Dark states of a moving mirror in the single-photon strong-coupling regime

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We investigate theoretically an optomechanical system in which a cavity with a moving mirror is driven by two external fields of different frequencies. Under a resonance condition in the resolved-sideband limit, we show that there are motional states of the mirror such that the system is decoupled from the external fields if the single-photon optomechanical coupling strength is sufficiently strong. The decoupling is a quantum interference effect associated with the coherence in the mirror's mechanical degrees of freedom. We discuss the properties of such dark states and indicate how they can be generated by optical pumping due to amplitude damping of the cavity field.

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I. INTRODUCTION

Quantum effects of a moving mirror interacting with electromagnetic fields in a cavity via radiation pressure have been a subject of considerable research interest recently [1–5]. At the heart of this research is the coupling between the mechanical and field degrees of freedom, which is a basis for some novel applications in quantum-information processing, such as the storage of optical information as a mechanical excitation [6] and optomechanical transducers for long-distance quantum communication [7]. In addition, the interaction can serve as a tool to explore fundamental quantum phenomena at macroscopic scales. These include, for example, quantum entanglement [8–13], Schrödinger cat states [14–17], and the modification of uncertainty relations due to quantum gravity [18].

An important general issue in optomechanics is the quantum response of a moving mirror to external driving fields. In this paper we address the issue for an optomechanical cavity driven by two laser fields in the single-photon strong-coupling regime. Our study can be considered as an overlap of two areas in current optomechanics, namely, the single-photon strong-coupling regime where the radiation pressure from a single cavity photon can displace the mirror with a displacement comparable to the zero-point fluctuations [19–24], and the coherence effects of two driving fields [25–30]. These two areas have been studied separately by many authors. For example, with a single driving field, theoretical investigations have shown that a single-photon strong coupling can lead to photon blockade effect [19], multiple mechanical sidebands [20], and mechanical backaction effects on photon statistics [31]. In addition, experimental progress [32–40] has been made in pursuit of this regime, although it remains a challenge at present. On the other hand, coherence effects of two driving fields have mainly been studied in the weak-coupling regime, where the effect of squeezing and cooling [41], quantum state transfer [42–44], and optomechanically induced transparency (OIT) [25–30,45] have been studied. Recently, the configuration of using multimode fields and a strong coupling was explored [46,47]. In particular, Stannigel *et al.* [46] indicated how single photons and phonons can be coupled nonlinearly in a controlled manner.

In this paper we show that when two laser fields are applied to a cavity with a strong optomechanical coupling, the mirror can evolve into a dark state such that the cavity-field mode cannot absorb energy from the external fields. In other words, the optomechanical cavity is decoupled from the external fields. Such a decoupling effect is due to quantum interference, and it is similar to the mechanism of dark states in Λ-type atomic systems for coherent population trapping [48] and electromagnetically induced transparency [49]. However, the main difference here is the involvement of coherent superposition of mechanical states of the mirror instead of electronic states, and this also distinguishes our work from that of Dong *et al.* [45] who have used superposition of optical modes to achieve a decoupling.

We should point out that although the decoupling effect described in this paper can be regarded as one kind of OIT phenomena, the transparency in previous studies of OIT refers to the nonabsorption of a probe field, and the cavity actually contains a large number of photons due to the presence of a control field [25–30]. In our case, we show that by exploiting the single-photon strong coupling, there can be almost no photons in the driven cavity-mode when the mirror is in the dark state. In addition, we shall see that the dark states are non-Gaussian states as they can exhibit sub-Poisson phonon number statistics.

The organization of the paper is as follows. After introducing our system model in Sec. II, we derive an effective Hamiltonian in Sec. III. Such an effective Hamiltonian is based on single-photon blockade mechanism and resonance approximation. In addition, we show that there exists a set of optomechanical coupling strengths that allows us to confine the system in a finite-number phonon-number state. In Sec. IV, we present the dark states and discuss their properties. In Sec. V, we discuss how cavity-field damping can be used to prepare dark states by optical pumping. This is confirmed numerically by solving the evolution of the system governed by the master equation. Section VI is devoted to our conclusions.

II. THE MODEL

We consider an optomechanical cavity formed by a harmonically bounded, movable end mirror and a fixed end mirror [Fig. 1(a)], in which the cavity field and the movable end

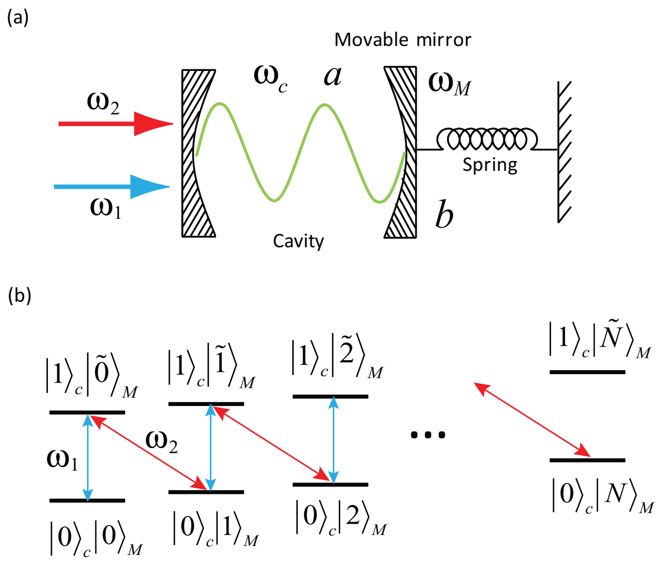

FIG. 1. (Color online) (a) Schematic diagram of an optomechanical system consisting of a fixed end mirror and a movable end mirror with two driving fields. (b) The coupling scheme between energy levels of the optomechanical system (only zero- and one-photon states are shown). Here each laser is used to establish a set of resonant transitions, and $|p\rangle_M = |\tilde{p}(0)\rangle_M$ and $|\tilde{p}\rangle_M = |\tilde{p}(1)\rangle_M$ for simplicity. By choosing $g = g_N$, there is no transition between $|0\rangle_c|N\rangle_M$ and $|1\rangle_c|\tilde{N}\rangle_M$.

mirror are coupled with each other via radiation pressure. The optomechanical cavity is driven by two lasers with frequencies ω_1 and ω_2. The Hamiltonian of the system is given by

$$\begin{aligned} H &= \omega_c a^\dagger a + \omega_M b^\dagger b - g a^\dagger a (b^\dagger + b) \\ &\quad + [(\Omega_1 e^{-i\omega_1 t} + \Omega_2 e^{-i\omega_2 t}) a^\dagger + \text{H.c.}], \end{aligned} \tag{1}$$

where a (b) and ω_c (ω_M) are, respectively, the annihilation operator and resonant frequency of the cavity field (mechanical) modes, and we have set $\hbar = 1$. The Ω_1 and Ω_2 are proportional to the amplitudes of the external fields. Single-photon coupling strength $g = \omega_c x_{zpf}/L$, where $x_{zpf} = \sqrt{\hbar/(2M\omega_M)}$ is the zero-point fluctuation of the mirror with mass M and L is the cavity rest length. In the frame rotating at the frequency ω_c, the transformed Hamiltonian reads

$$\begin{aligned} H_r &= \omega_M b^\dagger b - g a^\dagger a (b^\dagger + b) \\ &\quad + [(\Omega_1 e^{-i\Delta_1 t} + \Omega_2 e^{-i\Delta_2 t}) a^\dagger + \text{H.c.}], \end{aligned} \tag{2}$$

where the detunings $\Delta_1 = \omega_1 - \omega_c$ and $\Delta_2 = \omega_2 - \omega_c$ are defined.

The first two terms of H_r correspond to the Hamiltonian H_0 without driving, and it can be diagonalized by introducing a displaced oscillator basis as

$$H_0 = \omega_M b^\dagger b - g a^\dagger a (b^\dagger + b) = \sum_{n,p} \varepsilon_{n,p} |\psi_{n,p}\rangle\langle\psi_{n,p}|. \tag{3}$$

Here the states $|\psi_{n,p}\rangle$ are eigenvectors of H_0 given by

$$|\psi_{n,p}\rangle = |n\rangle_c \otimes D(ng/\omega_M)|p\rangle_M = |n\rangle_c \otimes |\tilde{p}(n)\rangle_M, \tag{4}$$

where $n(p)$ is the cavity photon (phonon) number, and $D(ng/\omega_M) = \exp[\frac{ng}{\omega_m}(b^\dagger - b)]$ is the displacement operator. Therefore $|\tilde{p}(n)\rangle_M$ denotes the n-photon displaced Fock state of the mirror. The energy eigenvalues of H_0 are $\varepsilon_{n,p} = p\omega_M - n^2 g^2/\omega_M$, which depend nonlinearly on photon number n and linearly on phonon number p.

By using the eigenbasis of H_0, H_r becomes

$$\begin{aligned} H_r &= \sum_{n,p} \varepsilon_{n,p} |\psi_{n,p}\rangle\langle\psi_{n,p}| + \sum_{n,p,p'} [A^{(n)}_{p,p'} (\Omega_1 e^{-i\Delta_1 t} \\ &\quad + \Omega_2 e^{-i\Delta_2 t}) |\psi_{n,p'}\rangle\langle\psi_{n-1,p}| + \text{H.c.}], \end{aligned} \tag{5}$$

where we have expressed the annihilation operator a in the eigenbasis as

$$\begin{aligned} a &= \sum_{n,p} \sum_{n',p'} |\psi_{n,p}\rangle\langle\psi_{n,p}|a|\psi_{n',p'}\rangle\langle\psi_{n',p'}| \\ &= \sum_{n,p,p'} A^{(n)}_{p,p'} |\psi_{n-1,p}\rangle\langle\psi_{n,p'}|, \end{aligned} \tag{6}$$

with the coefficients $A^{(n)}_{p,p'} = \sqrt{n}\langle p|D(g/\omega_M)|p'\rangle$. Specifically, the coefficients are given by [50]

$$A^{(n)}_{p,p'} = \begin{cases} \sqrt{n}\sqrt{\frac{p!}{p'!}} e^{-\frac{\xi^2}{2}} (-\xi)^{p'-p} L_p^{p'-p}(\xi^2), & p \leqslant p', \\ \sqrt{n}\sqrt{\frac{p'!}{p!}} e^{-\frac{\xi^2}{2}} (\xi)^{p-p'} L_{p'}^{p-p'}(\xi^2), & p > p', \end{cases} \tag{7}$$

where $\xi = g/\omega_M$ and $L_r^s(x)$ are associated Laguerre polynomials. For later discussions, it is convenient to go to the interaction picture in which the Hamiltonian is transformed as

$$\begin{aligned} H_r' &= e^{iH_0 t} H_r e^{-iH_0 t} - H_0 = \sum_{n,p,p'} \big[A^{(n)}_{p,p'} (\Omega_1 e^{i(\varepsilon_{n,p'} - \varepsilon_{n-1,p} - \Delta_1)t} \\ &\quad + \Omega_2 e^{i(\varepsilon_{n,p'} - \varepsilon_{n-1,p} - \Delta_2)t}) |\psi_{n,p'}\rangle\langle\psi_{n-1,p}| + \text{H.c.}\big], \end{aligned} \tag{8}$$

where H_0 is the Hamiltonian of the system without driving, as shown in Eq. (3).

III. EFFECTIVE HAMILTONIAN IN A CONFINED HILBERT SPACE

In this section, we show how the Hamiltonian can be reduced to a form that effectively operates in a finite-dimensional Hilbert space. The key idea is to utilize resonance transitions and the dependence of transition matrix elements on coupling strength g. First of all, assuming that the cavity-field decay rate κ is much smaller than ω_M (i.e., the resolved-sideband limit), we choose the frequencies ω_1 and ω_2 of the driving fields such that the detunings Δ_1 and Δ_2 satisfy the resonance conditions:

$$\Delta_1 = \varepsilon_{1,p} - \varepsilon_{0,p} = -g^2/\omega_M, \tag{9}$$

$$\Delta_2 = \varepsilon_{1,p} - \varepsilon_{0,p+1} = -\omega_M - g^2/\omega_M. \tag{10}$$

In this way, the Hamiltonian can be decomposed into two parts:

$$H_r' = \tilde{H}_r' + V, \tag{11}$$

where $\tilde{H}_r'$ describes the resonant transitions

$$\begin{aligned} \tilde{H}_r' &= \sum_p \Big[\big(A^{(1)}_{p,p} \Omega_1 |\psi_{1,p}\rangle\langle\psi_{0,p}| \\ &\quad + A^{(1)}_{p+1,p} \Omega_2 |\psi_{1,p}\rangle\langle\psi_{0,p+1}| \big) + \text{H.c.} \Big], \end{aligned} \tag{12}$$

while V describes the off-resonance transitions

$$V = {\sum_{n,p,p'}}' \big[A^{(n)}_{p,p'}(\Omega_1 e^{i\delta_1(n,p,p')t} + \Omega_2 e^{i\delta_2(n,p,p')t}) |\psi_{n,p'}\rangle\langle\psi_{n-1,p}| + \text{H.c.}\big]. \quad (13)$$

Here the primed summation in V excludes those terms in $\tilde{H}'_r$, i.e., $V = H'_r - \tilde{H}'_r$, and the off-resonance detunings are

$$\delta_1(n,p,p') = (p' - p)\omega_M - \frac{2g^2}{\omega_M}(n-1), \quad (14)$$

$$\delta_2(n,p,p') = (p' - p + 1)\omega_M - \frac{2g^2}{\omega_M}(n-1). \quad (15)$$

$\tilde{H}'_r$ describes resonant transitions between zero- and one-photon states, which are shown in Fig. 1(b). One can see that the laser with amplitude Ω_1 and frequency ω_1 can resonantly couple the states $|\psi_{0,p}\rangle$ and $|\psi_{1,p}\rangle$ (vertical arrows), while the other laser can resonantly couple the states $|\psi_{0,p+1}\rangle$ and $|\psi_{1,p}\rangle$. On the other hand, V describes off-resonance transitions and $\delta_j(n,p,p')$ $(j = 1,2)$ are understood as the corresponding detunings. If $\delta_j(n,p,p')$ are sufficiently large compared with the driving strengths Ω_j, then the transitions described by V can be neglected in the spirit of the rotating wave approximation. Consequently, if initially the cavity is in the vacuum state, then the system is confined to the zero- and single-photon states [Fig. 1(b)]. This can also be understood as a photon blockade effect [19], which is based on the fact that the excitation to two-photon space from one-photon states is far off-resonance.

For the reason explained above, we make the approximation $H_r \approx \tilde{H}'_r$. Specifically, in order to neglect V, we need to block the transitions from one-photon states to two-photon states, and this requires $\delta_j(2,p,p') \gg A^{(2)}_{p,p'}\Omega_j$. Since $A^{(2)}_{p,p'}$ is of order 1, the condition of neglecting V is

$$\Omega_i \ll |2g^2/\omega_M - K\omega_M|, \quad (16)$$

for $i = 1,2$, and here K is the nearest integer to $2(g/\omega_M)^2$. For example, if $g < \omega_M/2$ then $K = 0$.

Furthermore, the Hamiltonian can be truncated within phonon number N by setting the optomechanical coupling strength specifically at $g = g_N$, where g_N is the smallest positive value to have the coefficient vanish:

$$A^{(1)}_{N,N} = \exp\left(-\frac{g_N^2}{2\omega_M^2}\right) L^0_N\left(g_N^2/\omega_M^2\right) = 0 \quad (17)$$

for a given positive integer N. Notice that with coupling strength $g = g_N$, the transition matrix element $\Omega_1 A^{(1)}_{N,N}$ from the state $|\psi_{0,N}\rangle$ to $|\psi_{1,N}\rangle$ is exactly zero. Therefore the mirror's states are confined within phonon number N in the sense that they cannot be excited beyond the phonon number N. Note that $A^{(1)}_{N,N} = \langle N|D(g/\omega_M)|N\rangle$ is actually the overlap between a displaced number state $D(g/\omega_M)|N\rangle$ and an undisplaced number state $|N\rangle$, and the zero of this overlap is understood as a quantum interference effect. A more detailed discussion of properties of displaced number states, and particularly the interference in phase space related to the behavior of $A^{(1)}_{N,N}$, can be found in Ref. [50].

To illustrate how g_N scales with N, we plot in Fig. 2 the value of g_N as a function of N. We see that g_N decreases as N increases. In the large N limit, we find that g_N decreases as

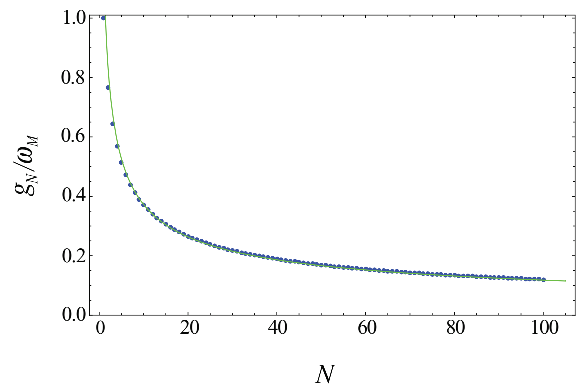

FIG. 2. (Color online) Exact solution of g_N satisfying Eq. (17) as a function of N (points) and the approximate solution based on Eq. (18) (solid line).

$N^{-1/2}$ according to the asymptotic behavior of the associated Laguerre polynomial. Specifically, we have

$$g_N \approx \frac{3\pi\omega_M}{8\sqrt{N}}, \quad (18)$$

which is plotted in the solid line as a comparison. Formula (18) provides a good approximation for $N > 5$.

Let us summarize the result in this section, we have obtained the effective Hamiltonian at $g = g_N$ under the conditions (9), (10), and (16):

$$H_{\text{eff}} = \sum_{p=0}^{N-1} \big[\big(A^{(1)}_{p,p}\Omega_1|\psi_{1,p}\rangle\langle\psi_{0,p}| + A^{(1)}_{p+1,p}\Omega_2|\psi_{1,p}\rangle\langle\psi_{0,p+1}|\big) + \text{H.c.}\big], \quad (19)$$

which governs the quantum dynamics in the zero- and one-photon subspace with phonon number $p \leqslant N$.

IV. DARK STATES

In this section, we put forward a class of dark states on the basis of the effective Hamiltonian with a finite dimension and then discuss some properties of the dark states. It can be shown that the effective Hamiltonian H_{eff} has an eigenvector $|D\rangle$ with a zero eigenvalue, i.e., $H_{\text{eff}}|D\rangle = 0$. The explicit form of this eigenvector is given by

$$|D\rangle = C\sum_{p=0}^{N}\beta_p|p\rangle_M \otimes |0\rangle_c, \quad (20)$$

where $\beta_0 = 1$, and for $1 \leqslant p \leqslant N$, the coefficients are

$$\beta_p = (-1)^p\left(\frac{\Omega_1}{\Omega_2}\right)^p \prod_{i=0}^{p-1}\frac{A^{(1)}_{i,i}}{A^{(1)}_{i+1,i}}, \quad (21)$$

and C is a normalization constant. Such an eigenvector is a coherent superposition of phonon number states with a vacuum cavity field. In such a state, the destructive interference fully forbids any excitation of the cavity field even though the cavity is constantly driven by the two external fields. Therefore it is a dark state of the optomechanical system induced by quantum coherence of the mirror. Note that, in the frame where the

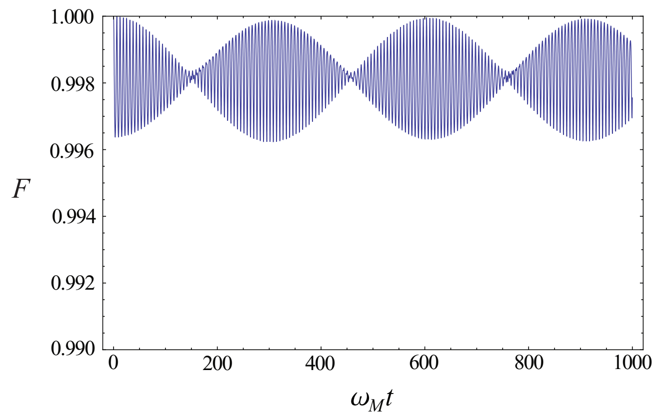

FIG. 3. (Color online) Overlap probability $F = |\langle D(t)|\Psi(t)\rangle|^2$ as a function of time. The initial state $|\Psi(0)\rangle$ is a dark state with $N = 10$, and the parameters are $g_N/\omega_M = 0.37$, $\Omega_1/\omega_M = 0.01$, $\Omega_2/\omega_M = 0.03$, $\Delta_1/\omega_M = -0.14$, and $\Delta_2/\omega_M = -1.14$.

system is governed by the Hamiltonian given in Eq. (2), the dark state is freely evolving as

$$|D(t)\rangle = e^{-iH_0 t}|D\rangle = C\sum_{p=0}^{N}\beta_p e^{-ip\omega_M t}|p\rangle_M \otimes |0\rangle_c \qquad (22)$$

under the approximations made in the last section.

As a numerical test of the validity of the dark state as well as the effective Hamiltonian, we solve the evolution of the system state $|\Psi(t)\rangle$ based on the Schrödinger equation with the Hamiltonian (2) without making the approximations in Sec. III, i.e., $H_r|\Psi(t)\rangle = i|\dot{\Psi}(t)\rangle$. Specifically we choose the initial state as a dark state $|\Psi(0)\rangle = |D(0)\rangle$ and calculate the fidelity $F = |\langle D(t)|\Psi(t)\rangle|^2$ between $|\Psi(t)\rangle$ and $|D(t)\rangle$. If the effective Hamiltonian H_{eff} is valid, then $|\Psi(t)\rangle$ should be well described by $|D(t)\rangle$, i.e., $F \approx 1$. We find that this is indeed the case when condition (16) is satisfied, and a numerical example is given in Fig. 3, where the fidelity $F > 0.99$ over a long period of time. Note that the off-resonant transitions, which are not included in H_{eff}, are responsible for the high frequencies pattern.

A. Phonon number statistics

The phonon number distribution of the dark states is complicated by the Laguerre functions in Eq. (21). While the details of $|\beta_p|^2$ require a numerical evaluation of Eq. (21), we find that $|\beta_p|^2$ is mainly controlled by the ratio of the strengths of driving fields. Such a feature is illustrated in Fig. 4(a) for various Ω_2/Ω_1. For example, when $\Omega_2/\Omega_1 = 3$, the probability decreases quickly with the increase of phonon number m. In the case $\Omega_2/\Omega_1 = 1$, there is a peak that appears in the probability distribution. If the ratio is further decreased to $\Omega_2/\Omega_1 = 1/3$, the peak is shifted towards higher phonon numbers.

To further study the statistical property of the distributions, we plot in Fig. 4(b) the ratio $\langle(\Delta n)^2\rangle/\langle n\rangle$ as a function of Ω_1/Ω_2, which shows that phonon number distributions of dark states exhibit a sub-Poissonian statistics. We see that $\langle(\Delta n)^2\rangle/\langle n\rangle$ decreases with Ω_1/Ω_2 and is always less than 1 (i.e., sub-Poisson distribution) except at the small region

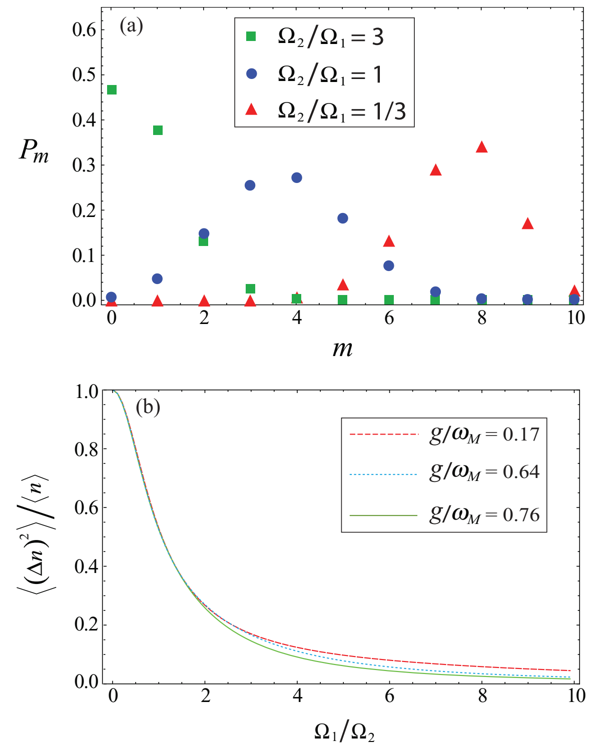

FIG. 4. (Color online) An illustration of the phonon statistics of dark states. (a) Phonon number probability distribution $P_m = C^2|\beta_m|^2$ of dark states for different ratios of driving strengths, with $g_N = 0.37\omega_M$ and $N = 10$. (b) $\langle(\Delta n)^2\rangle/\langle n\rangle$ as a function of Ω_1/Ω_2 for $g/\omega_M = 0.17$, 0.64, and 0.76 corresponding to $N = 20$, 3, and 2, respectively.

Ω_1/Ω_2 near zero. Besides, we also find that the curves are quite insensitive to different values of g_N.

B. Remarks on small deviations from g_N

The optomechanical coupling strength g_N has been used in order to obtain the effective Hamiltonian H_{eff} applicable to mirror states of phonon numbers in the range $0 \leqslant p \leqslant N$. When $g \neq g_N$, $A^{(1)}_{N,N}$ is nonzero, then the Hamiltonian cannot be exactly truncated. Let us consider the system with g slightly deviated from g_N. If the initial state is a dark state $|D\rangle$, then the system will make a transition to $|1\rangle_c|\tilde{N}\rangle_M$ by the driving field of frequency ω_1, and subsequent interactions with the driving fields could excite the mirror to phonon number states higher than N. However, we point out that since the transition rate from to $|1\rangle_c|\tilde{N}\rangle_M$ is proportional to $|\beta_N|^2 \times |A^{(1)}_{N,N}|^2$ according to first-order perturbation theory, the transition probability out of $|D\rangle$ would be negligible in a finite time duration as long as the product of $|\beta_N|^2$ and $|A^{(1)}_{N,N}|^2$ is sufficiently small. If such a condition is satisfied, then $|D\rangle$ may still be treated as a dark state approximately even though g is slightly deviated from g_N. Indeed, as we have illustrated the phonon number distribution of $|D\rangle$ in Fig. 4(a), $|\beta_N|^2$ can be highly suppressed by using $\Omega_2 > \Omega_1$, such that the dark state can still be valid with a small deviation of g from g_N.

V. PREPARATION OF DARK STATES BY CAVITY-FIELD DAMPING

In this section we discuss how the system can be optically pumped into the dark states by cavity-field damping. In the frame where the system Hamiltonian is given by Eq. (2), the evolution of the system under cavity-field damping is governed by the master equation

$$\frac{d\rho}{dt} = -i[H_r,\rho] - \frac{\kappa}{2}(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a), \tag{23}$$

where ρ is the density matrix of the photon-mirror system, and κ is the cavity-field decay rate. Here we do not include the damping of the mirror's motion in Eq. (23), and this is justified as long as the mechanical damping rate γ_M is much smaller than κ and the time interval is restricted to $t \ll 1/\gamma_M$. Specifically there is a time interval $\kappa^{-1} \ll t \ll \gamma_M^{-1}$ where the optical pumping is almost completed before the mechanical decoherence becomes significant. In the later part of this section, we discuss the effect of mechanical damping on the dark-state preparation.

It should be noted that if the approximation $H_r' \approx H_{\rm eff}$ is exact, then by going back to the original frame governed by the system Hamiltonian H_r in Eq. (2), $\rho = |D(t)\rangle\langle D(t)|$ is already a solution of the master equation, Eq. (23), because the cavity-field-damping term has no effect on $|D\rangle$ (which has zero photons). However, in order to study the evolution of the system without relying on the approximations made in Sec. III, we use H_r directly in the master equation.

The master equation, Eq. (23), is equivalent to a set of coupled ordinary differential equations, and they can be solved numerically by MATHEMATICA for an initial ground state of the system. To perform numerical calculations, we truncate the dimension of the density matrix, which is sufficiently larger than that of the dark-state density matrix. For the parameters used in the figures in this section, $\rho(t)$ appears to be converging when the photon and phonon number states are kept up to 2 and 15, respectively. Specifically, we are interested in the fidelity F defined by

$$F = \mathrm{Tr}[|D(t)\rangle\langle D(t)|\rho(t)], \tag{24}$$

which measures the probability of the system in the dark state. Some examples are given in Fig. 5 in which F increases with time and approaches a steady value close to 1 in a finite time. For the three cases shown in Fig. 5, the fidelities can reach $F \approx 0.99$ with $g_N/\omega_M = 0.37$ $(N = 10)$ at time $T \approx 8000\omega_M^{-1}$.

The increase of F is understood as a coherent population trapping effect [48], because when a photon leaks out of the cavity, the mirror can have a nonzero transition probability going to the dark state. Since the dark state is decoupled from the driving fields, it can no longer be excited, and hence the occupation of the dark state accumulates as time increases. However, we remark that during the optical pumping process, there is a loss due to the cavity-field decay that triggers the mirror to make a transition to phonon number states higher than N. Such a loss, which amounts to about 1% in Fig. 5, can be reduced if N is chosen to be large enough.

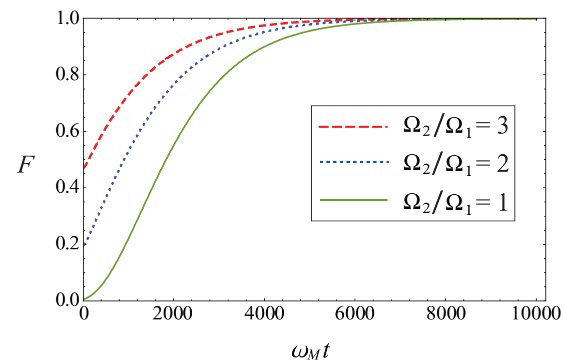

FIG. 5. (Color online) Time evolution of the fidelity F for systems operating at various Ω_2/Ω_1 ratios. The parameters are $g_N/\omega_M = 0.37$, $N = 10$, $\kappa/\omega_M = 0.05$, $\Omega_2/\omega_M = 0.01$, $\Delta_1/\omega_M = -0.14$, and $\Delta_2/\omega_M = -1.14$.

A. Sensitivity to g_N

In Sec. IV B, we have indicated that, if g is slightly deviated from g_N, then the quantity $|\beta_N|^2 \times |A_{N,N}^{(1)}|^2$ would characterize the degradation of $|D\rangle$ as a dark state. This is because the quantity is proportional to the leakage rate out of the state $|D\rangle$ when $g \neq g_N$. In addition, we have pointed out that the condition $\Omega_2 \geqslant \Omega_1$ is useful in order to make $|\beta_N|^2$ insignificant at phonon number N. This is relevant to the state generation process considered here, because once the system is optically pumped into the state $|D\rangle$, it will be decoupled from the driving field approximately as long as $|\beta_N|^2 \times |A_{N,N}^{(1)}|^2$ is small. With $\Omega_2 \geqslant \Omega_1$, we have tested numerically the sensitivity of the fidelity to small variations of g values. As illustrated in Fig. 6, the fidelities still reach about $F \approx 0.99$ in the presence of about 3% deviations from the g_N value.

B. Effects of mechanical damping

Generally speaking, the presence of mechanical damping will damage the coherence of the quantum state of the mirror. To quantitatively address the influence of mechanical damping on the preparation of dark states at a temperature, here we

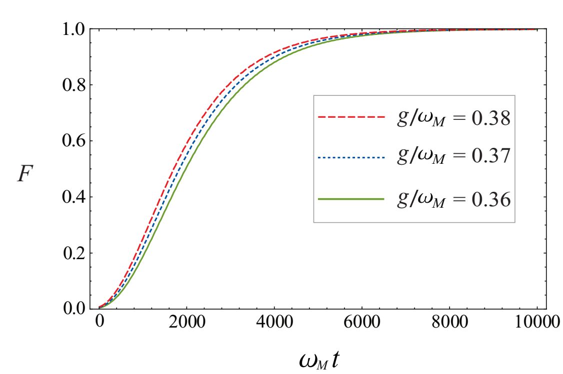

FIG. 6. (Color online) Fidelity F as a function of time for various g in the vicinity of $g_N = 0.37\omega_M$ corresponding to $N = 10$. The parameters are $\kappa/\omega_M = 0.05$, $\Omega_1 = \Omega_2 = 0.01\omega_M$, $\Delta_1/\omega_M = -0.14$, and $\Delta_2/\omega_M = -1.14$.

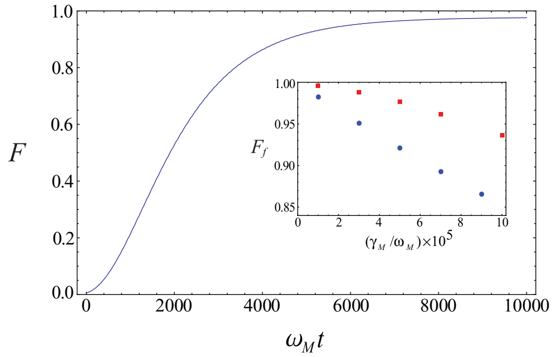

FIG. 7. (Color online) Fidelity F as a function of time for the system with a mechanical damping rate of $\gamma_M/\omega_M = 5 \times 10^{-5}$ and thermal phonon number $\bar{n}_M = 0$. The inset figure shows the final fidelity F_f (at $\omega_M t = 10^4$) for various γ_M, with thermal phonon numbers $\bar{n}_M = 0$ (red squares) and $\bar{n}_M = 1$ (blue circles). The parameters are $g/\omega_M = 0.37$, $\kappa/\omega_M = 0.05$, $\Omega_1 = \Omega_2 = 0.01\omega_M$, $\Delta_1/\omega_M = -0.14$, and $\Delta_2/\omega_M = -1.14$.

include the mechanical damping into the master equation:

$$\begin{aligned}\frac{d\rho}{dt} &= -i[H_r,\rho] - \frac{\kappa}{2}(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) \\ &\quad - \frac{\gamma_M(\bar{n}_M+1)}{2}(b^\dagger b\rho - 2b\rho b^\dagger + \rho b^\dagger b) \\ &\quad - \frac{\gamma_M \bar{n}_M}{2}(bb^\dagger\rho - 2b^\dagger\rho b + \rho bb^\dagger), \qquad (25)\end{aligned}$$

where γ_M is the mechanical damping rate, $\bar{n}_M$ is the average phonon number at thermal equilibrium, and we have assumed that the damping is due to coupling to a Markovian bath.

By solving Eq. (25) numerically, we plot in Fig. 7 the evolution of fidelity with a mechanical damping rate of $\gamma_M/\omega_M = 5 \times 10^{-5}$, while the other parameters and the size of the Hilbert space are kept identical to the case of the solid green curve in Fig. 5. It is shown that the fidelity grows to about 0.97 finally, which is slightly lower than that of Fig. 5. This numerical result indicates that a mechanical damping at this magnitude has a small influence on the preparation of dark states. In addition, we provide the dependence between final dark-state fidelity and mechanical damping rate for thermal phonon numbers $\bar{n}_M = 0$ and $\bar{n}_M = 1$ in the inset figure, which shows that the final fidelity declines as the mechanical damping rate increases and the average thermal phonon number increases. From these results, we see that a mechanical damping rate of $\gamma_M/\omega_M < 10^{-4}$ is essential to the successful preparation of the dark state at the time scale of $\omega_M t = 10^4$. In other words, the lifetime of the phonons should be longer than the preparation time of the dark states.

VI. CONCLUSION

To conclude, we have studied theoretically a quantum interference effect arising from mechanical coherence in cavity optomechanics. Specifically, we have discovered a class of dark states of a moving mirror that make the cavity field decoupled from two external driving fields under the conditions (9), (10), and (16) in the resolved-sideband limit. For dark states involving a low number of phonons (e.g., $N < 20$), g has to be comparable to the mechanical frequency ω_M, and hence the system operating in the single-photon strong-coupling regime is required. In this paper we provide an analytical expression of the dark states, which indicates the dependence of phonon number distribution on the ratio of the driving amplitudes Ω_2/Ω_1 and the optomechanical coupling strength. In particular, we found that the phonon number distributions of the dark states exhibit a sub-Poissonian statistics. Finally, we indicate that dark states can be prepared with high fidelity by optical pumping due to cavity-field damping as long as the lifetime of phonons is sufficiently longer than the preparation time, and our numerical simulations suggest that the scheme is insensitive to small deviations of g values if $\Omega_2 \geqslant \Omega_1$.

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