Critical power for self-focusing in the case of ultrashort laser pulses

P. Polynkin[1,*] and M. Kolesik[1,2]
[1]*College of Optical Sciences, The University of Arizona, Tucson, Arizona 85721, USA*
[2]*Department of Physics, Constantine the Philosopher University, Nitra, Slovakia*
(Received 19 October 2012; published 23 May 2013)

We attempt to evaluate the applicability of the concept of the critical power for self-focusing, originally developed for intense quasi-continuous-wave (cw) beams, to the case of ultra-intense and ultrashort laser pulses propagating in air. Our results show that, unlike in the cw case, no particular value of peak pulse power can be viewed as a sharp demarcation line between linear and nonlinear propagation regimes. Our analysis further reveals the important role played by chromatic dispersion in the propagation dynamics of the laser pulse.

DOI: 10.1103/PhysRevA.87.053829 PACS number(s): 42.65.Jx, 42.65.Sf

Self-focusing of intense optical beams in transparent media is arguably one of the most important phenomena in nonlinear optics. Following the invention of Q-switched laser sources, for thirty years self-focusing was studied using laser pulses with durations in the nanosecond to millisecond range [1]. The dominant effects that govern beam evolution in this case are Kerr self-focusing and diffraction. The role of the medium dispersion is essentially negligible.

The key concept that emerged as a result of the early studies of self-focusing is that of the critical power for self-focusing beam collapse. The critical power in the quasi-continuous-wave(quasi-cw) case is a sharp demarcation line between two qualitatively different propagation regimes: For the laser power below critical, linear optics prevails and the beam diverges due to diffraction. For power above critical, the nonlinearity wins, and the beam undergoes a transverse collapse until an additional highly nonlinear effect, such as multiphoton absorption or photoionization, arrests the collapse from progressing all the way to a singularity.

The development of ultrafast laser systems in the early 1990s enabled the extension of self-focusing studies to the case of gaseous media. In gases, the optical power level sufficient for the observation of self-focusing is in the multi-GW range. Typical optical pulse duration in this case is in the subpicosecond range, and a nontrivial temporal pulse evolution brings the complexity of beam propagation to a new level relative to that in the case of quasi-cw laser beams [2–4]. Notwithstanding that, the concept of the critical power for self-focusing has migrated from the quasi-cw case to the case of ultrashort laser pulses essentially unchanged. The validity of such a generalization, which has been a norm for almost twenty years, is not *a priori* justified.

We are not suggesting that cw propagation models are seriously considered by anybody as capable of describing the propagation of intense ultrashort laser pulses and that the fact that they are not adequate in the ultrashort-pulse case requires any additional experimental proof. Our goal is to show that the concept of a critical power for self-focusing loses its precise meaning in the case of ultrashort laser pulses. More specifically, the fact that the peak power of an ultrashort laser pulse is above the value given by the cw formula [5]

$$P_{\text{cr}} = \frac{3.79\lambda_0^2}{8\pi n_0 n_2} \tag{1}$$

does not necessarily mean that the pulse will experience self-focusing collapse, as routinely implied by many contemporary authors [6]. (The formula above is derived for a cw beam with a perfect Gaussian transverse intensity profile; λ_0 is the laser wavelength in vacuum, n_0 and n_2 are the linear and nonlinear refractive indices of the propagation medium, respectively.)

From the practical standpoint, self-focusing of femtosecond laser pulses with collimated or nearly collimated beam profiles is of a particular interest, because such a case is relevant to potential standoff applications, e.g., in remote sensing. Laboratory investigations, however, are limited to the propagation distances of several to several tens of meters, in which case weak external focusing of the beam is conventionally employed. A natural question that arises in this context is to what degree the concept of the critical power for self-focusing collapse is applicable to weakly focused ultrafast laser beams?

In the present work, we address the above question through the attempt to realize a "nearly ideal" situation of self-focusing collapse of a femtosecond laser beam in air, in the regime of very weak external focusing. The results of our experiments and numerical simulations fall into the category of observations that seem to be counterintuitive before the measurement is performed, but suddenly appear quite natural after the results of the measurement and computer simulation are known. Our results show that the generalization of the concept of the critical power for self-focusing to the case of femtosecond laser beams is far from being straightforward. Slightly above threshold, chromatic dispersion of air, which is commonly considered to be a weak contributor to the pulse propagation dynamics, plays a key role and may prevent the self-focusing collapse from happening.

Among prior works, the publication [7] is most relevant to the present investigation. Numerical simulations reported in Ref. [7] show that slightly above the nominal (cw) self-focusing threshold, dispersion, together with plasma defocusing, can stabilize the transverse collapse of the beam and result in the generation of a spatial soliton-like structure which is capable of propagating over long distances.

In our experimental setup, transform-limited femtosecond laser pulses with 36 femtosecond FWHM pulse width are

*ppolynkin@optics.arizona.edu

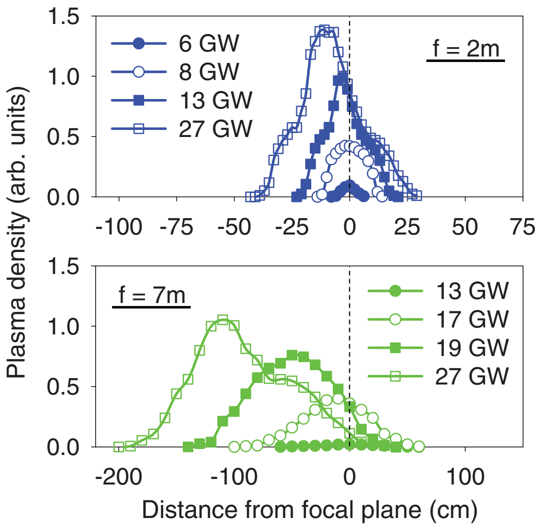

FIG. 1. (Color online) Time-integrated plasma density, on an arbitrary unit scale, vs distance from the linear focal plane of the focusing telescope. The FWHM intensity diameter of the input beam is 5 mm, and the effective focal lengths of the focusing telescope are indicated in the individual figures. Different curves correspond to different values of the input peak power of the laser pulse, as indicated in the legends. Note that the scales of the horizontal axes in the two figures are different.

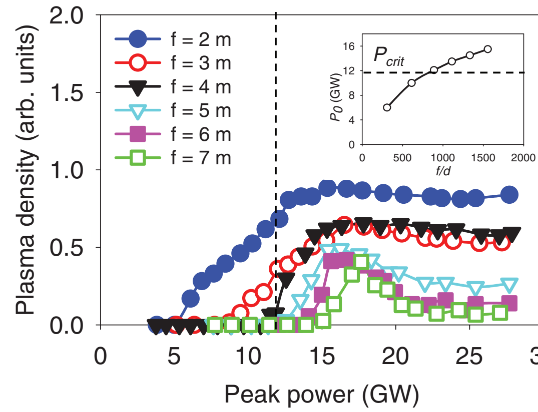

FIG. 2. (Color online) Measured plasma density at linear focal plane of optical system vs peak power of femtosecond laser beam under various focusing conditions. The cw threshold power for self-focusing, which equals approximately 12 GW [16], is marked by the vertical dashed line. Inset: Experimental peak laser power corresponding to the onset of plasma generation vs linear confocal parameter of optical system. The cw critical power for self-focusing is shown with the horizontal dashed line. For very weak focusing, the onset of plasma generation occurs at the peak pulse power noticeably above the cw critical power.

generated by a commercial chirped-pulse amplification system operating at 800 nm center wavelength. No multipass amplification is used, and the output beam profile has an essentially ideal Gaussian beam shape defined by the cavity mode of the regenerative amplifier. The output beam with the full width at half maximum (FWHM) intensity diameter of 5 mm is weakly focused in ambient air by a spherical mirror telescope operated very slightly off axis in order to minimize aberrations. Keeping focusing conditions fixed, we measure the density of plasma generated along the beam path, using the capacitive plasma probe described, e.g., in Refs. [8,9]. This measurement technique is perfectly suitable for our purposes, as we are interested only in detecting the level of input peak pulse power at which plasma just starts being generated at the focal plane of the telescope. It has been shown [10] that the measurement technique based on the application of the capacitive plasma probe of the kind that we use, being much simpler than alternative approaches based on interferometry [11] or plasma fluorescence [12], provides for a reliable measurement of plasma density in laser filaments on an arbitrary unit scale.

Numerical simulations to support out experiments are conducted using our standard ultrafast pulse propagator code based on the unidirectional pulse propagation equation (UPPE) [13]. The code has been previously successfully benchmarked against experiments in various cases of propagation and filamentation of ultra-intense laser pulses with complex beam shapes such as Bessel [8] and Airy [14] beams.

In Fig. 1, we show plasma density measured along the optical path for two values of the focal length of the focusing telescope. Different curves correspond to different values of input peak power of the laser pulses. The indicated values of peak pulse power are derived from the pulse energy measured with a pyroelectric energy meter and the temporal pulse profile measured with a commercial frequency-resolved optical gating (FROG) system [15]. Each data point is the result of averaging over 100 laser shots. It is evident from the data that, for both focusing conditions, as the input pulse energy increases, plasma generation first starts exactly at the focal plane of the focusing telescope. With the pulse energy further increased, the plasma string becomes longer and moves towards the laser source. The longitudinal shift of the plasma distribution is more pronounced for the case of weaker external focusing.

The combined data for plasma density vs peak power of the input laser pulses, for various effective focal lengths of the mirror telescope, is shown in Fig. 2. The goal of this measurement is to detect the onset of plasma generation. As suggested by the data shown in Fig. 1, plasma always starts being generated at the linear focal plane, as the input pulse energy is increased. Accordingly, for each focusing condition, the plasma is detected at a fixed location—the linear focal plane of the focusing system.

The dashed vertical line in Fig. 2 shows the nominal critical power of 12 GW, which is calculated using the standard continuous-wave formula (1). For n_2 we use the value from the recent independent measurement [16]. Since for ultrashort pulses with the duration of less than ~50 fs the nonlinear optical response is essentially instantaneous, the nonlinear refractive index n_2 is a well-defined physical quantity. However, the precise meaning of the nominal critical power P_{cr}, calculated according to the formula (1), for the case of ultrashort laser pulses, is not *a priori* clear.

It is evident from the data shown in Fig. 2 that for tighter focusing, the onset of plasma generation occurs at the peak laser power below the nominal critical power, while for looser focusing, the plasma starts being generated at the peak pulse power noticeably above the nominal critical power. For all curves, plasma density first sharply increases with the input pulse power and then declines. The decline in the plasma density, which in this case is measured at a particular location (the linear focal plane), corresponds to the entire plasma channel shifting towards the laser source as the input laser power is increased.

In the inset in Fig. 2, we show the peak pulse power for the onset of plasma generation, vs confocal parameter of the focusing telescope. (The confocal parameter is defined as the ratio of the linear focal length to the FWHM input beam diameter. In all cases, the FWHM input beam diameter is fixed at 5 mm.) Data points comprising the curve are derived from the data curves shown in the main Fig. 2 and correspond to the peak power values at which plasma just starts to appear for a particular focusing condition. It is evident that the power level at which a sharp onset of plasma generation occurs is a smooth and monotonically growing function of the input peak power of the laser pulse. The input peak power equal to the nominal critical power calculated according to the formula (1) does not correspond to any outstanding feature in this data plot.

Note that all focusing conditions that we use are normally considered "weak" focusing. However, for the case of the shortest focal length of 2 m, the FWHM waist size of the beam at the focal plane, if the beam propagated linearly, would be about 140 μm. That is comparable to the diameter of the femtosecond laser filament at 800 nm wavelength, which has been previously measured to be between 100 and 200 μm [17,18]. According to the data shown in Fig. 2, the onset of plasma generation in that case occurs at the peak pulse power of about one half of the nominal cw critical power, i.e., significantly before the self-focusing could possibly occur. Under these conditions, plasma is generated due to the significant intensity buildup at the focal plane through the external linear focusing alone. For the case of sufficiently lose focusing, when linear focusing alone would produce a beam waist significantly larger than the transverse size of a common filament in air, the sharp onset of plasma generation in the focal plane is indicative of the beam self-focusing.

Also note that the linear dispersion length corresponding to our 36-fs-long pulses in air is about 35 m [2], which is much longer than the maximum propagation length used in our experiments (7 m). Thus the pulses are not expected to be significantly temporally broadened on the way to the filamentation zone by the linear air dispersion alone. In fact, the conventional paradigm of femtosecond laser filamentation in gases is based on the notion of a dynamic balance between beam self-focusing and its defocusing by free electrons generated on the beam axis through multiphoton ionization. Chromatic dispersion is conventionally considered to play an essentially negligible role in the beam propagation inside the filamentation zone, as spectral broadening of the laser pulse, averaged over the beam cross section, is usually quite moderate. As we will show below, that simplified picture may easily fail and lead to significant errors. In the presence of plasma generation, whether due to the significant intensity buildup through the linear focusing alone in the case of tight focusing, or through self-focusing in the case of loose focusing, the peak intensity of the laser pulse in the filamentation zone is limited to about 5×10^{13} W/cm^2 due to the effect of intensity clamping [19]. Using this value for a meaningful estimate of the total plasma generated across the beam, at a particular longitudinal position, is problematic due to the complex temporal and spectral reshaping of the pulse inside the filamentation zone. These effects have been reported to cause the generation of intense ultrashort temporal features with the duration in the range of several optical cycles [20]. Numerical simulations are necessary to gain insight into the complex dynamics of the laser pulse propagating in the highly nonlinear regime.

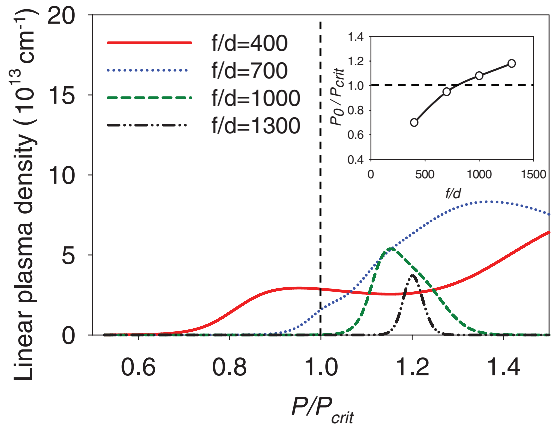

FIG. 3. (Color online) Numerical simulations corresponding to the experimental data shown in Fig. 2. Linear confocal parameters for different curves are shown in the legend. The peak pulse power is measured in the units of the nominal cw threshold power for self-focusing. Inset: Combined simulation data for peak power at the onset of plasma generation vs linear confocal parameter of the focusing system.

The results of numerical simulations of our experiments are shown in Fig. 3. As in the experiments, the input peak power of the laser pulses equal to the nominal cw critical power for self-focusing does not correspond to any outstanding feature in the plot for the power onset of plasma generation vs confocal parameter. For tighter focusing, plasma generation occurs at the peak pulse power significantly below the nominal cw critical power, while for weaker focusing, the onset of plasma generation is noticeably above the nominal cw critical power. The experimental and numerical data for the power onset of plasma generation vs confocal parameter (the curves shown in the insets in Figs. 2 and 3) are in quantitative agreement with each other. The agreement between experiments and simulations alleviates the concerns that our experimental observations are influenced by the lack of control over experimental conditions or by the uncertainty of the nominal critical power for self-focusing. Both experiments and numerical simulations consistently show that the nominal cw value of the critical power determined through the application of the conventional formula (1), unlike in the cw case, cannot serve as a sharp dividing line between the essentially linear and the strongly nonlinear propagation regimes.

What makes the case of ultrashort laser pulses so different from the quasi-cw case is the nontrivial temporal evolution of the pulse, coupled to the spatial reshaping of the laser beam undergoing self-focusing collapse. Different effects participating in the beam propagation dynamics are interrelated and their roles cannot be straightforwardly isolated and separately quantified. The role of chromatic dispersion,

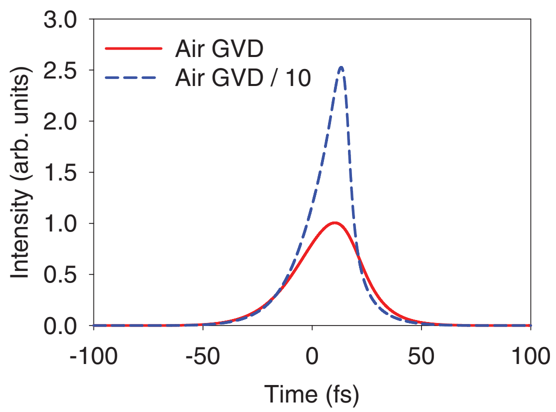

FIG. 4. (Color online) Simulation results for temporal intensity profile of pulse on beam axis at linear focal plane of focusing system, for cases of actual chromatic dispersion of air and of air dispersion artificially reduced by a factor of 10. The input peak pulse power is 30% above the nominal self-focusing threshold and the confocal parameter of the focusing system equals 1300.

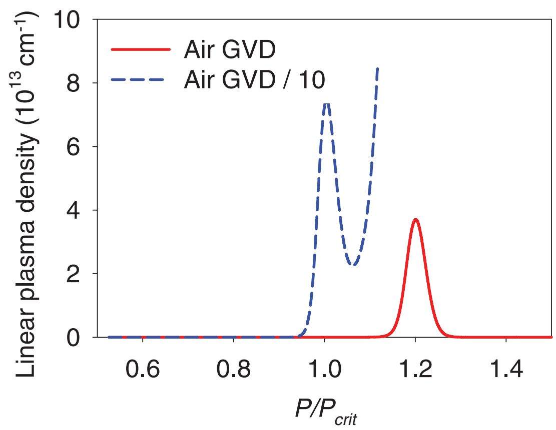

FIG. 5. (Color online) Simulated plasma density at linear focal plane vs peak pulse power, for the same conditions as those used in Fig. 4.

which is an enabling effect in the formation of laser filaments in condensed media [21,22], is often underappreciated in the case of self-focusing and filamentation in gases. However, as our simulation results will show, the role of dispersion in that case can be very significant.

To illustrate the importance of the role played by dispersion, we compare the results of numerical simulations of the pulse propagation in the common air with that in an artificial gaseous medium in which all medium response parameters are the same as those in air, but the chromatic dispersion is reduced by a factor of 10. (Note that eliminating chromatic dispersion completely, for the purpose of making this comparison, would eliminate the interaction between different temporal slices of the pulse. That would prevent plasma defocusing, which is a time-integrated effect that is only directly operative on the trailing edge of the pulse, from assisting multiphoton absorption in arresting self-focusing collapse on the leading pulse edge.)

The results of these simulations are shown in Figs. 4 and 5. The confocal parameter of the external linear focusing used in the simulations equals 1300. The input peak power of the laser pulses in the case of Fig. 4 is 30% above the nominal critical power for self-focusing. It is evident that reducing the already very weak chromatic dispersion of air by a factor of 10 results in a significant difference in the temporal pulse shape at the focal plane and in an even more significant difference in the generated plasma density. This analysis shows (yet again) that straightforward intuition based on the concepts borrowed from intense cw nonlinear optics may easily fail to yield even qualitatively correct results in the case of ultrashort laser pulses undergoing self-focusing and filamentation.

In conclusion, we have analyzed, both experimentally and numerically, the applicability of the conventional concept of critical power for self-focusing, which is commonly applied to the propagation of intense continuous-wave laser beams, to the case of femtosecond laser pulses. Our results show that the critical power concept is not straightforwardly applicable to the ultrashort-pulse case. Our analysis further shows that chromatic dispersion of air, which is commonly considered to be a weak effect, plays an important role in the arrest of the self-focusing collapse in the ultrashort-pulse case.

This work was supported by The United States Air Force Office of Scientific Research (US AFOSR) under programs FA9550-12-1-0143, FA9550-11-1-0144, and FA9550-10-1-0561.

[1] *Self-focusing: Past and Present*, edited by R. W. Boyd, S. G. Lukishova, and Y. R. Shen (Springer, New York, 2009).
[2] A. Couairon and A. Mysyrowicz, Phys. Rep. **441**, 47 (2007).
[3] L. Berge, S. Skupin, R. Nuter, J. Kasparian, and J.-P. Wolf, Rep. Prog. Phys. **70**, 1633 (2007).
[4] S. L. Chin, *Femtosecond Laser Filamentation* (Springer, Berlin, 2010).
[5] G. Fibich and A. L. Gaeta, Opt. Lett. **25**, 335 (2000).
[6] M. Durand, A. Jarnac, A. Houard, Y. Liu, S. Grabielle, N. Forget, A. Durecu, A. Couairon, and A. Mysyrowicz, Phys. Rev. Lett. **110**, 115003 (2013).
[7] L. Berge and A. Couairon, Phys. Rev. Lett. **86**, 1003 (2001).
[8] P. Polynkin, M. Kolesik, A. Roberts, D. Faccio, P. Di Trapani, and J. Moloney, Opt. Express **16**, 15733 (2008).
[9] S. Henin, Y. Petit, D. Kiselev, J. Kasparian, and J.-P. Wolf, Appl. Phys. Lett. **95**, 091107 (2009).
[10] D. Abdollahpour, S. Suntsov, D. G. Papazoglou, and S. Tzortzakis, Opt. Express **19**, 16866 (2011).
[11] Y.-H. Chen, S. Varma, T. M. Antonsen, and H. M. Milchberg, Phys. Rev. Lett. **105**, 215005 (2010).
[12] A. Iwasaki, N. Akozbek, B. Ferland, Q. Luo, G. Roy, C. M. Bowden, and S. L. Chin, Appl. Phys. B **76**, 231 (2003).
[13] M. Kolesik and J. V. Moloney, Phys. Rev. E **70**, 036604 (2004).

[14] P. Polynkin, M. Kolesik, J. V. Moloney, G. A. Siviloglou, and D. N. Christodoulides, Science **324**, 229 (2009).

[15] Grenouille 8-2-USB pulse measurement system by Swamp Optics, LLC.

[16] J. K. Wahlstrand, Y.-H. Cheng, and H. M. Milchberg, Phys. Rev. A **85**, 043820 (2012).

[17] A. Braun, G. Korn, X. Liu, D. Du, J. Squier, and G. Mourou, Opt. Lett. **20**, 73 (1995).

[18] J.-C. Diels, J. Yeak, D. Mirell, R. Fuentes, S. Rostami, D. Faccio, P. Di Trapani, Laser Phys. **20**, 1101 (2010).

[19] S. Xu, J. Bernhardt, M. Sharifi, W. Liu, and S. L. Chin, Laser Phys. **22**, 195 (2012).

[20] J. H. Odhner, D. A. Romanov, and R. J. Levis, Phys. Rev. Lett. **103**, 075005 (2009).

[21] P. Chernev and V. Petrov, Opt. Lett. **17**, 172 (1992).

[22] D. Faccio, M. Clerici, A. Averchi, A. Lotti, O. Jedrkiewicz, A. Dubietis, G. Tamosauskas, A. Couairon, F. Bragheri, D. Papazoglou, S. Tzortzakis, and P. Di Trapani, Phys. Rev. A **78**, 033826 (2008).