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Investigation of the collapse of quantum states using entangled photons

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We propose a scheme to investigate the time scale of wave-function collapse by using polarization-entangled photon pairs. The setup is similar to those employed to study quantum correlations, but in the present case, synchronization is essential at all stages. We find that it is possible to discriminate between the scenarios of instantaneous collapse and finite-time reduction via a large number of double measurements of polarization. The quantities to be recorded present distinct behaviors in each scenario, the deviations being small but distinguishable from pure statistical fluctuations. The connection between the presented formalism and the theory of weak measurements is discussed as well as the possible consequences for experimental tests of nonlocality.

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I. INTRODUCTION

The wave function and its collapse remain in controversial positions in the general framework of quantum theory. Nevertheless, for long periods in the development of wave mechanics these issues were put aside by most users of the quantum formalism as, perhaps, an underlying discomfort. One of the reasons for this is the fact that there were plenty of more direct questions to be coped with regarding, e.g., atomic and particle physics. In the last few decades, however, experiments reached a remarkable sophistication and textbook illustrations became feasible in the laboratory. This allowed for objective discussions on, until then, purely academic matters, such as for example in the experimental tests [1] of Bell's inequalities [2].

Since then, part of the focus has started to migrate from operational aspects to more foundational ones. This ongoing move is so important that, justifiably, it has been termed the *second quantum revolution* [3]. Examples of this process are the debate on the "reality" of quantum states, which has received special attention in the last year [4] (see also [5]), and the many facets of the measurement problem, in particular, the collapse of the state vector [6–11]. These two topics are intimately related since there is no collapse problem in the epistemic view, where a state is regarded as the experimenter's information on some aspects of reality. In particular, in the statistical interpretation [12], where the basic entity is an ensemble (nothing being said about single particles), the decoherence program [8] alone seems to solve the remaining puzzle, namely, the lack of superposition states in the macroscopic world. However, if one admits that the quantum state of a single object has a physical reality, the ontic view, then the collapse problem persists. In such a case it is hard to accept that any kind of instantaneous evolution can happen. In this work we take this observation earnestly, and argue that if the state vector is of ontological nature, then the collapse should not be instantaneous. In what follows we show that it is possible to check this hypothesis experimentally via a large number of synchronized polarization measurements of two correlated photons.

In a previous work [10] we presented some basic assumptions that we will adopt here, along with a formal example, constituting a proof of principle of how one could investigate the collapse time of quantum states. This example involved a single, completely unspecified, system with a two-dimensional Hilbert space, where two almost simultaneous measurements of *incompatible* observables were required, a conceptual and practical difficulty.

In the present paper we study a specific composite system, two entangled photons, for which the need of incompatible measurements is removed. A description of a feasible experiment to investigate the collapse time is given along with a detailed theoretical analysis, where all calculations and estimates are based on realistic figures, with imperfect synchronization taken into account.

The paper is organized as follows: In the next section we state our basic hypothesis, discuss its interpretation in the specific case of photodetection, and analyze the possible connections with the theory of weak measurements. In Sec. III, we give a description of the proposed experiment along with the calculation of the quantities to be measured. In Sec. IV we give our conclusions and some final remarks.

Finally, it is worth mentioning that nonvanishing collapse times have been considered before in different circumstances, e.g., in the search for stochastic terms which, added to the Schrödinger equation, produce a reduction dynamics consistent with Born's rule [13].

II. FINITE-TIME REDUCTION

In a scenario of noninstantaneous collapse the measurement postulate must be recast in some way. A recent proposal [10] that we will adopt here, with some modifications, reads as follows:

(I) *Measurement duration and* "*hits*." We take into account the fact that any actual measurement has a duration, which we denote by Δt, and, most importantly, we assume that the collapse is a process initiated by a random hit (we borrow this terminology from [6] in a distinct context) occurring at $t^{(h)}$, which is taken as a stochastic variable obeying some probability distribution $f(t)$, defined in the window $[t_0,t_0+\Delta t]$. For $t<t^{(h)}$, the system remains uncoupled to external degrees of freedom. The exact nature of the distribution and the duration Δt depend on the specific system and measurement method, as will be exemplified later.

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(II) *Finite-time collapse*. The quantum state takes a short time δt, starting from $t^{(h)}$, to complete the reduction. We do not make any specific statements about the nonunitary time evolution in the interval $[t^{(h)},t^{(h)}+\delta t]$. But we do assume that during the reduction the state of the system is still contained in a ket belonging to the enlarged Hilbert space $\mathcal{E}_T=\mathcal{E}\otimes\mathcal{E}_X$, where $\mathcal{E}$ concerns the system of interest and $\mathcal{E}_X$ refer to all relevant degrees of freedom that couple with it. In addition, we assume that δt is a decreasing function of $|\langle\Psi_0|\Psi_f\rangle|$ (the inner product between the initial and final states in $\mathcal{E}$), such that, if the states are orthogonal $\delta t\to\infty$, while $\delta t=0$ for coincident initial and final states.

Regarding (I) we remark that it is very important to realize that $t^{(h)}$ must not be confused with the moment when the pointer comes to a definite position, that is, when the avalanche photodiode (APD) delivers a macroscopic current, in the case of photodetection. This time scale has been studied from different perspectives [14]. It is an essential part of our hypothesis that this macroscopic phenomenon is preceded by a microscopic event that triggers the collapse of the state ket at $t=t^{(h)}$.

We now give the form of $f(t)$ for the detection of photons. We recall that, according to standard quantum mechanics, the probability density associated with the detection of a photon with a given frequency is proportional to the intensity profile of the corresponding mode of the quantized electromagnetic field. That is, $|\varphi(\mathbf{r},t)|^2\propto|\mathbf{E}(\mathbf{r},t)|^2$, where φ is the wave function and $\mathbf{E}$ is the electric field. Since we admit that the macroscopic detection of a photon is an "echo" of a microscopic hit, a one-to-one relation, we must have $f(t)\propto|\mathbf{E}|^2$. In words, *we associate the probability distribution for the occurrence of a hit with the temporal intensity profile of the photons that reach the detector*.

A. Connection with weak measurements

Let us briefly analyze how the previous hypotheses fit in the framework of measurement theory. The general measurement postulate can be written as follows [15]: Let $\{\hat{M}_j\}$ be a set of operators, with $\sum_j \hat{M}_j^\dagger\hat{M}_j=\hat{I}$, and a state $|\psi\rangle$ describing an arbitrary system immediately before measurement. Then the probability of obtaining a result j is $p(j)=\langle\psi|\hat{M}_j^\dagger\hat{M}_j|\psi\rangle$, and the state of the system after the measurement satisfies

$$|\psi\rangle\to\frac{\hat{M}_j|\psi\rangle}{\sqrt{p(j)}}. \tag{1}$$

In this postulate any discussion of the duration of the partial collapse is also missing. If $\hat{M}_j=|u_j\rangle\langle u_j|$, with $\{|u_j\rangle\}$ being an orthonormal basis, one recovers the usual postulate for projective measurements.

An important and nontrivial situation encompassed by the general postulate is that of weak measurements [16–18]. In this case the operators $\{\hat{M}\}$ are not projectors and the state after the measurement changes slightly with respect to $|\psi\rangle$. It has been shown that weak measurements are universal in the sense that any generalized measurement, including projective ones, can be formally seen as a series of infinitesimal partial collapses [19]. This process can be pictorially written as

$$\begin{aligned}\left|\psi\left(x_\nu^{(0)}\right)\right\rangle&\to\left|\psi\left(x_\nu^{(1)}\right)\right\rangle\to\left|\psi\left(x_\nu^{(2)}\right)\right\rangle\to\cdots\\&\to\left|\psi\left(x_\nu^{(n)}\right)\right\rangle\cdots\to\left|\psi\left(x_\nu^{(f)}\right)\right\rangle,\end{aligned} \tag{2}$$

where variations of the parameters $\{x_\nu\}$ over a specified range make the ket $|\psi(x_\nu)\rangle$ span a particular region in the Hilbert space. For example, in the case of a single qubit one can take $\{x_1,x_2\}$ as the angles defining a state in the Bloch sphere. Each step is assumed to be infinitesimal, $x_\nu^{(n)}-x_\nu^{(n-1)}=dx_\nu^{(n)}$ and it is usual to assume the time evolution of $\{x_\nu\}$ to be a stochastic process.

In this framework it is natural to think of δt as the time taken in going from $|\psi(x_\nu^{(0)})\rangle$ to $|\psi(x_\nu^{(f)})\rangle$. The hit time $t^{(h)}$ would be the moment associated with the first weak measurement. In our model $t^{(h)}$ is a stochastic parameter too, such that the process itself and the moment it starts will be random.

One may ask how, in practice, a single weak measurement can be performed. One possible answer, already experimentally implemented in superconducting qubits [20], is by *not detecting* with some probability. The qubit is prepared in a superposition of the ground state $|0\rangle$ and the excited state $|1\rangle$, say, $|\psi\rangle=(|0\rangle+|1\rangle)/\sqrt{2}$. The qubit well potential is such that, in the ground state, the tunneling probability is vanishingly small, while state $|1\rangle$ presents a probability p to tunnel (when the potential height is adiabatically lowered) and reach the detector. Thus, if we get a click, we have a projective collapse $|\psi\rangle\to|1\rangle$. However, if in a particular realization no detection occurs, then the state becomes

$$|\psi\rangle\to\sqrt{\frac{1+p}{2}}|0\rangle+e^{-i\phi_M}\sqrt{\frac{1-p}{2}}|1\rangle, \tag{3}$$

where ϕ_M is a phase due to the adiabatic change in the potential. This characterizes a partial collapse towards $|0\rangle$. We note that, if one resorts to weak measurements to interpret a finite-time collapse, the above notions must be extended to entangled states. This was recently considered in the context of quantum information theory [21]. Whether or not a complete collapse is physically, and not only mathematically, a composition of partial collapses is a relevant issue. We believe that the experiment we propose here may shed some light on this question.

III. TWO CORRELATED PHOTONS

The system we address here is composed of two spatially separated photons which are simultaneously generated, and led to the entangled polarization state

$$\begin{aligned}|\Psi_0\rangle&=\alpha|+\rangle_L\otimes|-\rangle_R+\beta|-\rangle_L\otimes|+\rangle_R\\&\equiv\alpha|+-\rangle+\beta|-+\rangle,\end{aligned} \tag{4}$$

where $\alpha\neq0$ and $\beta\neq0$, and the subscripts L and R refer to the photons sent to the "left" and "right" detectors, respectively (Fig. 1). The generation can be achieved with a nonlinear crystal via a parametric down-conversion process, which, in general, gives synchronization and may also produce entanglement in polarization [22,23]. Since the photons follow distinct optical paths, a delay between them is likely to be introduced. While this is not critically relevant in evaluating violations of Bell's inequalities, it may hinder the phenomenon we intend to investigate. Up to this point, the synchronization of the pair can be restored with the help of a Hong-Ou-Mandel (HOM) apparatus [24] and delay lines coupled with translation stages. From this point to the detectors the synchronization is technically nontrivial, but can be handled, in principle. For

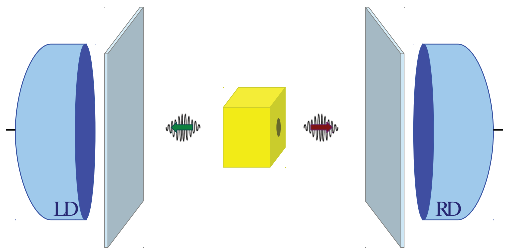

FIG. 1. (Color online) Pictorial representation of the experimental setup. The box contains a nonlinear crystal that generates pairs of synchronized photons via parametric down-conversion. The crystal is pumped by a pulsed laser (not shown).

a recent proposal of a scheme to measure ultrashort delays see [25]. As the photons reach the detectors their polarizations are measured with the filters set in the *same* direction, for which $\{|+\rangle,|-\rangle\}$ are eigenstates.

To be more realistic, we assume that the two wave packets attain the detectors simultaneously (in the laboratory frame), except for a delay $\mathcal{T}$ that eventually persists (Fig. 2). We stress that our model encompasses the expected situation in which the residual delay is typically much larger than δt. Note carefully that the procedure ensures that the centroids of each wave packet will reach the detectors approximately at the same time, and not that the (unpredictable) hits themselves will be simultaneous. Note also that the spatiotemporal profile to be considered is not that of the generated photons, but rather of the photons just before detection (with the spreading and deformation taken into account). Finally, the quantities we suggest to be measured are subtle statistical deviations, so we need a robust sampling. This demand naturally leads us to consider that a pulsed laser (in our case a titanium-sapphire laser) with a high repetition rate is employed as the primary source of photons.

From basic quantum mechanics we immediately infer the statistical distribution resulting from a series of N coincidence polarization measurements on state (4); see Table I. Fluctuations with magnitude $\Delta\mathcal{N} \sim \sqrt{N}/2$ naturally show up for any finite number of repetitions. Since in any standard interpretation of quantum mechanics the collapse is assumed to be instantaneous, the second measurement of polarization (in the same direction) will not play any role.

Now we address the same question, this time considering the possibility of noninstantaneous collapse. In this case we must analyze the development of the events more carefully. We start by assuming that the intensity profile of the electromagnetic field associated with the photons is already characterized. We denote the distributions for a hit in the left and right

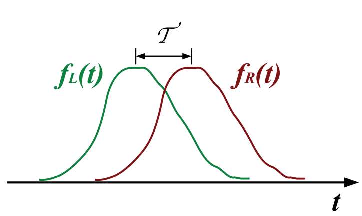

FIG. 2. (Color online) Probability densities for the occurrence of a hit at the left and right photons. The residual delay, due to imperfections in the synchronization process, is denoted by $\mathcal{T}$.

TABLE I. Outputs and relative frequencies of two sequential polarization measurements in the same direction according to quantum mechanics. The second measurement plays no role since, after the first one, the state collapses instantly.

Result	Frequency
Left +, right −	$\lvert\alpha\rvert^2$
Left −, right +	$1-\lvert\alpha\rvert^2$

detectors by $f_L(t)$ and $f_R(t)$, respectively, and, for definiteness, we assume the left photon to be delayed with respect to the right one. Apart from this we consider the two packets as having the same shape, that is,

$$f_R(t) = f_L(t-\mathcal{T}). \tag{5}$$

In this scenario two distinct situations may happen. If, as we suppose here, δt is smaller than any other time scale in the problem, then, with high probability, when the second hit takes place, the reduction due to the first one is already completed. For these realizations we obtain exactly the results shown in Table I. However, in a small number of nontrivial events, according to our hypothesis, the second hit catches the state ket while it is still collapsing. As soon as the first hit happens, no matter in what detector (the two filters are parallel), the state starts to collapse following one of the two kinematic routes

$$|\Psi_1(t)\rangle = a_1(t)|+-\rangle \otimes |\Phi^{(1)}_{+-}\rangle + b_1(t)|-+\rangle \otimes |\Phi^{(1)}_{-+}\rangle, \tag{6}$$

$$|\Psi_2(t)\rangle = a_2(t)|+-\rangle \otimes |\Phi^{(2)}_{+-}\rangle + b_2(t)|-+\rangle \otimes |\Phi^{(2)}_{-+}\rangle, \tag{7}$$

with boundary conditions

$$\begin{aligned} a_1[t^{(h)}] &= \alpha, \quad a_1[t^{(h)} + \delta t^{(1)}] = 1,\\ b_1[t^{(h)}] &= \beta, \quad b_1[t^{(h)} + \delta t^{(1)}] = 0, \end{aligned} \tag{8}$$

and

$$\begin{aligned} a_2[t^{(h)}] &= \alpha, \quad a_2[t^{(h)} + \delta t^{(2)}] = 0,\\ b_2[t^{(h)}] &= \beta, \quad b_2[t^{(h)} + \delta t^{(2)}] = 1, \end{aligned} \tag{9}$$

where $\delta t^{(1)}$ and $\delta t^{(2)}$ denote the collapse times for routes (6) and (7), respectively. We denote the instant when the first hit happens by $t^{(h)}$. The kets $\{|\Phi\rangle\}$, whose time dependence was suppressed, correspond to the microscopic states of the degrees of freedom that couple to the system. To be consistent with Born's rule we assume that route (6) happens with relative frequency $|\alpha|^2$ and the second route, Eq. (7), with relative frequency $|\beta|^2 = 1-|\alpha|^2$. We use the terminology "kinematic route" because we are not providing, or trying to provide, the dynamical equations that are satisfied by $a(t)$ and $b(t)$ during the reduction. Instead, we use only the fact that at the end of the process Born's postulate should be verified. We also remark that by excluding terms proportional to $|++\rangle$ and $|--\rangle$ in the intermediate states (6) and (7), with vanishing coefficients for $t = t^{(h)}$ and $t = t^{(h)} + \delta t^{(i)}$, we assume angular momentum conservation during each individual process. Setting $y = t^{(h)}_L - t^{(h)}_R$, where $t^{(h)}_L$ ($t^{(h)}_R$) is the time when a hit occurs in the left (right) detector, the probability $P_<$ for the occurrence of both hits in a time interval shorter than $\delta t^{(i)}$ when the first hit leads to route i is

$$P_< = |\alpha|^2 P_<^{(1)} + |\beta|^2 P_<^{(2)}, \tag{10}$$

where

$$P_<^{(i)} = \int_{-\Delta t}^{+\Delta t} \int_{t''=t'-\delta t^{(i)}}^{t''=t'+\delta t^{(i)}} f_L(t')\, f_R(t'') dt' dt''. \tag{11}$$

The probability density for the relative variable y for each route is given by $p^{(i)}(y) = dP_<^{(i)}/d(\delta t^{(i)})|_y$. It is clear from the previous relation that $p^{(1)}(y) = p^{(2)}(y) = p(y)$.

Let us then consider the rare event of a second hit happening between t and $t+dt$ with $t^{(h)} < t < t^{(h)} + \delta t^{(i)}$. In this situation, if the state is caught collapsing via route (6), the outcomes after the second reduction is completed are $|+-\rangle|\Phi_{+-}^{1f}\rangle$ with probability $|a_1(t)|^2$ and $|-+\rangle|\Phi_{-+}^{1f}\rangle$ with probability $|b_1(t)|^2$. If the state is collapsing through route (7), the possible results are $|+-\rangle|\Phi_{+-}^{2f}\rangle$ with probability $|a_2(t)|^2$ and $|-+\rangle|\Phi_{-+}^{2f}\rangle$ with probability $|b_2(t)|^2$. The final states $\{|\Phi^f\rangle\}$ need not be macroscopic pointers at this stage. Rather, we assume that they contain the state of the apparatus which, after some extra time, will describe a definite macroscopic pointer position. We, thus, have a composite von Neumann chain. From the previous reasoning, the probability of getting $+-$ between t and $t+dt$ is proportional to

$$[|\alpha|^2|a_1(t)|^2 + (1-|\alpha|^2)|a_2(t)|^2]dt. \tag{12}$$

The probability of obtaining $+-$ for any t, satisfying $t^{(h)} < t < t^{(h)} + \delta t^{(1)}$ or $t^{(h)} < t < t^{(h)} + \delta t^{(2)}$, depending on the route, is given by the integral

$$\begin{aligned} P(+-\mid \delta t) &= \frac{|\alpha|^2}{P_<}\int_{-\delta t^{(1)}}^{\delta t^{(1)}} |a_1(y)|^2 p(y)dy \\ &\quad + \frac{|\beta|^2}{P_<}\int_{-\delta t^{(2)}}^{\delta t^{(2)}} |a_2(y)|^2 p(y)dy \\ &= \frac{|\alpha|^2(\Gamma + P_<) + |\beta|^2\Lambda}{P_<}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \Lambda &= \int_{-\delta t^{(2)}}^{\delta t^{(2)}} |a_2(y)|^2 p(y)dy, \\ \Gamma &= \int_{-\delta t^{(1)}}^{\delta t^{(1)}} |a_1(y)|^2 p(y)dy - P_<. \end{aligned} \tag{14}$$

We can write the unconditional probability of getting the result $+-$ as

$$\begin{aligned} P(+-) &= (1-P_<)|\alpha|^2 + P_<\, P(+-|\delta t) \\ &= |\alpha|^2 + (|\alpha|^2\Gamma + |\beta|^2\Lambda). \end{aligned} \tag{15}$$

Therefore, if the collapse is not instantaneous, within our hypothesis, the outcomes of two well-synchronized polarization measurements should be characterized by Table II.

Table II might give the impression that we are suggesting a correction to Born's postulate. This is not the case, since the postulate refers to the likelihood of each possible result of a single measurement. If collapse were indeed instantaneous, a second measurement of the same observable would be innocuous. What we have just shown is that, if the collapse takes a finite time, then a close consideration of Born's rule leads to the probabilities in Table II. Once a sufficiently large number N of repetitions is made, the numerical difference between the results $+-$ and $-+$ according to Table I is $\Delta N_I = (2|\alpha|^2 - 1)N$, while the same quantity, according to Table II, is $\Delta N_{II} = [2(|\alpha|^2\Gamma + |\beta|^2\Lambda) + 2|\alpha|^2 - 1]N$. Thus, the deviation between the two scenarios is given by

TABLE II. Outputs and relative frequencies of two sequential, accurately synchronized, polarization measurements in a scenario of finite-time reduction.

Result	Frequency
Left +, right −	$\|\alpha\|^2 + (\|\alpha\|^2\Gamma + \|\beta\|^2\Lambda)$
Left −, right +	$1 - \|\alpha\|^2 - (\|\alpha\|^2\Gamma + \|\beta\|^2\Lambda)$

$$\Delta N = \Delta N_{II} - \Delta N_I = 2(|\alpha|^2\Gamma + |\beta|^2\Lambda)N. \tag{16}$$

For a maximally entangled state, with $|\alpha| = 1/\sqrt{2}$, implying $\delta t^{(1)} = \delta t^{(2)}$, it would not be possible to reveal a potentially nonvanishing collapse time, since this would lead to $\Delta N = 0$. This is due to the fact that, in this completely symmetrical case, we must have $|a_1(y)|^2 = |b_2(y)|^2$. Therefore, the initial state (4) has to be unbalanced. As we will see next, Γ and Λ are typically very small numbers and the difference (16) is subtle. The immediate question that arises is, since $\Delta N/N$ is small, can we safely distinguish it from pure statistical fluctuations ($\Delta\mathcal{N}$) that surely occur in an actual experiment? Fortunately, the deviation in the worse scenario, where $\Delta\mathcal{N}_I \sim \sqrt{N}/2$ and $\Delta\mathcal{N}_{II} \sim -\sqrt{N}/2$ tend to minimize $|\Delta N|$, is $\Delta\mathcal{N} = \sqrt{N}$, while $\Delta N \sim N$, so that for a sufficiently large number of realizations one can reach a ratio $\Delta N/\Delta\mathcal{N}$ as large as needed. In fact, $\Delta N/\Delta\mathcal{N} = 2(|\alpha|^2\Gamma + |\beta|^2\Lambda)\sqrt{N}$, and the number of realizations must satisfy

$$N > K^2[2(|\alpha|^2\Gamma + |\beta|^2\Lambda)]^{-2}, \tag{17}$$

for a statistical significance of K standard deviations ($\Delta N > K\Delta\mathcal{N}$).

Once the general framework is set, let us go back to our proposed experiment in more specific terms. Suppose the source of light is a pulsed titanium-sapphire laser whose normalized temporal profile of intensity reads

$$f(t) = \frac{1}{2\sigma_t}\mathrm{sech}^2\left(\frac{t}{\sigma_t}\right), \tag{18}$$

where the pulse width σ_t provides the coherence time. By using (5) and (11) we get

$$\begin{aligned} P_<^{(i)} = \frac{1}{4}\sum_{n=0,1}(-1)^n \Bigg\{ &\mathrm{csch}^2\big(A_n^{(i)}\big)\ln\left[\frac{\cosh\big(A_n^{(i)} + \Delta t/\sigma_t\big)}{\cosh\big(A_n^{(i)} - \Delta t/\sigma_t\big)}\right] \\ &-2\coth\big(A_n^{(i)}\big)\tanh\left(\frac{\Delta t}{\sigma_t}\right)\Bigg\}, \end{aligned} \tag{19}$$

with $A_n^{(i)} = [\mathcal{T} + (-1)^{n+1}\delta t^{(i)}]/\sigma_t$. Assuming that $\delta t^{(i)}$ and $\mathcal{T}$ are much smaller than the coherence time, the above result simplifies to

$$P_{<}^{(i)} \approx \frac{1}{2}\tanh\left(\frac{\Delta t}{\sigma_t}\right)\left[\tanh\left(\frac{\mathcal{T}+\delta t^{(i)}}{\sigma_t}\right)-\tanh\left(\frac{\mathcal{T}-\delta t^{(i)}}{\sigma_t}\right)\right], \quad (20)$$

leading to the probability distribution

$$p(y) \approx \frac{1}{2\sigma_t}\tanh\left(\frac{\Delta t}{\sigma_t}\right)\operatorname{sech}^2\left(\frac{\mathcal{T}+y}{\sigma_t}\right). \quad (21)$$

A typical repetition rate of a pulsed laser is 100 MHz; however, this is not the frequency at which the correlated pairs are detected in coincidence. We assume that this rate is diminished by three orders of magnitude, giving one detection in coincidence per 10 μs, on average. Furthermore, the residual delay is of the order of $\delta L/c$, where δL is the step of the translation stage in the delay line and c is the velocity of light. Usually $\delta L \approx 1$ μm, so $\mathcal{T} \approx 3.3$ fs. The duration Δt of each measurement is set to ensure that the detected photons belong to the same pair. We can safely consider the window of coincidence to be $\Delta t \approx 1$ ns. Finally, for the sake of illustration let us assume that the final shape of the wave packets at detection is still given by Eq. (18), with a relatively large spreading of $\sigma_t \approx 1$ ps (the coherence time soon after the generation is, say, 200 fs).

Consider that we intend to investigate the compatibility of experimental data with a collapse in the range of $\delta t \sim 0.1$ fs, with $\alpha = \sqrt{3}/2$. This would lead to $P_< \approx 10^{-4}$, corresponding to ten nontrivial detections per second. Of course, in order to get numbers we must assume some functional form for $a_1(t)$ and $a_2(t)$ before calculating Γ and Λ. The quantitative results weakly depend on this choice, but the qualitative features remain unchanged. By choosing an exponential dependence for $|a_1(t)|$ and $|a_2(t)|$, satisfying the appropriate boundary conditions, we get $|\alpha|^2\Gamma + |\beta|^2\Lambda \approx 10^{-4}$ [26]. If one adopts the sequential weak-measurement view, $a(t)$ should probably be seen as a smoothed version of the actual random-walk-type dynamics followed by the coefficients [19].

Suppose that we obtain a reliable statistics characterized by $\Delta N/\Delta\mathcal{N} \sim 6$ (six standard deviations), corresponding to a 12-h-long experiment ($N \approx 10^9$ realizations). This result alone would be a strong evidence for finite-time collapse. Of course, further experimentation would be necessary, varying $\mathcal{T}$, α, and the orientation of the filters, to investigate the actual time dependence of $a(t)$. It would be especially important to repeat the same procedure with the filters set in orthogonal directions for, in this situation, there should be no measurable difference between the two scenarios for any pair α,β. Conversely, if in the original experiment we obtain $\Delta N/\Delta\mathcal{N} \sim 1$, then 0.1 fs would be an upper bound for an exponential reduction in the system studied. Lastly, the above estimates would not change appreciably for $\mathcal{T} = 0$, showing that the scheme is robust for delays of the order of femtoseconds.

IV. CONCLUSIONS

It might be considered insufficient to assert that if the state is ψ-ontic, in the sense adopted in the literature [27], then the wave function is a real thing [28]. A possibly reasonable extra requirement would be that the microscopic collapse should not be instantaneous. We stress that this has nothing to do with the condition of locality, since the collapse $(|a\rangle|b\rangle - |b\rangle|a\rangle)/\sqrt{2} \xrightarrow{\delta t} |a\rangle|b\rangle$ is, in general, nonlocal, provided that the subsystems are sufficiently far apart. By employing minimal statements (of kinematic nature) about this finite-time collapse and assuming that Born's rule remains valid during the nonunitary evolution, we claim that it is possible to probe the collapse duration experimentally on the scale of subfemtoseconds. Although decoherence is not a logical necessity of our model, the previous results may not be incompatible with it. In fact, it has been suggested by Schlosshauer [7] that a combination of dynamical localization models and the effects of environment is a promising strategy to approach the collapse problem.

An important point is the lack of covariance of our results, which is not an exception in dealing with entanglement. If we admit that the two hits are not causally related, their ordering may be swapped for some inertial frame. Alternatively, if one assumes that the hits are causally connected it is necessary to admit the existence of an "ether" in which the spatially separated subsystems exchange information via a supraluminal signaling. A stringent lower bound for the velocity of this signal has been placed [29], $v_{\text{signal}} \sim 10^4 c$ for a continuous set of referentials whose relative velocity with respect to earth is as large as $0.1c$. Although a detailed discussion of this point is outside the scope of the present work, we believe that the dynamics of collapse, and its time scale, deserve investigation in either case.

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