Erratum: Gravitationally coupled Dirac equation for antimatter [Phys. Rev. A 87, 032101 (2013)]

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The central result of our paper [Phys. Rev. A **87**, 032101 (2013)] is given by the particle-antiparticle symmetry relation [see Eq. (47) of our paper],

$$E \leftrightarrow -E, \quad f(r) \leftrightarrow g(r), \quad \varkappa \leftrightarrow -\varkappa, \tag{1}$$

valid for eigensolutions of the Dirac equation in a generalized Schwarzschild metric. This result is not affected by the current Erratum, but a number of typographical errors in intermediate steps of the derivation need to be corrected. First, we have to point out that Eq. (40) of our paper should have read

$$\overline{\gamma}^0 \overline{\gamma}^\mu \Gamma_\mu = -\frac{\widetilde{\gamma}^0 \vec{\widetilde{\gamma}} \cdot \hat{r}}{v(r) w(r)} G(r), \tag{2a}$$

$$G(r) = \frac{2v'(r)w(r) + v(r)w'(r)}{2v(r)w(r)}, \tag{2b}$$

where $\hat{r} = \vec{r}/|\vec{r}|$ is the position unit vector. Furthermore, $\widetilde{\gamma}^0$ is the Dirac β matrix,

$$\widetilde{\gamma}^0 = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}, \quad \vec{\widetilde{\gamma}} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \tag{3}$$

and $\vec{\widetilde{\gamma}}$ is the vector of flat-space Dirac γ matrices. For the record, we would like to clarify that Eq. (40) of our paper remains valid if the curved-space $\overline{\gamma}^\mu$ matrices on the left-hand side of Eq. (40) of our paper are replaced by flat-space $\widetilde{\gamma}^\mu$ matrices. The correction from Eq. (2) entails the following change for Eq. (46) of our paper,

$$\left(\frac{\partial}{\partial r} + \frac{1-\varkappa}{r} + G(r)\right) g(r) = v(r)\left(m - \frac{E}{w(r)}\right) f(r), \tag{4a}$$

$$\left(\frac{\partial}{\partial r} + \frac{1+\varkappa}{r} + G(r)\right) f(r) = v(r)\left(m + \frac{E}{w(r)}\right) g(r), \tag{4b}$$

where the invariant line element is $ds^2 = w(r)^2 dt^2 - v(r)^2 d\vec{r}^2$. Furthermore, Eq. (45) of our paper also needs to be modified (we suppress one intermediate step),

$$\frac{i\partial_t \psi(\vec{r})}{w^2(r)} = \begin{pmatrix} \frac{m}{w(r)} & \frac{\vec{\sigma}\cdot\hat{r}}{v(r)w(r)}\left(-i\frac{\partial}{\partial r} + i\frac{\vec{\sigma}\cdot\vec{L}}{r} - iG(r)\right) \\ \frac{\vec{\sigma}\cdot\hat{r}}{v(r)w(r)}\left(-i\frac{\partial}{\partial r} + i\frac{\vec{\sigma}\cdot\vec{L}}{r} - iG(r)\right) & -\frac{m}{w(r)} \end{pmatrix} \begin{pmatrix} f(r)\chi_{\varkappa\mu}(\hat{r}) \\ ig(r)\chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix}$$

$$= \begin{pmatrix} \left[\frac{1}{v(r)w(r)}\left(-\frac{\partial}{\partial r} + \frac{1}{r}(\varkappa - 1) - G(r)\right)g(r) + \frac{m}{w(r)} f(r)\right]\chi_{\varkappa\mu}(\hat{r}) \\ i\left[\frac{1}{v(r)w(r)}\left(\frac{\partial}{\partial r} + \frac{1}{r}(\varkappa + 1) + G(r)\right)f(r) - \frac{m}{w(r)} g(r)\right]\chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix} = \frac{E}{w^2(r)} \begin{pmatrix} f(r)\chi_{\varkappa\mu}(\hat{r}) \\ ig(r)\chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix}. \tag{5}$$

For completeness, we also indicate that the vector of $\vec{\alpha}$ matrices in Eq. (41) should be understood as $\alpha^i = \overline{\gamma}^0 \overline{\gamma}^i$, where $\overline{\gamma}^0 = \widetilde{\gamma}^0/w(r)$ and $\overline{\gamma}^i = \widetilde{\gamma}^i/v(r)$. We reemphasize that the symmetry relation (47) of our paper, on which all further considerations in the paper and all conclusions are based, remains valid.

