Parametric amplification in the field of incoherent light

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A parametric amplification of input signal wave in the field of broadband incoherent light is analyzed at group-velocity matching of pump and idler waves. It is shown that such group-velocity matching is possible both in Type-I and Type-II phase-matching nonlinear crystals. The appropriate conditions at which an evolution of the signal wave in the field of the incoherent pump coincides with an evolution in the field of the monochromatic pump wave are demonstrated. The large parametric gain of the signal wave which is not sensitive to pump fluctuations is feasible in Type-II phase-matching nonlinear crystals.

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I. INTRODUCTION

A coherent amplification excited by the incoherent light in the nonlinear medium is of particular importance. The driving of a single signal mode by combining action of mutually incoherent longitudinal pump modes in parametric oscillator was predicted in Refs. [1,2] and experimentally observed in Ref. [3]. This feature of parametric interaction also takes place in parametric oscillators and amplifiers pumped by several intersecting beams of the same or different frequencies [4–7]. In this case one signal wave is coupled to several pump beams through separate idler beams, and an incoherence of the pump beams is transferred to the corresponding idler beams. The cumulative pump action is also typical for parametric processes excited by Bessel (or more generally by conical) beams [8,9]. The wave propagating along the pump cone axis can be phase matched with an infinite set of pump plane waves of the same frequencies whose wave vectors are lying on the pump cone. The pump azimuthal incoherence is transferred to the idler conical beam.

It was predicted in Ref. [10] that a phase modulation of the pump pulse is transferred to the idler wave in optical parametric amplifier (OPA) if the group velocities of the pump and idler waves are equal (group-velocity matched). In such conditions the parametric generation process excited by the incoherent pump wave allows the signal wave to grow efficiently with a high degree of coherence from the quantum noise level, and the incoherence of the pump is absorbed by the idler wave [11–20]. The presence of group-velocity dispersion of interacting waves supports this phenomenon, provided that the dispersion parameter of the pump matches the dispersion parameter of the idler wave [15]. It was shown in Ref. [21] that the generation of a coherent wave by two incoherent waves is a characteristic feature of three-wave interaction in a quadratic nonlinear medium when the angular dispersion of both incoherent waves is properly chosen.

In this paper we analyze an evolution of input signal wave in OPA being pumped by a broadband incoherent wave at the group-velocity matching of pump and idler waves. We demonstrate that the condition of group-velocity matching of two (pump and idler) interacting waves is possible at Type-I and Type-II phase-matching in nonlinear crystals. The correlation functions and spectra of signal and idler waves are obtained in analytical form in the case of the first-order dispersion approximation. The spectra and parametric gain of signal and idler waves in the second-order dispersion approximation are obtained by numerical simulation of governing equations. It is found that in this case due to phase-matching conditions the frequency bandwidth of the idler wave at the output of OPA is essentially determined by the frequency bandwidth of signal wave.

The paper is organized as follows. In Sec. II we present the solution of the equations which describe the parametric down-conversion of incoherent light in the first-order dispersion approximation. The analysis of the correlation function and spectrum of signal wave (Sec. III) as well as idler wave (Sec. IV) in the case of signal seed into OPA is carried out. The variation of the mean intensity of signal wave being amplified in OPA by incoherent pump is analyzed in Sec. V. The results of the numerical simulations of the governing equations in the second-order dispersion approximation are presented in Sec. VI. An evolution of the spectrum as well as parametric gain of signal and idler waves with propagation in a nonlinear medium is analyzed.

II. GOVERNING EQUATIONS AND THEIR SOLUTION IN THE FIRST-ORDER DISPERSION APPROXIMATION

We consider the spatiotemporal evolution (z,t) of three interacting optical waves in a quadratic nonlinear medium in the first-order dispersion approximation. In this case the amplitudes $A_j(z,t)$, $j = 1,2,3$, of three waves obey the coupled partial differential equations:

$$\frac{\partial A_1}{\partial z} + \frac{1}{u_1}\frac{\partial A_1}{\partial t} = \sigma_1 A_2^* A_3, \tag{1a}$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{u_2}\frac{\partial A_2}{\partial t} = \sigma_2 A_1^* A_3, \tag{1b}$$

$$\frac{\partial A_3}{\partial z} + \frac{1}{u_3}\frac{\partial A_3}{\partial t} = -\sigma_3 A_1 A_2, \tag{1c}$$

where u_j and $\sigma_j = d_{ef} k_j / n_j^2$ stand for group-velocity and nonlinear coupling coefficient of the j wave, respectively, and are calculated for central frequency ω_{j0} of the wave. d_{ef} is the effective second-order susceptibility, $k_j = \omega_j n_j / c$ is a wave number, $\omega_j = \omega_{j0} + \Omega_j$ is a frequency, n_j is a refractive index. For definiteness we call the first, second, and third waves as the signal, idler, and pump waves, respectively. We assume that an initial condition of Eq. (1) at $z = 0$ for the envelope $A_{j0}(t) = A_j(0,t)$ is the stationary Gaussian stochastic process

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with zero mean and Gaussian autocorrelation function,

$$B_{j0}(T) = \langle A_{j0}(t+T)A_{j0}^*(t)\rangle = \langle a_{j0}^2\rangle \exp\left(-\frac{T^2}{\tau_j^2}\right), \quad (2)$$

where τ_j is the correlation time of the j wave. In further consideration $\langle a_{j0}^2\rangle = \langle |A_{j0}|^2\rangle$ stands for the normalized mean intensity of the j wave at the input of the nonlinear medium. We restrict our analysis to the linear regime of the parametric interaction and assume that group velocities of the idler and pump waves are equal ($u_2 = u_3$). Then in the reference frame of the pump wave $t \to t - z/u_3$ Eq. (1) can be written as

$$\frac{\partial A_1}{\partial z} + \nu\frac{\partial A_1}{\partial t} = \sigma_1 A_2^* A_{30}(t), \quad (3a)$$

$$\frac{\partial A_2}{\partial z} = \sigma_2 A_1^* A_{30}(t), \quad (3b)$$

where $\nu = 1/u_1 - 1/u_2$ characterizes the walk-off between the signal and idler (or pump) waves. The exact solution of Eq. (3) was obtained in Ref. [10]:

$$A_1(z,t) = A_{10}(t-\nu z) + \int_0^{\nu z}\left[A_{10}(t-t_1)\frac{2r}{s}I_1(s) + \sigma_1 A_{30}(t-t_1)A_{20}^*(t-t_1)\frac{I_0(s)}{\nu}\right]dt_1, \quad (4a)$$

$$A_2(z,t) = A_{20}(t) + A_{30}(t)\int_0^{\nu z}\left[\sigma_1\sigma_2 A_{20}(t-t_1)A_{30}^*(t-t_1) \times \frac{2(\nu z - t_1)}{\nu^2 s}I_1(s) + \sigma_2 A_{10}^*(t-t_1)\frac{I_0(s)}{\nu}\right]dt_1, \quad (4b)$$

where $s = 2\sqrt{(\nu z - t_1)r}$, $r = \frac{\sigma_1\sigma_2}{\nu^2}\int_{t-t_1}^{t}|A_{30}(t_2)|^2 dt_2$, and I_0, I_1 are the modified Bessel functions. We provide the analysis for a broadband incoherent pump whose correlation time τ_3 obeys an inequality $\tau_3 \ll |\nu|z$. It means that the frequency bandwidth of the pump wave should exceed the frequency bandwidth $\Delta\Omega_{\rm OPA}$ of OPA pumped by a monochromatic wave at small gain, $\Delta\Omega_{\rm OPA} \sim 1/(|\nu|z)$. In this case $r \approx \sigma_1\sigma_2 t_1\langle a_{30}^2\rangle/\nu^2$ and $s \approx 2\sqrt{\sigma_1\sigma_2\langle a_{30}^2\rangle t_1(\nu z - t_1)}/\nu$. We introduce into consideration in Eq. (4) new variable $\xi = t_1/(\nu z)$ and the characteristic length of the nonlinear interaction $L_n = (\sigma_1\sigma_2\langle a_{30}^2\rangle)^{-1/2}$. Then in the case of signal seed into OPA ($A_{10} \neq 0$, $A_{20} = 0$) Eq. (4) for $z \gg \tau_3/|\nu|$ can be rewritten as

$$A_1(z,t) = A_{10}(t-\nu z) + b\int_0^1 A_{10}(t-\nu z\xi)F_1(\xi)d\xi, \quad (5a)$$

$$A_2(z,t) = b\sqrt{\frac{\sigma_2}{\sigma_1}}\frac{A_{30}(t)}{\sqrt{\langle a_{30}^2\rangle}}\int_0^1 A_{10}^*(t-\nu z\xi)F_0(\xi)d\xi, \quad (5b)$$

where $b = z/L_n$ and

$$F_0(\xi) = I_0[2b\sqrt{\xi(1-\xi)}], \quad (6a)$$

$$F_1(\xi) = \sqrt{\frac{\xi}{1-\xi}}I_1[2b\sqrt{\xi(1-\xi)}]. \quad (6b)$$

Furthermore we present some integrals with $F_0(\xi)$ and $F_1(\xi)$ which are found below:

$$\int_0^1 F_0(\xi)d\xi = \frac{1}{b}\sinh(b), \quad (7a)$$

$$\int_0^1 F_1(\xi)d\xi = \frac{1}{b}[\cosh(b) - 1]. \quad (7b)$$

The obtained solution [Eq. (5)] permits an analytical treatment of the parametric amplification of a coherent as well as an incoherent signal wave in the field of the broadband incoherent pump field.

We assume that at the input of nonlinear medium the pump and signal waves are uncorrelated. In this case calculation of correlators $\langle A_1(z,t)A_{30}^*(t)\rangle$, $\langle A_2(z,t)A_{30}^*(t)\rangle$, and $\langle A_1(z,t)A_2^*(z,t)\rangle$ by use of Eq. (5) yields zero. So, the correlation between the interacting waves cannot arise under propagation in a nonlinear medium if these waves were uncorrelated at the input. We note that the correlation between the idler and pump waves appears only in some special cases, for example, when the pump wave exhibits only pure random phase fluctuations [13], or group-velocity dispersion is taken into account [15].

The possibility of group-velocity matching of the pump and idler waves was analyzed in various nonlinear crystals by numerical simulation of the equations,

$$\omega_1 + \omega_2 = \omega_3, \quad k_1 + k_2 = k_3(\theta), \quad u_2 = u_3(\theta), \quad (8a)$$

and

$$\omega_1 + \omega_2 = \omega_3, \quad k_1(\theta) + k_2 = k_3(\theta), \quad u_2 = u_3(\theta), \quad (8b)$$

for Type-I and Type-II collinear phase matching, respectively. The Selmeier equations from Ref. [22] were adopted. As an example, the obtained results for KDP crystal are presented in Fig. 1, where the corresponding signal wavelengths (λ_1) are also shown. We note that for fixed wavelength of the pump wave there can exist two different idler wavelengths and corresponding two phase-matching angles (θ) at which group-velocity matching is possible. The phase-matching curves are limited by the absorption in KDP crystal at $\lambda \approx 1.5$ μm. In the case of Type-II phase matching the limitation occurs also due to noncritical phase matching at $\theta = 90°$. We note that in the case of Type-I phase matching the walk-off parameter ν is small and for this reason the condition $|\nu|z \gg \tau_3$ can be fulfilled only for quite small values of pump correlation time. In this case the group-velocity dispersion of interacting waves should be taken into account (Sec. VI).

III. PARAMETRIC AMPLIFICATION OF SIGNAL WAVE

Let us calculate the correlation function $B_1(T) = \langle A_1(z,t+T)A_1^*(z,t)\rangle$ of the signal wave being amplified by an incoherent pump in the nonlinear medium. By use of Eq. (5a) we find

$$B_1(T) = \langle a_{10}^2\rangle\Big[\exp\left(-\frac{T^2}{\tau_1^2}\right) + b\int_0^1\left(\exp\{-[T+\nu z(\xi-1)]^2/\tau_1^2\} + \exp\{-[T-\nu z(\xi-1)]^2/\tau_1^2\}\right)F_1(\xi)d\xi + b^2\int_0^1\int_0^1 \exp\{-[T-\nu z(\xi_1-\xi_2)]^2/\tau_1^2\} \times F_1(\xi_1)F_1(\xi_2)d\xi_1 d\xi_2\Big]. \quad (9)$$

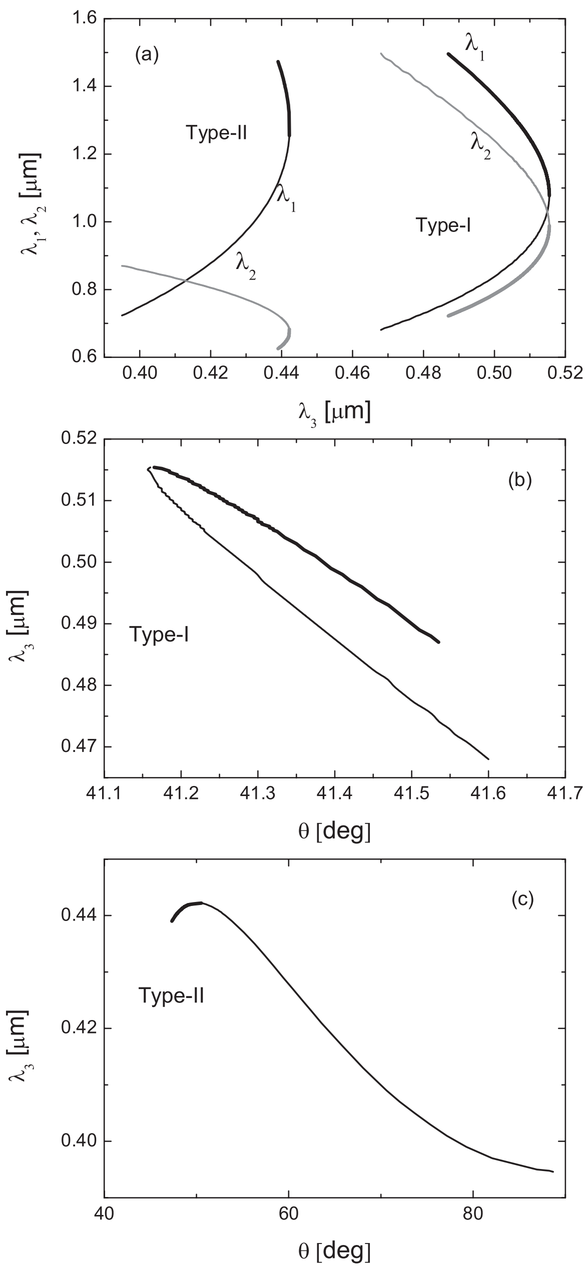

FIG. 1. Group-velocity-matching curves of pump (λ_3) and idler (λ_2) waves in KDP crystal (a) and corresponding phase-matching angles (θ) for Type-I (b) and Type-II (c) phase matching. Thick and thin parts of the curves correspond to different phase-matching angles.

The spectrum of the signal wave $G_1(\Omega) = 2\pi \int_{-\infty}^{\infty} B_1(T)\exp(i\Omega T)dT$ is obtained by the Fourier transform of Eq. (9), where $\Omega = \omega_1 - \omega_{10}$ is the frequency shift with respect to the central frequency ω_{10}. Furthermore we omit rather long calculations and present only the exact final result:

$$G_1(\Omega) = G_{10}(\Omega)|1 + D_1(\Omega)|^2, \tag{10}$$

where

$$D_1 = b\exp(-i\Omega\nu z)\int_0^1 F_1(\xi)\exp(i\Omega\nu z\xi)d\xi, \tag{11}$$

and $G_{10}(\Omega) = 2\pi^{3/2}\tau_1\langle a_{10}^2\rangle \exp(-\Omega^2\tau_1^2/4)$ is the spectrum of the signal wave at the input of the nonlinear medium. An integration in Eq. (11) yields

$$D_1 + 1 = \exp(-i\Omega\nu z/2)\left[\cosh\sqrt{b^2 - \frac{(\Omega\nu z)^2}{4}} - i\frac{\Omega\nu z/2}{\sqrt{b^2 - \frac{(\Omega\nu z)^2}{4}}}\sinh\sqrt{b^2 - \frac{(\Omega\nu z)^2}{4}}\right]. \tag{12}$$

The substitution of D_1 into Eq. (10) gives

$$G_1(\Omega) = G_{10}(\Omega) + G_{10}(\Omega)b^2\frac{\sinh^2\sqrt{b^2 - \frac{(\Omega\nu z)^2}{4}}}{b^2 - \frac{(\Omega\nu z)^2}{4}}. \tag{13}$$

So, the spectrum of the signal wave [Eq. (13)] as well as its correlation function $B_1(T)$ [Eq. (9)] are the same as in the case of monochromatic pump with a constant amplitude equal to $\sqrt{\langle a_{30}^2\rangle}$. The evolution of the signal wave in the field of the broadband incoherent pump ($\tau_3 \ll |\nu|z$) coincides with the evolution in the field of the coherent monochromatic pump wave and the parametric gain for $z \gg \tau_3/|\nu|$ is not sensitive to the pump fluctuations. This property of incoherent pump was also noticed in Ref. [20]. We note that the obtained condition $z \gg \tau_3|\nu|$ is necessary but not sufficient (see Sec. VI).

IV. PARAMETRIC AMPLIFICATION OF IDLER WAVE

The correlation function $B_2(T)$ of idler wave is

$$B_2(T) = \frac{\sigma_2}{\sigma_1}b^2\langle a_{10}^2\rangle\exp\left(-T^2/\tau_3^2\right) \times \int_0^1\int_0^1 \exp\left(-[T - \nu z(\xi_1 - \xi_2)]^2/\tau_1^2\right) \times F_0(\xi_1)F_0(\xi_2)d\xi_1 d\xi_2. \tag{14}$$

The Fourier transform of Eq. (14) yields the spectrum of the idler wave:

$$G_2(\Omega) = \frac{\sigma_2}{\sigma_1}\frac{\tau_{13}}{\tau_1}G_{10}(0)\exp\left(-\Omega^2\tau_{13}^2/4\right)b^2\int_0^1\int_0^1 F_0(\xi_1)F_0(\xi_2) \times \exp\left(-\nu^2 z^2(\xi_1 - \xi_2)^2/\left(\tau_1^2 + \tau_3^2\right) + i\Omega\nu z(\xi_1 - \xi_2)\tau_3^2/\left(\tau_1^2 + \tau_3^2\right)\right)d\xi_1 d\xi_2, \tag{15}$$

where $\tau_{13} = \tau_1\tau_3/\sqrt{\tau_1^2 + \tau_3^2}$. Furthermore we analyze the spectrum of the idler wave for $\tau_1 \gg \tau_3$. In this case we find

$$G_2(\Omega) \approx \frac{\sigma_2}{\sigma_1}\frac{\tau_3}{\tau_1}G_{10}(0)\exp\left(-\Omega^2\tau_3^2/4\right)b^2\int_0^1\int_0^1 F_0(\xi_1)F_0(\xi_2) \times \exp\left(-\nu^2 z^2(\xi_1 - \xi_2)^2/\tau_1^2\right)d\xi_1 d\xi_2. \tag{16}$$

So, the idler wave acquires the profile of the pump spectrum and its bandwidth does not depend on the propagation length in the nonlinear medium. For the narrow-band signal ($\tau_1 \gg |\nu|z$)

we obtain

$$G_2(\Omega) \approx \frac{\sigma_2}{\sigma_1}\frac{\tau_3}{\tau_1} G_{10}(0) \exp\left(-\Omega^2\tau_3^2/4\right)\sinh^2(z/L_n). \quad (17)$$

For the broadband signal ($\tau_1 \ll |\nu|z$) the exponent exp $(-\nu^2 z^2(\xi_1-\xi_2)^2/\tau_1^2)$ in Eq. (16) can be evaluated as $\sqrt{\pi}\frac{\tau_1}{|\nu|z}\delta(\xi_1-\xi_2)$, where δ is a Dirac delta function and, as a result, we find

$$G_2(\Omega) \approx \sqrt{\pi}\frac{\sigma_2}{\sigma_1}\frac{zL_p}{L_n^2} G_{10}(0)\exp\left(-\Omega^2\tau_3^2/4\right)\int_0^1 F_0^2(\xi)d\xi, \quad (18)$$

where $L_p = \tau_3/|\nu|$ is the characteristic correlation length of the pump wave. In the case of large gain ($b \gg 1$) we find $\int_0^1 F_0^2(\xi)d\xi \approx \frac{1}{4\sqrt{\pi}}b^{-3/2}\exp(2b)$. Then Eq. (18) can be written as

$$G_2(\Omega) \approx \frac{1}{4}\frac{\sigma_2}{\sigma_1}\frac{L_p}{\sqrt{L_n z}} G_{10}(0)\exp\left(-\Omega^2\tau_3^2/4\right)\exp(2z/L_n), \quad (19)$$

and the ratio of the spectral intensities of the idler and signal waves is

$$\frac{G_2(0)}{G_1(0)} = \frac{\sigma_2}{\sigma_1}\frac{L_p}{\sqrt{L_n z}}. \quad (20)$$

V. MEAN INTENSITIES OF SIGNAL AND IDLER WAVES

The normalized mean intensities of the signal and idler waves $\langle a_1^2\rangle = \langle|A_1|^2\rangle$ and $\langle a_2^2\rangle = \langle|A_2|^2\rangle$ obey the relation,

$$\sigma_2\langle a_1^2\rangle - \sigma_1\langle a_2^2\rangle = \sigma_2\langle a_{10}^2\rangle - \sigma_1\langle a_{20}^2\rangle, \quad (21)$$

which does not depend on the properties of the pump wave. This relation can be directly obtained from Eq. (3). By elimination of pump amplitude $A_{30}(t)$ in Eq. (3) we find $\sigma_2(\frac{\partial a_1^2}{\partial z} + \nu\frac{\partial a_1^2}{\partial t}) = \sigma_1\frac{\partial a_2^2}{\partial z}$. This equation can be rewritten as $\sigma_2\frac{\partial}{\partial z}a_1^2(z,t') = \sigma_1\frac{\partial}{\partial z}a_2^2(z,t)$, where $t' = t + \nu z$. In the case of stationary stochastic processes the averaging of intensities a_1^2, a_2^2 and an integration yields Eq. (21).

The mean intensity $\langle a_1^2\rangle$ of the signal wave can be found by use of Eq. (9) for $T = 0$. We have

$$\frac{\langle a_1^2\rangle}{\langle a_{10}^2\rangle} - 1 = 2b\int_0^1 \exp\left(-\nu^2 z^2(\xi-1)^2/\tau_1^2\right)F_1(\xi)d\xi + b^2\int_0^1\int_0^1 \exp\left(-\nu^2 z^2(\xi_1-\xi_2)^2/\tau_1^2\right) \times F_1(\xi_1)F_1(\xi_2)d\xi_1 d\xi_2. \quad (22)$$

So, the mean intensity of signal wave depends on the correlation time of signal wave τ_1. In the case of the narrow-band signal ($\tau_1 \gg |\nu|z$) its intensity gain is

$$\frac{\langle a_1^2\rangle}{\langle a_{10}^2\rangle} - 1 = \sinh^2\left(\frac{z}{L_n}\right). \quad (23)$$

For the broadband signal ($\tau_1 \ll |\nu|z$) we find

$$\frac{\langle a_1^2\rangle}{\langle a_{10}^2\rangle} - 1 = \sqrt{\pi}\frac{zL_s}{L_n^2}\left[1 + \int_0^1 F_1^2(\xi)d\xi\right], \quad (24)$$

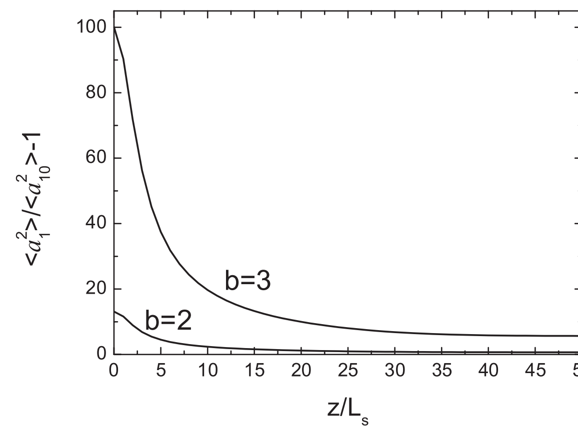

FIG. 2. Dependence of mean intensity gain of signal wave on ratio z/L_s for different values $b = z/L_n$.

where $L_s = \tau_1/|\nu|$ is the characteristic correlation length of the signal wave. The dependence of mean intensity gain of the signal wave on ratio z/L_s for different values of $b = z/L_n$ was obtained numerically and is shown in Fig. 2. The parametric gain of mean intensity depends on the correlation length L_s. The largest gain is obtained for the narrow-band signal when $L_s \gg z$. The mean intensity of the idler wave can be found by use of Eq. (21).

VI. NUMERICAL SIMULATION OF THE GOVERNING EQUATIONS IN THE SECOND-ORDER DISPERSION APPROXIMATION

Here we simulate the system of equations which includes the group-velocity dispersion coefficients g_j:

$$\frac{\partial A_1}{\partial z} + \frac{1}{u_1}\frac{\partial A_1}{\partial t} + i\frac{g_1}{2}\frac{\partial^2 A_1}{\partial t^2} = \sigma_1 A_2^* A_3, \quad (25a)$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{u_2}\frac{\partial A_2}{\partial t} + i\frac{g_2}{2}\frac{\partial^2 A_2}{\partial t^2} = \sigma_2 A_1^* A_3, \quad (25b)$$

$$\frac{\partial A_3}{\partial z} + \frac{1}{u_3}\frac{\partial A_3}{\partial t} + i\frac{g_3}{2}\frac{\partial^2 A_3}{\partial t^2} = -\sigma_3 A_1 A_2. \quad (25c)$$

The amplitudes of input signal ($j = 1$) as well as pump ($j = 3$) waves which obey the correlation function of Eq. (2) can be written as [23]

$$A_{j0} = \sqrt{\langle a_{j0}^2\rangle}\frac{1}{\sqrt{N_s}}\sum_{s=1}^{N_s}\exp[i\omega_{sj}t + i\varphi_{sj}], \quad (26)$$

where ω_{sj} are the normally distributed random numbers with variance $\Delta\Omega_{j0}/\sqrt{2}$, where $\Delta\Omega_{j0} = 2/\tau_j$ stands for the frequency bandwidth of the jth wave at the input. φ_{sj} are the random phases and quantity N_s has to be sufficiently large. In our calculations $N_s = 200$. Equations (25) were simulated $N \gg 1$ times and the averaged values were fixed. The simulations were performed by the use of the symmetrized split-step Fourier method [24]. Where it was possible, the obtained solutions were compared to the theoretical ones.

First, we verified the theoretical results presented in Secs. II–V which essentially were based on the assumption that an inequality $\tau_3 \ll |\nu|z$ is valid. The results of theoretical analysis as well as of numerical simulations are shown in Fig. 3 in the case of Type-II KDP crystal for pump and signal wavelengths $\lambda_{30} = 0.41\ \mu$m and $\lambda_{10} = 0.806\ \mu$m, respectively. The correlation time τ_3 of the pump wave was chosen rather large ($\tau_3 = 100$ fs) in order to avoid the group-velocity dispersion of the pump. For $l = 2$-cm-long Type-II KDP crystal we have $|\nu|l \approx 3$ ps and the inequality $\tau_3 \ll |\nu|l$ is valid. A good agreement between the theoretical and numerical results was obtained (Fig. 3). The spectral intensity of the amplified signal wave in the field of incoherent pump can considerably exceed the spectral intensity of the pump wave (Fig. 4). The depletion of the pump wave is observed and for this reason curve (1) if Fig. 4(b) differs from the theoretical curve (2) which was obtained by use of Eq. (13) in the case of the undepleted pump wave.

The condition $\tau_3 \ll |\nu|z$ is necessary but not sufficient to obtain a good agreement between the theoretical and numerical results. A numerical simulation of Eq. (25) has shown that an inequality $\tau_3 \ll |\nu|L_n$ also should be valid. That is demonstrated in Fig. 5. All parameters are the same as in the case of Fig. 3 with one exception for nonlinear interaction length L_n. We have $|\nu|L_n = 0.75$ ps (Fig. 5) and $|\nu|L_n = 3$ ps (Fig. 3). The averaging of the pump fluctuations is not sufficient in the case of the smaller value of L_n. For this reason an increase of spectral intensity of the signal and idler waves due to pump fluctuations is observed. It means that in this case at the output of the nonlinear medium an increase of signal wave fluctuations in comparison with the ones at the input is unavoidable. So, a large parametric gain of the signal wave which is not sensitive to pump fluctuations is possible at $z \gg L_n \gg \tau_3/|\nu|$ if the group-velocity dispersion of the pump wave can be neglected.

Next, we shall obtain the phase-matching curve of three interacting waves in nonlinear medium when group-velocity dispersion is taken into account. In the case of group-velocity matching of pump and idler waves ($u_2 = u_3$) this curve is a solution of the equation system:

$$\omega_1 + \omega_2 = \omega_3, \quad k(\omega_1) + k(\omega_2) = k(\omega_3), \tag{27}$$

where $\omega_j = \omega_{j0} + \Omega_j$, $j = 1,2,3$. We assume that an orientation of nonlinear crystal corresponds to the phase matching of the central frequencies ω_{j0} of interacting waves, so $\omega_{10} + \omega_{20} = \omega_{30}$, $k_{10} + k_{20} = k_{30}$, where $k_{j0} = k_j(\omega_{j0})$. By use of Taylor series in the second-order dispersion approximation Eq. (27) yields

$$\Omega_1 + \Omega_2 = \Omega_3, \tag{28a}$$

$$2\nu\Omega_1 + g_1\Omega_1^2 + g_2\Omega_2^2 = g_3\Omega_3^2, \tag{28b}$$

where $\nu = 1/u_1 - 1/u_2$. In the first-order dispersion approximation the group-velocity dispersion coefficients g_j can be neglected, and a solution of Eq. (28) is $\Omega_1 = 0$, $\Omega_2 = \Omega_3$. It means that only the central frequency of the signal wave obeys the phase-matching conditions, and the idler wave acquires the spectrum profile of the pump wave. In this case the down-conversion of an incoherent pump stimulates the generation of coherent signal wave [13,14].

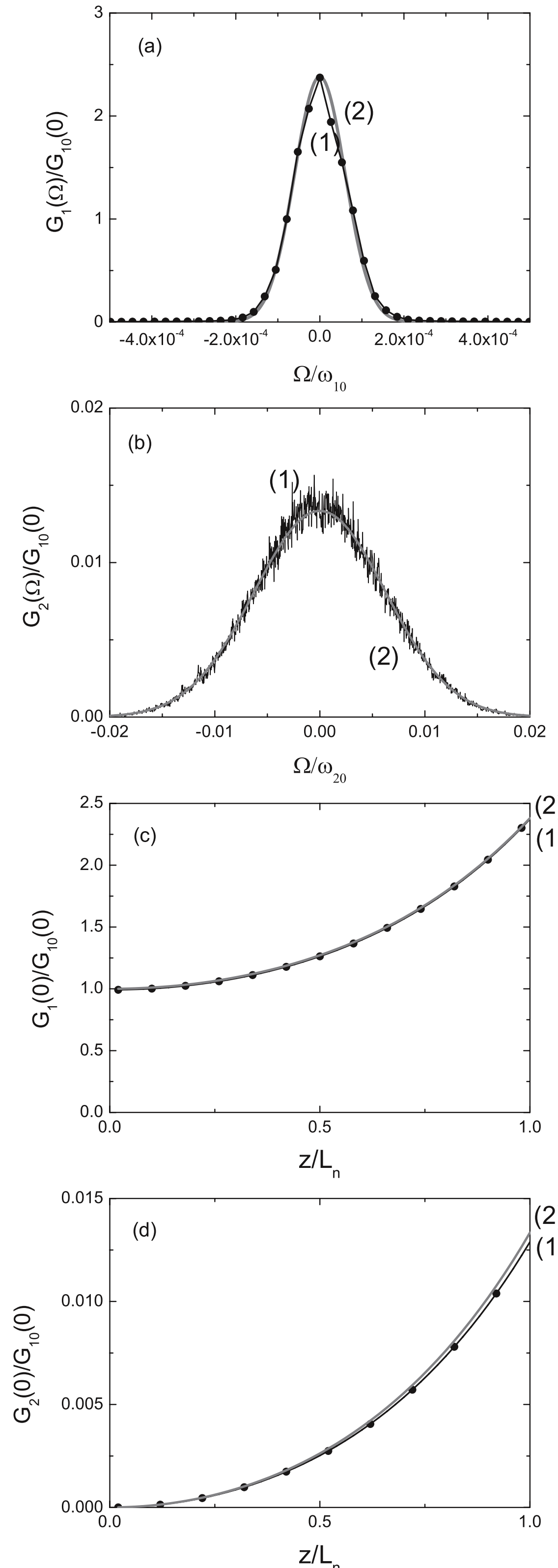

FIG. 3. Output spectra of the signal (a) and idler (b) waves at $z = 2$ cm. Dependence of spectral intensity of the signal (c) as well as idler (d) wave on the propagation length. Average of $N = 500$ numerical simulations [curves (1), black]. Theoretical results are represented by gray curves (2). Type-II KDP crystal, $\lambda_{30} = 0.41\ \mu$m, $\lambda_{10} = 0.806\ \mu$m ($u_2 = u_3$). $L_n = 2$ cm, crystal length $l = 2$ cm, $\tau_1 = 10$ ps, $\tau_3 = 100$ fs, $\langle a_{10}^2\rangle/\langle a_{30}^2\rangle = 10^{-6}$, $|\nu|l = 3$ ps.

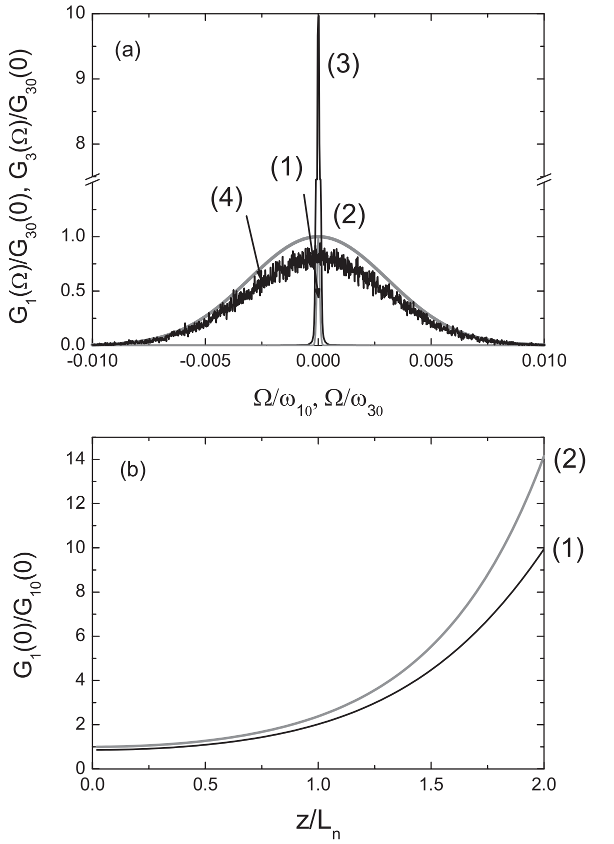

FIG. 4. (a) Input (gray, 1,2) and output (black, 3,4) spectra of the signal (1,3) and pump (2,4) waves at $z = 4$ cm. (b) Numerical (1) and theoretical (2) dependence of spectral intensity of the signal wave on the propagation length. Average of $N = 500$ numerical simulations (curves 3,4). Type-II KDP crystal, $\lambda_{30} = 0.41$ μm, $\lambda_{10} = 0.806$ μm ($u_2 = u_3$). $L_n = 2$ cm, $l = 4$ cm, $\tau_1 = 10$ ps, $\tau_3 = 100$ fs, $\langle a_{10}^2\rangle/\langle a_{30}^2\rangle = 10^{-2}$, $|\nu|l = 3$ ps. At $z = l$ ratio $\langle a_1^2\rangle/\langle a_{30}^2\rangle = 0.11$.

The insertion of Eq. (28a) into Eq. (28b) gives a phase-matching curve:

$$p_1\left(\frac{\Omega_1}{\Delta\Omega_{10}}\right)^2 + p_2\left(\frac{\Omega_2}{\Delta\Omega_{10}}\right)^2 + 2\frac{\Omega_1}{\Delta\Omega_{10}}\frac{\Omega_2}{\Delta\Omega_{10}} - 2p_3\frac{\Omega_1}{\Delta\Omega_{10}} = 0, \qquad (29)$$

where $p_1 = \frac{g_3 - g_1}{g_3}$, $p_2 = \frac{g_3 - g_2}{g_3}$, $p_3 = \frac{\nu}{g_3\Delta\Omega_{10}}$, and $\Delta\Omega_{10} = 2/\tau_1$ stands for the frequency bandwidth of the signal wave at the input. In the case of KDP crystal the Type-I phase-matching curve (ellipse) for pump and signal wavelengths $\lambda_{30} = 0.48$ μm and $\lambda_{10} = 0.73$ μm, respectively, is shown in Fig. 6 for two values of signal correlation time τ_1. The obtained curve is not symmetric with respect to the coordinate axes, and, as a result, the spectrum of the signal and idler waves being amplified in the field of incoherent pump should be asymmetric. The shape of the phase-matching curve [Eq. (29)] does not depend on the correlation time τ_3 of the pump wave. The pump frequency bandwidth $\Delta\Omega_{30} = 2/\tau_3$ can only limit in some cases the frequency bandwidth of signal and idler waves because the condition $-\Delta\Omega_{30} \leqslant \Omega_1 + \Omega_2 \leqslant \Delta\Omega_{30}$ should be fulfilled [see Eq. (28a)]. In general, the frequency bandwidth

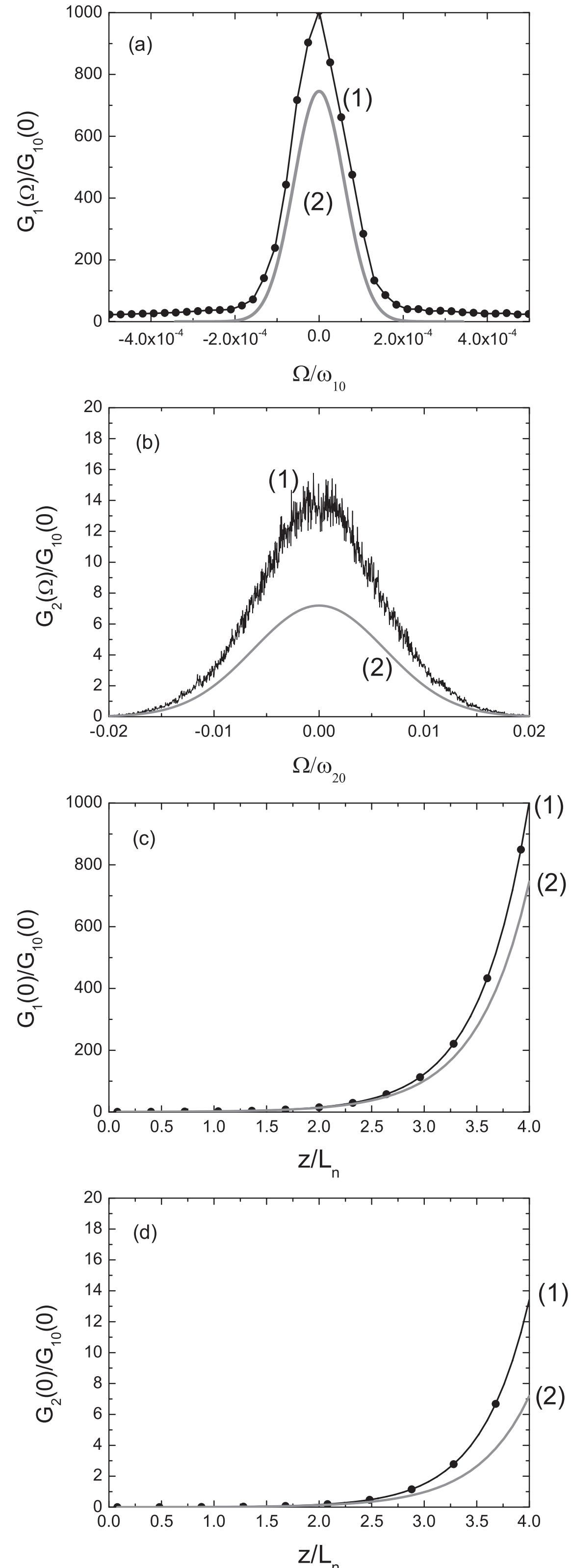

FIG. 5. Output spectra of signal (a) and idler (b) waves at $z = 2$ cm. Dependence of spectral intensity of the signal (c) as well as idler (d) wave on the propagation length. Average of $N = 500$ numerical simulations [curves (1), black]. Theoretical results are represented by gray curves (2). Type-II KDP crystal, $\lambda_{30} = 0.41$ μm, $\lambda_{10} = 0.806$ μm ($u_2 = u_3$). $L_n = 5$ mm, $l = 2$ cm, $\tau_1 = 10$ ps, $\tau_3 = 100$ fs, $\langle a_{10}^2\rangle/\langle a_{30}^2\rangle = 10^{-6}$, $|\nu|l = 3$ ps.

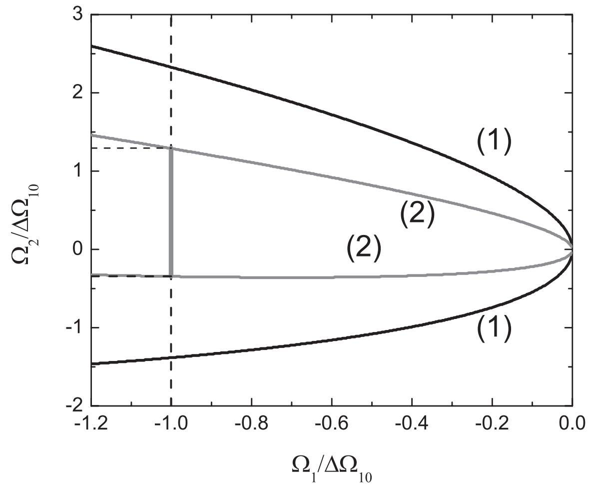

FIG. 6. Phase-matching curve. τ_1 [fs]: 100 (1), 20 (2). Type-I KDP crystal, $\lambda_{30} = 0.48\ \mu$m, $\lambda_{10} = 0.73\ \mu$m ($u_2 = u_3$). The possible values of phase-matched idler wave frequencies (thick straight line) are shown for $\tau_1 = 20$ fs.

of the idler wave in rather long crystal is determined by the frequency bandwidth of the signal wave. An increase of the coherence of the signal wave under propagation in the nonlinear crystal is possible only to some extent due to the filtration of the spectral components ($\Omega_1 > 0$) of the signal wave which do not obey the phase-matching condition (Fig. 6).

We note that in the special case $g_2 = g_3$ ($p_2 = 0$) the solution of Eq. (29) is $\Omega_1 = 0$, and the generation of a coherent signal wave by down-conversion of the incoherent pump becomes possible also under group-velocity dispersion of interacting waves [15].

The spectra of the signal and idler waves at the output of the OPA pumped by the incoherent wave with a short correlation time are shown in Fig. 7(a) (Type-I interaction). The correlation time $\tau_3 = 5$ fs of the pump wave is much smaller in comparison with the products $|\nu|l = 230$ fs and $|\nu|L_n = 57.5$ fs. So, the averaging of the pump fluctuations is proper [compare curves (2) and (3) in Fig. 7(b)]. In the case of group-velocity dispersion of the pump wave the phases of pump spectral components vary with propagation length z as $\varphi_3(\Omega,z) = \varphi_3(\Omega,0) - g_3\Omega^2 z/2$, and the mean value $\langle\varphi_3(\Omega,z)\rangle = -g_3\Omega^2 z/2$ is not zero. As a result, the optimum phase difference between the interacting waves is violated, and a small gain of the signal wave is observed [curve 1 in Fig. 7(b)]. The frequency bandwidths of signal as well as idler waves tend with a propagation to the constant values [Fig. 7(c)], and that is predetermined by the phase-matching curve of the signal and idler waves.

The parametric gain of the signal and idler waves increases with increase of the correlation time of the pump wave (Fig. 8). It was assumed that the spectral intensity $G_3(0)$ of the pump wave does not depend on its correlation time. In this case an increase of the correlation time τ_3 corresponds to a decrease of the pump frequency bandwidth $\Delta\Omega_{30} = 2/\tau_3$ as well as pump intensity $\langle a_{30}^2\rangle \propto G_3(0)\Delta\Omega_{30}$ and to an increase of the nonlinear interaction length $L_n \propto \langle a_{30}^2\rangle^{-1/2} \propto \tau_3^{1/2}$. The product $|\nu|L_n$ (in fs) is 41 (1), 58 (2), 70 (3); here the numbers in the parentheses number the curves shown in Figs. 8(a)–8(d).

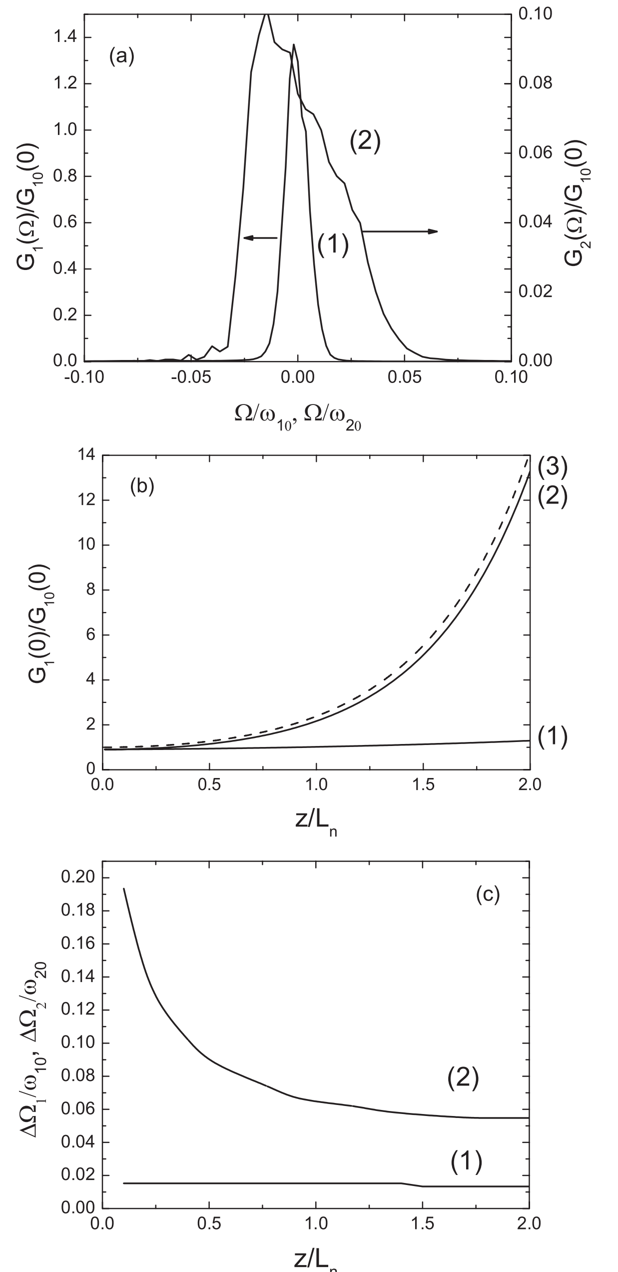

FIG. 7. (a) Output spectra of the signal (1) and idler (2) waves at $z = 4$ cm. (b) Numerical (1,2) and theoretical (3) dependencies of spectral intensity of the signal wave on the propagation length. Curve (2) is obtained neglecting group-velocity dispersion coefficients. (c) Spectral bandwidths of the signal (1) and idler (2) waves. Average of $N = 500$ numerical simulations. Type-I KDP crystal, $\lambda_{30} = 0.48\ \mu$m, $\lambda_{10} = 0.73\ \mu$m ($u_2 = u_3$). $L_n = 2$ cm, $l = 4$ cm, $\tau_1 = 100$ fs, $\tau_3 = 5$ fs, $\langle a_{10}^2\rangle/\langle a_{30}^2\rangle = 10^{-6}$, $|\nu|l = 230$ fs.

So, for a ratio $|\nu|L_n/\tau_3$ we obtain 4.1 (1), 2.9 (2), 2.3 (3). Obviously, the condition $\tau_3 \ll |\nu|L_n$ is not fulfilled, and the pump fluctuations cannot be averaged in a proper way. In this case an increase of the parametric gain of signal and idler waves observed in Fig. 8 is mainly caused by pump fluctuations and only partially by decrease of an influence of group-velocity dispersion of pump wave. In comparison, the parametric gain of signal wave in the case of monochromatic interacting waves

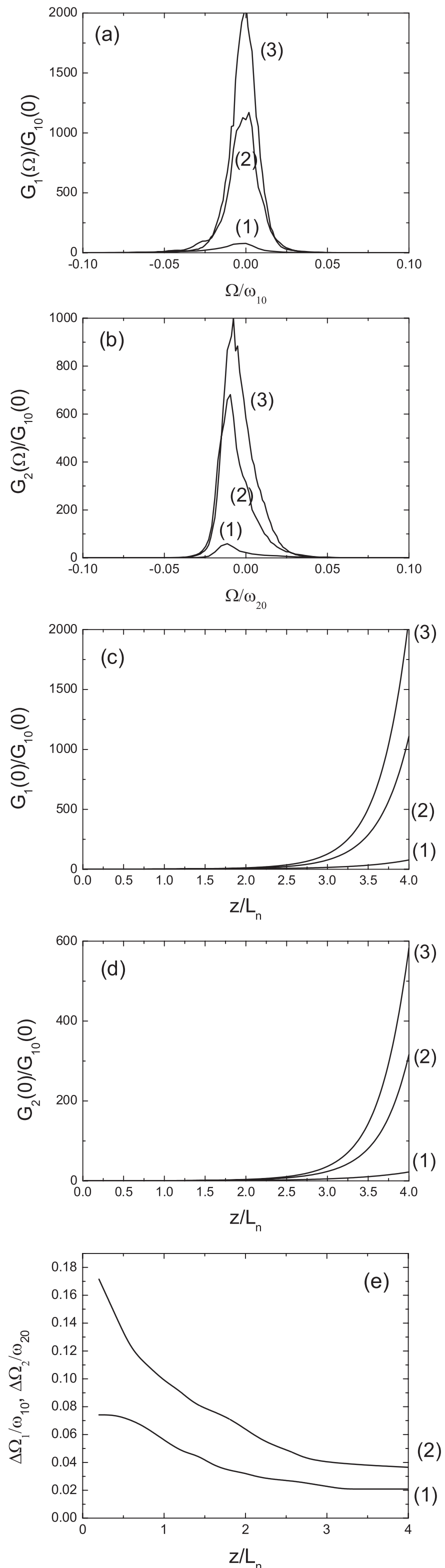

FIG. 8. Output spectra of the signal (a) and idler (b) waves at $z/L_n = 4$. Dependencies of the spectral intensities of the signal (c) and idler (d) waves on ratio z/L_n. Average of $N = 500$ numerical simulations. Type-I KDP crystal, $\lambda_{30} = 0.48$ μm, $\lambda_{10} = 0.73$ μm ($u_2 = u_3$). $\tau_1 = 20$ fs, $\langle a_{10}^2 \rangle / \langle a_{30}^2 \rangle = 10^{-6}$. τ_3 [fs]: 10 (1), 20 (2), 30 (3). L_n [cm]: $1/\sqrt{2}$ (1), 1 (2), $\sqrt{3/2}$ (3). (e) Dependence of the spectral bandwidth of signal (1) and idler (2) waves on the propagation length at $\tau_3 = 20$ fs.

is $G_1(0)/G_{10}(0) = \cosh^2(z/L_n)$ and at $z/L_n = 4$ we obtain $G_1(0)/G_{10}(0) \approx \exp(8)/4 \approx 750$; compare to parametric gain at $z/L_n = 4$ in Fig. 8(a) (curves 1–3).

The question arises: Is it possible to obtain in OPA pumped by incoherent wave a large gain of the signal wave which is not sensitive to pump fluctuations? The needed condition can be formulated as

$$L_d \gg L_n \gg \tau_3/|\nu|, \qquad (30)$$

where $L_d = \tau_3^2/(2g_3)$ is a characteristic dispersion length of a pump wave. Then Eq. (30) can be rewritten as $L_d \geqslant 100\tau_3/|\nu|$ and $\tau_3 \geqslant 200 g_3/|\nu|$. In the case of Type-I phase-matching KDP crystal (Figs. 7 and 8) we have $|\nu| = 57.5$ fs/cm, $g_3 = 7.9 \times 10^{-28}$ s^2/cm and $\tau_3 \geqslant 28$ ps, $L_n \geqslant 5$ m. So, the proper averaging of pump fluctuations at a large gain in the Type-I nonlinear crystal requires a very large nonlinear interaction length L_n. In the case of Type-II phase-matching KDP crystal (Figs. 3–5) the walk-off parameter ν is much larger. We have $|\nu| = 1.5$ ps/cm, $g_3 = 9.6 \times 10^{-28}$ s^2/cm and $\tau_3 \geqslant 130$ fs, $L_n \geqslant 8.6$ mm. As a result, the large parametric gain can be matched with an averaging of pump fluctuations in Type-II phase-matching nonlinear crystal.

VII. CONCLUSIONS

An evolution of the input signal wave in OPA pumped by broadband incoherent wave has been analyzed at group-velocity matching of pump and idler waves. We demonstrate that the group-velocity-matching of pump and idler waves is possible both for Type-I and Type-II phase matching in nonlinear crystals.

The correlation functions and spectra of signal and idler waves are obtained in analytical forms in the case of the first-order dispersion approximation. It is shown that when the parametric gain is not sensitive to the pump fluctuations the evolution of the signal wave in the field of incoherent pump coincides with an evolution in the field of the coherent monochromatic wave. That is possible if the pump correlation time τ_3 is much smaller than the products $|\nu|L_n$, $|\nu|l$, where ν, L_n, and l are the group-velocity mismatch parameter, the nonlinear interaction length, and crystal length, respectively. We demonstrate that the spectral intensity of the amplified signal wave can considerably exceed the spectral intensity of the pump wave.

In the case of second-order dispersion approximation the spectra of signal and idler waves were obtained by numerical simulation of governing equations. We demonstrate that the bandwidth of the idler wave at the output of OPA due to phase-matching conditions is mainly determined by the bandwidth of signal wave. In OPA pumped by incoherent wave a large gain of signal wave which is not sensitive to pump fluctuations preferably can be obtained in nonlinear crystals at Type-II phase matching. That is caused by a rather large group-velocity mismatch of signal and pump waves in comparison with a group-velocity mismatch in crystals at Type-I phase-matching.

We note that similar conditions of a three-wave interaction when the group velocity of one wave is mismatched with the group velocities of the other two waves occur in the case of stimulated Brillouin scattering (SBS) or stimulated Raman scattering (SRS); see Refs. [25,26]. So, the results obtained for OPA can be applied also to SBS and SRS excited by the incoherent broadband pump.

ACKNOWLEDGMENT

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