Transverse momentum transfer in atom-light scattering

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We predict a photon Hall effect in the optical cross section of atomic hydrogen, which is caused by the interference between an electric-quadrupole transition and an electric-dipole transition from the ground state to $3D_{3/2}$ and $3P_{3/2}$. This induces a magnetotransverse acceleration comparable to a fraction of g.

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I. INTRODUCTION

Light scattering exchanges momentum between matter and radiation, and thus induces a force on the matter. Classical light scattering is well known to be affected by a magnetic field. A specific feature, the photon Hall effect (PHE), was first predicted in multiple light scattering [1], and observed shortly afterwards [2] with typical changes in the magnetotransverse photon flux of order 10^{-5} per tesla of applied magnetic field. A Mie theory for the PHE [3] agreed quantitatively with the experiments. Given the wave number $\mathbf{k}$ of the incident photon flux and the magnetic field $\mathbf{B}$, the PHE induces an exchange of momentum between scatterer and radiation in the magnetotransverse ("upward") direction along $\mathbf{B} \times \mathbf{k}$. A light flux of 10^4 W/m^2 incident on a micron-sized particle with a relative PHE of 10^{-5}/T experiences a transverse force of 10^{-19} N/T, roughly equivalent to the Lorentz force on a charge e moving with a velocity of 1 m/s. The magnetotransverse acceleration for a 10 μm TiO_2 particle would be as small as 10^{-11} m/s^2 in a field of 10 T.

Atoms are strong light scatterers that can achieve elastic optical cross sections as large as the maximum unitary limit λ^2 near optical transitions, and with promising applications in mesoscopic physics [4]. When the typical Zeeman splitting $\omega_c/2$ ($\omega_c/2\pi = eB/2\pi m_e = 2.79$ MHz/G is the cyclotron circular frequency) equals the atomic linewidth (typically $\gamma \approx 100$ MHz, i.e., the decay rate $A = 2\gamma = 2 \times 10^8$ s^{-1}), the optical cross section is significantly altered by the magnetic field, typically true for a few gauss. Since atoms have small mass, the magnetotransverse recoil velocity will be much larger than for Mie particles. The magneto cross section of an atomic resonance with width γ and Zeeman splitting ω_c can be estimated as $\frac{1}{2}(\omega_c/\gamma)\lambda^2/\pi^2$. If we were to assume the Hall cross section to be of this order, the magnetotransverse acceleration would be as large as 4 (km/s^2)/G when tuned to the $5s^2$-$5s5p$ transition in strontium at 461 nm exposed to a small flux of 100 W/m^2. Unfortunately, no PHE can occur for pure electric-dipole (ED) transitions, since the ED imposes a symmetry between forward and backward scattering, as well as between upward and downward directions in the magneto cross section [3]. The PHE induced by the scattering from pairs of atoms in a cold ^{88}Sr gas is estimated to be a few percent [5].

II. PHOTON HALL EFFECT OF ATOM

Can the Photon Hall Effect of atom exist at all, and how large will the magnetotransverse momentum transfer to the atom be? Two striking differences exist between classical Mie scattering and light-atom scattering. First, given a monochromatic incident laser beam, the atom is usually subject to inelastic transitions to levels that are no longer excited by the same beam, thus preventing a stationary scattering process. Second, given the small mass of atoms, one must anticipate significant velocity recoils that change the resonant frequency via the Doppler effect and finally reduce the light scattering.

The optical cross section of an atom is expressed by the Kramers-Heisenberg formula [6],

$$\frac{d\sigma}{d\Omega}(\omega\mathbf{k}\varepsilon \to \omega_s\mathbf{k}_s\varepsilon_s) = \alpha^2\frac{\omega_s^3}{\omega^3}|f_{\rm ED}(\omega,\varepsilon,\varepsilon_s) + f_{\rm EQ}(\omega,\varepsilon,\varepsilon_s,\mathbf{k},\mathbf{k}_s) + \cdots|^2.$$

Here, α is the fine structure constant, ω and $\omega_s < \omega$ are the incident and scattered frequency, ε and ε_s are the polarization vectors of incident and scattered radiation, and $f(\omega)$ is the complex scattering amplitude associated with transitions in the atom, which can be either elastic or inelastic, and driven by either electric dipoles or quadrupoles (EQs). The above expression does not take into account stimulated emission. For this to be true we require that $W(\omega_s,\mathbf{k}_s,\varepsilon_s) < W_0(\omega_s)$, with W the radiation density per steradian, per bandwidth, per polarization, and $W_0 = \hbar\omega_s^3/(2\pi c_0)^3$ its value for the quantum vacuum.

We will focus on the simplest atom, atomic hydrogen, whose physics in a magnetic field has been studied in great detail [7,8]. This atom has the *unique* property that the fine-structure levels $3P_{3/2}$ and $3D_{3/2}$ strongly overlap, despite their hyperfine structure (hfs), The anomalous Zeeman effect of the latter is shown in Fig. 1. For not too large magnetic fields all levels are energetically close and can thus interfere constructively. It is instructive to first simply ignore the spin of both electron and proton, and to adopt a simple $1S$ ground state and excited levels $3P$ and $3D$ separated by the Lamb shift of $\Delta\omega/2\pi = 5.5$ MHz. The electronic transitions $1S \to 3P \to 1S$ and $1S \to 3D \to 1S$ are now both elastic. The $1S$-$3D$ transition, however, is ED forbidden and requires an EQ transition. The ED transition between the ground state $1S$ and the $3P$ level reads

$$f_{\rm ED}(\omega) = \frac{\omega^2}{c_0}\sum_{m=0,\pm1}\frac{\{\langle 1S|\mathbf{r}|3P_m\rangle\cdot\varepsilon_s\}\{\langle 3P_m|\mathbf{r}|1S\rangle\cdot\varepsilon\}}{\omega-\omega_m^P(B)+i\gamma_P} \equiv \frac{\omega^2 r_{3P}^2}{3c_0}[Q(\varepsilon_s\cdot\varepsilon)+R(\varepsilon_s\cdot\hat{\mathbf{z}})(\varepsilon\cdot\hat{\mathbf{z}})+Ti\varepsilon_s\cdot(\varepsilon\times\hat{\mathbf{z}})]. \quad (1)$$

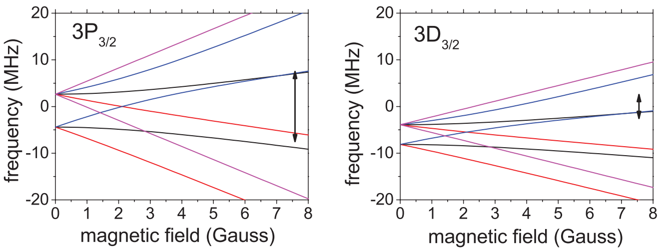

FIG. 1. (Color online) Hyperfine structure of the $3P_{3/2}$ (left) and $3D_{3/2}$ (right) levels of atomic hydrogen, as a function of magnetic field. Equal colors indicate equal values for the hyperfine magnetic quantum number m. The height of the vertical bar on the right indicates the linewidth $\gamma/2\pi$. The zero in frequency is chosen at the fine-structure level of $3P_{3/2}$. That of $3D_{3/2}$ is $\Delta\omega/2\pi = 5.5$ MHz lower due to the Lamb shift [9].

Here $\langle i|\mathbf{r}|j\rangle$ is the ED matrix element between states i and j. It depends on the orbital momentum but has constant radial part $r_{3P} = 0.517a_0$. The second expression is obtained by inserting the orbital eigenfunctions. The complex amplitudes Q, R, T are defined in terms of the line profiles of the three Zeeman levels $P_m(\omega) = 1/[\omega - \omega_m(B) + i\gamma_P]$, with $\gamma_P = 84$ MHz, according to $Q = P_0$, $R = \frac{1}{2}(P_{-1} + P_1)$, $T = \frac{1}{2}(P_{-1} - P_1)$. In our simplified picture, the Zeeman effect behaves normally (energy shift $m\omega_c/2$ linearly proportional to magnetic quantum number). We choose $\hat{\mathbf{k}} = \hat{\mathbf{x}}$, $\hat{\mathbf{B}} = \hat{\mathbf{z}}$, and let $\hat{\mathbf{B}} \times \hat{\mathbf{k}} = \hat{\mathbf{y}}$ be the Hall direction. For the elastic EQ transition via the $3D$ level we find

$$f_{\rm EQ}(\omega) = \frac{\omega^2}{c_0} \sum_{m=0,\pm1,\pm2} \times \frac{\{\mathbf{k}_s \cdot \langle 1S|\frac{1}{2}\mathbf{rr}|3D_m\rangle \cdot \varepsilon_s\}\{\mathbf{k} \cdot \langle 3D_m|\frac{1}{2}\mathbf{rr}|1S\rangle \cdot \varepsilon\}}{\omega - \omega_m^D(B) + i\gamma_D}$$

with $\gamma_D = 32$ MHz the natural linewidth of the $3D$ level. This expression can again be developed by inserting the orbital eigenfunctions associated with the $3D$ level, at fixed radial matrix element $q_{3D} = 0.867a_0^2$.

The differential cross section for incident unpolarized, broadband light is obtained from the interference between the two transitions, averaged over incident polarization, and summed over outgoing polarization. The Hall terms are defined by the difference in flux up and down along the vector $\hat{\mathbf{y}}$, and are all characterized by a factor $i(\hat{\mathbf{k}}_s \cdot \hat{\mathbf{y}})$ that emerges in the cross product of the two scattering amplitudes. We shall write this as

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \alpha^2 \frac{1}{\Delta} \int_\Delta d\omega \sum_{\varepsilon,\varepsilon_s} \mathrm{Re}\, f_{\rm ED}^* f_{\rm EQ} \\ &= \frac{d\sigma^0}{d\Omega} + \alpha^2 \frac{\omega^6}{\Delta c_0^4} r_{3P}^2 q_{3D}^2 \mathrm{Re}\, i(\hat{\mathbf{k}}_s \cdot \hat{\mathbf{y}}) \\ &\quad \times \sum_{m,m'} \bar{F}_{mm'}(B) A_{mm'}(\mathbf{k}, \hat{\mathbf{k}}_s, \hat{\mathbf{B}}). \end{aligned} \qquad (2)$$

The Hall cross section is a sum over $3 \times 5 = 15$ cross products among the magnetic sublevels. The factor $\bar{F}_{mm'}$ is the frequency-averaged cross product of the complex line profiles.

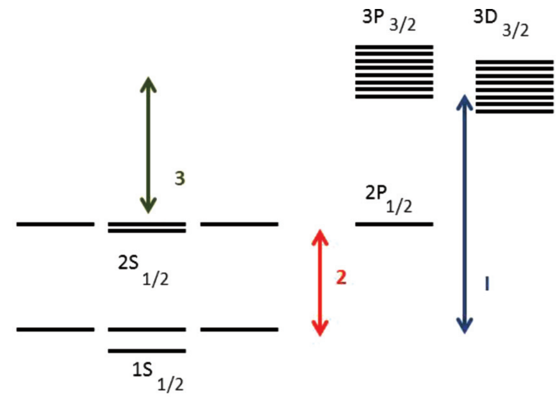

FIG. 2. (Color online) Three different broadband laser beams are necessary to generate a stationary process with magnetotransverse recoil. Laser 1 induces the transition to the $3P_{3/2}$ and $3D_{3/2}$ levels that generate the PHE. Laser beam 2, propagating opposite to laser beam 1, compensates for the longitudinal photon recoil produced by laser 1. Finally, laser beam 3 assures that the inelastic decay to $2S_{1/2}$ is pumped back to $3P_{3/2}$.

If the bandwidth Δ greatly exceeds the linewidths then

$$\begin{aligned}\bar{F}_{mm'}(B) &= \int_{-\infty}^{\infty} d\omega \left(\frac{1}{\omega - \omega_P(m) - i\gamma_P}\right)\left(\frac{1}{\omega - \omega_D(m') + i\gamma_D}\right) \\ &= \frac{2\pi i}{\omega_P(m) - \omega_D(m') + i(\gamma_D + \gamma_P)}. \end{aligned} \qquad (3)$$

It can be checked that only six functions $A_{mm'}$ actually generate a PHE, with $A_{0,m'=\pm1} = m'[1 - 2(\hat{\mathbf{k}}_s \cdot \hat{\mathbf{z}})^2]/60$ and $A_{m=\pm1,m'=\pm2} = [m + \frac{1}{2}m' + (-2m + \frac{1}{2}m')(\hat{\mathbf{k}}_s \cdot \hat{\mathbf{x}})^2 - \frac{1}{2}m'\frac{1}{2}(\hat{\mathbf{k}}_s \cdot \hat{\mathbf{y}})^2]/60$. Note that this simplified picture highlights the PHE as a "which-way" event inside the hydrogen atom. It is straightforward to calculate from Eq. (2) the total magnetotransverse recoil force (black line in Fig. 2). Inelastic scattering, photon recoil and hyperfine structure.

III. INELASTIC SCATTERING, PHOTON RECOIL AND HYPERFINE STRUCTURE

The present picture poses three problems. First, we know that excited $3P$ atoms will have a significant probability to decay inelastically to the metastable state $2S$ so that the PHE process will rapidly become transient. Second, absorbed photons will transfer momentum to the atom which will rapidly become Doppler detuned from the incident laser. These two aspects could be experimentally circumvented by doing the experiment in a pulsed mode, leaving sufficient time between light pulses for the atomic momentum and the $S_{1/2}$ to return to their equilibrium (i.e., dark) values. Finally, the inclusion of hyperfine structure considerably complicates the above picture. As we will show below, all these complications can be remedied to allow for a cw experimental observation.

The longitudinal photon recoil to the atom of the first laser can be compensated by a *second* laser beam (intensity I_2) opposite to the first, and exciting the atom to the $2P$ transition. If $I_2 \approx 3I_1$ the average recoil rate is equal to zero (see Fig. 2). The occurrence of inelastic decay to $2S$ must be compensated

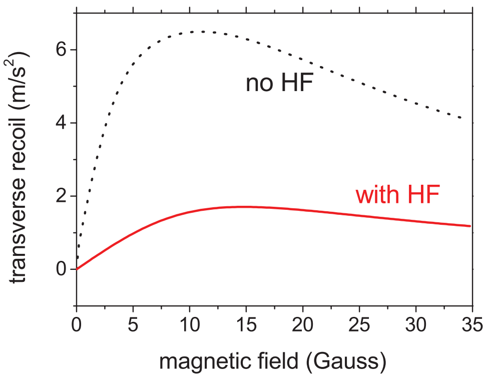

FIG. 3. (Color online) Magnetotransverse recoil acceleration of the hydrogen atom in the presence of a broadband laser beam with flux $I = 10\,\text{kW/m}^2$ and bandwidth $\Delta/2\pi = 1.59$ GHz (thus resolving the hf ground state). The black dotted curve follows from the model with neglect of (hyper)fine structure; the red solid curve takes into account the full hyperfine splitting of the $1S_{1/2}$ ground state and the $3D_{3/2}$ and $3P_{3/2}$ levels, which results in a smaller PHE recoil although with the same sign.

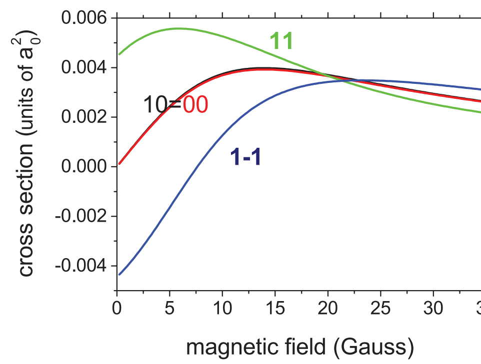

FIG. 4. (Color online) Magnetotransverse cross section (in units of the Bohr radius a_0 squared) of the hydrogen atom, as in the previous figure, here specified separately for transitions from the four hf ground states $|f = 0,1,m = -f,+f\rangle$. The red solid line in Fig. 3 is the sum of the four when converted to m/s^2. The cross sections for the two levels $f = 0,1$, $m = 0$ are equal. At zero magnetic field the cross sections for the opposite-spin $f = 1,m = \pm 1$ are opposite.

by a *third* laser beam (intensity I_3) that pumps $2S$ atoms back to $3P$. A straightforward analysis shows that detailed balance results in $N_{2S}/N_{1S} = (\omega_{23}/\omega_{13})^3 I_3/I_1$. If the two laser intensities are roughly equal we infer that $N_{2S} \ll N_{1S}$, so that the PHE with $1S$ as initial state is maintained. Note that the $2S$-$(3P,3D)$ transitions also induce a PHE which we will not discuss in view of its much smaller transverse recoil.

The inclusion of hf structure is a straightforward process that we shall not discuss in detail. The hf eigenfunctions $|3P(D)_{j=3/2}, f = 1,2, m = -f,\ldots,f\rangle$ can be constructed from the product states of orbital momentum and electron and proton spins with appropriate Clebsch-Gordan coefficients. In the presence of a magnetic field the magnetic sublevels $m = 0,\pm 1$ of the hf levels $f = 1,2$ mix, thus giving the eight sublevels whose Zeeman effect is shown in Fig. 1. The $1S_{1/2}$ ground state splits into one singlet and a triplet at $\Delta\omega/2\pi =$ 1.4 GHz higher in energy. The PHE can be determined by collecting all cross products among the transitions from the four $1S_{1/2}$ to the eight $3D_{3/2}$ and $3P_{3/2}$ levels. The result of this cumbersome task is shown as the red line in Fig. 3. The hf splitting decreases the recoil because the overlap between the levels decreases, roughly by a factor of 4. In this calculation it has been assumed that all hf levels are equally populated, as a result of the presence of the two additional laser beams and the broadband incident beam. In Fig. 4 we show the individual contributions of the four hf $1S_{1/2}$ states to the total recoil. The spin-polarized states $|f = 1,m = \pm 1\rangle$ have each a nonzero, though opposite, PHE at zero field that vanishes for equal level populations (a "spin Hall effect," unobservable in the present configuration with inelastic transitions that mix up all contributions). In addition, the PHE recoils from the unpolarized singlet state $|f = 0,m = 0\rangle$ and the unpolarized triplet state $|f = 1,m = 0\rangle$ are equal.

The calculation for the PHE discussed above is very specific for atomic hydrogen. Yet the basic mechanism may apply to other atoms or even molecules with nearly degenerate transitions.

IV. CONCLUSION

In conclusion, we have quantified the magnetotransverse scattering of *broadband* light from unpolarized atomic hydrogen. It is caused by the interference of an electric-dipole transition and an electric-quadrupole transition. A transverse recoil of several m/s^2 is predicted, i.e., a fraction of g. The experiment seems to be feasible in pulsed mode, but is rather involved in a cw implementation, and the generalization to other atoms is possible only if one has overlapping transitions with different (orbital) symmetry. These are often excluded by (hyper)fine splitting. The application of sufficiently high magnetic fields will induce level crossing of remote transitions, thus causing a PHE. It could be interesting to study the atomic spin Hall effect in the spin-polarized S state of atomic hydrogen [8], and to make a link with previous predictions [10].

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