Reconstruction of the surface-height autocorrelation function of a randomly rough dielectric surface from incoherent light scattering

Sangeeta Chakrabarti and A. A. Maradudin*
Department of Physics and Astronomy, University of California, Irvine, California 92697, USA

E. R. Méndez
División de Física Aplicada, Centro de Investigación Científica y de Educación Superior de Ensenada, Carretera Ensenada—Tijuana No. 3913, Ensenada, BC 22360, México
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An analytic approach is developed for obtaining the normalized surface height autocorrelation function of a one-dimensional randomly rough dielectric surface from experimental scattering data. It is based on the contribution to the mean differential reflection coefficient, obtained in the Kirchhoff approximation, from the light scattered incoherently. The incident light is *s* polarized, and its plane of incidence is perpendicular to the generators of the surface. Good agreement with numerically generated experimental data is obtained.

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While the main goal of studies of the inverse problem in the scattering of electromagnetic waves from rough surfaces is arguably the reconstruction of the profile of the rough surface from measurements of angular, wavelength, and polarization dependencies of the field scattered from [1,2] or transmitted through [3] it, there are other, statistical, properties of the surface profile function that are very useful and easier to obtain. These are the normalized surface height autocorrelation function [4], the power spectrum of the surface roughness [5], the probability density function of surface heights [6], and the rms height of the surface [7].

Here we present an approach to the determination of the normalized surface height autocorrelation function of a one-dimensional randomly rough dielectric surface, from measurements of the angular dependence of the contribution to the mean differential reflection coefficient from the light scattered incoherently (diffusely) from it. This approach also allows us to obtain an estimate of the rms height of the surface from these data.

This problem was studied earlier by Chandley [4]. There are several differences between his work and ours. Chandley's approach uses scalar diffraction theory and a thin phase screen approximation [8] to model the interaction of light with the randomly rough surface. A thin phase screen may be visualized as a layer of negligible thickness that introduces phase variations in the scattered wave without introducing amplitude variations. It is derived from simple optical path length arguments and geometrical optics concepts. Our approach, on the other hand, is based on an expression for the field scattered by a one-dimensional randomly rough dielectric surface provided by the Kirchhoff approximation. This approximation is based on the assumption that the scattered field is produced by the reflection of the incident light from the plane tangent to the surface at each point of it. We use this expression due to its simplicity, and because it is able to reconstruct well the surface profile function of a one-dimensional rough Dirichlet surface from experimental scattering data [1].

Chandley [4] used the angular dependence of the mean intensity of the scattered light in the far field as the experimental quantity to be inverted, while we use the angular dependence of the mean differential reflection coefficient, obtained from the incident and scattered fluxes for this purpose. The use of properly normalized far-field data permits the estimation of the standard deviation of heights. Another difference is that the dielectric constant of the scattering medium does not appear explicitly in Chandley's theory, but it does in ours. This means that it is not possible to use Chandley's theory to recover the dielectric constant of the scattering medium from experimental scattering data, but it is possible to do so with our approach, as we will see below.

Rough surfaces appear in natural situations and are introduced on purpose for some applications. Assuming that the surface profile function is a Gaussian random process, our approach permits the estimation of its most basic statistical properties, the standard deviation of heights and the height-height correlation function. Among other things, the spectral content of the surfaces and its standard deviation of slopes are determined by these quantities. The method described here could find applications in situations as varied as the characterization of ocean waves and the surface of antiglare screens and efficiency-enhanced structured solar cells.

The physical system we consider consists of vacuum in the region $x_3 > \zeta(x_1)$ and a dielectric medium in the region $x_3 < \zeta(x_1)$. We assume that the dielectric medium is characterized by a dielectric constant ϵ that is real, positive, and frequency independent. The surface profile function $\zeta(x_1)$ is assumed to be a single-valued function of x_1 that is differentiable. It also constitutes a stationary zero-mean, Gaussian random process, defined by

$$\langle \zeta(x_1)\zeta(x_1')\rangle = \delta^2 W(|x_1 - x_1'|), \tag{1a}$$

$$\langle \zeta^2(x_1)\rangle = \delta^2, \tag{1b}$$

where the angle brackets denote an average over the ensemble of realizations of $\zeta(x_1)$, δ is the rms height of the surface, and $W(|x_1|)$ is the normalized surface height autocorrelation function. From Eqs. (1a) and (1b) we obtain the result that $W(0) = 1$.

*aamaradu@uci.edu

The surface $x_3 = \zeta(x_1)$ is illuminated from the vacuum by an s-polarized plane wave of frequency ω, whose plane of incidence, and the plane of scattering, is the $x_1\,x_3$ plane, and whose angle of incidence, measured counterclockwise from the positive x_3 axis is θ_0. The expression for the contribution to the mean differential reflection coefficient from the light scattered incoherently from this surface is given in the Kirchhoff approximation by [9]

$$\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle(\theta_s,\theta_0)_{\text{incoh}} = \frac{1}{L_1}\left(\frac{\omega}{2\pi c}\right)\frac{\left|R_s\left(\frac{1}{2}(\theta_s+\theta_0)\right)\right|^2}{\cos\theta_0}\frac{\cos^2\frac{1}{2}(\theta_s+\theta_0)}{\cos^2\frac{1}{2}(\theta_s-\theta_0)} \times\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}dx_1' e^{-i(q-k)(x_1-x_1')} \times\left\{\left\langle e^{-i(\alpha_0(q)+\alpha_0(k))(\zeta(x_1)-\zeta(x_1'))}\right\rangle - \left\langle e^{-i(\alpha_0(q)+\alpha_0(k))\zeta(x_1)}\right\rangle\left\langle e^{i(\alpha_0(q)+\alpha_0(k))\zeta(x_1')}\right\rangle\right\}. \quad (2)$$

In this expression L_1 is the length of the x_1 axis covered by the rough surface, and θ_s is the angle of scattering measured clockwise from the positive x_3 axis. The function $R_s[\frac{1}{2}(\theta_s+\theta_0)]$ is defined by

$$R_s\left(\frac{1}{2}(\theta_s+\theta_0)\right) = \frac{\cos\left[\frac{1}{2}(\theta_s+\theta_0)\right]-\left\{\epsilon-\sin^2\left[\frac{1}{2}(\theta_s+\theta_0)\right]\right\}^{\frac{1}{2}}}{\cos\left[\frac{1}{2}(\theta_s+\theta_0)\right]+\left\{\epsilon-\sin^2\left[\frac{1}{2}(\theta_s+\theta_0)\right]\right\}^{\frac{1}{2}}}, \quad (3)$$

while the function $\alpha_0(q)$ is

$$\alpha_0(q) = [(\omega/c)^2 - q^2]^{\frac{1}{2}}, \quad \text{Re}\alpha_0(q) > 0, \quad \text{Im}\alpha_0(q) > 0, \quad (4)$$

where c is the speed of light in vacuum. The wave numbers k and q are related to the angles of incidence and scattering by

$$k = (\omega/c)\sin\theta_0, \quad q = (\omega/c)\sin\theta_s, \quad (5)$$

respectively.

The expression given by Eq. (2) simplifies considerably in the case of normal incidence $\theta_0 = 0$, $k = 0$, where it has the form

$$\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle(\theta_s,0)_{\text{incoh}} = \frac{1}{L_1}\left(\frac{\omega}{2\pi c}\right)\left[\frac{\cos\frac{1}{2}\theta_s-\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}{\cos\frac{1}{2}\theta_s+\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}\right]^2 \times\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}dx_1' e^{-iq(x_1-x_1')}\left\{\left\langle e^{-i\frac{2\omega}{c}\cos^2\frac{\theta_s}{2}(\zeta(x_1)-\zeta(x_1'))}\right\rangle - \left\langle e^{-i\frac{2\omega}{c}\cos^2\frac{\theta_s}{2}\zeta(x_1)}\right\rangle\left\langle e^{i\frac{2\omega}{c}\cos^2\frac{\theta_s}{2}\zeta(x_1')}\right\rangle\right\}. \quad (6)$$

Since we have assumed that $\zeta(x_1)$ is a Gaussian random process, the ensemble averages in this expression can be evaluated analytically. Then, with the change of variable $x_1 = u + x_1'$, Eq. (6) becomes

$$\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle(\theta_s,0)_{\text{incoh}} = \left(\frac{\omega}{2\pi c}\right)\left[\frac{\cos\frac{1}{2}\theta_s-\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}{\cos\frac{1}{2}\theta_s+\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}\right]^2 \times e^{-(2\omega\delta/c)^2\cos^4\frac{\theta_s}{2}}\int_{-\infty}^{\infty}du\, e^{-iqu}\left[e^{(2\omega\delta/c)^2\cos^4\frac{\theta_s}{2}W(|u|)}-1\right]. \quad (7)$$

We see from Eq. (7) that if the factor $\cos^4(\theta_s/2)$ in the exponent in the integrand were a constant the integral would be a Fourier transform and therefore readily inverted. Now $\cos^4(\theta_s/2)$ decreases from unity at $\theta_s = 0$ to $1/4$ at $\theta_s = \pi/2$. We will make the approximation of replacing it by unity in what follows. This approximation is very good when the scattering is limited to small angles. For example, $\cos^4(\theta_s/2) = 0.9$ for $\theta_s = 26.19^\circ$, and $\cos^4(\theta_s/2) = 0.8$ for $\theta_s = 37.92^\circ$. With this approximation we obtain the result

$$e^{(2\omega\delta/c)^2W(|u|)} - 1 = \frac{2c}{\omega}\int_0^{\omega/c}dq\cos uq\, e^{(2\omega\delta/c)^2\cos^4\frac{\theta_s}{2}} \times\left[\frac{\cos\frac{1}{2}\theta_s+\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}{\cos\frac{1}{2}\theta_s-\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}\right]^2\left\langle\frac{\partial R_s}{\partial\theta_s}\right\rangle(\theta_s,0)_{\text{incoh}}. \quad (8)$$

The limits of integration in this expression are dictated by the fact that the mean differential reflection coefficient is defined only in the interval $|q| < \omega/c$, and that for normal incidence $\langle\partial R_s/\partial\theta_s\rangle(\theta_s,0)_{\text{incoh}}$ is an even function of θ_s. If we now use the fact that $W(0) = 1$, we obtain from Eq. (8)

$$e^{(2\omega\delta/c)^2} - 1 = 2\frac{c}{\omega}\int_0^{\omega/c}dq\, e^{(2\omega\delta/c)^2\cos^4\frac{\theta_s}{2}} \times\left[\frac{\cos\frac{1}{2}\theta_s+\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}{\cos\frac{1}{2}\theta_s-\left(\epsilon-\sin^2\frac{1}{2}\theta_s\right)^{\frac{1}{2}}}\right]^2\left\langle\frac{\partial R}{\partial\theta_s}\right\rangle(\theta_s,0)_{\text{incoh}}, \quad (9)$$

from which δ can be determined. Once δ has been determined, Eq. (8) can be used to determine $W(|u|)$. In evaluating the integrals in Eqs. (8) and (9) it is convenient to make the change of variable $q = (\omega/c)\sin\theta_s$, Eq. (5).

To illustrate this approach to the determination of $W(|x_1|)$ and δ, we apply it to the reconstruction of these functions from data for $\langle\partial R_s/\partial\theta_s\rangle(\theta_s,0)_{\text{incoh}}$ for three different one-dimensional randomly rough dielectric surfaces. These data were obtained by means of rigorous computer simulations [10]. In these simulations, a realization of the surface profile function of one of these types of randomly rough surfaces is generated by an approach that is based on the power spectrum of the surface roughness $g(Q)$ [10]. In our notation, the power

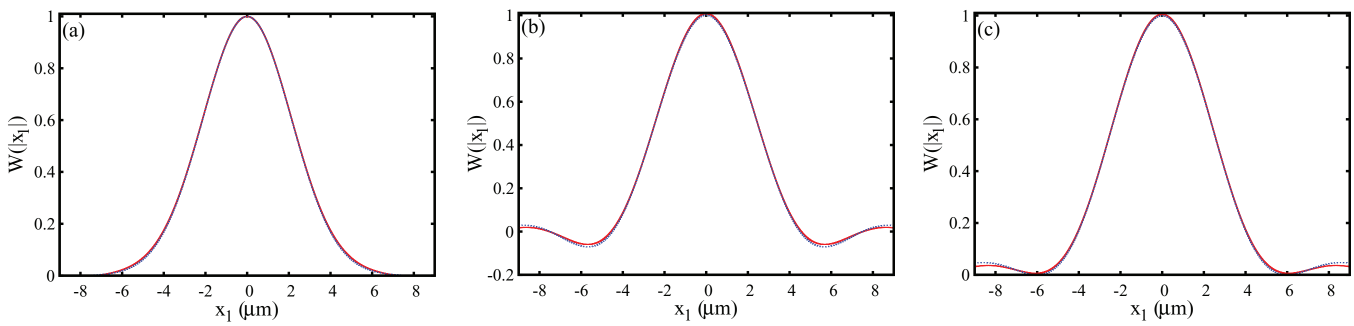

FIG. 1. (Color online) A plot of $W(|x_1|)$ as a function of x_1. The input surface height autocorrelation function is depicted by the dotted (blue) curve, and the reconstructed function is depicted by the solid (red) curve. Here $\theta_0 = 0^\circ$, $\lambda = 632.8$ nm, $\epsilon = 2.25$, $\delta = 60$ nm, and $a = 3\ \mu$m. (a) Gaussian power spectrum. (b) Cosine power spectrum. (c) Triangular power spectrum.

spectrum can be defined by the expression [11]

$$g(Q) = \frac{1}{\delta^2} \lim_{L\to\infty} \frac{\langle|\hat{\zeta}_L(Q)|^2\rangle}{L}, \tag{10}$$

where $\hat{\zeta}_L(Q)$ is the Fourier transform of a section of the random profile of the surface of length L. According to the Wiener-Khinchin theorem, $g(Q)$ and $W(|x_1|)$ constitute a Fourier transform pair [11]:

$$g(Q) = \int_{-\infty}^{\infty} dx_1 W(|x_1|)\exp(-iQx_1). \tag{11}$$

The field scattered from this realization of the surface is given in terms of the values of the single nonzero component of the electric field in the vacuum and of its normal derivative, both evaluated on the rough surface. These two source functions satisfy a pair of coupled one-dimensional inhomogeneous integral equations, obtained with the help of Green's second integral identity in the plane [12]. These equations are solved by converting them into a pair of coupled inhomogeneous matrix equations by the use of a numerical quadrature scheme. The results are used to calculate the differential reflection coefficient for that realization of the surface profile function. An arithmetic average of the results for the differential reflection coefficient obtained from an ensemble of 2000 realizations of the surface profile function yields the mean differential reflection coefficient. The use of data for the mean differential reflection coefficient obtained in this way allows an assessment of the quality of the reconstruction of $W(|x_1|)$ to be made.

The three choices for the power spectra and the corresponding expressions for $W(|x_1|)$ are:

(1) a Gaussian power spectrum

$$g(Q) = \sqrt{\pi}a\exp(-a^2Q^2/4) \tag{12a}$$
$$W(|x_1|) = \exp\left(-x_1^2/a^2\right); \tag{12b}$$

(2) a cosine power spectrum

$$g(Q) = \frac{a\pi}{2}\cos\left(\frac{aQ}{2}\right) \qquad |Q| < \pi/a$$
$$= 0 \qquad |Q| > \pi/a \tag{13a}$$

$$W(|x_1|) = \frac{a^2}{4}\frac{\cos(\pi x_1/a)}{(a/2)^2 - x_1^2}; \tag{13b}$$

(3) a triangular power spectrum

$$g(Q) = 2a\left(1 - \frac{a|Q|}{\pi}\right) \qquad |Q| \leqslant \pi/a$$
$$= 0 \qquad |Q| > \pi/a \tag{14a}$$
$$W(|x_1|) = \left[\mathrm{sinc}\left(\frac{\pi x_1}{2a}\right)\right]^2. \tag{14b}$$

In these expressions a is a parameter that defines the lateral scale of the roughness. The latter two power spectra are unlikely to be found in naturally occurring or fabricated randomly rough surfaces, but the reconstruction of $W(|x_1|)$ from scattering data obtained on the basis of such spectra tests the versatility of our inversion approach.

The reconstructed $W(|x_1|)$ and the input $W(|x_1|)$ are plotted for the random surface defined by Eqs. (12) in Fig. 1(a), for the surface defined by Eqs. (13) in Fig. 1(b), and for the surface defined by Eqs. (14) in Fig. 1(c). The wavelength of the incident light is $\lambda = 632.8$ nm; the dielectric constant of the scattering medium is $\epsilon = 2.25$. The roughness of each of the three randomly rough surfaces is characterized by the values $\delta = 60$ nm and $a = 3\ \mu$m. In each case, the reconstructed and input values of $W(|x_1|)$ are plotted for x_1 in the interval $-9\ \mu\text{m} < x_1 < 9\ \mu$m. It is seen that on the scale of these figures the reconstructed and input $W(|x_1|)$ are indistinguishable for $|x_1| \lesssim 4\ \mu$m and are in good agreement for the larger values of x_1. In particular, the oscillations of the input $W(|x_1|)$ given by Eqs. (13b) and (14b) are well reproduced in the reconstructed $W(|x_1|)$. The values of δ obtained from Eq. (9) are (1) 60.18 nm, (2) 59.81 nm, and (3) 59.45 nm, all very close to the input value $\delta = 60$ nm. Thus, for the values of the experimental, material, and roughness parameters employed in obtaining the results presented in Figs. 1(a)–1(c), the approach to the reconstruction of $W(|x_1|)$ and δ developed here has yielded excellent results.

To explore how the quality of the reconstruction is affected by varying the experimental, material, and roughness parameters we have carried out several additional calculations. The majority of these calculations have been carried out on the basis of the mean differential reflection coefficients of surfaces defined by the Gaussian power spectrum, Eq. (12a), which seems the most physical of the three power spectra considered here.

In Figs. 2(a) and 2(b) we have plotted the reconstructed and input $W(|x_1|)$ for the surface defined by Eq. (12) when the

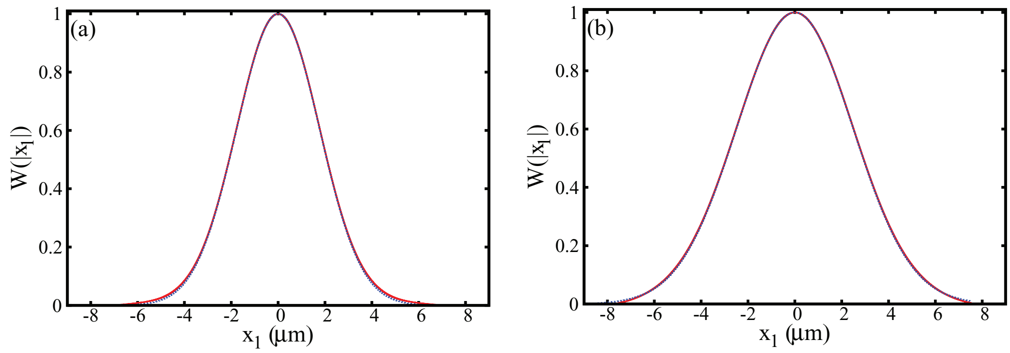

FIG. 2. (Color online) A plot of $W(|x_1|)$ as a function of x_1 for the random surface defined by the Gaussian power spectrum. The input surface height autocorrelation function is depicted by the dotted (blue) curve, and the reconstructed function is depicted by the solid (red) curve. Here $\theta_0 = 0^\circ$, $\lambda = 632.8$ nm, $\epsilon = 2.25$, and $\delta = 60$ nm. (a) $a = 2.5$ μm. (b) $a = 3.5$ μm.

parameter a has the values 2.5 and 3.5 μm, respectively, while λ, ϵ, and δ retain the values assumed in obtaining Fig. 1(a). It is seen from a comparison of the results in Fig. 2 with the results in Fig. 1(a) that varying the parameter a from 2.5 to 3.5 μm for fixed values of the other parameters does not degrade the quality of the reconstructions of $W(|x_1|)$. The reconstruction presented in Fig. 2(b) is marginally better in the wings than the one presented in Fig. 2(a). The reconstructed values of δ for the surfaces leading to Figs. 2(a) and 2(b) are (1) 59.91 nm and (2) 60.04 nm, respectively, which are in very good agreement with the input value $\delta = 60$ nm.

In Fig. 3 we have plotted the reconstructed and input $W(|x_1|)$ for the surface defined by Eq. (12) when the rms height δ is increased to $\delta = 100$ nm. a assumes the values 3, 6, 10, 14, and 18 μm, while λ and ϵ have the values obtained in Fig. 1(a). There is no significant worsening of the quality of the reconstruction arising from a near doubling of the rms height of the surface for $|x_1| \lesssim 5$ μm. In the wings, the reconstructed $W(|x_1|)$ begins to display oscillations instead of monotonically decreasing to zero. The reconstructed values of δ obtained from Eq. (9) for these five surfaces are (1) 101.3 nm, (2) 104.2 nm, (3) 106.1 nm, (4) 105.8 nm, and (5) 104.6 nm. These values are to be compared with the input value $\delta = 100$ nm.

We have also considered the effect on the quality of the reconstruction of varying the wavelength of the incident light. In Figs. 4(a) and 4(b) we present the reconstructed and input $W(|x_1|)$ for the surface defined by Eq. (12) when the wavelength λ has the values $\lambda = 458$ and 680 nm, respectively, while ϵ, δ, and a have the values assumed in obtaining Fig. 1(a). The reconstruction at the shorter wavelength presented in Fig. 4(a), while good overall, is poorer than the one presented in Fig. 1(a) in that it becomes negative at large values of $|x_1|$, in the

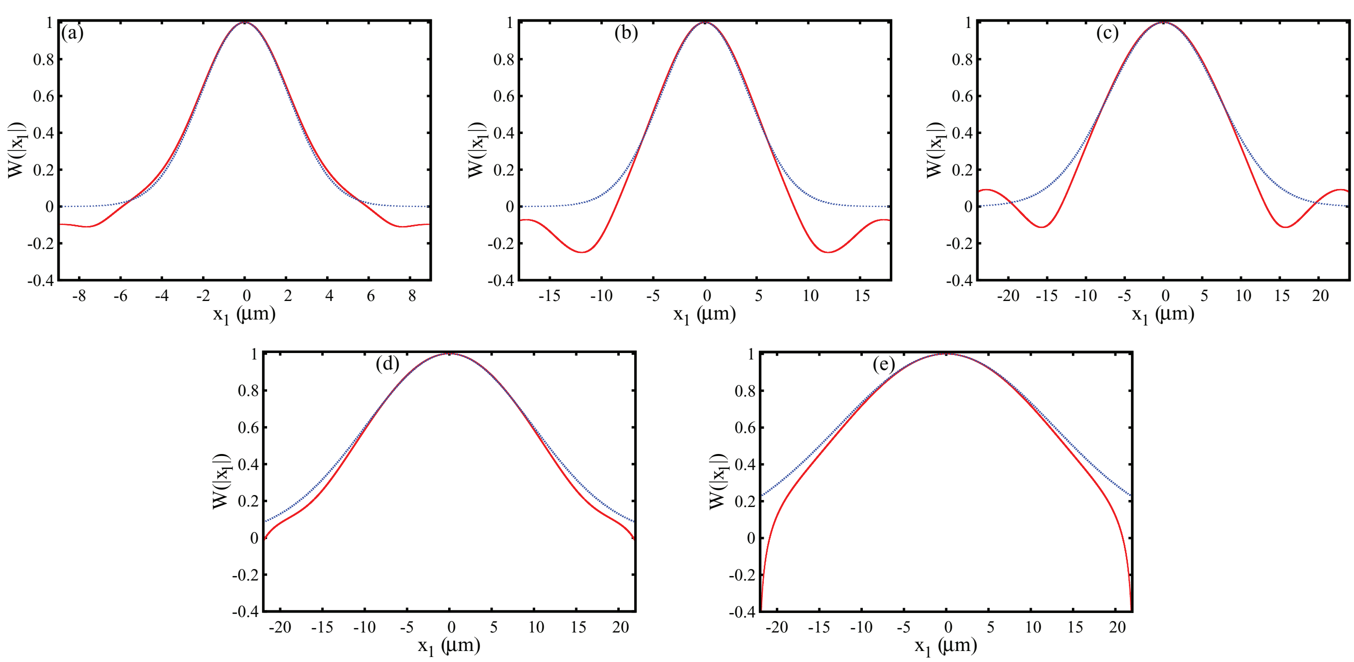

FIG. 3. (Color online) A plot of $W(|x_1|)$ as a function of x_1 for the random surface defined by the Gaussian power spectrum. The input surface height autocorrelation function is depicted by the dotted (blue) curve, and the reconstructed function is depicted by the solid (red) curve. Here $\theta_0 = 0^\circ$, $\lambda = 632.8$ nm, $\epsilon = 2.25$, and $\delta = 100$ nm. (a) $a = 3$ μm. (b) $a = 6$ μm. (c) $a = 10$ μm. (d) $a = 14$ μm. (e) $a = 18$ μm.

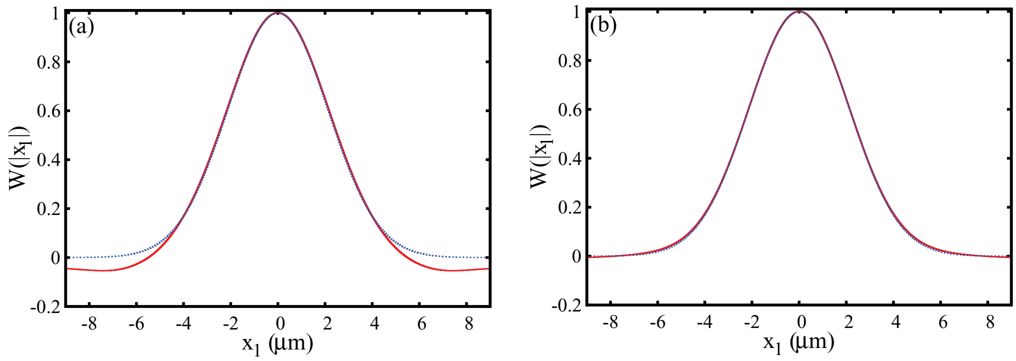

FIG. 4. (Color online) A plot of $W(|x_1|)$ as a function of x_1 for the random surface defined by the Gaussian power spectrum. The input surface height autocorrelation function is depicted by the dotted (blue) curve, and the reconstructed function is depicted by the solid (red) curve. Here $\theta_0 = 0^\circ$, $\epsilon = 2.25$, $\delta = 60$ nm, and $a = 3$ μm. (a) $\lambda = 458$ nm. (b) $\lambda = 680$ nm.

wings. In contrast, the reconstruction at the longer wavelength presented in Fig. 4(b) is as good as the one presented in Fig. 1(a). The reconstructed values of δ obtained from Eq. (9) for these surfaces are (1) 60.62 nm and (2) 59.92 nm, in good agreement with the input value $\delta = 60$ nm.

Finally, we have considered the effect on the quality of the reconstructions when the dielectric constant of the scattering medium is changed, with all of the remaining experimental, material, and roughness parameters keeping the values used in obtaining Fig. 1. In Figs. 5(a)–5(c) we present the reconstructed and input $W(|x_1|)$ for the random surfaces defined by Eqs. (12)–(14), respectively, obtained for a value of $\epsilon = 2.7225$, so that these surfaces are more reflective than the ones leading to Fig. 1. The reconstructed curves are seen to lie on top of the input curves. The values of δ obtained from Eq. (9) for the surfaces generated by the Gaussian, cosine, and triangular power spectra are (1) 59.9 nm, (2) 59.89 nm, and (3) 60.18 nm, respectively, again very close to the input value $\delta = 60$ nm.

Until now we have carried out the reconstruction of $W(|x_1|)$ and the determination of δ knowing the value of the dielectric constant ϵ of the scattering medium. In practical situations we often do not know the value of ϵ, and it becomes one of the quantities to be determined by the inversion procedure. To determine the value of ϵ from data for the dependence of the mean differential reflection coefficient, we assume a sequence of values for ϵ. Since the refractive index of glass is about 1.5, we assumed values of ϵ starting at 2.2 and extending up to 3.0. For each assumed value of ϵ, values of δ and the function $W(|x_1|)$ were determined from Eqs. (9) and (8), respectively. The angular dependence of the mean differential reflection coefficient corresponding to each set of ϵ, δ, and $W(|x_1|)$ determined in this manner was then calculated by the use of Eq. (7). The dielectric constant closest to the actual value of ϵ was obtained by minimizing the cost function

$$\chi = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} d\theta_s \left[\left\langle \frac{\partial R_s}{\partial \theta_s} \right\rangle (\theta_s, 0)_{\mathrm{input}} - \left\langle \frac{\partial R_s}{\partial \theta_s} \right\rangle (\theta_s, 0)_{\mathrm{calc}} \right]^2 \tag{15}$$

with respect to variations of ϵ. A plot of χ as a function of ϵ possesses a clear minimum at the value of ϵ that describes the scattering medium best and, together with the corresponding results for δ and $W(|x_1|)$, defines the statistical properties of the one-dimensional randomly rough dielectric surface. For example, if we consider the surface defined by the Gaussian surface height correlation function and the values $\delta = 60$ nm, $a = 3$ μm, and $\epsilon = 2.7225$ assumed in obtaining the results presented in Fig. 5(a), this method yields $\epsilon = 2.725$ and $\delta = 59.87$ nm.

The transverse correlation length T of a randomly rough surface is defined as the distance over which the normalized surface height autocorrelation function $W(|x_1|)$ decreases significantly from its value of unity at $x_1 = 0$ [13]. A precise definition of this length does not appear to have been given in the literature. In fact, for a one-dimensional randomly rough surface several definitions have been proposed [13]. One that

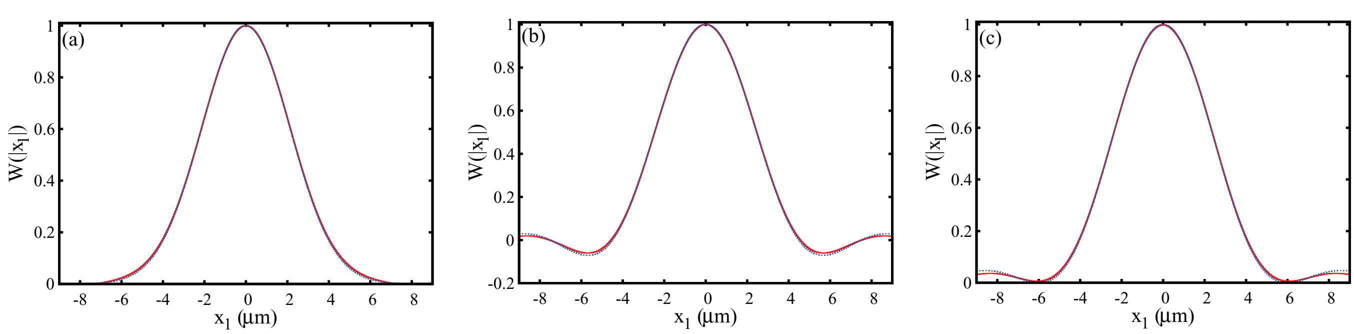

FIG. 5. (Color online) The same as Fig. 1, but with $\epsilon = 2.7225$.

is expressed in terms of $W(|x_1|)$ can be written

$$\left|\frac{d^2}{dx_1^2}W(|x_1|)\right|\bigg|_{x_1=0} = \frac{2}{T^2}. \tag{16}$$

This definition is a modification of the one given by Bass and Fuks [14], which lacks the factor of 2 on the right-hand side. It has the advantage over the latter definition in that for a surface defined by the Gaussian surface height autocorrelation function, Eq. (11b), where the characteristic length a is normally called the transverse correlation length, it yields the result $T = a$. On applying this definition to the reconstructed surface height autocorrelation functions obtained for the surfaces derived from the Gaussian, cosine, and triangular power spectra and depicted in Figs. 1(a)–1(c) and 5(a)–5(c), for values of ϵ equal to 2.25 and 2.7225, respectively, we obtain the values (1) $T = 3.079$ and 3.082 μm, (2) $T = 3.125$ and 3.154 μm, and (3) $T = 3.338$ and 3.366 μm. Since $W(|x_1|)$ is independent of ϵ, the near equality of the results for the two different values of ϵ for each of the three types of surfaces is an indication of the accuracy of our inversion method, especially for x_1 in the vicinity of $x_1 = 0$.

In this paper we have presented a simple approach to the reconstruction of the normalized surface height autocorrelation function $W(|x_1|)$ of a one-dimensional randomly rough dielectric surface from data for the mean differential reflection coefficient obtained for normally incident s-polarized light. This approach, which is based on the Kirchhoff approximation, also enables the rms height of the surface to be determined and provides an estimate of the transverse correlation length of the surface roughness. It also enables the determination of the dielectric constant of the scattering medium in cases in which this is not known *a priori*. The agreement between the reconstructed and input $W(|x_1|)$ has been found to be excellent for values of the experimental, material, and roughness parameters characterizing the scattering system in the ranges $458 \leqslant \lambda \leqslant 680$ nm, $2.25 \leqslant \epsilon \leqslant 2.7225$, $60 \leqslant \delta \leqslant 100$ nm, and $2.5 \leqslant a \leqslant 18$ μm. The domain of validity of this approach to inversion is undoubtedly larger than this, but its determination requires additional calculations.

We have been able to assess the quality of our reconstructions of $W(|x_1|)$ because we know the $W(|x_1|)$ used in generating the mean differential reflection coefficients used in our inversion approach. In an actual experimental situation we do not have this information. How, then, can one determine whether a particular reconstruction of $W(|x_1|)$ is a reliable one? The first step is to find out if Eq. (9) has a solution for δ. If it does not, it means that the surface is one for which the Kirchhoff approximation is inapplicable. If a solution exists, and a result for $W(|x_1|)$ is obtained from Eq. (8), it should have the properties that the transverse correlation length T obtained from it is significantly larger than the wavelength λ and that the rms height of the surface δ is smaller than T. If these conditions are met, the reconstructed $W(|x_1|)$ should be a good representation of the actual normalized surface height autocorrelation function of the surface. In any case, it is clear from the results presented here that for weakly rough surfaces the Kirchhoff approximation provides a simple and accurate method for inverting experimental data for the mean differential reflection coefficient to obtain the normalized surface height autocorrelation function and rms height of a one-dimensional randomly rough dielectric surface.

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