Conditions for two-photon interference with coherent pulses

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We study the conditions for two-photon classical interference with coherent pulses. We find that the temporal overlap between optical pulses is not required for interference. However, coherence within the same inputs is found to be essential for the interference.

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Interference is one of the most interesting phenomena in nature for many physicists. Since the first experimental demonstration of optical interference by Young [1], it has been considered one of the most important notions for understanding optics [2]. Classically, it is understood as a coherent superposition of electromagnetic waves, and it explains, in classical terms, many interesting phenomena. For example, one of the outputs of a Mach-Zehnder (MZ) interferometer shows a sinusoidal oscillation with respect to the relative phase difference between two inputs, and this phenomenon can be fully explained by classical theory.

Classical physics, however, sometimes cannot completely describe interference. Let us consider a Hong-Ou-Mandel (HOM) interference between two identical photons [3–5]. When two identical photons enter into a beam splitter (BS) at the same time, the coincidences between two detectors at the outputs of the BS are completely suppressed, a characteristic referred as a HOM dip. The visibility, V, of the HOM dip is defined as the relative depth of the dip compared to the noninterfering cases. Using single-photon states, the coincidences can be completely suppressed, so the visibility can reach up to $V = 1$. The classical theory of the coherent superposition of electromagnetic waves, however, can only explain a HOM dip with $V \leqslant 0.5$ [6]. Thus, a HOM dip with $V > 0.5$ should be considered as a nonclassical phenomenon, and therefore a quantum effect described by a superposition of indistinguishable probability amplitudes [7]. Note that the classical HOM interference has been studied specifically for the investigation of temporal and/or spectral properties of light sources [8–12].

In many experimental demonstrations with optical pulses, classical and quantum interference are measured when pulses have temporal overlap at a BS [10–15]. This often leads to a common misconception that classical and/or quantum interference requires the optical pulses to be temporally overlapping. However, it has been shown that temporal overlapping between optical pulses is not a requirement for quantum interference; both single- and two-photon quantum interference can occur from temporally nonoverlapping single-photon states [16–20]. In these papers, the authors clearly explain the phenomena in terms of quantum physics and the superposition of probability amplitudes with Feynman diagrams [7].

In classical physics, however, the superposition of probability amplitudes and Feynman diagrams are not applicable. Instead, the classical electromagnetic wave superposition theory should be employed to describe the interference. It is easy to think that there would be no interference between two temporally nonoverlapping optical pulses because it seems that the electromagnetic waves do not exist without an optical pulse. Counter-intuitively, however, classical interference does not require the temporal overlap of optical pulses [8,9,21].

In this paper, we study the conditions for two-photon interference between two classical optical pulses. In particular, we investigate the HOM-type two-photon interference with coherent pulses. We found that classical two-photon interference requires coherence within each input rather than the temporal overlap of optical pulses from the inputs. The result can be explained by the classical theory of wave superposition. We also provide a quantum analogy to this phenomenon for more intuitive understanding.

Figure 1 shows the schematic of our two-photon interference experiment with weak coherent pulses, $I_A(i)$, $I_A(j)$, $I_B(i)$, and $I_B(j)$. Here, i,j denote the timing labels and the subscripts A,B are the input modes. The intervals between $I_k(i)$ and $I_k(j)$ for both $k = A,B$ are the same, T. The optical delay between two inputs, $\Delta l = I_A(i) - I_B(i)$, can be scanned for the interference measurement. Note that the interval between $I_B(i)$ and $I_B(j)$ is fixed at T during the scanning. We will only consider the case in which the scanning of Δl is much smaller than T, so it does not provide temporal overlap between two different labeling pulses, e.g., between $I_A(i)$ and $I_B(j)$. If the correlation measurement between two outputs C and D is of interest, the electronic delay $\tau_d = 0$ or $\pm T$ at C introduces the intensity correlation measurement between pulses at various delays. Note that the electronic delay τ_d is not interchangeable with the optical delay Δl. The intensities at the BS outputs $I_C(i)$ and $I_D(i)$ are

$$\begin{aligned} I_C(i) &= \tfrac{1}{2}I_A(i) + \tfrac{1}{2}I_B(i) - \sqrt{I_A(i)I_B(i)}\sin\Delta\phi(i) \\ I_D(i) &= \tfrac{1}{2}I_A(i) + \tfrac{1}{2}I_B(i) + \sqrt{I_A(i)I_B(i)}\sin\Delta\phi(i), \end{aligned} \quad (1)$$

where $\Delta\phi(i)$ denotes the relative phase between two pulses $I_A(i)$ and $I_B(i)$. The relative phase can be represented as

$$\Delta\phi(i) = \Delta\phi_{AB}(i) + \frac{2\pi\,\Delta l}{\lambda}, \quad (2)$$

where $\Delta\phi_{AB}(i)$ represents the inherent phase difference between two pulses $I_A(i)$ and $I_B(i)$ and λ is the wavelength of the light.

When two pulses $I_A(i)$ and $I_B(i)$ are coherent, that is $\Delta\phi_{AB}(i)$ has a fixed definite value, Eq. (1) shows sinusoidal interference which corresponds to a single-photon

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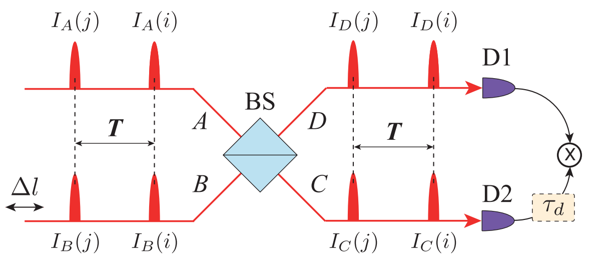

FIG. 1. (Color online) The schematic of Hong-Ou-Mandel interference with four weak coherent pulses.

interference, i.e., Mach-Zehnder-like interference. However, if the two pulses are incoherent, and thus $\Delta\phi_{AB}(i)$ varies randomly, the single-photon interference will be washed out since $\langle \sin\Delta\phi(i)\rangle = 0$, where $\langle x\rangle$ represents the average of x over many events.

The coincidences between D1 and D2 correspond to the correlation measurement between I_C and I_D for low input intensities I_A and I_B. Let us first consider the correlation measurement between two pulses at the same timing, $\langle I_C(i)I_D(i)\rangle$. Note that this case is equivalent to a standard HOM interferometer with coherent pulses as the two pulses meet at the BS. It is easily accomplished in the experiment by putting a zero electronic delay at D2, $\tau_d = 0$. Since $\langle \sin\Delta\phi(i)\rangle = 0$ for a randomized $\Delta\phi(i)$, the correlation measurement is represented by [6]

$$\langle I_C I_D\rangle = \tfrac{1}{4}\langle I_A^2\rangle + \tfrac{1}{4}\langle I_B^2\rangle + \left(\tfrac{1}{2} - \langle \sin^2\Delta\phi\rangle\right)\langle I_A\rangle\langle I_B\rangle. \quad (3)$$

Here, we omitted the label i in Eq. (3). The $\langle \sin^2\Delta\phi\rangle$ term vanishes for the interference free case. In the interference case, $\langle \sin^2\Delta\phi\rangle = 1/2$, thus the whole last term of Eq. (3) disappears. Therefore, the visibility of the two-photon interference in classical physics is

$$V_c = \frac{2\langle I_A\rangle\langle I_B\rangle}{\langle I_A^2\rangle + \langle I_B^2\rangle + 2\langle I_A\rangle\langle I_B\rangle}. \quad (4)$$

For constant intensities $\langle I_k^2\rangle = \langle I_k\rangle^2$, the maximum classical visibility is $V_c^{\max} = 0.5$ when $\langle I_A\rangle = \langle I_B\rangle$. This result is the classical limit of a HOM interference.

Now, let us consider the correlation measurement between pulses which did not exist at the same time, i.e., $\langle I_C(i)I_D(j)\rangle$ where $i \neq j$, thus, $\tau_d = T$. Noting that $\langle \sin\Delta\phi(i)\rangle = \langle \sin\Delta\phi(j)\rangle = 0$, the correlation measurement can be represented by

$$\begin{aligned}\langle I_C(i)I_D(j)\rangle &= \tfrac{1}{4}\langle I_A(i)I_A(j)\rangle + \tfrac{1}{4}\langle I_A(i)I_B(j)\rangle + \tfrac{1}{4}\langle I_B(i)I_A(j)\rangle \\ &\quad + \tfrac{1}{4}\langle I_B(i)I_B(j)\rangle - \sqrt{\langle I_A(i)I_A(j)I_B(i)I_B(j)\rangle} \\ &\quad \times \langle \sin\Delta\phi(i)\sin\Delta\phi(j)\rangle. \end{aligned} \quad (5)$$

In general, $\langle \sin\Delta\phi(i)\sin\Delta\phi(j)\rangle = 0$, thus, Eq. (5) does not show interference. However, let us consider the case when intensities of two pulses are the same at the same input, e.g., $I_A(i)$ and $I_A(j)$, and also they are coherent. Then, one can find the following conditions are satisfied:

$$I_k(i) = I_k(j), \quad \Delta\phi(i) = \Delta\phi(j) + \Delta\phi_{ij}, \quad (6)$$

where $\Delta\phi_{ij}$ is a constant phase. Note that the second condition is satisfied if $\Delta\phi_{AB}(i) = \Delta\phi_{AB}(j) + \Delta\phi_{ij}$. It is important to remember that, although $\Delta\phi(i)$ and $\Delta\phi(j)$ are related, they are still randomly varying. This condition can be obtained from, for example, mode-locked laser pulses. Note that, for the mode-locked laser pulses, $\Delta\phi_{ij} = 0$. With these conditions, Eq. (5) transforms to Eq. (3), and therefore the two-photon interference with a visibility $V = 0.5$ will be measured.

It is notable that all the events registered as coincidences for this case come from two temporally separated coherent pulses: one at i and the other at j. It seems natural that, when there is no overlap between optical pulses, the electromagnetic waves are also not overlapped, i.e., no superposition. This intuition, however, is incorrect as indicated by the last term of Eq. (5). This interference term contains all four intensities of input pulses, and thus shows that all electromagnetic waves interfere although there is no optical pulse overlapping. In short, in the classical view of interference, the electromagnetic waves are responsible for the interference, not the photons [21].

Because classical physics is a subset of quantum physics, one should be able to explain the interference with temporally nonoverlapping coherent pulses with quantum descriptions. Moreover, quantum descriptions are usually more intuitive, so one can more easily understand the underlying physics. Therefore, let us consider quantum interpretations of the phenomenon with Feynman diagrams. Since the coincidences are registered by two photons separated by $T = \tau_d$, there are four possible biphoton amplitudes as depicted in Fig. 2. Here, a and b denote the annihilation operators at input A and B, and the subscripts i and j are the labeling parameters. Although the Feynman diagrams are depicted with single-photon states,

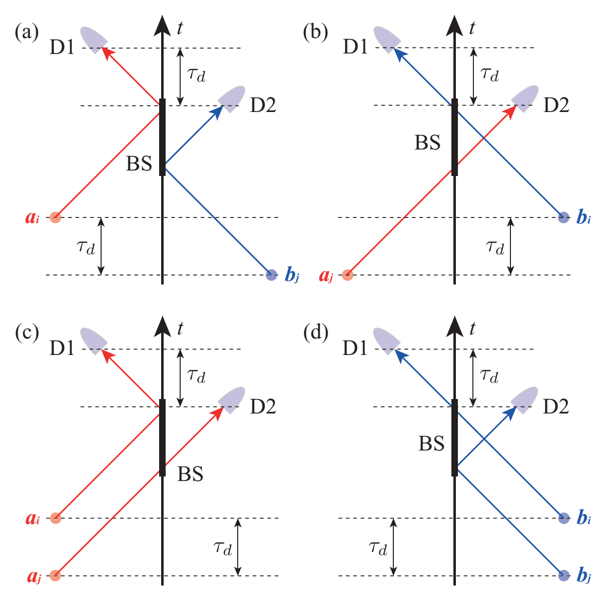

FIG. 2. (Color online) Feynman diagrams for $\Delta l = 0$. In general, all cases (a)–(d) are distinguishable, so they do not interfere. However, when pulses in the same inputs (between a_i and a_j, for example) are coherent, (a) and (b) become indistinguishable, so they interfere. For single-photon pulses, one can effectively suppress (c) and (d) while maintaining the coherence, so one can measure a $V = 1$ HOM dip with temporally nonoverlapping pulses. For classical optical pulses such as coherent states, however, one cannot remove cases (c) and (d) without disturbing the coherence, so the maximum visibility is limited to 0.5.

they are still applicable for our case since the coherent pulses are so weak that they mostly contain only a single photon; also, when more than two photons exist at the same time, they do not lead to relevant coincidences.

In general, all four cases are distinguishable, so they do not interfere. However, when a_i and a_j are coherent and b_i and b_j are also coherent, Figs. 2(a) and 2(b) become indistinguishable, so they do interfere. Because half of the cases interfere while the other half do not, the expected visibility will be 0.5.

This quantum description raises an interesting question: If coherent pulses only exist in the cases shown in Figs. 2(a) and 2(b), do they show $V = 1$ two-photon interference? The answer to this question is that it is impossible to remove the other cases [Figs. 2(c) and 2(d)] while maintaining the coherence between pulses in the same inputs unless the optical pulses are single-photon states [22,23]. Thus, even with the quantum description, the classical visibility is limited to 0.5.

It is interesting to compare the visibility limitation scenario to that of Franson interference [24]. When the coincidence detection does not distinguish between the "long-short" and "long(short)-long(short)" cases, the visibility is limited 0.5 [25,26]. This visibility limitation in the Franson interferometer can be overcome once the measurement apparatus can distinguish them [27] while our visibility limitation is inherent.

Figure 3 shows our experimental setup. Femtosecond laser pulses from a mode-locked Ti:sapphire laser are used for the experiment. Note that the pulse train and the electronic delay $\tau_d = T = mT_p$ where T_p and m are the pulse period and an integer, respectively, will implement Fig. 1. The central wavelength and the spectral bandwidth of the pulses are 780 nm and 15 nm, respectively. The repetition rate of the pulses is 85 MHz, so the interval between adjacent pulses $T_p \approx$ 11.8 ns, which corresponds to 3.5 m in space. The attenuator is introduced to reduce the average photon number per pulse.

A BS splits the incoming pulses into two paths. Each pulse enters into acousto-optical modulators (AOM1 and AOM2) and the deflected pulses are collected by single-mode optical fibers at inputs A and B. After the fiber polarization controllers (FPC), that make the polarization identical, the incoming pulses interfere at the fiber beam splitter (FBS). The optical path delay Δl is scanned by a translation stage placed at input B. A typical scanning range for Δl is hundreds of μm which is much smaller than T_p, so the scanning of Δl does not provide temporal overlap between adjacent pulses. Silicon avalanche photodiode based-single photon detectors D1 and D2 are placed at the outputs of the FBS, and a variable electronic delay τ_d is introduced at D2.

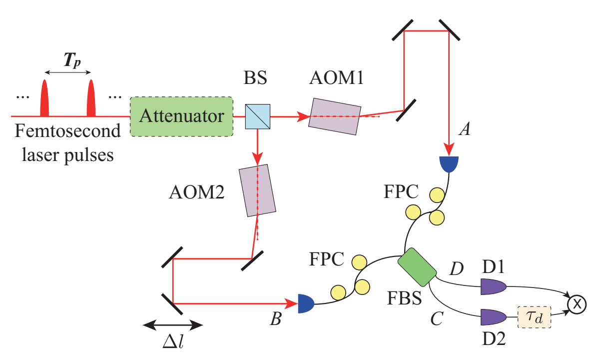

FIG. 3. (Color online) Experimental setup. BS: beam splitter; AOM: acousto-optic modulator; FPC: fiber polarization controller; FBS: fiber beamsplitter; D1 and D2: single photon detectors. The AOMs are used for the phase randomization between two arms.

The phase randomization between inputs A and B can be accomplished with the help of two AOMs modulated by two independent radio frequency (RF) drivers. An AOM adds additional phase to the deflected beam relative to the driving RF signal. Thus, if the RF signals are unsynchronized, two AOMs will wash out the phase relation between the two inputs. It is experimentally verified by applying either synchronized [see Fig. 4(a)] or independent [see Fig. 4(b)] RF signals. While the synchronized RF signals maintain the single-photon interference as they conserve the phase relation between A and B, the independent RF signals completely suppress the interference. Despite the independency of RF signals, the frequencies are still almost the same: 40 MHz. Note that the RF signals do not disturb the coherence within each input, e.g., between $I_A(i)$ and $I_A(j)$.

After we confirmed the phase randomization between A and B, we measured coincidence counts between D1 and D2. The result with $\tau_d = 0$ is shown in Fig. 4(c). It shows a clear HOM interference with visibility 0.52 ± 0.09, which is consistent to the classical limit of HOM interference visibility. Note that this case corresponds to a standard HOM interference with temporally overlapped coherent pulses.

Figure 4(d) shows the two-photon interference between temporally nonoverlapped coherent pulses. Here, the electronic delay $\tau_d = 212$ ns, so $m = 18$ was chosen. The data show a clear two-photon interference with $V = 0.50 \pm 0.09$. Since the mode-locked laser pulse trains satisfy the coherence condition within the same inputs, Eq. (6), we can still observe the same two-photon interference as the temporally overlapped pulses.

In order to disturb the coherence within the same inputs, that is, to prevent the pulses satisfying Eq. (6), we input fast random noise to the frequency modulation (FM) input of one of the AOM RF drivers. The random noise produces random frequency deviations of $\pm 50\%$ to the RF signal, so the coherence between pulses at the same input will be degraded when τ_d is sufficiently large. This will cause $\Delta\phi_{AB}(i) \neq \Delta\phi_{AB}(j)$, thus $\Delta\phi(i) \neq \Delta\phi(j)$. Note that the amount of the frequency deviation is much smaller than the spectral bandwidth of the coherent pulses, so the interference degradation due to the frequency mismatch is negligible.

The coincidence counts with FM noise input are depicted in Figs. 4(e) and 4(f) for $\tau_d = 0$ and 212 ns, respectively. For the case of $\tau_d = 0$, the experiment corresponds to a standard HOM interferometer with two temporally overlapping coherent pulses, so we observe the HOM dip with classical visibility limit. The measured visibility is 0.46 ± 0.09. When $\tau_d =$ 212 ns, however, the two-photon interference is eliminated as the FM noise diminishes the coherence between pulses at the same input.

To summarize, we studied the conditions for two-photon interference with weak coherent pulses. While single-photon interference was erased by two AOMs modulated by independent RF signals, two-photon interference with

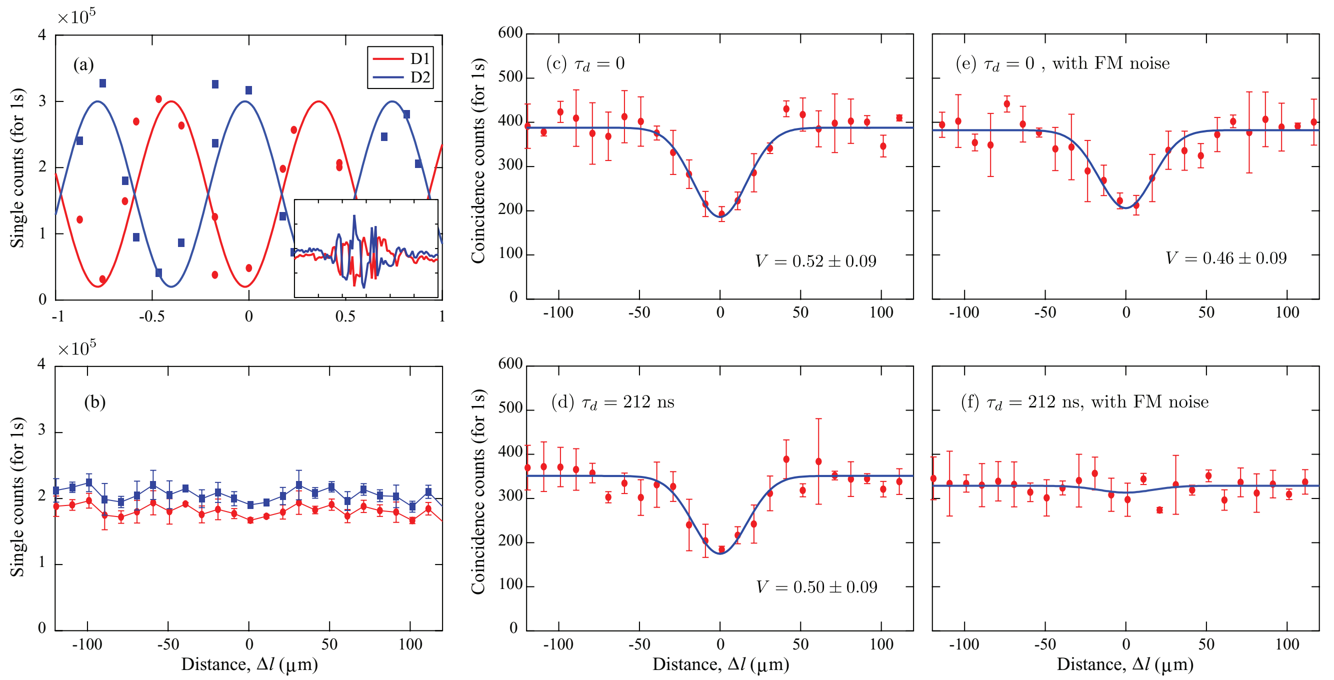

FIG. 4. (Color online) Single and coincidence counts for various conditions. Single and coincidence counts are proportional to $\langle I_C \rangle$ ($\langle I_D \rangle$) and $\langle I_C I_D \rangle$, respectively. Error bars are the experimentally obtained standard deviations. The single counts when RF signals to AOM1 and AOM2 are (a) synchronized and (b) independent. The inset of (a) shows the envelope of the single-photon interference. (c)–(f) Coincidence counts between D1 and D2. (c) $\tau_d = 0$. (d) $\tau_d = 212$ ns. (e),(f) $\tau_d = 0$ and $\tau_d = 212$ ns with an additional noise input to the FM input of the RF driver to AOM1. Estimated visibilities are also shown. (c) and (e) correspond to a HOM interference with classical pulses so a standard HOM dip is observed. (d) and (f) correspond to the two-photon interference between temporally nonoverlapping optical pulses.

$V = 0.5$ was still measured. We found that, counterintuitively, *classical* two-photon interference requires coherence within each input rather than the temporal overlap of optical pulses from the inputs.

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