of a photon in a waveguide with finite cross section is more like a quadratic one near the cutoff frequency, quite different from the linear regime, we approximate the exact dispersion relation by a quadratic function of the wave vector of the photon by expanding it to second order in the wave vector. In such a quadratic waveguide, we find that there is a bound state and two quasibound states for each scattering channel defined by a certain transverse mode. We note that this bound state does not exist in the usual linear waveguides.

There are three kinds of resonance phenomena, which correspond to the complete reflection of the photon incident in a given channel. One occurs at the single-photon resonance, namely the incident photon energy is resonant with the TLS without coupling to the higher transverse mode. Once the incident photon couples to the higher transverse mode, this resonance phenomenon is replaced by a photonic Feshbach resonance, namely a complete reflection occurs when the incident energy of the photon equals the bound-state energy of the higher transverse mode. The third type of resonance always occurs at the minimum frequency of the quadratic waveguide, whether or not the singe photon is coupled to a higher transverse mode. This resonance phenomenon is called cutoff-frequency resonance. We also notice that the transverse mode will lead to an incident photon loss as a result of scattering into other higher channels. We also compare in detail the results obtained by the linear and quadratic dispersion relations, respectively.

This paper is organized as follows. In Sec. II, we describe the system and the effective Hamiltonian, including two transverse modes. We also derive the second-order dispersion relation. Then, we calculate the single-photon transport in the higher transverse mode without coupling to the incident mode in Sec. III. We find a bound state and two quasi-bound states [5,41,42] by utilizing the quadratic dispersion relation. In Sec. IV, we obtain the single-photon reflection and transmission coefficients with coupling to the higher transverse mode through the Lippmann-Schwinger equation. The transverse effect in both linear and quadratic waveguides are discussed in Sec. V. Finally, we present our conclusions in Sec. VI.

II. MODEL

The setup under consideration is a waveguide-QED system (see Fig. 1) consisting of a quasi-1D rectangular waveguide with inner dimensions L_x and L_y and a two-level atom. The waveguide supports quantum fields of transverse electric waves TE$_{mn}$, which are described by the annihilation (creation) operators $a_{m,n,k}^{(\dagger)}$. Here the natural numbers m and n are, respectively, the transverse quantum numbers in the x and y directions, while the continuous variable k denotes the wavevector along the z axis. The eigenmode function of the electric fields in the waveguide can be expressed as [43]

$$\begin{aligned}\widetilde{u}_{m,n,k}^{(x)}(\mathbf{r}) &= -i\varepsilon_k \frac{2n\pi}{k_{\mathrm{cut}}L_y}\cos\left(\frac{m\pi}{L_x}x\right)\sin\left(\frac{n\pi}{L_y}y\right)e^{ikz},\\ \widetilde{u}_{m,n,k}^{(y)}(\mathbf{r}) &= i\varepsilon_k \frac{2m\pi}{k_{\mathrm{cut}}L_x}\sin\left(\frac{m\pi}{L_x}x\right)\cos\left(\frac{n\pi}{L_y}y\right)e^{ikz},\end{aligned} \tag{1}$$

where we introduce the cutoff wave number

$$k_{\mathrm{cut}} = \sqrt{(m\pi/L_x)^2 + (n\pi/L_y)^2}, \tag{2}$$

and the electric field per photon $\varepsilon_k = \sqrt{\hbar\omega_{m,n,k}/(2\epsilon_0 V_k)}$, with frequency

$$\omega_{m,n,k} = c\sqrt{(m\pi/L_x)^2 + (n\pi/L_y)^2 + k^2}, \tag{3}$$

and the effective volume $V_k = L_x L_y 2\pi/|k|$ of a segment (with length $2\pi/|k|$) of the waveguide. The parameter ϵ_0 is the vacuum permittivity and c is the speed of light in vacuum.

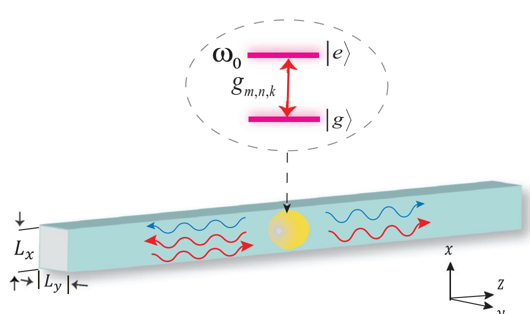

FIG. 1. (Color online) Schematic diagram for single-photon transport in a quasi-one-dimensional waveguide coupled to a TLS with transition frequency ω_0. The cross-section size of the waveguide is L_x (L_y) along the x (y) direction.

When a two-level atom is placed in the waveguide, it will couple to these quantum fields via the dipole interaction. Denoting the ground and excited states of the atom as $|g\rangle$ (with energy 0) and $|e\rangle$ (with energy ω_0), we can define the atomic transition operators as $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$, and then the Hamiltonian (with $\hbar = 1$) of the waveguide-QED system reads

$$\begin{aligned}H = \omega_0|e\rangle\langle e| &+ \int_{-\infty}^{+\infty} dk \sum_{m,n} \omega_{m,n,k} a_{m,n,k}^{\dagger} a_{m,n,k}\\ &+ \int_{-\infty}^{+\infty} dk \sum_{m,n} (g_{m,n,k}\sigma_+ a_{m,n,k} + \mathrm{H.c.}).\end{aligned} \tag{4}$$

Here, the coupling strength is $g_{m,n,k} = -d_{e,g}^{(x)}\widetilde{u}_{m,n,k}^{(x)}(\mathbf{r}_0) - d_{e,g}^{(y)}\widetilde{u}_{m,n,k}^{(y)}(\mathbf{r}_0)$.

Keeping the coupling between photons and atoms $g_{m,n,k}$ nonzero requires $mn \neq 0$. If $m = 0$, namely the coupling along y direction is zero, then the transverse mode quantum number n should be nonzero, namely $n = 1,2,3,\ldots$; otherwise, if $n = 0$, namely the coupling along x direction is zero, then the transverse mode quantum number m should be nonzero, namely $m = 1,2,3,\ldots$. When the transverse sizes satisfy $L_x = L_y$, then the modes TE$_{10}$ and TE$_{01}$, bearing the same cutoff frequencies, are degenerate. To mainly show our idea, namely the effect induced by transverse size of the waveguide, we will choose two modes with different cutoff frequencies. The relation $\omega_{m,n}^{\mathrm{cut}} \equiv ck_{\mathrm{cut}}$ gives the exact cutoff frequency $\omega_{m,n}^{\mathrm{cut}}$ for the transverse mode (m,n).

As a result of $g_{0,0,k} = 0$, we do not consider the TE$_{00}$ mode with $\omega_{0,0,k} = c|k|$. To reduce the energy distribution in the transverse mode of the transport photon, we assume that the photons are in the lowest transverse mode TE$_{01}$, which is the main transport channel we will consider here. However, the transverse mode TE$_{11}$ with a little higher energy is very close to the lowest transverse mode TE$_{01}$ for a finite cross section of the waveguide, while other transverse modes are far away from TE$_{01}$. Therefore, the finite cross-section effect of the quasi-1D waveguide on photon transport can be mainly characterized by the two transverse modes TE$_{01}$ and TE$_{11}$. Then the Hamiltonian (4) reduces to

$$H = H_0 + V \tag{5}$$

with the free Hamiltonian H_0 of the photon and the two-level atom

$$H_0 = H_w + \omega_0|e\rangle\langle e|, \tag{6}$$

where

$$H_w = \int_{-\infty}^{+\infty} dk(\omega_{a,k}a_k^{\dagger}a_k + \omega_{b,k}b_k^{\dagger}b_k), \tag{7}$$

and the interaction Hamiltonian V between the photon and the atom

$$V = \int_{-\infty}^{+\infty} dk\sigma_+(g_{1k}a_k + g_{2k}b_k) + \mathrm{H.c.} \tag{8}$$

by defining TE_{01} as the a mode, and TE_{11} as the b mode, that is

$$a_k \equiv a_{0,1,k}, \quad b_k \equiv a_{1,1,k}, \tag{9}$$

$$g_{0,1,k} = 2id_{e,g}^{(x)}\sqrt{\frac{\hbar\omega_{ak}}{2\epsilon_0 V_k}}\sin\left(\frac{\pi}{L_x}y_0\right) \equiv g_{1k}, \tag{10}$$

$$g_{1,1,k} = id_{e,g}^{(x)}\sqrt{\frac{\hbar\omega_{bk}}{\epsilon_0 V_k}}\cos\left(\frac{\pi}{L_x}x_0\right)\sin\left(\frac{\pi}{L_x}y_0\right) \equiv g_{2k}, \tag{11}$$

and

$$\omega_{a,k} \equiv \omega_{0,1,k}, \quad \omega_{b,k} \equiv \omega_{1,1,k}, \tag{12}$$

$$\omega_a^{\text{cut}} \equiv \omega_{0,1}^{\text{cut}}, \quad \omega_b^{\text{cut}} \equiv \omega_{1,1}^{\text{cut}}. \tag{13}$$

In many works related to 1D waveguides, the dispersion relation of the photon is approximated up to the first order of the photon wave vector [11,13–15,18–23]. However, the exact dispersion relation (3) near the cutoff frequency is more like a quadratic one, so we expand the frequency $\omega_{a,k}$ around (k_0,ω_0) with $\omega_0 = \omega_{a,k_0} = c\sqrt{\pi^2 L_y^{-2} + k_0^2}$, and $\omega_{b,k}$ around (k_0',ω_0) with $\omega_0 = \omega_{b,k_0'} = c\sqrt{\pi^2 L_x^{-2} + \pi^2 L_y^{-2} + k_0'^2}$, up to second order in k. After introducing $p = k - k_0$ (for $\omega_{a,k}$), and $p = k - k_0'$ (for $\omega_{b,k}$), the two dispersion relations can be rewritten as

$$\omega_{s,p} \simeq \omega_0 + v_{s1}p + v_{s2}p^2 \quad (s = a,b), \tag{14}$$

with the first- and the second-order coefficients given by

$$v_{a1} = c\delta/\omega_0, \quad v_{a2} = \frac{\omega_0 v_{a1}^2}{2\delta^2} - \frac{v_{a1}}{2\omega_0}, \tag{15}$$

$$v_{b1} = |v_{a1}|\sqrt{2\delta^2 - \omega_0^2}/\delta, \quad v_{b2} = 2v_{a2}. \tag{16}$$

Here, we have introduced $\omega \equiv c\pi/L_x$ and $\delta = \pm\sqrt{\omega_0^2 - \omega^2}$, which is proportional to the size L_x of the cross section. The $\pm$ sign represents the sign of k_0 (k_0'). The approximated quadratic dispersion relation (14) shifts the cutoff frequency from ω_s^{cut} (exact) to $\omega_s^{\text{min}} = (4v_{s2}\omega_0 - v_{s1}^2)/(4v_{s2})$ (approximated). Here $s = a$, b.

We assume the photons are entering from the left end of the waveguide in the a mode; thus for the right-moving photons, $k_0,k_0' > 0$, and δ takes the "+"sign, while for the left-moving photons, $k_0,k_0' < 0$, and δ takes the "−"sign. Therefore, the dispersion relations (14) can be rewritten as

$$\omega_{s,k} \simeq \begin{cases} \omega_0 + |v_{s1}|k + v_{s2}k^2, & k_0,\ k_0' > 0, \\ \omega_0 - |v_{s1}|k + v_{s2}k^2, & k_0,\ k_0' < 0, \end{cases} \tag{17}$$

for $s = a,b$. We note that the terms in the dispersion relation (14) that depend on the photon wave vector p describe the frequency detuning of the photon from the atom. Later on, we will use the dispersion relations (14) in our derivations.

III. SCATTERING AND BOUND STATES IN THE SINGLE *b* MODE

Since the photon scattering process in the so-called b mode may contribute to the photon transport in the a mode, we first consider the photon scattering in a single b mode. We inject the photon in the b mode with the atom only coupled to the transverse-mode b mode ($g_1 = 0$). By employing the Lippmann-Schwinger equation, we calculate the scattering state of the photon in the b mode. The bound state is also obtained by the poles of the T matrix [44].

Under the above consideration, the Hamiltonian is directly obtained by setting $g_1 = 0$ and $\omega_{a,k} = 0$ in the Hamiltonian (5) $H^b = H_0^b + V^b$, which includes the free Hamiltonian

$$H_0^b = H_w^b + \omega_0|e\rangle\langle e|, \tag{18}$$

with $H_w^b = \int_{-\infty}^{+\infty} dk\, \omega_{b,k} b_k^\dagger b_k$, and the interaction part

$$V^b = \int_{-\infty}^{+\infty} dk (g_{2k}\sigma_+ b_k + \text{H.c.}). \tag{19}$$

We assume the single photon is initially input from the left end of the waveguide in the b mode $b_k^\dagger|\emptyset\rangle$ with energy $\omega_{b,k}$, while the atom is in the ground state $|g\rangle$, then the scattering state is given by the Lippmann-Schwinger equation [44,45]

$$|\varphi_{bk}^{(+)}\rangle = b_k^\dagger|\emptyset\rangle|g\rangle + \frac{1}{\omega_{b,k} + i0^+ - H_0^b}V^b|\varphi_{bk}^{(+)}\rangle. \tag{20}$$

Here, the input state $b_k^\dagger|\emptyset\rangle|g\rangle$ is the eigenstate of the free Hamiltonian H_0^b with eigenenergy $\omega_{b,k}$,

$$H_0^b b_k^\dagger|\emptyset\rangle|g\rangle = \omega_{b,k} b_k^\dagger|\emptyset\rangle|g\rangle, \tag{21}$$

and $|\varphi_{bk}^{(+)}\rangle$ is the eigenstate of the total Hamiltonian H^b with the same eigenenergy $\omega_{b,k}$.

We assume that the solution of the scattering state $|\varphi_{bk}^{(+)}\rangle$ is in the form

$$|\varphi_{bk}^{(+)}\rangle = |\phi_{b,k}\rangle|g\rangle + \beta_{b,k}|\emptyset\rangle|e\rangle. \tag{22}$$

Here, $|\phi_{b,k}\rangle$ is the single-photon state after being scattered, and $\beta_{b,k}$ is the probability amplitude for the atom to be in its excited state. Substituting this solution into Eq. (20), the scattering state is obtained,

$$|\varphi_{bk}^{(+)}\rangle = b_k^\dagger|\emptyset\rangle|g\rangle + \beta_{b,k}|\emptyset\rangle|e\rangle + G_{bw}^0(\omega_{b,k} + i0^+)\beta_{bk} \times \int_{-\infty}^{+\infty} dk' g_{2k'}^* b_{k'}^\dagger|\emptyset\rangle|g\rangle, \tag{23}$$

where $G_{bw}^0(z) = (z - H_w^b)^{-1}$ is the free Green operator for the b-mode photon, and

$$\beta_{bk} = \frac{g_{2k}}{\omega_{b,k} + i0^+ - \omega_0 - \Sigma_b(\omega_{b,k})}, \tag{24}$$

with the self-energy defined by

$$\Sigma_b(E) \equiv \int_{-\infty}^{+\infty} dk \frac{|g_{2k}|^2}{E + i0^+ - \omega_{b,k}} \tag{25}$$

$$\approx -\frac{i\gamma_b v_{b1}}{\sqrt{v_{b1}^2 + 4v_{b2}(E - \omega_0)}}. \tag{26}$$

Here, if we directly substitute the exact coupling expression (11) into Eq. (25), the divergence of self-energy Σ_b occurs. In obtaining the result (26), we have assumed g_{2k} to be independent of k, namely $g_{2k} = g_2$. This assumption is equivalent to the Markov approximation [46].

It follows from Eq. (26) that, when $E \geqslant (4v_{b2})^{-1}(4v_{b2}\omega_0 - v_{b1}^2) = \omega_{bk}^{\min}$, $\Sigma_b(E)$ is purely imaginary, while $E < \omega_{bk}^{\min}$, $\Sigma_b(E)$ is real. Using the scattering state, the T-matrix elements are given by

$$t_{k'k}(\omega_{b,k}) = \langle g|\langle\emptyset|b_{k'}V^b|\varphi_{bk}^{(+)}\rangle = \beta_{bk}g_{2k'}^*. \tag{27}$$

The bound state can be obtained by solving the transcendental equation $[t_{k'k}(E_{bs})]^{-1} = 0$. We directly obtain the bound-state-energy transcendental equation,

$$E_{bs} = \omega_0 - \frac{i\gamma_b v_{b1}}{\sqrt{v_{b1}^2 + 4v_{b2}(E_{bs} - \omega_0)}}, \tag{28}$$

by using the result (26). Here we have defined the decay rate for the atom induced by the b-mode $\gamma_b = 2\pi|g_2|^2/v_{b1}$. Later on, we use γ_b to denote the coupling strength g_2.

It follows from this result (28) that if $v_{b2} \to 0$, which corresponds to a linear waveguide, the bound-state-energy solution is

$$E_{bs} = \omega_0 - i|\gamma_b|. \tag{29}$$

The fact that there is no real solution means that there is no bound state in the linear waveguide. However, for the quadratic waveguide, the transcendental equation (28) gives one real solution

$$E_{bs} = \Delta_a^{\mathrm{F}} + \omega_0, \tag{30}$$

with

$$\Delta_a^{\mathrm{F}} \equiv \frac{u_b^2 - u_b v_{b1}^2 + v_{b1}^4}{12u_b v_{b2}}. \tag{31}$$

This real solution denotes a bound state. Two complex solutions,

$$E_{bs} = \bigg[(-1 + i\sqrt{3})u_b - 2v_{b1}^2 + (-1 - i\sqrt{3})\frac{v_{b1}^4}{u_b} + 24v_{b2}\omega_0\bigg](24v_{b2})^{-1} \tag{32}$$

and

$$E_{bs} = \bigg[(-1 - i\sqrt{3})u_b - 2v_{b1}^2 + (-1 + i\sqrt{3})\frac{v_{b1}^4}{u_b} + 24v_{b2}\omega_0\bigg](24v_{b2})^{-1}, \tag{33}$$

correspond to two quasibound states. Each of these is a metastable state that decays on a very long time scale and appears to be a localized bound state in real space [41,42]. The parameters therein are defined by

$$\begin{aligned} u_b^3 &= l_b, \\ l_b &= -v_{b1}^6 - 216v_{b1}^2v_{b2}^2\gamma_b^2 \\ &\quad + 12\sqrt{3}\sqrt{v_{b1}^4v_{b2}^2\gamma_b^2\left(v_{b1}^4 + 108v_{b2}^2\gamma_b^2\right)}. \end{aligned} \tag{34}$$

We note that l_b is always real, thus there are three values for $u_b = l_b^{1/3}, l_b^{1/3}e^{i\frac{2\pi}{3}}$, and $l_b^{1/3}e^{i\frac{4\pi}{3}}$. However, we can choose u_b to be real. Then there is always a real solution (30) for Eq. (28), which describes a bound state with energy (30). In addition, when the detuning $\Delta_{ak} \equiv \omega_{ak} - \omega_0$ satisfies

$$\Delta_{ak} = \Delta_a^{\mathrm{F}} \tag{35}$$

or, equivalently,

$$k = \frac{-v_{a1} \pm \sqrt{v_{a1}^2 + 4v_{a2}\Delta_a^{\mathrm{F}}}}{2v_{a2}} \equiv k_{\mathrm{F}}, \tag{36}$$

the input photon energy ω_{ak} is resonant with the bound state in the b mode. This is the Feshbach resonance. Moreover,

$$\lim_{\gamma_b \to 0} \Delta_a^{\mathrm{F}} = -\frac{v_{b1}^2}{4v_{b2}} = \Delta_{\max}^{\mathrm{F}}. \tag{37}$$

This maximum value $\Delta_{\max}^{\mathrm{F}}$ of Δ_a^{F} versus the coupling strength γ_b between the transverse mode and the TLS denotes the maximum value of the bound-state energy in this transverse mode $E_{bs}^{\max} = \Delta_{\max}^{\mathrm{F}} + \omega_0$.

IV. PHOTON TRANSMISSION AND REFLECTION IN THE *a* MODE WHILE THE ATOM IS COUPLED TO THE *b* MODE

Now we consider the photon injected in the a mode with the atom coupled to the a and b modes at the same time. We calculate the scattering state of the a-mode photon. Using the Lippmann-Schwinger equation, the scattering state is

$$|\psi_k^{(+)}\rangle = a_k^\dagger|\emptyset\rangle|g\rangle + \frac{1}{\omega_{a,k} + i0^+ - H_0}V|\psi_k^{(+)}\rangle. \tag{38}$$

By a similar procedure to the last section, the scattering state is obtained as

$$\begin{aligned} |\psi_k^{(+)}\rangle &= a_k^\dagger|\emptyset\rangle|g\rangle + \beta_k|\emptyset\rangle|e\rangle + G_w^0(\omega_{a,k} + i0^+)\beta_k \\ &\quad \times \int_{-\infty}^{+\infty} dk'(g_{1k'}^*a_{k'}^\dagger + g_{2k'}^*b_{k'}^\dagger)|\emptyset\rangle|g\rangle, \end{aligned} \tag{39}$$

where the similar free Green operator is $G_w^0(z) = (z - H_w)^{-1}$, and the excited probability amplitude of the atom is

$$\beta_k = \frac{g_{1k}}{\omega_{a,k} + i0^+ - \omega_0 - \Sigma_a(\omega_{a,k}) - \Sigma_b(\omega_{a,k})} \tag{40}$$

with the self-energy for the a mode defined by

$$\Sigma_a(E) \equiv \int_{-\infty}^{+\infty} dk \frac{|g_{1k}|^2}{E + i0^+ - \omega_{a,k}} \tag{41}$$

$$\simeq -\frac{i\gamma_a v_{a1}}{\sqrt{v_{a1}^2 + 4v_{a2}(E - \omega_0)}}, \tag{42}$$

and $\Sigma_b(E)$ defined by Eq. (25). Similarly, if we directly substitute the exact coupling expression (10) into Eq. (41), the divergence of self-energy Σ_a also occurs. In obtaining the result (42), we have also assumed g_{1k} to be independent of k, namely $g_{1k} = g_1$. Here, the decay rate induced by the a mode $\gamma_a = 2\pi|g_1|^2/v_{a1}$ is introduced. Later on, we also use γ_a to denote the coupling strength g_1.

By using the scattering state (39), we obtain the matrix elements of the scattering operator S in k space,

$$S_{k',k} = \delta(k'-k) - 2\pi i\delta(\omega_{ak'} - \omega_{ak})t_{k'k}(\omega_{a,k} + i0^+), \quad (43)$$

where the T-matrix elements are directly obtained as

$$t_{k'k}(\omega_{a,k} + i0^+) = \langle g|\langle\emptyset|a_{k'}V|\psi_k^{(+)}\rangle = \beta_k g_{1k'}^*, \quad (44)$$

and the δ function here is defined as $\delta(x) = 1$ at $x = 0$; otherwise, $\delta(x) = 0$.

Through the relation

$$S_{k',k} = r\delta(k+k') + t\delta(k-k'), \quad (45)$$

we obtain the reflection amplitude

$$r(k) = -i\frac{1}{|2v_{a2}k + |v_{a1}||}\frac{\gamma_a v_{a1}}{\Delta_{ak} - \Sigma_a(\omega_{a,k}) - \Sigma_b(\omega_{a,k})} \quad (46)$$

for the input single photon in the a mode. Note that, in obtaining the result (46), we have discarded the term proportional to $\bar{\delta}[\Delta_{ak} - \Sigma_a(\omega_{a,k}) - \Sigma_b(\omega_{a,k})]$ and the principle value label $\mathcal{P}$ when using the formula $1/(x + i0^+) = \mathcal{P}/x - i\pi\bar{\delta}(x)$. Here the Dirac δ function is defined as $\bar{\delta}(x) = \infty$ if $x = 0$; otherwise, $\bar{\delta}(x) = 0$. This procedure is reasonable because the definition of detuning Δ_{ak} already restricts its regime to $\Delta_{ak} \geqslant -v_{a1}^2/(4v_{a2}) = \Delta_{ak}^{\min}$, which contradicts the basic condition $\Delta_{ak} < -v_{a1}^2/(4v_{a2})$, under which the δ term $\bar{\delta}[\Delta_{ak} - \Sigma_a(\omega_{a,k}) - \Sigma_b(\omega_{a,k})]$ may contribute.

In terms of the detuning Δ_{ak}, the reflection amplitude is

$$r(\Delta_{ak}) = -i\frac{1}{\sqrt{v_{a1}^2 + 4v_{a2}\Delta_{ak}}} \times \frac{\gamma_a v_{a1}}{\Delta_{ak} - \Sigma_a(\Delta_{ak} + \omega_0) - \Sigma_b(\Delta_{ak} + \omega_0)}. \quad (47)$$

Then the reflection coefficient $R = |r|^2$ can be directly obtained.

The transmission amplitude t is directly obtained through $t = 1 + r$ and the transmitted coefficient is straightforwardly obtained as $T = |t|^2$. Interestingly, we find three resonance points where the single-photon transmission amplitude is zero: (1) $t(\Delta_{ak} = \Delta_a^{\mathrm{F}}) = 0$, where $\Delta_{ak} = \Delta_a^{\mathrm{F}}$ means that the input single-photon energy is resonant with the bound-state energy in the transverse mode $\omega_{ak} = E_{bs}$, namely the photonic Feshbach resonance; (2) $t(\Delta_{ak} = 0, \gamma_b = 0) = 0$, or in terms of the wave vector $t(k = k_{\mathrm{res}}, \gamma_b = 0) = 0$, with $k_{\mathrm{res}} = 0, -v_{a1}/v_{a2}$ for $v_{a1} > 0$. This resonance is denoted as single-photon resonance. (3) $\lim_{\Delta_{ak}\to\Delta_{ak}^{\min}} t(\Delta_{ak}) = 0$. We call this resonance the cutoff (minimum) frequency resonance. Under these three resonances, the transmission $T = 0$.

For comparison, we also obtain the reflection amplitude

$$r_1 = \frac{-i\gamma_a}{\Delta_{ak} + i(\gamma_a + \gamma_b)}, \quad (48)$$

and the transmission amplitude

$$t_1 = 1 + r_1 = \frac{\Delta_{ak} + i\gamma_b}{\Delta_{ak} + i(\gamma_a + \gamma_b)} \quad (49)$$

for linear waveguides and add a subscript "1" to denote that this result only applies to linear waveguides. This result is in agreement with Refs. [13,23] when $\gamma_b = 0$. Correspondingly, the reflection and transmission coefficients are

$$R_1 = |r_1|^2 = \frac{\gamma_a^2}{\Delta_{1ak}^2 + (\gamma_a + \gamma_b)^2} \quad (50)$$

and

$$T_1 = |t_1|^2 = \frac{\Delta_{1ak}^2 + \gamma_b^2}{\Delta_{1ak}^2 + (\gamma_a + \gamma_b)^2}. \quad (51)$$

Here, $\Delta_{1ak} = v_{a1}k$ is the detuning of the single photon for the a mode in the linear waveguide from the two-level atom.

For linear waveguides, it follows from Eq. (50) that the transverse mode will reduce the reflection of the single photon. For the transmission of the photon, the transverse mode will increase the transmission of the single photon when $\gamma_a\gamma_b > 2\Delta_{ak}^2$; otherwise, it will decrease its transmission. In addition, as a result of the transverse mode, i.e., $\gamma_b \neq 0$, the single-photon probability is not conserved in its input mode, that is $T_1 + R_1 < 1$. The photon is scattered into the transverse mode with probability

$$P_{1L} \equiv 1 - R_1 - T_1 = \frac{2\gamma_a\gamma_b}{\Delta_{ak}^2 + (\gamma_a + \gamma_b)^2}. \quad (52)$$

This probability loss has a Lorentz shape centered at the single-photon resonance $\Delta_{ak} = 0$ with width $\gamma_a + \gamma_b$. Under the resonance condition $\Delta_{ak} = 0$ and identical coupling to both scattering mode (a mode) and transverse mode (b mode), namely, $\gamma_a = \gamma_b$, the loss probability reaches $P_{1L} = 0.5$.

V. TRANSVERSE EFFECT IN LINEAR AND QUADRATIC WAVEGUIDES

A. Transverse effect in linear waveguides

To show the transverse effect on single-photon transport, we first consider its effect in linear waveguides. We plot the transmission coefficient T_1 and P_{1L} versus detuning Δ_{1ak} ($= k$ with $v_{a1} = 1$) under different transverse coupling strengths $\gamma_b/\omega_0 = 0$, 0.01, and 0.1 in Figs. 2(a) and 2(c), respectively, and versus the coupling strength γ_b under the single-photon resonance condition $\Delta_{1ak} = 0$ in Figs. 2(b) and 2(d), respectively.

It follows from Fig. 2(a) that, at the single-photon resonance condition, the perfect reflection ($T_1 = 0$) of the single photon is damaged by the transverse mode, and the width of the transmission energy band increases as the transverse-mode coupling strength increases. When the transverse-mode coupling strength is strong enough, the perfect reflection becomes perfect transmission [Fig. 2(b)]. Furthermore, the transverse mode forces the single photon to leave the input mode if the input photon is near resonance with the atom. Especially, exactly at the single-photon resonance, the loss probability reaches its largest value. However, this largest value at the single-photon resonance not always increases as the transverse-mode coupling strength increases, as shown in Fig. 2(d). It first increases rapidly to 0.5 at $\gamma_b = \gamma_a$, then decreases gradually as the transverse-mode coupling strength increases and finally reaches zero when γ_b is strong enough.

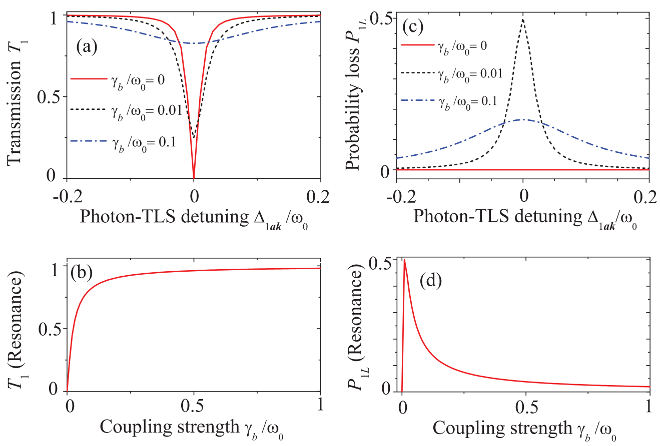

FIG. 2. (Color online) Results for linear waveguides. (a) Transmission coefficient T_1 versus detuning Δ_{1ak} and (b) versus the coupling strength γ_b between the transverse mode and the TLS at the single-photon resonance $\Delta_{1ak} = 0$. (c) The single-photon loss probability P_{1L} versus detuning Δ_{1ak} and (d) versus the coupling strength γ_b between the transverse mode and the TLS at the single-photon resonance $\Delta_{1ak} = 0$. Other parameters are $\gamma_a/\omega_0 = 0.01$, $\delta/\omega_0 = 0.8$, and $v_{a1} = 1$. All the parameters are in units of ω_0.

B. Transverse effect in quadratic waveguides

Now we illustrate the transverse effect on the single-photon transport properties in a quadratic waveguide. We plot the transmission coefficient T versus wave vector k in Fig. 3 and versus the detuning Δ_{ak} in Fig. 4(a), and the loss probability $P_{2L} = 1 - T - R$ versus the detuning Δ_{ak} in Fig. 4(b).

Figure 3(a) shows that the single photon is perfectly reflected at $k = k_{\rm res}$ and $k = k_C$; otherwise, it is completely transmitted without coupling to the transverse mode. However, once the TLS is coupled to the transverse mode, the original perfect-reflection points $k = k_{\rm res}$ have been shifted to $k = k_{\rm F}$ with some probability loss at $k = k_{\rm res}$. When increasing the coupling strength of the transverse mode, the two sides of the perfect reflection peaks at $k = k_{\rm F}$ move toward the center peak at $k = k_C$, while the probability loss at $k = k_{\rm F}$ is reduced. We also note that the center perfect-reflection peak at $k = k_C$ is not dependent on the transverse-mode coupling; It is decoupled from the transverse mode. This is because it is only determined by the minimum detuning $\Delta_{ak}^{\rm min}$ between the photon and the TLS. Compared with the linear waveguide, this phenomenon is more robust against the finite cross-section effect of the waveguide. Also, there are two additional perfect-reflection peaks. Between the peaks $k_{\rm F} < k < k_C$ (or $k_C < k < k_{\rm F}$), there is a perfect transmission band.

Figure 4 shows the single-photon transport properties in terms of the input energy. Without coupling to the tranverse mode, the single photon is perfectly reflected at $\Delta_{ak} = \Delta_{ak}^{\rm min}$, $\Delta_{ak} = 0$ (single-photon resonance). However, as a result of the coupling to the transverse mode, the perfect reflection at the single-photon resonance disappears but it is replaced by another perfect reflection at $\Delta_{ak} = \Delta_a^{\rm F}$, which denotes that the input single-photon energy is resonant with the bound-state energy in the transverse mode. This is the photonic Feshbach resonance [17,47,48]. Moreover, the position of the perfect reflection as a result of photonic Feshbach resonance moves away from the single-photon resonance position. Figure 4(b) shows that the photon loss probability only occurs in the regime $\Delta_{ak} > \Delta_{\rm max}^{\rm F}$. This is because

$$\lim_{\Delta_{ak}\to\Delta_{\rm max}^{\rm F}} P_{2L} = 0. \tag{53}$$

When $\Delta_{ak} \leqslant \Delta_{\rm max}^{\rm F}$, the loss probability becomes zero $P_{2L} = 0$. Therefore, when the single-photon input energy satisfies $\Delta_{ak}^{\rm min} \leqslant \Delta_{ak} \leqslant \Delta_{\rm max}^{\rm F}$, the transverse mode cannot exert a negative effect on the single-photon transport. We point out that the features for T and P_{2L} versus the b mode photon-atom coupling γ_b at the single-photon resonance remain similar with that in the linear waveguide [Figs. 2(b) and 2(d)].

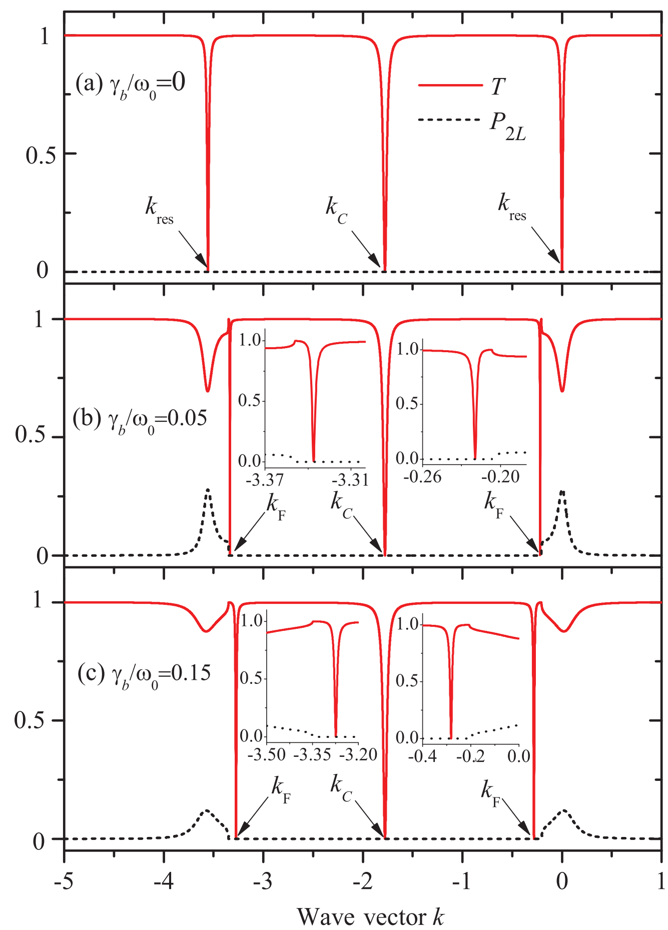

FIG. 3. (Color online) Results for quadratic waveguides: transmission coefficient T and probability loss P_{2L} versus wave vector k when (a) $\gamma_b/\omega_0 = 0$, (b) $\gamma_b/\omega_0 = 0.05$, and (c) $\gamma_b/\omega_0 = 0.15$. Other parameters are the same as in Fig. 2.

To show this more explicitly, how the transverse mode plays a role in the photonic Feshbach resonance, we plot the photonic Feshbach resonance peak position $\Delta_a^{\rm F}$ in Fig. 4 versus the transverse mode coupling strength γ_b in Fig. 5. As the curve shows, $\Delta_a^{\rm F}$ is nearly a linear curve and decreases when increasing the transverse-mode coupling strength. This phenomenon agrees with the properties of T shown in Fig. 4. Since $\Delta_a^{\rm F}$ is also a component of the bound-state energy (30) in the transverse mode, except for a constant ω_0, this curve also shows the bound-state-energy dependence on the transverse-mode coupling strength.

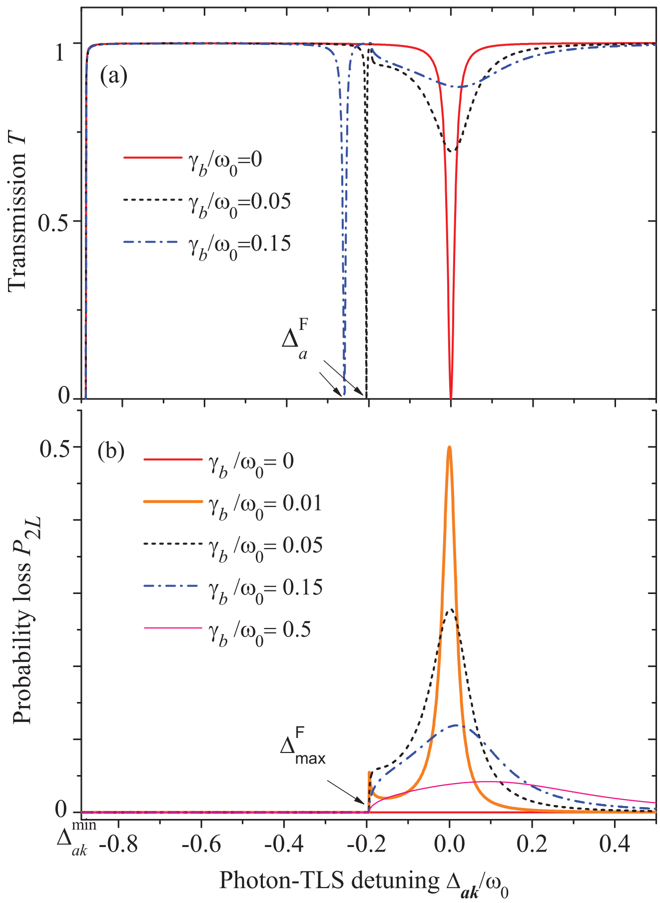

FIG. 4. (Color online) Results for quadratic waveguides. (a) Transmission coefficient T versus detuning Δ_{ak} in quadratic waveguide. (b) The single-photon loss probability P_{2L} versus detuning Δ_{ak}. Other parameters are the same as in Fig. 2.

We also find the line shape for the photonic Feshbach resonance is very close to Fano line shape [49], which is

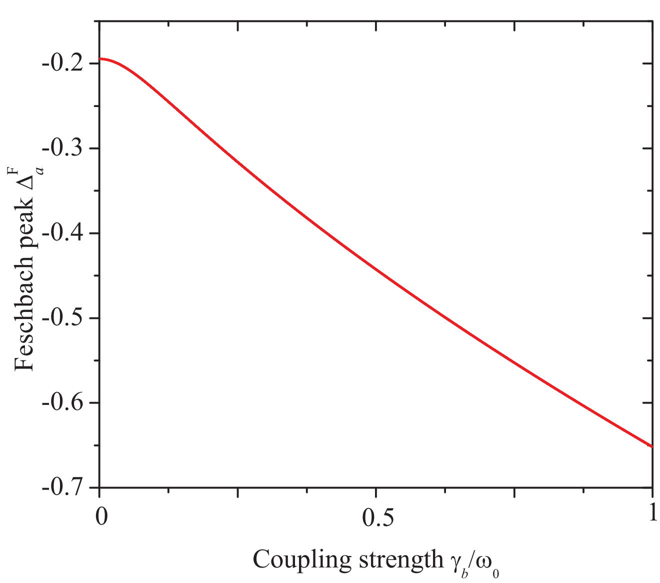

FIG. 5. (Color online) Feschbach peak Δ_a^{F} versus the transverse-mode coupling strength γ_b for waveguides in the quadratic regime. Other parameters are the same as in Fig. 2.

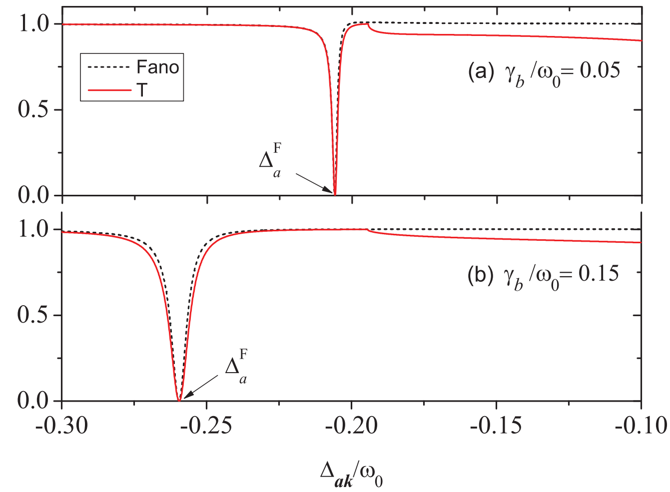

FIG. 6. (Color online) Comparison between the quasi-Fano line shape (red solid line) for transmission coefficient T around Feshbach resonance and the Fano line shape (black dashed line) given by Eq. (54). (a) $q = 10^{-4}$, $d = 10^{-3}$; (b) $q = 10^{-4}$, $d = 10^{-5/2}$. Other parameters are the same as in Fig. 2.

compared with the Fano line in Fig. 6 by defining the Fano function

$$f = \frac{\left(\Delta_{ak} - \Delta_a^{\mathrm{F}} + q\right)^2}{\left(\Delta_{ak} - \Delta_a^{\mathrm{F}}\right)^2 + d^2}. \tag{54}$$

We call our line shape quasi-Fano line. Here, we would like to point out that the similar resonance originated from a bound state in higher transverse modes has also been discovered in an electronic quasi-1D waveguide [50,51]. The resonance line shape is Fano type for electrons in their results [51]. However, compared with the resonance found in Refs. [50,51] for electrons, the similar resonance induced by the bound state for photons we find occurs exactly at the bound-state energy, while the resonance position for electrons acquires a shift in electronic waveguides [50,51].

Finally, we would like to estimate some parameters for the conditions when the additional transverse mode is involved for the study of single-photon transport. Usually, we can ignore the influence of the b mode, when the a mode is close to resonance of an atomic transition while the b mode is off resonance. We now estimate the quantitative condition by assuming that the effect of mode a is 100 times that of mode b. Namely, when

$$100 \times \frac{g_2}{|\omega_{bk} - \omega_0|} \leqslant \frac{g_1}{|\omega_{ak} - \omega_0|} \tag{55}$$

or

$$L_x \leqslant c\frac{(\sqrt{2}-1)\pi}{\omega_0} \equiv L_c, \tag{56}$$

the transverse mode b cannot affect the single-photon transport. To obtain Eq. (56), we have used $|\omega_b^{\mathrm{cut}} - \omega_a^{\mathrm{cut}}| \geqslant |\omega_{bk} - \omega_{ak}|$ and $g_2 = g_1$. However, when the transverse size L_x of the waveguide is larger than the critical size L_c, $L_x > L_c$, the transverse mode should be taken into account. For a 1D circuit system with $\omega_0 \simeq 10$ GHz [26], then $L_c \simeq 3.9$ cm. For a 3D optical cavity system with $\omega_0 \simeq 2.21 \times 10^6$ GHz [52], then $L_c \simeq 176.6$ nm.

In addition, when the photon frequency $\sim 10^9$ GHz, such as x rays [39,40], the TLS with transition energy 14.4 keV (the nuclear transition of $^{57}\mathrm{F_e}$), corresponding to $\omega_0 \simeq 3.48 \times 10^9$ GHz, then the critical size becomes $L_c \simeq 1.12$ Å. Experimentally, this 1.12 Å looks too difficult. Therefore, it is very necessary to consider the transverse-mode effect in the single-photon transport in a waveguide with finite cross section. A finite-cross-section waveguide is closer to our experimental quantum coherent device design and fabrication. Taking advantage of the finite-cross-section waveguide will ease the stringent requirements on realizing quantum-coherent devices.

VI. CONCLUSIONS AND DISCUSSIONS

We studied the finite cross-sectional effect of the waveguide on single-photon transport. To mainly characterize the finite cross-section effect of the waveguide, we pick out one of the numerous transverse modes, whose eigenfrequency is closest to that of the transport mode. We consider the transport properties of a single photon in such a finite cross-section waveguide by calculating the transmission, reflection coefficients, and the single-photon loss probability. By using a quadratic dispersion relation, we find a bound state and two quasibound states [5,41,42] emerging in such a waveguide with a finite cross section, which will not occur in the usual linear waveguide. Moreover, when the input photon energy is resonant with the bound-state energy in the transverse mode, the photon will be completely reflected. This is the photonic Feshbach resonance. In addition, the input photon is also completely reflected when the input energy of it is at a single-photon resonance with the TLS or at the cutoff frequency allowed by the approximated quadratic waveguide. The photonic Feshbach resonance and the cutoff frequency resonance phenomena do not occur in a linear waveguide even in an infinitely idealized 1D waveguide.

Furthermore, as a result of transverse-mode coupling, the photon will be lost when the input energy is above the maximum bound-state energy regulated by the coupling strength between the transverse mode and the TLS. Therefore, only the input energy is below this maximum bound-state energy; the single photon can safely pass through or be completely reflected by the TLS instead of lost in some other transverse mode even though in a finite cross-section waveguide.

ACKNOWLEDGMENTS

We would like to thank P. Zhang and D. Z. Xu for helpful discussions. This work is supported by National Natural Science Foundation of China under Grants No. 11121403, No. 10935010, and No. 11074261, and National 973 program (Grant No. 2012CB922104). T.S. has been supported by the EU under the IP project AQUTE. F.N. acknowledges partial support from the Army Research Office, JSPS-RFBR Grant No. 12-02-92100, Grant-in-Aid for Scientific Research (S), MEXT Kakenhi on Quantum Cybernetics, and the JSPS-FIRST program.

[1] H. J. Kimble, Nature (London) **453**, 1023 (2008).
[2] Y. Xiao, M. Klein, M. Hohensee, L. Jiang, D. F. Phillips, M. D. Lukin, and R. L. Walsworth, Phys. Rev. Lett. **101**, 043601 (2008).
[3] D. E. Chang, A. S. Sørensen, E. A. Demler, and M. D. Lukin, Nature Phys. **3**, 807 (2007).
[4] L. Zhou, Z. R. Gong, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. Lett. **101**, 100501 (2008).
[5] Z. R. Gong, H. Ian, L. Zhou, and C. P. Sun, Phys. Rev. A **78**, 053806 (2008).
[6] J. Q. Liao, J. F. Huang, Y. X. Liu, L. M. Kuang, and C. P. Sun, Phys. Rev. A **80**, 014301 (2009).
[7] J. Q. Liao, Z. R. Gong, L. Zhou, Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **81**, 042304 (2010).
[8] Y. Chang, Z. R. Gong, and C. P. Sun, Phys. Rev. A **83**, 013825 (2011).
[9] F. M. Hu, L. Zhou, T. Shi, and C. P. Sun, Phys. Rev. A **76**, 013819 (2007).
[10] T. Shi and C. P. Sun, arXiv:0907.2776.
[11] J. Q. Liao and C. K. Law, Phys. Rev. A **82**, 053836 (2010).
[12] Z. H. Wang, Y. Li, D. L. Zhou, C. P. Sun, and P. Zhang, Phys. Rev. A **86**, 023824 (2012).
[13] J. T. Shen and S. Fan, Opt. Lett. **30**, 2001 (2005); Phys. Rev. Lett. **95**, 213001 (2005).
[14] J. T. Shen and S. Fan, Phys. Rev. Lett. **98**, 153003 (2007); Phys. Rev. A **76**, 062709 (2007).
[15] T. S. Tsoi and C. K. Law, Phys. Rev. A **78**, 063832 (2008); **80**, 033823 (2009).
[16] T. Shi and C. P. Sun, Phys. Rev. B **79**, 205111 (2009).
[17] D. Z. Xu, H. Ian, T. Shi, H. Dong, and C. P. Sun, Sci. China Phys. Mech. Astron. **53**, 1234 (2010).
[18] D. Roy, Phys. Rev. B **81**, 155117 (2010); Phys. Rev. Lett. **106**, 053601 (2011).
[19] T. Shi, S. Fan, and C. P. Sun, Phys. Rev. A **84**, 063803 (2011).
[20] H. Zheng, D. J. Gauthier, and H. U. Baranger, Phys. Rev. A **85**, 043832 (2012).
[21] E. Rephaeli and S. Fan, Phys. Rev. Lett. **108**, 143602 (2012).
[22] T. Shi and S. Fan, arXiv:1208.1258.
[23] J. F. Huang, J. Q. Liao, and C. P. Sun, Phys. Rev. A **87**, 023822 (2013).
[24] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. **81**, 3611 (1998).
[25] J. Hwang, M. Pototschnig, R. Lettow, G. Zumofen, A. Renn, S. Götzinger, and V. Sandoghdar, Nature (London) **460**, 76 (2009).
[26] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Jr., Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Science **327**, 840 (2010).
[27] C. P. Sun, L. F. Wei, Y. X. Liu, and F. Nori, Phys. Rev. A **73**, 022318 (2006).
[28] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003); J. Q. You, J. S. Tsai, and F. Nori, *ibid.* **68**, 024510 (2003).
[29] I. Chiorescu, P. Bertet, K. Semba, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Nature (London) **431**, 159 (2004).
[30] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).

[31] M. A. Sillanpää, J. I. Park, and R. W. Simmonds, Nature (London) **449**, 438 (2007).
[32] J. Q. You and F. Nori, Phys. Today **58**, 42 (2005).
[33] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).
[34] I. Buluta, S. Ashhab, and F. Nori, Rep. Prog. Phys. **74**, 104401 (2011).
[35] G. Wendin and V. S. Shumeiko, arXiv:cond-mat/0508729.
[36] S. Hughes, Opt. Lett. **29**, 2659 (2004).
[37] S. Hughes and H. Kamada, Phys. Rev. B **70**, 195313 (2004).
[38] D. Englund, A. Faraon, I. Fushman, N. Stoltz, P. Petroff, and J. Vučković, Nature (London) **450**, 857 (2007).
[39] R. Röhlsberger, K. Schlage, B. Sahoo, S. Couet, and R. Rüffer, Science **328**, 1248 (2010).
[40] R. Röhlsberger, H. C. Wille, K. Schlage, and B. Sahoo, Nature (London) **482**, 199 (2012).
[41] H. Nakamura, N. Hatano, S. Garmon, and T. Petrosky, Phys. Rev. Lett. **99**, 210404 (2007).
[42] N. Hatano, K. Sasada, H. Nakamura, and T. Petrosky, Prog. Theor. Phys. **119**, 187 (2008).
[43] W. R. Raudorf, Am. J. Phys. **46**, 35 (1978).
[44] J. R. Taylor, *Scattering Theory: The Quantum Theory on Nonrelativistic Collisions* (Wiley, New York, 1972).
[45] P. Zhang, L. Zhang, and W. Zhang, Phys. Rev. A **86**, 042707 (2012).
[46] C. W. Gardiner and M. J. Collett, Phys. Rev. A **31**, 3761 (1985).
[47] H. Feshbach, Ann. Phys. (NY) **5**, 357 (1958).
[48] E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, Phys. Rep. **315**, 199 (1999).
[49] U. Fano, Phys. Rev. **124**, 1866 (1961).
[50] S. A. Gurvitz and Y. B. Levinson, Phys. Rev. B **47**, 10578 (1993).
[51] J. U. Nöckel and A. D. Stone, Phys. Rev. B **50**, 17415 (1994).
[52] J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, and H. J. Kimble, Science **303**, 1992 (2004).