Spontaneous emission from a medium with elliptic and hyperbolic dispersion

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(Received 6 February 2013; published 28 May 2013)

We analyze the properties of spontaneous emission of a two-level atom in anisotropic media where the geometry of physical dispersion relations is characterized by an ellipsoid or a hyperboloid. Within the framework of quantum optics the rate of spontaneous emission in the above media is explicitly given with the orientation of the dipole transition matrix element taken into account. It indicates that for the ellipsoid case the intensity of the photons coupled into different modes can be tuned by changing the direction of the matrix element relative to the optical axis. For the hyperboloid case it is found that spontaneous emission in the hyperbolic medium (HM) can be dramatically enhanced compared with the case in the background medium. Moreover, in the HM the spontaneous emission exhibits strong directivity and gets the maximum in the asymptote direction.

DOI: 10.1103/PhysRevA.87.053836 PACS number(s): 42.25.Gy, 78.20.Ci, 41.20.Jb

I. INTRODUCTION

There have been many works devoted to the quantization of the electromagnetic field and the spontaneous emission of the atom in various media which may be inhomogeneous, nonlinear, absorptive, anisotropic, or dispersive [1–11]. Many of these works are especially concerned with the anisotropy of the media besides some additional properties such as bianisotropic, inhomogeneous, and dispersive [7–11]. When the diagonal elements of permittivity tensor "$\overleftrightarrow{\epsilon}$" are all positive the geometry of the physical dispersion relation is an ellipsoid [three-dimensional (3D)] or an ellipse [two-dimensional (2D)], e.g., the uniaxial anisotropic media. However, the rate of spontaneous emission of the atom in the media with elliptic dispersion has not been given in detail with two factors taken into account. The first is the orientation of the dipole matrix element vector relative to the optical axis and the second is the difference between the contributions to the decay rate from the different permitted modes. On the other hand, metamaterials which are composed of the periodic dielectric element arrays, e.g., the split-ring resonators (SRRs), are realized experimentally in a narrow microwave frequency region [12,13] or in the near-visible light region [14]. The materials have the left-handed property because of negative permittivity and permeability simultaneously. It causes refocusing and phase compensation [15], which provide a new manipulating space for the design of quantum optical devices [16–18]. In the sense of the effective medium, metamaterials are also anisotropic as well as left handed [19–21]. When some diagonal elements of the effective permittivity tensor are negative the geometry of the dispersion relation in metamaterials is a hyperboloid (3D) or a hyperbola (2D), which are called hyperbolic metamaterials (HMM), as shown in the inset of Fig. 1. Note that for the HMM hyperbolic dispersion only holds for the wave vectors k in a small finite range due to the origin of the effective medium. We define the hyperbolic medium (HM) as the medium where hyperbolic dispersion holds for the full wave vector, which is used as the ideal model for the HMM. The HMM have been recently found to have many novel properties such as the superlens effect [22–24], broadband thermal emission beyond the blackbody limit in the near field [25,26], the slow-light effect [27], the "big flash" of the photons in the HMM by an optical metric signature phase transition [28], or an optical topological transition [29]. Moreover, the HMM provide a new model for the study of the early universe as an analog of vacuum in the strong magnetic field [30]. Besides the properties and applications in the framework of classical electromagnetic theory, some interesting quantum optical properties (QOPs) of the HMM have also been found experimentally [31] and theoretically [32,33], for instance, controlling spontaneous emission with the HMM [31], the broadband Purcell effect [32], and the dipole radiation and its enhancement near the surface of the HMM [34]. However, the above theoretical analyses of spontaneous emission in the HMM are within the framework of the macroscopic electromagnetic wave theory where the atomic emission from the quantum transition is approximated as a dipole radiation [32,34,35] and the radiated power is exacted from the Green's function of the system. The rate of spontaneous emission for a two-level atom in the anisotropic medium with elliptic and hyperbolic dispersion has not been provided in detail within the framework of the quantum optics. In this paper we will explicitly give the expression of the decay rate for a two-level atom in the foregoing media under the Weisskopf-Wigner approximation. It indicates that for the ellipsoid case the intensity of the photons coupled into the different modes can be tuned by changing the direction of the matrix element vector. For the hyperboloid case it is found that the spontaneous emission in the HM can be dramatically enhanced in comparison with the dielectric background; meanwhile, the spontaneous emission exhibits strong directivity and gets the maximum in the asymptote direction.

II. MODEL AND FORMULA

To deal with the problem some theories of the quantum optics of dielectric media are needed. There are many schemes

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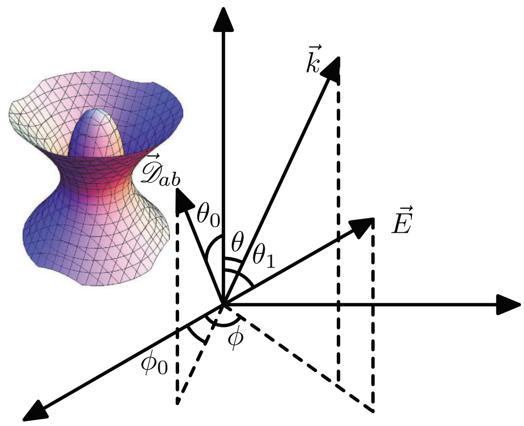

FIG. 1. (Color online) Schematic diagram of the three vectors $\vec{\mathscr{D}}_{ab},\vec{k},\vec{E}$. Inset: The geometries of the dispersion relation in hyperbolic and elliptic mediums.

of the electromagnetic quantization for the different types of media. One of them is that the physical quantities characterizing the medium such as polarization field, magnetization field, and their combinations are involved in the quantization procedure where the interaction between their quanta and the photon is taken into account [1–3]. Another scheme is that the permittivity and permeability tensor are regarded as the parameters in the field equation with different gauges (gauge conditions) for the different media [4,5,8]. In our case the medium is considered as anisotropic, homogeneous, nondispersive, and lossless for simplicity. The quantization scheme in Ref. [8] is used where the medium is characterized by the constitutive equations $\mathbf{D}(r) = \overset{\leftrightarrow(1)}{\epsilon}(r)\cdot\mathbf{E}(r) + \overset{\leftrightarrow(2)}{\epsilon}(r)\cdot\mathbf{B}(r)$; $\mathbf{H}(r) = \overset{\leftrightarrow(1)}{\mu}(r)\cdot\mathbf{E}(r) + \overset{\leftrightarrow(2)}{\mu}(r)\cdot\mathbf{B}(r)$. For our case $\overset{\leftrightarrow(2)}{\epsilon}(r) = \overset{\leftrightarrow(1)}{\mu}(r) = 0$; $\overset{\leftrightarrow(2)}{\mu}(r) = 1$ and

$$\overset{\leftrightarrow(1)}{\epsilon}(r) \equiv \overset{\leftrightarrow}{\epsilon} = \epsilon_0\begin{pmatrix} \epsilon_t & 0 & 0\\ 0 & \epsilon_t & 0\\ 0 & 0 & \epsilon_L\end{pmatrix}, \tag{1}$$

where $\epsilon_L > 0$ and $\epsilon_t > 0$ for the uniaxial anisotropic medium or $\epsilon_L > 0$ and $\epsilon_t < 0$ for the HM. According to Maxwell's equations the dispersion relation for the medium characterized by Eq. (1) in the principal axis coordinate system is expressed as

$$\frac{k_z^2}{\epsilon_t} + \frac{k_x^2+k_y^2}{\epsilon_L} = \left(\frac{\omega}{c}\right)^2. \tag{2}$$

Equation (2) describes a hyperboloid or an ellipsoid, which depends on the sign of ϵ_t with $\epsilon_L > 0$ assumed. After the photon is emitted from an atom it is subsequently coupled into the field modes permitted by the dielectric environment. In the dielectric system the photon propagates in the fashion of the classical field modes of the system and inversely interacts with the atom. In the spirit of Einstein's original model the total energy of the electromagnetic field mode in the dielectric should be $\hbar\omega_k$ [36], which can be used to determine the field amplitude included in the factor g_k. According to Ref. [8] the eigenvector field in the dielectric should satisfy the following gauge condition and the eigenequations:

$$\vec{\nabla}\cdot[\overset{\leftrightarrow}{\epsilon}(r)\cdot\vec{F}_k(r)] = 0, \tag{3}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{F}_k(r) = \left(\frac{\omega_k}{c}\right)^2\overset{\leftrightarrow}{\epsilon}(r)\cdot\vec{F}_k(r). \tag{4}$$

The quantized electric field can be expressed as $\hat{\vec{E}}(r,t) = \sum_k \vec{e}_k F_k(r)e^{-i\omega_k t}\hat{a}_k + \text{H.c.}$ and the total Hamiltonian of the photon and the atom under the rotating-wave approximation is

$$\hat{\mathscr{H}} = \sum_k \hbar\omega_k\hat{a}_k^\dagger\hat{a}_k + \frac{1}{2}\hbar\nu\hat{\sigma}_z + \hbar\sum_k g_k(\hat{\sigma}_+\hat{a}_k + \hat{\sigma}_-\hat{a}_k^\dagger), \tag{5}$$

where $\hat{\sigma}_z = |a\rangle\langle a| - |b\rangle\langle b|$, $\hat{\sigma}_+ = |a\rangle\langle b|$, $\hat{\sigma}_- = |b\rangle\langle a|$, and $|a\rangle,|b\rangle$ are the excited and ground states of the atom with eigenvalues E_a,E_b and

$$g_k = -\frac{\vec{\mathscr{D}}_{ab}\cdot\vec{e}_k F_k(0)}{\hbar}. \tag{6}$$

$\vec{\mathscr{D}}_{ab}$ is the matrix element of the atom's dipole between states $\vec{\mathscr{D}}_{ab} \equiv -\langle a|e\hat{\vec{R}}|b\rangle = -\langle b|e\hat{\vec{R}}|a\rangle$.

For simplicity the Hamiltonian equation (5) can be expressed as in the interaction [37] picture

$$\mathscr{H}_I = \hbar\sum_k\left[g_k^*\hat{\sigma}_+\hat{a}_k e^{i[\omega_0-\omega(\vec{k})]t} + g_k\hat{\sigma}_-\hat{a}_k^\dagger e^{-i[\omega_0-\omega(\vec{k})]t}\right], \tag{7}$$

where $E_a - E_b = \hbar\omega_0$. The state vector of the composite system of the photon and the atom is expressed as $|\psi(t)\rangle = c_a(t)|a,0\rangle + \sum_{\vec{k},s} c_{b,\vec{k}}(t)|b,1_{\vec{k}}\rangle$, where subscript s denotes the different modes for a fixed k, e.g., the extraordinary wave and ordinary wave. Solving the equation of the motion $i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = \mathscr{H}_I|\psi(t)\rangle$ we get $\frac{dc_a}{dt} = -\sum_{\vec{k}s}|g_k|^2\int_0^t dt'\, e^{i[\omega_0-\omega(\vec{k})](t-t')}c_a(t')$. Change the summation to the integration in spherical coordinate systems:

$$\begin{aligned}\frac{dc_a}{dt} &= -\frac{V}{(2\pi)^3}\sum_s\int |g_k|^2k^2\sin\theta\, e^{i[\omega_0-\omega(\vec{k})](t-t')}c_a(t')\\ &\quad\times d\theta\, d\phi\, dk\, dt'\\ &= -\frac{V}{(2\pi)^3}\sum_s\int |g_k|^2k^2\sin\theta\,\frac{\partial\omega}{\partial k}^{-1}e^{i[\omega_0-\omega(\vec{k})](t-t')}\\ &\quad\times c_a(t')d\theta\, d\phi\, d\omega\, dt'\\ &= -\frac{V}{(2\pi)^3}\sum_s\int G(\omega,\theta,\phi)e^{i[\omega_0-\omega(\vec{k})](t-t')}\\ &\quad\times c_a(t')d\theta\, d\phi\, d\omega\, dt',\end{aligned} \tag{8}$$

where the coordinate transformation in the second line is $\omega = \omega(k,\theta,\phi)$, $\theta' = \theta$, $\phi' = \phi$, $t' = t'$, and the Jacobian $J = \frac{\partial(\omega,\theta',\phi',t')}{\partial(k,\theta,\phi,t')} = \frac{\partial\omega}{\partial k}$. It is defined $G(\omega,\theta,\phi) \equiv |g_k|^2k^2\sin\theta\frac{\partial\omega}{\partial k}^{-1}$. In the Weisskopf-Wigner approximation $G(\omega,\theta,\phi)$ is substituted by $G(\omega_0,\theta,\phi)$ and the decay rate of the atom can be expressed as the following integral:

$$\Gamma = \frac{V}{2\pi^2}\int G(\omega_0,\theta,\phi)d\theta\, d\phi. \tag{9}$$

Here the eigenmode of electric field components is assumed as $\vec{E}_k(r) = \vec{F}_k(r)$, $\vec{H}_k = \frac{1}{i\mu_0\omega}\vec{\nabla}\times\vec{E}_k$. The consideration that a single photon is coupled into the classical eigenmode of the medium requires the total energy of the mode $U = \frac{1}{2}\int_V(\vec{E}_k\cdot\overset{\leftrightarrow}{\epsilon}\cdot\vec{E}_k + \mu_0|\vec{H}_k|^2)d^3x = \hbar\omega$, which determines the amplitude $F_k(0)$ under the box normalization condition.

A. Spontaneous decay rate in the medium with dispersion geometry of ellipsoid

In order to check the validity of the formulism, the case $\epsilon_t > 0$ is first considered where the medium becomes uniaxial anisotropic with an ellipsoidal dispersive geometry. In the medium there are usually two types of eigenmodes, called extraordinary waves and ordinary waves, which accommodate the emitted photons from the atom. To this end we have to explore the amplitudes and the energy of the two modes in detail. The eigenmode of the electric field component is supposed as the plane wave $\vec{E}_k(r) = \vec{E}_{k0} e^{i\vec{k}\cdot\vec{r}} e^{i\omega(\vec{k})t}$ and $\vec{H}_k(r) = \vec{H}_{k0} e^{i\vec{k}\cdot\vec{r}} e^{i\omega(\vec{k})t}$, where the complex vector amplitudes are to be determined. Substituting the expression into the Maxwell equations we get the two sets of dispersion and polarization relations

$$k = \frac{\omega}{c}\sqrt{\epsilon_t} \tag{10}$$

with $E_z = 0$ and $k_x E_{k0x} + k_y E_{k0y} = 0$, and

$$\frac{k_x^2 + k_y^2}{\epsilon_L} + \frac{k_z^2}{\epsilon_t} = \left(\frac{\omega}{c}\right)^2 \tag{11}$$

with $H_z = 0$ and $\frac{E_{k0x}}{E_{k0y}} = \frac{k_x}{k_y}$.

In the spherical coordinate system the dispersion relations can be uniformly written as $\omega(k) = f(\theta)ck$, where $f(\theta) = \frac{1}{\sqrt{\epsilon_t}}$ for the transversal mode, Eq. (10), and $f(\theta) = \sqrt{\frac{\sin^2\theta}{\epsilon_L} + \frac{\cos^2\theta}{\epsilon_t}}$ for the longitudinal mode, Eq. (11). For the transverse mode the total energy and the corresponding amplitude are, respectively,

$$U_T = \epsilon_0 \left(E_{k0}^T\right)^2 \epsilon_t V = \hbar\omega_0/2, \quad E_{k0}^T = \sqrt{\frac{\hbar\omega_0}{2V\epsilon_0\epsilon_t}}. \tag{12}$$

For the longitudinal mode the corresponding quantities are

$$\begin{aligned} U_L &= \frac{V\epsilon_0^2\mu_0\omega_0^2\left(E_{k0}^L\right)^2\epsilon_L^2\epsilon_t^2}{k_z^2\epsilon_L^2 + \epsilon_t^2\left(k_x^2 + k_y^2\right)} = \omega_0\hbar/2, \\ E_{k0}^L &= \left[\frac{\omega_0\hbar\left[\cos(2\theta)\left(\epsilon_L^2 - \epsilon_t^2\right) + \epsilon_L^2 + \epsilon_t^2\right]}{4f^2V\epsilon_0\epsilon_L^2\epsilon_t^2}\right]^{1/2}, \end{aligned} \tag{13}$$

where the factor $\omega_0\hbar/2$ in right-hand side of the equation occurs because there are two modes averagely carrying the energy of the photon. In the principal axis system of the tensor $\overleftrightarrow{\epsilon}$ there are three vectors which should be distinguished. They are the vectorial transition matrix element $\vec{\mathscr{D}}_{ab} = (\mathscr{D}_{ab},\theta_0,\phi_0)$, the electric field vector $\vec{E}_{k0} = (E_{k0},\theta_1,\phi_1)$, and the wave vector $\vec{k} = (k,\theta,\phi)$, which is shown in Fig. 1. The factor in Eq. (9) is explicitly given:

$$|g_k|^2 = \frac{\mathscr{D}_{ab}^2 E_{k0}^2 \cos^2\theta_{0,1}}{\hbar^2}, \tag{14}$$

where $\theta_{0,1}$ is the angle between $\vec{\mathscr{D}}_{ab}$ and $\vec{E}_{k0}$. According to the geometrical relations of the vectors $\vec{\mathscr{D}}_{ab}$ and $\vec{E}_{k0}$ and the gauge condition $\vec{k}\cdot\overleftrightarrow{\epsilon}\cdot\vec{E}_{k0} = 0$, we get the equations $\cos\theta_{0,1} = \sin\theta_0\sin\theta_1\cos(\phi_0 - \phi_1) + \cos\theta_0\cos\theta_1$ and $\epsilon_L\cos\theta_1\cos\theta + \epsilon_t\sin\theta_1\sin\theta\cos(\phi_1 - \phi) = 0$. It is noted that $\phi_1 = \phi + \frac{\pi}{2}$ for the transversal mode and $\phi_1 = \phi$ for the longitudinal mode. With these conditions we get the factor $\cos^2\theta_{0,1} \equiv c_q^{T(L)}$ for the different modes

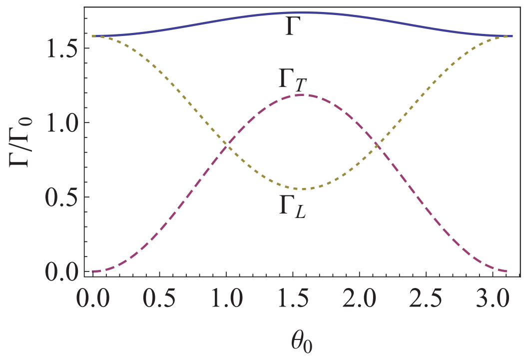

FIG. 2. (Color online) The decay rate of the atom in the medium with $\epsilon_t = 2.5$, $\epsilon_L = 3.5$ relative to the case for the vacuum $\tilde{\Gamma} = \Gamma/\Gamma_0$. Dotted line: the decay rate for the transverse mode. Dashed line: the decay rate for the longitudinal mode. Solid line: the total decay rate.

$$c_q^T = \sin^2\theta_0\sin^2(\phi_0 - \phi), \tag{15}$$

$$c_q^L = \frac{[\epsilon_t\cos\theta_0 - \epsilon_L\sin\theta_0\cot\theta\cos(\phi_0 - \phi)]^2}{\epsilon_L^2\cot^2\theta + \epsilon_t^2}. \tag{16}$$

After the implementation of Eq. (9) we get the decay rate for the two modes:

$$\Gamma^T = \frac{3\sqrt{\epsilon_t}\sin^2\theta_0}{4}\Gamma_0, \tag{17a}$$

$$\Gamma^L = \frac{\epsilon_L + 4\epsilon_t - \cos(2\theta_0)(\epsilon_L - 4\epsilon_t)}{8\sqrt{\epsilon_t}}\Gamma_0, \tag{17b}$$

$$\Gamma = \frac{\cos(2\theta_0)(\epsilon_t - \epsilon_L) + \epsilon_L + 7\epsilon_t}{8\sqrt{\epsilon_t}}\Gamma_0, \tag{17c}$$

where $\Gamma_0 = \frac{\mathscr{D}_{ab}^2\omega_0^3}{3\pi c^3\epsilon_0\hbar}$. When $\epsilon_L = \epsilon_t > 0$, Eq. (17c) reduces as $\sqrt{\epsilon_t}\Gamma_0 = \Gamma(\epsilon_t)$, which gives the decay rate of the atom in the homogeneous isotropic medium with permittivity ϵ_t. The decay rate relative to the vacuum $\Gamma/\Gamma_0 \equiv \tilde{\Gamma}$ in the case $\epsilon_t = 2.5$, $\epsilon_L = 3.5$ is shown in Fig. 2 for the different modes. From Fig. 2 it follows that when the matrix element vector $\vec{\mathscr{D}}_{ab}$ is parallel to the optical axis ($\theta_0 = 0$) the maximal coupling between the atom and the longitudinal mode of the system where $E_z \neq 0$ can be obtained, and many more photons emitted from the atom are coupled into the mode. Meanwhile, the transverse mode gets the no coupling with $\vec{\mathscr{D}}_{ab}$ for $E_z = 0$ and $\tilde{\Gamma}_T = 0$. In this case, $\tilde{\Gamma} = \tilde{\Gamma}_L = 1.58 = \sqrt{\epsilon_t}$, the anisotropic medium behaves as the isotropic medium with permittivity ϵ_t for the atom's spontaneous emission. With the increase of θ_0, $\tilde{\Gamma}_T$ increases due to the enhancement of the coupling with the transverse mode and $\tilde{\Gamma}_L$ decreases due to the reduction of the coupling with the longitudinal mode. When $\theta_0 = \frac{\pi}{2}$ where $\vec{\mathscr{D}}_{ab}$ is perpendicular to the optical axis $\tilde{\Gamma}_T$ gets the maximum and $\tilde{\Gamma}_L$ gets the minimum. According to Eq. (17c), Γ gets the maximum ($\epsilon_t < \epsilon_L$) or the minimum ($\epsilon_t > \epsilon_L$): $\Gamma_m = \frac{\mathscr{D}_{ab}^2\omega_0^3(2\epsilon_L + 6\epsilon_t)}{24\pi c^3\epsilon_0\hbar\sqrt{\epsilon_t}}$ at $\theta_0 = \frac{\pi}{2}$. These results can be used to control the intensity of the different modes from the spontaneous emission by tuning θ_0.

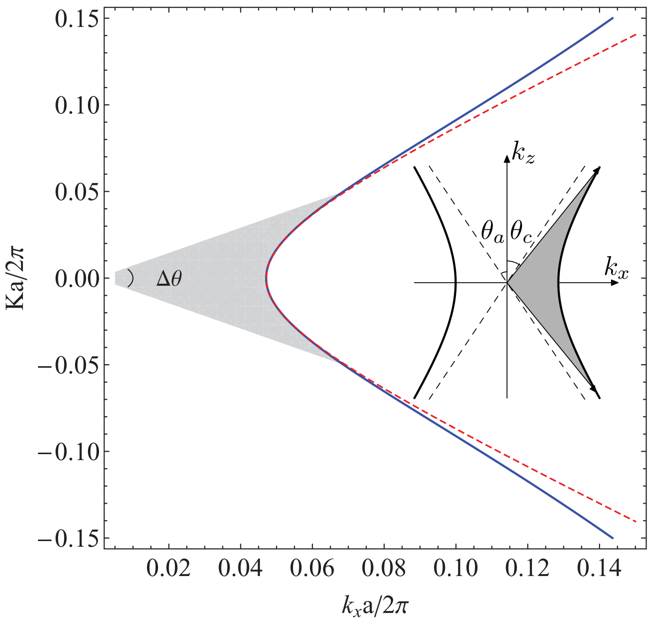

FIG. 3. (Color online) Thick solid line: the equifrequency contour for the one-dimensional periodic system with parameters $\epsilon_1 = -92.4$, $\epsilon_2 = 2.34$, $d_1 = 0.05a$, $d_2 = 0.95a$, $\omega = 0.03\frac{2\pi c}{a}$, where a is the lattice constant. Dashed line: the perfect hyperbolic dispersion curve in the long wavelength limit $\frac{k_x^2}{\epsilon_L(0)} + \frac{k_z^2}{\epsilon_t(0)} = (\frac{\omega}{c})^2$. Inset: The schematic diagram of the integration range for the cutoff in Eq. (18) with upper and lower limits $\pm x_c (= \pm\cos\theta_c)$

B. Spontaneous decay rate in the medium with dispersion geometry of hyperboloid

When $\epsilon_t < 0$, $\epsilon_L > 0$, Eq. (1) indicates the hyperboloid geometry of the dispersion relation. Due to $\epsilon_t < 0$, the branch $k = \frac{\omega}{c}\sqrt{\epsilon_t}$ corresponds to the evanescent wave. The mode cannot carry the energy away from the atom. The contribution to the decay rate from the branch $k = \frac{\omega}{c}\sqrt{\epsilon_t}$ is ignored. For the branch $\frac{k_x^2+k_y^2}{\epsilon_L} + \frac{k_z^2}{\epsilon_t} = (\frac{\omega}{c})^2$ the decay rate Γ is proportional to the following expression:

$$\Gamma = \beta\int_{-1}^{1} dx \frac{x^2\epsilon_L^2\sin^2\theta_0 - 2(x^2-1)\epsilon_t^2\cos^2\theta_0}{(x^2-\epsilon_u)^{5/2}}, \tag{18}$$

where $x = \cos\theta$, $\epsilon_u = \frac{\epsilon_t}{\epsilon_t - \epsilon_L}$, and $\beta = \frac{D^2\omega^3(\frac{\epsilon_L\epsilon_t}{\epsilon_L-\epsilon_t})^{5/2}}{8\pi c^3\epsilon_0\hbar\epsilon_L^2\epsilon_t^2}$.

It is noted that under the Weisskopf-Wigner approximation the integration Eq. (9) is actually calculated on the equal frequency surface. For an ellipsoid the variable θ in Eq. (9) integrates over the range $[0,\pi]$, while for the hyperboloid θ integrates over the range $[\theta_a, \pi - \theta_a]$, as shown in the inset of Fig. 3 where θ_a is the polar angle of the asymptote. In terms of Eq. (18) there are two poles $\pm\sqrt{\epsilon_u}$, of which the position on the axis depends on the parameters ϵ_t, ϵ_L. When $\epsilon_t > \epsilon_L > 0$, $\epsilon_u > 1$ and $\epsilon_L > \epsilon_t > 0$, $\epsilon_u < 0$ the poles $\pm\sqrt{\epsilon_u}$ are out of the range $[-1,1]$ or on the imaginary axis. This fact enables Eq. (18) to be calculated out and the result is given in Eq. (17). However, in the hyperboloid case where $\epsilon_t < 0$, $\epsilon_L > 0$, $0 < \epsilon_u < 1$ the poles lie in the range $[-1,1]$ which cause the integral to diverge. This divergence is the manifestation of the change of the topology from the ellipsoid to the hyperboloid [29]. To this point, we can also understand the mechanism from the point of view of the density of states. The spontaneous emission process is the transition of the combined atom and field system from the initial state $|\psi_i\rangle = |a\rangle \bigotimes |0\rangle$ with energy level $E_i = E_a$ into a continuum of final states $|\psi_f\rangle = |b\rangle \bigotimes |1_k\rangle$ with energy level $E_f = E_b + \hbar\omega_0$. Because the final energy level E_f is highly degenerate and the conservation of probability $|c_a|^2 = e^{-\Gamma t} = 1 - \sum_k |c_{b,k}|^2$, the decay rate Γ can be approximately represented by $\Gamma \propto \ln[1 - tP_{if}\rho(E_f)]^{-1}$, where $P_{if} = |\langle\psi_f|\mathscr{H}_I|\psi_i\rangle|^2 (\approx |c_{b,k}|^2)$ is the transition probability between the states $|\psi_i\rangle$ and $|\psi_f\rangle$. For the homogeneous medium with the dispersion relation $\omega = \omega(\vec{k})$ the photon density of states is obtained by a surface (line) integral on an isofrequency surface (contour) for the three- (two-) dimensional case. $\omega(\vec{k}) = \omega_0$: $\rho(\omega_0) \propto \int \frac{dS(dl)}{|\nabla_{\vec{k}}\omega(\vec{k})|}$. Due to the different topological characteristics from the ellipsoid (ellipse) the integral $\rho(\omega_0)$ diverges for the hyperbolic geometry, which leads to the divergence of Γ_H. It also indicates that the medium in which the dispersion relation geometry is characterized by a hyperboloid is the perfect approximation for some composite materials such as photonic crystals, metamaterials in the long-wavelength limit. Therefore, some kinds of cutoff have to be introduced for the calculation of Eq. (18) in the case $\epsilon_t < 0$, $\epsilon_L > 0$. To this end it is defined that $x_a = \cos\theta_a = \sqrt{\epsilon_u}$ and $\cos\theta_c = x_c \equiv \alpha x_a$, where $\pm x_c$ are the new integral limits for Eq. (18) with condition $x_c < x_a$. The integral range of the variable θ for the cutoff limits $\pm x_c$ is marked by the shadowed region in Fig. 3. When $\alpha = 1$, the integral limit approaches the two polar poles $\pm x_a$. Under the cutoff approximation we get the decay rate Γ for the hyperbolic medium:

$$\Gamma_H = \Gamma_0 \frac{\alpha^3\left[\epsilon_L^3 - \epsilon_L\cos^2\theta_0\left(-4\epsilon_L\epsilon_t + \epsilon_L^2 + 6\epsilon_t^2\right)\right] - 6\alpha\epsilon_L\epsilon_t\cos^2\theta_0(\epsilon_L - \epsilon_t)}{4(\alpha^2-1)^{3/2}(\epsilon_L - \epsilon_t)^2\sqrt{\frac{\epsilon_L\epsilon_t}{\epsilon_L-\epsilon_t}}}. \tag{19}$$

In Fig. 4 we give the $\tilde{\Gamma}_H = \Gamma/\Gamma_0$ vs the parameter α for the different θ_0 with $\epsilon_t = -2.5$, $\epsilon_L = 3.5$. It is obviously found that when θ_c approaches $\theta_a(\alpha \to 1)$ $\tilde{\Gamma}_H$ increases monotonously; in particular, $\tilde{\Gamma}_H$ increases more sharply for the α in the neighborhood of unit. This is because the larger α, the more modes with high k vectors are involved in the spontaneous emission. In the HM the modes with large k usually have a high density of states and more photons emitted from the atom can be accommodated by these modes. Moreover, since the photons are mainly coupled into the longitudial mode, $\tilde{\Gamma}_H$ increases when the matrix element vector $\vec{\mathscr{D}}_{ab}$ gets the small angle θ_0 with optical

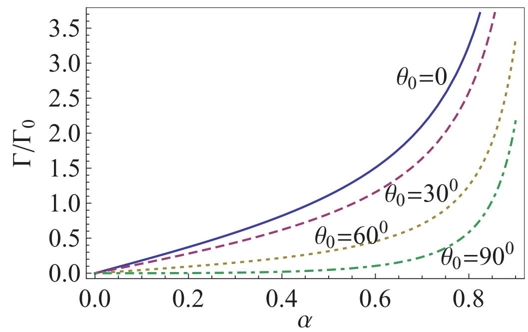

FIG. 4. (Color online) The relative decay rate of the atom in the medium with $\epsilon_t = -2.5$, $\epsilon_L = 3.5$ relative to the case for the vacuum according to parameter α for different angles θ_0.

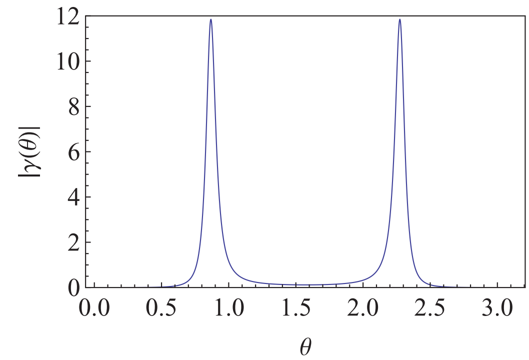

FIG. 5. (Color online) The amplitude of the integrand $\gamma(\theta)$ corresponding to θ with parameters $\epsilon_t = -2.5 + 0.5i$, $\epsilon_L = 3.5$.

axis, which enhances the coupling factor g_k. According to Eq. (19) there are $\lim_{\alpha\to 1} \tilde{\Gamma}_H = \infty$, which implicates that the perfect hyperbolic medium is an idealization model of some real composite materials; arbitrarily large $\tilde{\Gamma}_H$ can be achieved theoretically only if α is sufficiently large in spite of the difference of θ_0, which means that the more like the strict hyperbolic medium the real materials behave, the more enhancement of the spontaneous emission can be obtained. However, the strict hyperbolic medium is the ideal model for the real medium such as metamaterials, photonic crystals. Therefore an estimation of spontaneous emission for the real case is necessary for understanding the physical meanings of hyperbolic medium approximation. Let us consider a one-dimensional multilayer periodic composite (also called metamaterials) based on the two-layer unit cell where the frequency, the permittivity, and the thickness of the sublayers are chosen as $\omega = 0.03\frac{2\pi c}{a}$, $\epsilon_2 = 2.34$ (glass), $\epsilon_1 = -92.4$, $d_1 = 0.05a$, $d_2 = 0.95a$ [38]. a is the lattice constant. The case ($\epsilon_1 = -92.4$, $\omega = 0.03\frac{2\pi c}{a}$) corresponds to the wavelength $\lambda = 1600$ nm for silver with $a = 48$ nm [39]. According to the Bloch wave and effective plane-wave theory [38] [$\phi(z) = u_K(z)e^{iKz+ik_x x}$] we get the equifrequency contour of the system and the dispersion curve in the long-wave limit $\frac{k_x^2}{\epsilon_L^0} + \frac{K^2}{\epsilon_t^0} = (\frac{\omega}{c})^2$, where the Bloch wave vector K corresponds to k_z in Eq. (2) and $\epsilon_L^0 = \frac{a\epsilon_1\epsilon_2}{d_1\epsilon_2+d_2\epsilon_1} = 2.47$ (harmonic mean), $\epsilon_t^0 = \frac{d_1\epsilon_1+d_2\epsilon_2}{a} = -2.40$ (arithmetic mean) which is obtained by keeping the second-order terms in the Taylor expansion of the dispersion relation of the periodic system. In Fig. 3 the equifrequency contour and the hyperbola in the long-wavelength limit are given with a solid line and a dashed line, respectively. It is noted that the contour has no asymptote but the latter does. To estimate the rate of spontaneous emission in the composite with Eq. (19) it is reasonable that the overlapped part between the curves should be employed to evaluate the parameter α which provides a measure of the comparability between the metamaterials and the hyperbolic medium. It follows from Fig. 3 that in the range $k_x < 0.065\frac{2\pi c}{a}$ the two curves are coincident and the point $k_x^c = 0.065\frac{2\pi c}{a}$ gives $\alpha = \cos\theta_c/\cos\theta_a = 0.9$. Substituting ϵ_L^0,ϵ_t^0,α into Eq. (19) the typical rates of spontaneous emission of the atom in the above metamaterials are $\Gamma_H|_{\theta_0=0} = 6.9\Gamma_0 = 4.4\Gamma(\epsilon_L^0)$ and $\Gamma_H|_{\theta_0=\pi/2} = 2.2\Gamma_0 = 1.4\Gamma(\epsilon_L^0)$. It is noted that the above results are not the exact value of the decay rate Γ_H but the lower limit. In fact there are surely emitted photons, the wave-vector component k_x of which falls into the range $k_x > 0.065\frac{2\pi c}{a}$ as shown in Fig. 3. What is different is that the wave vectors with $k_x > 0.065\frac{2\pi c}{a}$ are on the contour not on the dashed line. In Fig. 3, the point of intersection of the contour and the asymptote is denoted by (k_x^{cro},K^{cro}). If we ignore the difference between the two curves for the part where $0.065 < k_x < k_x^{\text{cro}}$, the parameter α can reach 0.99 when $k_x^c = 0.11\frac{2\pi c}{a}$, where $k_x^c < k_x^{\text{cro}}$. The corresponding decay rates are $\Gamma_H|_{\theta_0=0} = 92.7\Gamma_0 = 59.1\Gamma(\epsilon_L^0)$ and $\Gamma_H|_{\theta_0=\pi/2} = 36.4\Gamma_0 = 57.1\Gamma(\epsilon_L^0)$. The hyperbolic tendency of the composite dramatically enhances the spontaneous emission of the atom. Besides the enhancement of the spontaneous emission the directivity of the emission is also worthy of being noted. Equation (18) can be rewritten as $\Gamma = \beta\int_0^\pi \gamma(\theta)d\theta$. To prevent the divergence of the integral we add a small imaginary part to ϵ_t. Figure 5 gives the amplitude of the integrand $\gamma(\theta)$ with $\epsilon_t = -2.5 + 0.5i$, $\epsilon_L = 3.5$. There are obviously two peaks which correspond to the directions of the asymptotes. This strong directivity of the spontaneous emission has been noted experimentally and used to design the single gun [23,40].

III. LOCAL-FIELD CORRECTION

The analysis in the above sections is based on the assumption $\lambda \gg a \gg l$, where l is the dimension of the quantum system and a is the lattice constant. When the dimension l of the system is comparable to the lattice constant a, the correction of the mode by the scattering cannot be ignored. The excited atom is assumed to feel the local electric field inside the an empty spherical cavity which is cut out of the homogeneous dielectric medium. The cavity is assumed to have a radial dimension R_0 and $R_0 \ll \lambda(= 2\pi c/\omega_0)$. According to the scheme in Ref. [4] the eigenfunction $\vec{F}_k(r)$ satisfies the wave equation

$$\vec{\nabla}\times\vec{\nabla}\times\vec{F}_k(r) - k_0^2\overleftrightarrow{\epsilon}\cdot\vec{F}_k(r) = (\overleftrightarrow{I} - \overleftrightarrow{\epsilon})\Theta(R_0 - r)k_0^2\vec{F}_k(0), \tag{20}$$

where the unit step function $\Theta(R_0 - r)$ describes the cavity at $r = 0$ and $k_0 = \omega_0/c$, $\Theta(R_0 - r)\vec{F}_k(r) \simeq \Theta(R_0 - r)\vec{F}_k(0)$ due to $R_0 \ll \lambda$, and $\overleftrightarrow{I}$ is the unix matrix.

The solution of Eq. (20) is expressed as

$$\vec{F}_k(r) = E_{k0}e^{ikr} + k_0^2 \int \overleftrightarrow{G}(r,r')(\overleftrightarrow{I} - \overleftrightarrow{\epsilon}) \times \Theta(R_0 - r')\vec{F}_k(0)d^3r'. \quad (21)$$

Solving Eq. (21) with the Green's function $\overleftrightarrow{G}(r,r')$ in the uniaxial anisotropic medium we get the corrected local field $\vec{F}_k(0)$ at $r = 0$ which the atom feels in the cavity:

$$\begin{aligned}\vec{F}_k(0) &= \vec{E}_{k0} + \lim_{R_0,r\to 0}\lim_{\delta\to 0}\int\int \overleftrightarrow{G}(k,\epsilon_t + i\delta,\epsilon_L + i\delta)e^{ik(r-r')} \\ &\quad \times \Theta(R_0 - r')\cdot(\overleftrightarrow{I} - \overleftrightarrow{\epsilon})\vec{F}_k(0)d^3k\, d^3r' \\ &= \vec{E}_{k0} + \overleftrightarrow{M}\cdot k_0^2(\overleftrightarrow{I} - \overleftrightarrow{\epsilon})\vec{F}_k(0), \end{aligned} \quad (22)$$

where $\overleftrightarrow{M} = \lim_{R_0,r\to 0}\lim_{\delta\to 0}\int\int \overleftrightarrow{G}(k,\epsilon_t + i\delta,\epsilon_L + i\delta)e^{ik(r-r')}\Theta(R_0 - r')d^3k\, d^3r'$. After completing the integral and the limit process we get

$$\overleftrightarrow{M} = -k_0^{-2}\begin{pmatrix} \frac{1}{2\epsilon_t} & & \\ & \frac{1}{2\epsilon_t} & \\ & & \frac{1}{\epsilon_L} \end{pmatrix}. \quad (23)$$

Substituting Eq. (23) into Eq. (21) we get the corrected local field $\vec{F}_k(0)$:

$$\vec{E}_{\mathrm{corr}}(0) = \frac{2\epsilon_t}{\epsilon_t + 1}\vec{E}_{k0}^T + \epsilon_L\vec{E}_{k0}^L, \quad (24)$$

where $\vec{E}_{k0}^T,\vec{E}_{k0}^L$ are the transverse and longitudinal components of the amplitude $\vec{E}_{k0}$, respectively. For the transverse mode $E_z = 0$, the modified decay rate Γ_{corr}^T can be directly obtained by substituting $F_k(0)$ in Eq. (6) with $\frac{2\epsilon_t}{\epsilon_t+1}F_k(0)$ and it follows that $\Gamma_{\mathrm{corr}}^T = (\frac{2\epsilon_t}{\epsilon_t+1})^2\Gamma^T$. For the longitudinal mode in the anisotropic medium where $E_z \neq 0$ we can also get the modified $\Gamma_{\mathrm{corr}}^L(\Gamma_{\mathrm{corr}}^H)$ after instituting Eq. (24) into Eq. (6). In Fig. 6 the local-field corrected decay rate Γ_{corr}^H is given with the same parameters as in Fig. 4. By comparing Fig. 4 with Fig. 6 it follows that because of the enhancement of the local-field $E_k(0)$ the corresponding decay rate increases largely due to the enhancement of the interaction and the rule in which the decay rate varies when the relative parameter changes are invariant despite the local-field correction.

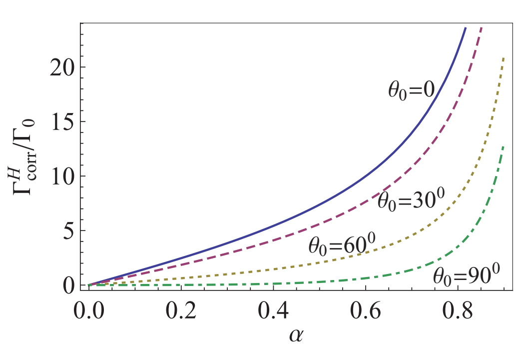

FIG. 6. (Color online) The local-field corrected decay rate Γ_{corr}^H with the same parameters as in Fig. 4.

IV. DISCUSSION

In the framework of quantum optics we investigate the spontaneous emission of the two-level atom in the homogeneous anisotropic medium where the dispersion geometry exhibits an ellipsoid or a hyperboloid. Under the Weisskopf-Wigner approximation the corresponding decay rate Γ is derived in detail. For the ellipsoid case, there are two kinds of modes that contribute to the decay rate Γ and to some degree the medium with ellipsoid dispersion provides two types of "mode spaces" to accommodate the emitted photons. Moreover, the polar angle θ_0 can also be used to change the intensity of the photons coupled into the different modes. When $\epsilon_L = \epsilon_t$ the obtained formula (17c) reduces to the case of the isotropic medium, which justifies the validity of our model.

When the above model is applied to the hyperboloid case the divergence is encountered and the cutoff of the wave vector k is introduced for the approximation of the real materials. When the topology of the dispersive geometry changes from ellipsoid to hyperboloid the transverse radiating wave mode in the ellipsoid case degenerates into the evanescent wave mode. The enhanced spontaneous emission relative to the background medium ϵ_L can still be obtained only if the dispersion geometry of the real materials is sufficiently close to the strict hyperboloid. This means that more high-k modes which correspond to a larger parameter α get involved in the coupling of the photons. In practice the HM is used as the ideal model for some real materials with periodic microstructure, whose band structure exhibits the approximative hyperbolic geometry in a small range of k, e.g., the hyperbolic metamaterials. In the long-wavelength limit the above materials can be approximated as the hyperbolic medium within the effective medium theory. It is expected that the smaller the lattice a, the closeer to the HMM the materials are. Therefore, more enhancement of the spontaneous emission can be achieved. The perspective of the density of states provides a physical explanation for the mechanism of the spontaneous emission enhancement. When the geometry of the physical dispersion relation of the medium changes from ellipsoid to hyperboloid, due to the change of the topological property the density of states diverges in the lossless continuous hyperbolic medium limit: $\rho(\omega) \approx \frac{K_{\mathrm{cut}}^3}{12\pi^2}|\frac{\epsilon_L}{\epsilon_t}(\frac{1}{\epsilon_t}\frac{d\epsilon_t}{d\omega} - \frac{1}{\epsilon_L}\frac{d\epsilon_L}{d\omega})|$, where K_{cut} is the momentum cutoff [28]. K_{cut} is defined by either metamaterial structure scale or by losses. It is the occurrence of the large ρ that enables the transition probability to be increased dramatically. Even when the loss and the dispersion of the $\epsilon(\omega)$ in the materials is taken into account the spontaneous emission is expected to be largely enhanced.

ACKNOWLEDGMENTS

This work was supported by the 863 Program of China (Project No. 2011AA010205), the National Natural Science Foundation of China (Grants No. 61131006, No. 61021064, No. 11174309, and No. 11204340), the Major National Development Project of Scientific Instrument and Equipment (Grant No. 2011YQ150021), the Important National Science and Technology Specific Projects (Grant No. 2011ZX02707), the major project (Project No. YYYJ-1123-1) and the International Collaboration and Innovation Program on High

Mobility Materials Engineering of the Chinese Academy of Sciences, and the Shanghai Municipal Commission of Science and Technology (Project No. 10JC1417000).

APPENDIX A: DERIVATIONS OF EQ. (5)

In the homogeneous isotropic medium where the permittivity is $\overleftrightarrow{\epsilon} = \epsilon_0 \left(\begin{smallmatrix} \epsilon_t & & \\ & \epsilon_t & \\ & & \epsilon_L \end{smallmatrix}\right)$, according to the minimal coupling principle the Hamiltonian of the atom in the electromagnetic field is written as

$$
\begin{aligned}
\hat{\mathscr{H}} &= V(\vec{r}) + \frac{1}{2m}(\hat{\vec{P}} - q\hat{\vec{A}})^2 \\
&= V(\vec{r}) + \frac{\hat{\vec{P}}^2}{2m} - \frac{q}{2m}(\hat{\vec{P}} \cdot \hat{\vec{A}} + \hat{\vec{A}} \cdot \hat{\vec{P}}) \\
&= V(\vec{r}) + \frac{\hat{\vec{P}}^2}{2m} - \frac{q}{m}\hat{\vec{P}} \cdot \hat{\vec{A}} - \frac{iq}{2m}\hbar\left(1 - \frac{\epsilon_L}{\epsilon_t}\right)\frac{\partial A_z}{\partial z} \\
&= \hat{\mathscr{H}}_{\text{atom}} + \hat{\mathscr{H}}_I + \hat{\mathscr{H}}_{II},
\end{aligned} \tag{A1}
$$

where $\hat{\mathscr{H}}_{\text{atom}} = V(\vec{r}) + \frac{\hat{\vec{P}}^2}{2m}$, $\hat{\mathscr{H}}_I = -\frac{q}{m}\hat{\vec{P}} \cdot \hat{\vec{A}}$, $\hat{\mathscr{H}}_{II} = -\frac{iq}{2m}\hbar (1 - \frac{\epsilon_L}{\epsilon_t})\frac{\partial A_z}{\partial z}$, and q is the charge of the particle. For the third line in Eq. (A1) the conditions $[\hat{\vec{P}},\hat{\vec{A}}] = -i\hbar\vec{\nabla} \cdot \hat{\vec{A}}$ and $\vec{\nabla} \cdot (\overleftrightarrow{\epsilon} \cdot \hat{\vec{A}}) = 0$ are used. For the electron $q = -e$, $\hat{\mathscr{H}}_I = \frac{e}{m}\hat{\vec{P}} \cdot \hat{\vec{A}} = e\vec{r} \cdot \hat{\vec{A}}(-i\omega) = e\vec{r} \cdot \hat{\vec{E}}$. Under the rotating-wave approximation the term reads [37]

$$
\hat{\mathscr{H}}_I = \hbar \sum_k g_{Ik}(\hat{\sigma}_+\hat{a}_k + \hat{\sigma}_-\hat{a}_k^\dagger), \tag{A2}
$$

where the quantized field $\hat{\vec{E}} = \sum_k \vec{e}_k E_k \hat{a}_k e^{ik\cdot r}e^{-i\omega t} + \text{H.c.}$ and $g_{Ik} = -\frac{\vec{\mathscr{D}}_{ab}\cdot\vec{e}_k E_k}{\hbar}$. $\vec{\mathscr{D}}_{ab}$ is the matrix element of the dipole between states $\vec{\mathscr{D}}_{ab} \equiv -\langle a|e\hat{\vec{R}}|b\rangle = -\langle b|e\hat{\vec{R}}|a\rangle$.

For $\hat{\mathscr{H}}_{II} = \frac{ie}{2m}\hbar(1 - \frac{\epsilon_L}{\epsilon_t})\frac{\partial A_z}{\partial z}$, $\vec{E}_k = \vec{e}_k E_k e^{ik\cdot r}e^{-i\omega t}$, $\vec{E} = -\frac{\partial \vec{A}}{\partial t}$; $\frac{\partial A_z}{\partial z} = \frac{k_z}{\omega}E_z$. After field quantization $\hat{\vec{E}} = \sum_k \vec{e}_k E_k \hat{a}_k e^{ik\cdot r}e^{-i\omega t} + \text{H.c.}$ the term is expressed as

$$
\hat{\mathscr{H}}_{II} = \hbar g_{IIk} \sum_k (\hat{a}_k^\dagger + \hat{a}_k), \tag{A3}
$$

where $g_{IIk} = \frac{ie}{2m}\frac{k_z}{\omega}(1 - \frac{\epsilon_L}{\epsilon_t})E_{kz}$. Because the term $\hat{a}_k^\dagger + \hat{a}_k$ in $\hat{\mathscr{H}}_{II}$ means the field fluctuation process, which is a small quantity compared with the spontaneous emission and $g_{Ik}/g_{IIk} \sim 2mcr/\hbar \gg 1$, $\hat{\mathscr{H}}_{II}$ can be ignored under the rotating-wave approximation.

APPENDIX B: GREEN'S FUNCTION FOR ANISOTROPIC MEDIA AND THE LOCAL-FIELD CORRECTION

The Green's function G for the uniaxial media is defined to satisfy the wave equation

$$
\vec{\nabla} \times \vec{\nabla} \times \overleftrightarrow{G}(\vec{r}',\vec{r}) - k_0^2 \overleftrightarrow{\epsilon} \cdot \overleftrightarrow{G}(\vec{r}',\vec{r}) = \delta^3(\vec{r} - \vec{r}') \tag{B1}
$$

and the Fourier transform of Eq. (B1) is

$$
\left[\Delta_i(k)^2 - k_i^2\right] G_{ij}(k) - \sum_{n\neq i} k_n k_i G_{nj}(k) = \delta_{ij}(2\pi)^{-3}, \tag{B2}
$$

where $k_0 = \omega_0/c$, $\overleftrightarrow{\epsilon} = \left(\begin{smallmatrix} \epsilon_1 & & \\ & \epsilon_2 & \\ & & \epsilon_3 \end{smallmatrix}\right)$, and $\Delta_i^2 = k^2 - \epsilon_i k_0^2 \equiv k^2 - k_\epsilon^{i2}$.

Solving Eq. (B2) we get the symmetric matrix $G_{ij}(k) = (A_{ij} + B_{ij})/C$, where $A_{ij} = (\Delta_l^2\Delta_n^2/2 - k_l k_n)(-1)^p \varepsilon_{iln}\delta_{ij}$, $B_{ij} = k_i k_j \Delta_n^2 (-1)^p \varepsilon_{ijn}$, and $C = (2\pi)^3[(\Delta_1\Delta_2\Delta_3)^2 - (-1)^p \varepsilon_{ijn}\Delta_i\Delta_j\Delta_n/2]$. The symbols ε_{ijn} and δ_{ij} are the Levi-Civita symbol and the Kronecker symbol, respectively. p is the number of permutations in ε_{ijn} and the repeated indices mean Einstein summation. For the uniaxial media $\epsilon_1 = \epsilon_2 = \epsilon_t$, $\epsilon_3 = \epsilon_L$, the matrix $G_{ij}(k)$ is expressed explicitly as

$$
\overleftrightarrow{G}(k) = \frac{1}{8\pi^3 C_2}\begin{pmatrix} (\Delta_L^2 - k_3^2) - \frac{\Delta_L^2}{\Delta_t^2}k_2^2 & \frac{\Delta_L^2}{\Delta_t^2}k_1k_2 & k_1k_3 \\ \frac{\Delta_L^2}{\Delta_t^2}k_1k_2 & (\Delta_L^2 - k_3^2) - \frac{\Delta_L^2}{\Delta_t^2}k_1^2 & k_2k_3 \\ k_1k_3 & k_2k_3 & \Delta_t^2 - k_1^2 - k_2^2 \end{pmatrix}, \tag{B3}
$$

where $C_2 = \Delta_t^2(\Delta_L^2 - k_3^2) - \Delta_L^2(k_1^2 + k_2^2)$, $\Delta_t^2 = k^2 - \epsilon_t k_0^2$, and $\Delta_L^2 = k^2 - \epsilon_L k_0^2$.

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