Near-threshold laser-modified proton emission in the nuclear photoeffect

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The change of the probability of proton emission in the nuclear photoeffect due to an intense coherent (laser) field is discussed near the threshold, where the hindering effect of the Coulomb field of the remainder nucleus is essential. The ratio of the laser-assisted and laser-free differential cross section is deduced and found to be independent of the polarization state of the γ field and the two types of initial nuclear state considered. The numerical values of this ratio are given at some characteristic parameters of the intense field.

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I. INTRODUCTION

The development of coherent electromagnetic sources of higher and higher intensity with increasing photon energy up to the hard x-ray range motivates the theoretical study of the change of the processes of strongly bound systems, such as nuclear processes, by these intense fields [1]. In this paper, the change of the nuclear photoeffect due to the presence of an intense coherent electromagnetic field is studied. This process is analogous to the laser-assisted x-ray photoeffect (x-ray absorption), a process which was already discussed [2] in the late 1980s taking into account gauge invariance [3,4]. The laser-assisted nuclear photoeffect (LANP) and the laser-assisted x-ray photoeffect (x-ray absorption) are laser-assisted bound-free transitions. The difference between them lies in the charged particle (proton or electron, respectively) which takes part in these processes. Although the LANP was recently investigated far from the threshold and neglecting the effect of the Coulomb field of the remainder nucleus [5], in the case of the laser-assisted x-ray absorption processes it was found that the most interesting changes due to the presence of the laser field appear near the threshold [6,7].

Thus, applying the results of [2], the LANP is reexamined in a gauge-invariant manner and near the threshold, where the hindering effect of the Coulomb field of the remainder nucleus is very large so that it must be taken into account. The effect of the Coulomb field of the remainder nucleus on the transition rate is approximately taken into account. The laser-modified differential cross section is compared to the laser-free differential cross section, and it is shown that their ratio does not depend on nuclear parameters in the two types of initial nuclear states investigated and on the state of polarization of the γ radiation, but it has only a laser parameter dependence.

The process investigated can be symbolically written as

$$\omega_\gamma + n\omega_0 + {}^{A+1}_{Z+1}Y \rightarrow {}^{A}_{Z}X + {}^{1}_{1}p, \qquad (1)$$

where ${}^{A+1}_{Z+1}Y$ denotes the target nucleus of mass number $A+1$ and of charge number $Z+1$. The target nucleus absorbs a γ photon symbolized by ω_γ, and n laser photons take part in the process which is symbolized by $n\omega_0$. $n<0$ and $n>0$ correspond to $|n|$ laser photon emission and absorption, respectively. As a result, a free proton ${}^{1}_{1}p$ is emitted and the remainder nucleus is ${}^{A}_{Z}X$.

The calculation is made in the radiation (pA) gauge, and in the long-wavelength approximation (LWA) of the electromagnetic fields, the recoil of the remainder nucleus and the initial momentum carried by the laser and γ fields are neglected. In the case of a circularly polarized monochromatic wave for the vector potential of a laser field, $\overrightarrow{A_L}(t) = A_0[\cos(\omega_0 t)\overrightarrow{e}_1 - \sin(\omega_0 t)\overrightarrow{e}_2]$ is used. ω_0 is the angular frequency of the laser. The amplitude of the corresponding electric field $E_0 = \omega_0 A_0/c$. The frame of reference is spanned by the unit vectors $\overrightarrow{e}_x = \overrightarrow{e}_1$, $\overrightarrow{e}_y = \overrightarrow{e}_2$, and $\overrightarrow{e}_z = \overrightarrow{e}_1 \times \overrightarrow{e}_2$. The vector potential describing the γ radiation is $\overrightarrow{A}_\gamma = \sqrt{2\pi\hbar/(V\omega_\gamma)}\,\overrightarrow{\varepsilon}\exp(-i\omega_\gamma t)$, with $\hbar\omega_\gamma$ the energy and $\overrightarrow{\varepsilon}$ the unit vector of the state of polarization of the γ photon, and V the volume of normalization.

II. GAUGE-INVARIANT S-MATRIX ELEMENT OF LASER-MODIFIED PROTON EMISSION IN THE NUCLEAR PHOTOEFFECT

It is shown in [3] that the electromagnetic transition amplitudes of a particle (proton) of rest mass m and of charge e in the presence of a laser field are determined by the matrix elements of the operator $-e\overrightarrow{r}\cdot\overrightarrow{E}$ with the eigenstates of the instantaneous energy operator

$$\varepsilon^g = \frac{1}{2m}\left(\overrightarrow{p} - \frac{e}{c}\overrightarrow{A^g}\right)^2 + V(r) \qquad (2)$$

in both (rE and pA) gauges. (e is the elementary charge and the superscript g refers to the gauge.) Accordingly, the gauge-independent S-matrix element can be written as

$$S_{\mathrm{fi}} = -\frac{i}{\hbar}\int dt \int d^3r\,\psi_f^*(-e\overrightarrow{r}\cdot\overrightarrow{E}(t))\psi_i, \qquad (3)$$

where ψ_i and ψ_f are the initial and final states of the proton in the same gauge and $\hbar$ is the reduced Planck constant.

Our calculation is carried out in the radiation (pA) gauge because of the choice of the final state of the proton (see below). The initial state of the proton has the form

$$\psi_i = e^{\left(i\frac{e\vec{r}\cdot\vec{A}}{\hbar c}\right)}\phi_0(\overrightarrow{r})e^{-i\frac{E_b}{\hbar}t}, \qquad (4)$$

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where $\phi_0(\vec{r})$ is a stationary nuclear state of separation energy E_b of the proton. The $\exp(i\frac{e\vec{r}\cdot\vec{A}}{\hbar c})$ factor, where $\vec{A} = \vec{A}_L(t) + \vec{A}_\gamma$, appears because of gauge transformation since ϕ_0 is the eigenfunction of the instantaneous energy operator,

$$\varepsilon^E = \frac{1}{2m}\vec{p}^{\,2} + V_N(r) + V_{iC}(r), \tag{5}$$

in the rE gauge. $V_N(r)$ is the nuclear potential and $V_{iC}(r)$ is the Coulomb potential felt by the proton initially, and the superscript E refers to the rE gauge. The modification of the initial state due to the laser field is neglected since the direct effect of the intense laser field on the nucleus has been found to be negligible [8] at the laser parameters discussed. It is also supposed that the initial nucleus does not have an excited state which is resonant or nearly resonant with the applied γ radiation.

If similarly to [5] the modification of the final state due to the strong interaction is neglected, then in the final state and in the pA gauge the instantaneous energy operator ε^R reads

$$\varepsilon^R = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A_L}(t)\right)^2 + V_C(r), \tag{6}$$

where the superscript R refers to the radiation (pA) gauge and $V_C(r)$ is the Coulomb potential of the remainder nucleus.

An approximated solution of (6), i.e., an approximated time-dependent state of a particle in the laser plus Coulomb fields, is the Coulomb-Volkov solution of a proton of wave number vector $\vec{Q}$ [9,10],

$$\psi_{\vec{Q}}(\vec{r},t) = V^{-1/2}e^{i\vec{Q}\cdot\vec{r}}\chi(\vec{Q},\vec{r})\exp(-i\widehat{E}t/\hbar)f(t). \tag{7}$$

Here $V^{-1/2}e^{i\vec{Q}\cdot\vec{r}}\chi(\vec{Q},\vec{r})$ is the Coulomb function, i.e., the wave function of a free proton in a repulsive Coulomb field of charge number Z, V denotes the volume of normalization, $\vec{r}$ is the relative coordinate of the two particles,

$$\chi(\vec{Q},\vec{r}) = e^{-\pi\eta/2}\Gamma(1+i\eta)\,{}_1F_1(-i\eta,1;i[Qr - \vec{Q}\cdot\vec{r}]), \tag{8}$$

where

$$\eta(Q) = Z\alpha_f\frac{mc}{\hbar Q}, \tag{9}$$

is the Sommerfeld parameter, with α_f the fine-structure constant, and it is supposed that m is much less than the rest mass of the remainder nucleus. ${}_1F_1$ is the confluent hypergeometric function and Γ is the Gamma function [11].

The function

$$f(t) = \exp[i\alpha\sin(\omega_0 t + \eta_0)], \tag{10}$$

where

$$\alpha = \alpha_\vartheta\sin(\vartheta)\ \text{ with }\ \alpha_\vartheta = \frac{eE_0Q}{m\omega_0^2}. \tag{11}$$

Here the polar angles of the wave number vector $\vec{Q}$ of the outgoing proton are ϑ and η_0, i.e., they are the polar angles of the direction in which the proton is ejected.

In the low-energy range ($QR \ll 1$, where R is the radius of a nucleon) and for $|\vec{r}| \leqslant R$, the long-wavelength approximation yields

$$|\chi(\vec{Q},\vec{r})|_{\vec{r}=0} = \chi_C(Q) = \sqrt{\frac{2\pi\eta(Q)}{\exp[2\pi\eta(Q)] - 1}}, \tag{12}$$

which is the square root of the so-called Coulomb factor. (The Coulomb factor $[\chi_C^2(Q)]$ describes well, e.g., the Coulomb correction to the spectrum shape of beta decay [12].)

For the final state of a proton of wave number vector $\vec{Q}$, the LWA of the nonrelativistic Coulomb-Volkov solution $\psi_{\vec{Q}}$ is used,

$$\psi_{\vec{Q},\mathrm{LWA}}(\vec{r},t) = \chi_C(Q)V^{-1/2}e^{i\vec{Q}\cdot\vec{r}}\exp(-i\widehat{E}t/\hbar)f(t) \tag{13}$$

with $\widehat{E} = \hbar^2Q^2/(2m) + U_p$, that is, the energy of the outgoing proton in the intense field, where $U_p = e^2E_0^2/(2m\omega_0^2)$ is the ponderomotive energy.

Substituting (4) and (13) into (3) and using $\vec{E} = -\frac{1}{c}\partial_t\vec{A}$, one can obtain the following form of the gauge-independent S-matrix element:

$$S_{\mathrm{fi}} = -\frac{\chi_C(Q)}{\sqrt{V}}\int \exp[i(\widehat{E} + E_b)t/\hbar]f^*(t)\frac{\partial}{\partial t}G[\vec{q}(t)]dt, \tag{14}$$

where

$$G(\vec{q}) = \int \phi_0(\vec{r})e^{-i\vec{q}\cdot\vec{r}}d^3r \tag{15}$$

is the Fourier transform of the initial stationary nuclear state $\phi_0(\vec{r})$ of the proton and

$$\vec{q}(t) = \vec{Q} - \frac{e}{\hbar c}\vec{A}. \tag{16}$$

[Equation (14) can be obtained directly with the aid of Eq. (27) of [2].]

Using the $\partial_t G = (\partial_q G)\sum_{j=1}^{j=3}(\partial_{A_j}q)(\partial_t A_j)$ identity,

$$\frac{\partial}{\partial t}G = \left(\frac{\partial}{\partial q}G\right)\frac{e}{\hbar q}\left(\vec{Q}\cdot\vec{E} - \frac{e}{\hbar c}\vec{A}\cdot\vec{E}\right) \tag{17}$$

with $\vec{E} = -\frac{1}{c}\partial_t\vec{A}$, i.e., $\vec{E} = \vec{E}_L(t) + \vec{E}_\gamma$.

The $\vec{Q}\cdot\vec{E}_L$ term of the last factor of (17) can be neglected if the pure intense field-induced proton stripping process is negligible since this term describes the process without the γ photon. Furthermore, the ratio of the amplitudes of $\vec{A}_\gamma\cdot\vec{E}_L$ and $\vec{A}_L\cdot\vec{E}_\gamma$ equals $\omega_L/\omega_\gamma \ll 1$. Therefore, the $\vec{Q}\cdot\vec{E} - \frac{e}{\hbar c}\vec{A}\cdot\vec{E} = \vec{Q}\cdot\vec{E}_\gamma - \frac{e}{\hbar c}\vec{A}_L\cdot\vec{E}_\gamma$ approximation is justified to use, where $\vec{E}_\gamma = i\sqrt{2\pi\hbar\omega_\gamma/V}\vec{\varepsilon}\exp(-i\omega_\gamma t)$. The relative strength of the $\vec{Q}\cdot\vec{E}_\gamma$ and $\frac{e}{\hbar c}\vec{A}_L\cdot\vec{E}_\gamma$ terms is characterized by the parameter $\delta = eA_0/(\hbar cQ)$. In the laser-free case, $Q = \sqrt{2m\Delta}/\hbar$, where $\Delta = \hbar\omega_\gamma - E_b$ is the difference of the photon energy and the proton separation energy. Numerical estimation shows that $\delta \simeq 0.05$ near $\Delta =$ 50 keV used here and in the case of laser photon energy and intensity values discussed. Therefore, the $\vec{Q}\cdot\vec{E}_\gamma$ term is the leading one in the last factor of (17). As to the radiation field dependence of $\partial_q G$, the effect of $\vec{A}_\gamma$ is negligible in $\vec{q}(t)$ and thus $\vec{q}(t) = \vec{Q} - \frac{e}{\hbar c}\vec{A}_L$. It was shown above that the amplitude of oscillation of $\vec{q}(t)$ due to the intense field can be neglected. Therefore, $q = Q$ can be used in $\partial_q G$, and (18) is obtained as

$$\frac{\partial}{\partial t}G = \left(\frac{\partial}{\partial q}G\right)_{q=Q}\frac{e}{\hbar}\frac{\overrightarrow{Q}\cdot\overrightarrow{E}_\gamma}{q}. \tag{18}$$

Using the Jacobi-Anger formula in the Fourier series expansion of $f^*(t)$ [13], the S-matrix element can be written as

$$S_{\mathrm{fi}} = \sum_{n=n_0}^{\infty}\frac{2\pi\delta[\omega_n(Q)]i}{V}\chi_C(Q) \times \left(\frac{\partial}{\partial q}G\right)_{q=Q}\frac{e\sqrt{2\pi\hbar\omega_\gamma}}{\hbar}\xi J_n(\alpha)e^{-in\eta_0}, \tag{19}$$

where $\xi = \overrightarrow{Q}\cdot\overrightarrow{\varepsilon}/Q$, $J_n(\alpha)$ is a Bessel function of the first kind, and

$$\omega_n(Q) = \frac{\hbar Q^2}{2m} + \frac{U_p + E_b}{\hbar} - \omega_\gamma - n\omega_0. \tag{20}$$

The terms that are small if the $\frac{eE_0}{\hbar\omega_0\beta_k} \ll 1$ condition is fulfilled (β_1 and β_2, see below), were neglected in the calculation.

III. GAUGE-INVARIANT DIFFERENTIAL CROSS SECTION OF LASER-MODIFIED PROTON EMISSION IN THE NUCLEAR PHOTOEFFECT

In this paper, two cases of initial nuclear energy $-E_b$ of the initial state having a different type of space-dependent part $\phi_0(\overrightarrow{r})$ are considered in order to show the general nature of the effect of the laser on the process. The one case is the ^8B one-proton halo isotope of separation energy $E_b = 0.137$ MeV [14] and of initial state $\phi_0(\overrightarrow{r}) = (2\pi)^{-1}\beta_1^{3/2}e^{-\beta_1 r}/(\beta_1 r)$, with $\beta_1 = \mu\sqrt{2mE_{b,1}}/\hbar$ and m the rest mass of the proton ($\mu = 1.84$, $\beta_1 = 1.495\times 10^{12}$ cm^{-1}). Although the proton rest mass is more than 12% of the total rest mass of ^8B and the approximation, the fact that m is much less than the rest mass of the remainder nucleus is not very good, but following [5] we investigate ^8B. In the other case, the initial state is the $\nu s_{1/2}$ shell model state [15] of the form $\phi_0(\overrightarrow{r}) = N_\nu e^{-\rho^2/2}F(-\nu,\frac{3}{2},\rho^2)$, with $N_\nu = \beta_2^{3/2}[\Gamma(\nu+\frac{3}{2})/(2\pi\nu!)]^{1/2}/\Gamma(\frac{3}{2})$, where $\nu = 0,1,2,\ldots$ is the quantum number of the nuclear shell model and $\Gamma(x)$ denotes the Gamma function. $F(-\nu,\frac{3}{2},\rho^2)$ is the confluent hypergeometric function, $\rho = \beta_2 r$, $\beta_2 = \sqrt{m\omega_{\mathrm{sh}}/\hbar}$, m is the nucleon rest mass, and ω_{sh} is the shell model angular frequency ($\hbar\omega_{\mathrm{sh}} = 40A^{-1/3}$ MeV, where A is the nucleon number [16], and $\beta_2 = 9.82\times 10^{12}A^{-1/6}$ cm^{-1}).

The differential cross section of LANP has the form

$$\frac{d\sigma}{d\Omega_q} = \sum_{n=n_0}^{\infty}\frac{d\sigma_n}{d\Omega_q}, \tag{21}$$

where $d\Omega_q$ is the differential solid angle around the direction of the outgoing proton. $n_0 < 0$ is the smallest integer fulfilling the $\Delta + n\hbar\omega_0 - U_p > 0$ condition and Δ is the same in both cases of the initial state. (The cases $n < 0$ and $n > 0$ correspond to $|n|$ laser photon emission and absorption, respectively.)

The partial differential cross section

$$\frac{d\sigma_n}{d\Omega_q} = \sigma_{0,n}(Q_n)|\xi|^2 J_n^2(\alpha_{\vartheta n}\sin\vartheta) \tag{22}$$

with

$$\sigma_{0,n}(Q_n) = \alpha_f\frac{k_\gamma Q_n}{2\pi\lambda_p}\chi_C^2(Q_n)[\partial_Q G(\overrightarrow{Q})]^2_{Q=Q_n}. \tag{23}$$

Here $Q_n = \sqrt{2m[\Delta + n\hbar\omega_0 - U_p]}/\hbar$, $\xi = \overrightarrow{Q}_n\cdot\overrightarrow{\varepsilon}/Q_n$, λ_p is the reduced Compton wavelength of the proton, and $k_\gamma = \omega_\gamma/c$,

$$Q_n = \frac{\varepsilon_n}{\lambda_p}\sqrt{\frac{2\Delta}{mc^2}} \text{ with } \varepsilon_n = \sqrt{1 + \frac{n\hbar\omega_0 - U_p}{\Delta}}. \tag{24}$$

$J_n(\alpha_{\vartheta n}\sin\vartheta)$ is a Bessel function of the first kind, with $\alpha_{\vartheta n} = eE_0Q_n/(m\omega_0^2)$.

In the variable ε_n, the Coulomb factor χ_C^2 reads

$$\chi_C^2(\varepsilon_n) = \frac{K_{\mathrm{Cb}}}{\varepsilon_n\left[\exp\left[\frac{K_{\mathrm{Cb}}}{\varepsilon_n}\right] - 1\right]}, \tag{25}$$

where $K_{\mathrm{Cb}} = 2\pi Z\alpha_f\sqrt{mc^2/(2\Delta)}$ with Z the charge number of the remainder nucleus. The Coulomb factor causes a strong hindering of the effect in both the laser-assisted and laser-free cases.

Near the threshold ($Q_n \ll \beta_1,\beta_2$), the $[\partial_Q G]^2_{Q_n} \propto \varepsilon_n^2$ in the case of the two types of initial state discussed, and

$$\frac{d\sigma_n}{d\Omega_q} = \sigma_0 S_n|\xi|^2. \tag{26}$$

Here σ_0 is a constant that depends on the form of the initial state and

$$S_n = \frac{\varepsilon_n^2 J_n^2(\alpha_{\vartheta n}\sin\vartheta)}{\exp(K_{\mathrm{Cb}}/\varepsilon_n) - 1}. \tag{27}$$

In the laser-free case ($\alpha_{\vartheta n} = 0$) using $\varepsilon_0(U_p = 0) = 1$, $J_0^2(0) = 1$, and $J_n^2(0) = 0$ at $n \neq 0$, the differential cross section near above the threshold

$$\frac{d\sigma^{\mathrm{th}}}{d\Omega_q} = \sigma_0 S_{\mathrm{th}}|\xi|^2 \tag{28}$$

with $S_{\mathrm{th}} = [\exp(K_{\mathrm{Cb}}) - 1]^{-1}$.

IV. NUMERICAL RESULTS

The ratio R of the laser-assisted and the laser-free differential cross sections,

$$R = \sum_{n=n_0}^{\infty} R_n, \tag{29}$$

where $R_n = S_n/S_{\mathrm{th}}$ [(26) divided by (28)], equals the ratio of the rates of the corresponding processes in an elementary solid angle in a given direction of the outgoing proton. The rate of change in one channel,

$$R_n = \frac{\exp(K_{\mathrm{Cb}}) - 1}{\exp(K_{\mathrm{Cb}}/\varepsilon_n) - 1}\varepsilon_n^2 J_n^2(\alpha_{\vartheta n}\sin\vartheta), \tag{30}$$

with

$$\alpha_{\vartheta n} = \varepsilon_n\hbar ceE_0(\hbar\omega_0)^{-2}\sqrt{2\Delta/(mc^2)} \tag{31}$$

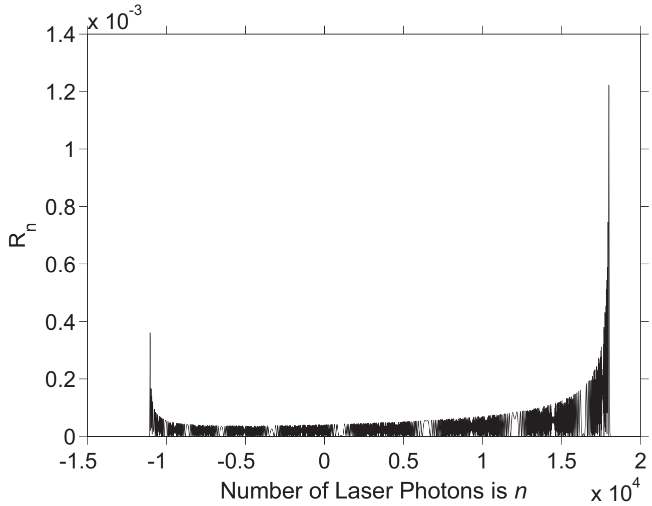

FIG. 1. Laser photon number dependence of R_n [see (30)], that is, the rate of change in one channel due to the presence of intense field. The cases $n < 0$ and $n > 0$ correspond to $|n|$ number laser photon emission and absorption, respectively. The charge number of the remainder nucleus $Z = 4$, $\Delta = \hbar\omega_\gamma - E_b$ is the difference of the gamma photon energy and the proton separation energy, $\vartheta = \pi/2$, the laser intensity $I = 10^{20}$ W cm^{-2} and the laser photon energy $\hbar\omega_0 = 1.65$ eV.

in the variable ε_n. So R_n and R describe the change caused by the intense coherent field independently of the state of γ polarization and the initial states applied.

In our numerical calculation, the laser photon energy $\hbar\omega_0 = 1.65$ eV. First the case in which the outgoing proton moves in the plane of polarization of the laser beam [$\vartheta = \pi/2$ ($\sin\vartheta = 1$)] is investigated in the case of $Z = 4$. Figure 1 shows the laser photon number dependence of R_n at $\Delta = 50$ keV with laser intensity $I = 10^{20}$ W cm^{-2}. The intensity dependence of R has been investigated with $\Delta = 50$ keV. R increases linearly from $R = 1.00$ at $I = 10^{18}$ W cm^{-2} up to $R = 1.12$ at $I = 10^{20}$ W cm^{-2}. Figure 2 depicts the Δ dependence of R with $I = 10^{20}$ W cm^{-2}. Figure 3 shows the ϑ dependence of R_n in the case $I = 10^{20}$ W cm^{-2} at (a) $n = 18\,000$, (b) $n = 14\,000$, (c) $n = 10\,000$, and (d) $n = 5000$. Finally, Fig. 4 depicts the ϑ dependence of R_n at $n = 10\,000$ with different laser intensities: (a) $I = 4 \times 10^{19}$ W cm^{-2}, (b) $I = 6 \times 10^{19}$ W cm^{-2}, and (c) $I = 10^{20}$ W cm^{-2}. The numerical calculation in all the cases discussed above has been repeated at $Z = 9$, and negligible change has been found.

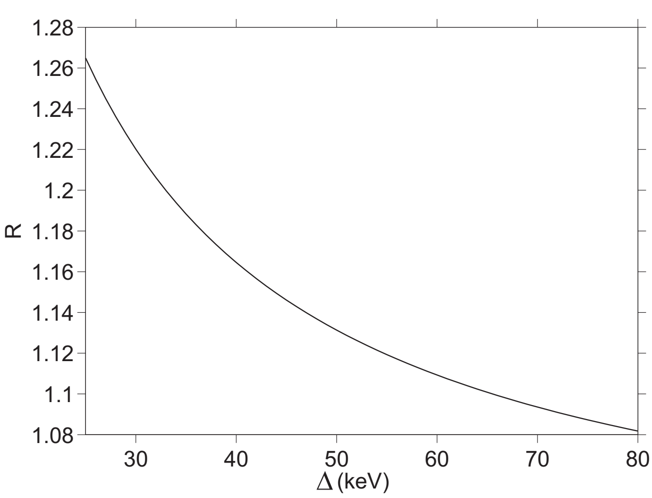

FIG. 2. The Δ dependence of R [see (29)] at $\vartheta = \pi/2$ in the case of $Z = 4$ with $I = 10^{20}$ W cm^{-2} and $\hbar\omega_0 = 1.65$ eV. $\Delta = \hbar\omega_\gamma - E_b$ is the difference of the gamma photon energy and the proton separation energy.

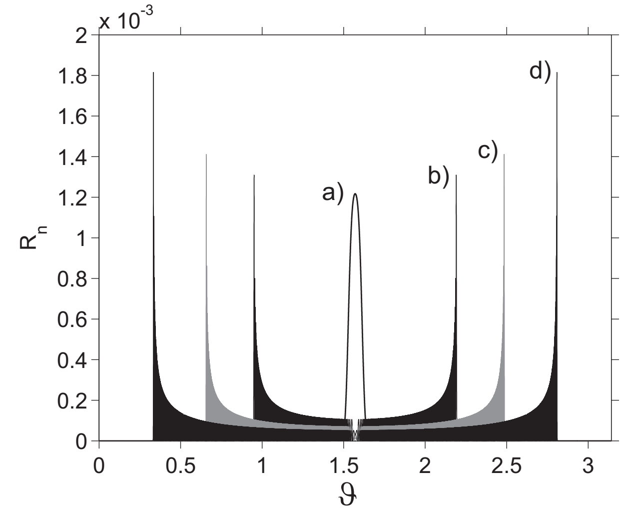

FIG. 3. The ϑ dependence of R_n [see (30)] with $I = 10^{20}$ W cm^{-2} and $\hbar\omega_0 = 1.65$ eV at (a) $n = 18\,000$, (b) $n = 14\,000$, (c) $n = 10\,000$, and (d) $n = 5000$. The charge number of the remainder nucleus $Z = 4$ and $\Delta = 50$ keV.

V. DISCUSSION AND SUMMARY

To compare our results and the results of [5], we investigate our formulas in the $\hbar\omega_\gamma \gg E_b$ limit. In this case, $\varepsilon_n \to 1$,

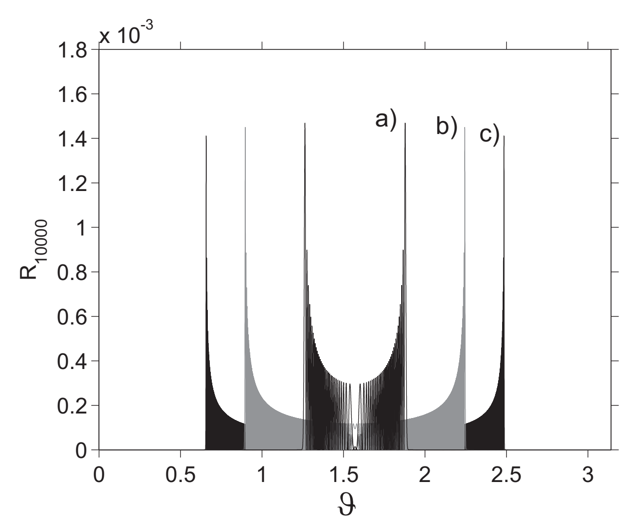

FIG. 4. The ϑ dependence of R_n [see (30)] with $n = 10\,000$ and $\hbar\omega_0 = 1.65$ eV at (a) $I = 4 \times 10^{19}$ W cm^{-2}, (b) $I = 6 \times 10^{19}$ W cm^{-2}, and (c) $I = 10^{20}$ W cm^{-2}. The charge number of the remainder nucleus $Z = 4$ and $\Delta = 50$ keV.

$\Delta \to \hbar\omega_\gamma$,

$$\alpha_{\vartheta n} \to \alpha_\vartheta = \hbar c e E_0 (\hbar\omega_0)^{-2} \sqrt{2\hbar\omega_\gamma/(mc^2)} \tag{32}$$

and

$$R_n \to J_n^2(\alpha_\vartheta \sin\vartheta). \tag{33}$$

Thus in this limit, the n dependence disappears from the argument of the Bessel function and

$$R \to \sum_{n=n_0}^{\infty} J_n^2(\alpha_\vartheta \sin\vartheta) \simeq 1. \tag{34}$$

The cross sections obtained in [5] are symmetric in n around $n = 0$ (see the figures of [5]) and the total cross section is asserted to be unaffected by the laser radiation. This corresponds to $R = 1$. In contrast, our result is significantly asymmetric in n (see Fig. 1), which is a consequence of the ε_n dependence of R_n [see (30)]. The change (increase) of the kinetic energy of the proton is manifested in the increase of ε_n from $\varepsilon_{-11\,000} = 0.761$ up to $\varepsilon_{18\,000} = 1.240$. The increase of ε_n with increasing n causes an asymmetry in the n dependence of Fig. 1. The sum of the changes (increments) in the different channels results in a moderate increment of R ($R < 1.28$), as can be seen in Fig. 2.

Summarizing, one can say that near the threshold, R, which measures the change of the rate of the LANP, has minor Δ and intensity dependence and negligible Z dependence. Furthermore, R_n, which is the rate of change in one channel (at a definite laser photon number), has a significant laser photon number and ϑ dependences. R and R_n are the same in the cases of the two different initial states considered. Since the numerical results obtained seem to be independent of the initial nuclear states chosen, it can be expected that R and R_n have a minor dependence from the form of the initial state in general.

Regarding the experimental situation at such a high intensity, it is hard to distinguish photoprotons from background protons that arise as a result of interaction with the hot electron plasma created by the intense laser field. We also have to mention that in an experiment, the γ ray pulse from an accelerator must be synchronized with an intense (e.g., attosecond) pulse of a laser system. Moreover, in the case of ^8B, which has a short half-life of about 770 ms, the ^8B nuclei must be created *in situ* in the laser beam by a nuclear reaction. Fortunately, most of the heavier nuclei have protons of a $\nu s_{1/2}$ shell model state (the other case investigated) in their stable ground state. The wispy target determined by the focal spot of the focused intense laser beam, the low repetition rate of the laser system, and the angular resolution of the proton detector together make it very challenging to carry out a successful near-threshold, laser-modified proton emission experiment that could produce significant counting statistics.

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