Dynamics of quantum correlations in colored-noise environments

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We address the dynamics of entanglement and quantum discord for two noninteracting qubits initially prepared in a maximally entangled state and then subjected to a classical colored noise, i.e., coupled with an external environment characterized by a noise spectrum of the form $1/f^\alpha$. More specifically, we address systems in which the Gaussian approximation fails, i.e., mere knowledge of the spectrum is not enough to determine the dynamics of quantum correlations. We thus investigate the dynamics for two different configurations of the environment: in the first case, the noise spectrum is due to the interaction of each qubit with a single bistable fluctuator with an undetermined switching rate, whereas in the second case we consider a collection of classical fluctuators with fixed switching rates. In both cases, we found analytical expressions for the time dependence of entanglement and quantum discord, which may also be extended to a collection of fluctuators with random switching rates. The environmental noise is introduced by means of stochastic time-dependent terms in the Hamiltonian, and this allows us to describe the effects of both separate and common environments. We show that the non-Gaussian character of the noise may lead to significant effects, e.g., environments with the same power spectrum, but different configurations give rise to the opposite behavior for quantum correlations. In particular, depending on the characteristics of the environmental noise considered, both entanglement and discord display either a monotonic decay or the phenomena of sudden death and revivals. Our results show that the microscopic structure of the environment, in addition to its noise spectrum, is relevant for the dynamics of quantum correlations and may be a valid starting point for the engineering of non-Gaussian colored environments.

I. INTRODUCTION

Entanglement and discord describe remarkable features of quantum systems. Indeed, they are closely related to the amount of quantum correlations contained in the system, coming either from nonseparability or the impossibility of local discrimination [1]. Beyond their specific role in fundamental physics, entanglement and discord have also been recognized as resources for quantum technology, e.g., for the processing of quantum information and for the effective implementation of quantum enhanced protocols. In particular, in the past decade it has been recognized [2–5] that separable mixed states may represent a resource if they show a nonzero quantum discord. In fact, mixed separable states with nonzero discord have been exploited to achieve a speed-up for certain computational tasks compared to classical states [6,7]. This is true also for continuous variable systems, where Gaussian quantum discord

has been introduced [8,9], measured [10], and exploited for quantum enhanced protocols [11,12].

An essential ingredient to exploit the quantumness of a physical system is the preservation of its coherent time evolution. On the other hand, the unavoidable interaction with its environment usually destroys coherence and quantumness [13,14], and in turn their use for quantum technology. For these reasons, much attention has been devoted to the analysis, characterization, and control of the dynamics of quantum correlations in different physical systems [15], including quantum optics [16–20], nuclear magnetic resonance [21,22], nanophysics [23,24], and biology [25,26].

For bipartite open quantum systems interacting with a quantum environment, entanglement and discord may exhibit peculiar features such as sudden death and transitions, revivals, and trapping [27–32]. Such phenomena have been linked either to direct [33] or indirect [34] effective two-qubit interactions. For noninteracting qubits they are due to the non-Markovian nature of the environment [35], which results in the transfer of correlations back and forth from the two-qubit system to the various parts of the total system. In this framework, it was shown [34] that for a common bosonic environment, the entanglement between two qubits is more robust against

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decoherence, and its dynamics is faster than for independent environments with the same spectrum. The effect of the noise spectrum on the dynamics of quantum discord has also been analyzed [36]. Recently, revivals of quantum correlations have been found also for a quantum system coupled to classical sources [37–39] and have been connected to a quantifier of non-Markovianity for the dynamics of a single qubit. Indeed, it was proven that a classical noise can mimic, without loss of generality, a quantum environment not affected by the system or influenced in a way that does not result in backaction [37].

Among the class of open quantum systems interacting with a classical environment, particular attention has been devoted to systems comprised of two qubits subjected to a classical source of random telegraph noise (RTN) [38,40–44], namely those interacting with a bistable fluctuator randomly switching between its two states with a given rate γ. Depending upon the ratio between the switching rate and the system-environment coupling, the dynamics of a quantum system may exhibit Markovian or non-Markovian behavior. In fact, the great interest in RTN is due to the fact that it is able to model environmental fluctuations appearing in many nanodevices based on semiconductors, metals, and superconductors [45–50]. Furthermore, it also represents the basic building block to describe noises of the type $1/f^\alpha$, which are responsible for decoherence in quantum solid-state devices [51–56]. These kinds of noise spectra stem from a collection of random telegraph sources with different switching rates, and they are usually referred to as *colored* spectra. The color of the noise depends upon the value of the parameter α [57]. For $\alpha = 1$, the so-called pink $1/f$ noise is found, which is obtained from a set of random telegraph fluctuators weighted by the inverse of the switching rate. Another interesting case is the $1/f^2$ spectrum, also called brown noise due to its relation to Brownian motion.

Environments characterized by $1/f^\alpha$ noise spectra usually arise when a system is coupled to a large number of bistable fluctuators, with a specific distribution of their switching rates. Upon considering a collection of fluctuators, the colored noise may be implemented by means of a linear combination of sources of RTN, each characterized by a specific switching rate chosen from a suitable distribution. On the other hand, the same spectrum can be obtained if we consider a single fluctuator with a random switching rate. As already observed by several authors [54,58,75], different microscopic configurations of the environment leading to the same spectra may correspond to different physical phenomena, e.g., different evolutions for the quantum correlations. In these cases, i.e., when knowledge of the noise spectrum is not sufficient to describe decoherence phenomena, the noise is referred to as non-Gaussian. In particular, we investigate the time evolution of entanglement and discord of two initially entangled noninteracting qubits coupled to a classical environment described either by means of a single random fluctuator or a collection of RTN sources. For the model of a single fluctuator with a random switching rate, the correlations decay with a damped oscillating behavior. In the configuration with many bistable fluctuators, pink noise leads to a monotonic decay of entanglement and discord, while the presence of brown noise induces phenomena of sudden death and revivals. We attribute these discrepancies to the different number of decoherence channels in the two

configurations. In other words, the time evolution of the system is determined not only by the spectrum of the environment, but also by its configurations, i.e., to its microscopic structure.

For both configurations, the dynamics of the two qubits is ruled by a stochastic Hamiltonian with time-dependent coupling. The average of the time-evolved states over the switching parameters describes the evolution of the two-qubit state under the effect of the noise. Upon a suitable choice of stochastic time-dependent terms in the Hamiltonian, we are able to describe the effects of both separate and common environments: in the former, each qubit is locally coupled to a random external signal, while in the latter both qubits are subject to the effect of a common classical environment. The interest in systems interacting with a common environment arises from recent experiments in cavity QED [59] and circuit QED [60,61] where reservoir-mediated interaction between two qubits is at the heart of entanglement generation.

Environmental effects due to a collection of bistable fluctuators have been experimentally observed in nanoscale electronic devices, where the single electron tunneling turns out to be affected by charge fluctuations [62,63]. Our results may thus be used as a valid guideline to analyze decoherence phenomena in Josephson circuits subject to noise stemming from fluctuating background charges and flux [64–67]. Other systems in which the noise is non-Gaussian and comes from a collection of two-level fluctuators include silicon [68] and magnetic ones [69], as well as vortex matter [70].

The paper is organized as follows: In Sec. II, we briefly review the definition of negativity as a measure of entanglement and of quantum discord. In Sec. III, we present the physical model for two qubits interacting with a classical environment, being represented either by a single random bistable fluctuator or a collection of fluctuators. Section IV reports results for the different configurations, and Sec. V closes the paper with a discussion and some concluding remarks.

II. ENTANGLEMENT AND QUANTUM DISCORD

In this section, we briefly review the concepts of entanglement and quantum discord between two qubits. In particular, we evaluate entanglement by means of negativity [71], which is given by

$$N = 2 \left| \sum_i \lambda_i^- \right|, \qquad (1)$$

where λ_i^- are the negative eigenvalues of the partial transpose of the system density matrix. Note that the negativity is bound between 0 for separable states, and 1 for maximally entangled states. The concept of negativity has recently been extended to the case of tripartite systems of identical particles [72].

The quantumness of correlations of a bipartite system may also be quantified by discord [2,73], namely the difference between the total and the classical correlations in a system. The quantum mutual information quantifies the total correlations in a system and is defined as $I = S(\rho^A) + S(\rho^B) - S(\rho)$, where $\rho^{A(B)}$ is the partial trace of the total bipartite system ρ and $S(\rho) = -\text{Tr}\rho \log_2(\rho)$ is the von Neumann entropy. The classical correlations are evaluated by means of the expression [73] $C = \max_{\{\Pi_j\}}[S(\rho^A) - S(\rho^A|\{\Pi_j\})]$, where $\{\Pi_j\}$ are

projective measurements on subsystem B and $S(\rho^A|\{\Pi_j\}) = \sum_j p_j S(\rho_j^A)$, with $\rho_j^A = \mathrm{Tr}_B[\Pi_j \rho \Pi_j]/\mathrm{Tr}[\Pi_j \rho \Pi_j]$ the remaining state of A after obtaining outcome j on B. Therefore, the quantum discord is the difference between the mutual information and the classical correlation:

$$Q = I - C. \tag{2}$$

Usually, the evaluation of quantum discord is not an easy task, since it involves an optimization problem. However, for two-qubit systems described by a density matrix of the form (from now on referred to as states with an X shape) $\rho = \frac{1}{4}(\mathbb{I} + \sum_{j=1}^{3} c_j \sigma_j \otimes \sigma_j)$, where σ_j are the three Pauli matrices, the optimization may be carried out analytically [74], leading to

$$
\begin{aligned}
Q = \frac{1}{4}[&(1 - c_1 - c_2 - c_3)\log_2(1 - c_1 - c_2 - c_3) \\
&+ (1 - c_1 + c_2 + c_3)\log_2(1 - c_1 + c_2 + c_3) \\
&+ (1 + c_1 - c_2 + c_3)\log_2(1 + c_1 - c_2 + c_3) \\
&+ (1 + c_1 + c_2 - c_3)\log_2(1 + c_1 + c_2 - c_3)] \\
&- \frac{1-c}{2}\log_2(1-c) - \frac{1+c}{2}\log_2(1+c), \tag{3}
\end{aligned}
$$

where $c := \max\{|c_1|, |c_2|, |c_3|\}$.

III. THE PHYSICAL MODEL

We address quantum correlations, both entanglement and discord, between two noninteracting qubits subjected to noisy environments. Our analysis is concerned with environments with noise spectra of the form $1/f^\alpha$, which are realized by different configurations of bistable fluctuators. The two qubits are initially prepared in the Bell state $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. The interaction between the qubits and the environment can be either local or global, i.e., we examine both the case of independent environments acting locally on each qubit and the case of a common environment affecting the two qubits. If we set $\hbar = 1$ and adopt the spin notation, the two-qubit Hamiltonian describing the interaction with a single fluctuator is given by

$$H(t) = H_A(t) \otimes \mathbb{I}_B + \mathbb{I}_A \otimes H_B(t), \tag{4}$$

where $H_{A(B)}$ is the Hamiltonian of a single qubit subject to a classical time-dependent noise which affects the transition amplitude parameter $c_{A(B)}(t)$:

$$H_{A(B)}(t) = \epsilon \mathbb{I}_{A(B)} + \nu c_{A(B)}(t)\sigma_{x\ A(B)}, \tag{5}$$

where ϵ is the qubit energy in the absence of noise (energy degeneracy is assumed), $\mathbb{I}_{A(B)}$ is the identity matrix for subspace $A(B)$, ν is the coupling constant between the system and the environment, and σ_x is the Pauli matrix. If the time-dependent coefficient $c_{A(B)}(t)$ can randomly flip between two values $c(t) = \pm 1$ with a fixed rate γ, then Eq. (5) describes a qubit subject to a random telegraph noise [38,40,43,44,75,76]. This Hamiltonian, extended to describe qudits, has also been used to analyze the time evolution of entanglement of a continuous-time quantum walk of two indistinguishable particles on a one-dimensional lattice [77]. The Hamiltonian (4) is stochastic due to the random nature of the noise parameter $c(t)$. For a specific choice of $c(t)$,

the total system evolves according to the evolution operator $e^{-i\int H(t')\,dt'}$, with positivity ensured by the very structure of the Hamiltonian in Eq. (5) [78]. By averaging the global state over different realizations of the sequences of $c(t)$, the two-qubit mixed state is obtained.

To reproduce the $1/f^\alpha$ spectrum, the single RTN frequency power density must be integrated over the switching rates γ with a proper distribution:

$$S_{1/f^\alpha}(f) = \int_{\gamma_1}^{\gamma_2} S_{\mathrm{RTN}}(f,\gamma) p_\alpha(\gamma)\,d\gamma, \tag{6}$$

where $S_{\mathrm{RTN}}(f,\gamma)$ is the random telegraph noise frequency spectral density with Lorentzian form $S_{\mathrm{RTN}}(f,\gamma) = 4\gamma/(4\pi^2 f^2 + \gamma^2)$. The integration is performed between a minimum and a maximum value of the switching rates, respectively γ_1 and γ_2. $p(\gamma)$ is the switching rate distribution and takes a different form depending on the kind of noise:

$$
p_\alpha(\gamma) = \begin{cases} \frac{1}{\gamma\,\ln(\gamma_2/\gamma_1)}, & \alpha = 1, \\ \frac{(\alpha-1)}{\gamma^\alpha}\left[\frac{(\gamma_1\gamma_2)^{\alpha-1}}{\gamma_2^{\alpha-1} - \gamma_1^{\alpha-1}}\right], & 1 < \alpha \leqslant 2. \end{cases} \tag{7}
$$

It follows that, in order to simulate a frequency spectrum proportional to $1/f^\alpha$, the switching rates must be selected from a distribution proportional to $1/\gamma^\alpha$. When the integration in Eq. (6) is performed, the spectrum has the requested $1/f^\alpha$ behavior in a frequency interval, so that every frequency belonging to such an interval satisfies the condition $\gamma_1 \ll f \ll \gamma_2$. Equation (6) can be obtained either considering a single bistable fluctuator whose switching rate is randomly chosen from a distribution $p_\alpha(\gamma)$ or from a collection of sources of RTN each with a switching rate taken from the same γ distribution. Even if the spectrum is the same, the physical systems described are indeed very different.

A. $1/f^\alpha$ noise from a single fluctuator

In the case of a single fluctuator, the noise parameter $c(t)$ can only flip between two values ± 1. The difference with the RTN case is that here the switching rate is not known *a priori*. This means that the bistable fluctuator is described by a statistical mixture whose elements are chosen from the ensemble $\{\gamma, p_\alpha(\gamma)\}$. This model describes the physical situation in which each of the two qubits interacts with a single fluctuator for separate baths, or with the same fluctuator in the case of a common bath. The qubits are affected only by one source of noise, and therefore only one decoherence channel is present.

The global system evolves according to the Hamiltonian (4), with a specific choice of both the parameter $c(t)$ and of its switching rate. In particular, for each value of the switching rate picked from a distribution $p_\alpha(\gamma)$, we create a large number of sequences $c(t)$. This means that we are actually introducing some uncertainty on the rate γ. This uncertainty gives rise to a $1/f^\alpha$ spectrum. For each selected switching rate, the evolution corresponds to that of a bistable fluctuator with a characteristic RTN phase shown to be

$$\varphi_{A(B)}(t) = -\nu \int_0^t c(t')dt' \tag{8}$$

and characterized by a distribution [54,75]

$$p(\varphi,t) = \frac{1}{2}e^{-\gamma t}\left\{[\delta(\varphi+\nu t)+\delta(\varphi-\nu t)]\right.$$
$$+\frac{\gamma}{\nu}[\Theta(\varphi+\nu t)+\Theta(\varphi-\nu t)]$$
$$\left.\times\left[\frac{I_1[\gamma t\sqrt{1-(\varphi/\nu t)^2}]}{\sqrt{1-(\varphi/\nu t)^2}}+I_0[\gamma t\sqrt{1-(\varphi/\nu t)^2}]\right]\right\},$$
$$\tag{9}$$

where $I_\nu(x)$ is the modified Bessel function and $\Theta(x)$ is the Heaviside step function. For a given γ, the global system is described by an X-shaped density matrix obtained by averaging over the noise phase the density matrix $\rho(\varphi,\gamma,t)$ corresponding to a specific choice of the parameter $c(t)$ [76]:

$$\rho(\gamma,t) = \int \rho(\varphi,\gamma,t)p(\varphi,t)d\varphi = \frac{1}{2}[(1+\beta_{de(ce)})|\phi^+\rangle\langle\phi^+|$$
$$+(1-\beta_{de(ce)})|\psi^+\rangle\langle\psi^+|], \tag{10}$$

where $|\psi^+\rangle = (|01\rangle+|10\rangle)/\sqrt{2}$ is a Bell state and $\beta_{de} = D_{2\nu}^2(t)$ and $\beta_{ce} = D_{4\nu}(t)$ are time-dependent coefficients. The function $D_{m\nu}(t)$ represents the average of the RTN phase factor, i.e.,

$$\langle e^{im\varphi(t)}\rangle = \int e^{im\varphi(t)}p(\varphi,t)d\varphi = D_{m\nu}(t), \tag{11}$$

where

$$D_{m\nu}(t) = \begin{cases} e^{-\gamma t}\left[\cosh(\kappa_{m\nu}t)+\frac{\gamma}{\kappa_{m\nu}}\sinh(\kappa_{m\nu}t)\right], \\ e^{-\gamma t}\left[\cos(\kappa_{m\nu}t)+\frac{\gamma}{\kappa_{m\nu}}\sin(\kappa_{m\nu}t)\right] \end{cases} \tag{12}$$

for $\gamma \geqslant m\nu$ and $\gamma \leqslant m\nu$, respectively, where $\kappa_{m\nu} = \sqrt{|\gamma^2-(m\nu)^2|}$ with $m \in \{2,4\}$. The two-qubit density matrix is obtained by averaging the density operator in Eq. (10) over the switching rates:

$$\overline{\rho}_{de}(t) = \int_{\gamma_1}^{\gamma_2}\int_{\gamma_1}^{\gamma_2}\rho_{de}(\gamma,t)\,p_\alpha(\gamma_A)p_\alpha(\gamma_B)\,d\gamma_A d\gamma_B, \tag{13}$$

$$\overline{\rho}_{ce}(t) = \int_{\gamma_1}^{\gamma_2}\rho_{ce}(\gamma,t)\,p_\alpha(\gamma)\,d\gamma. \tag{14}$$

Once the average over γ is performed, the time-evolved density matrix reads

$$\overline{\rho}_{de(ce)}(t) = \frac{1}{2}[(1+\Lambda_{de(ce)})|\phi^+\rangle\langle\phi^+|$$
$$+(1-\Lambda_{de(ce)})|\psi^+\rangle\langle\psi^+|], \tag{15}$$

where the time-dependent coefficient $\Lambda_{de(ce)}$ can be written as

$$\Lambda_{de} = \left[\int_{\gamma_1}^{\gamma_2}D_{2\nu}\,p_\alpha(\gamma)\,d\gamma\right]^2,$$
$$\Lambda_{ce} = \int_{\gamma_1}^{\gamma_2}D_{4\nu}\,p_\alpha(\gamma)\,d\gamma. \tag{16}$$

This means that the quantum system is again described by a density matrix with an X form.

B. $1/f^\alpha$ noise from a collection of fluctuators

The $1/f^\alpha$ noise spectrum can arise from the coupling of a system with a large number of fluctuators, each characterized by a specific switching rate, picked from the distribution $p_\alpha(\gamma)$

[55,79] in a range $[\gamma_1,\gamma_2]$. In this case, the random parameters in Eq. (5) describe a linear combination of bistable fluctuators $c(t) = \sum_{j=1}^{N_f}c_j(t)$, where N_f is the number of fluctuators and we drop the subscript $A(B)$ to simplify the notation. Each $c_j(t)$ has a Lorentzian power spectrum whose sum gives the power spectrum of the noise:

$$S(f) = \sum_{j=1}^{N_f}S_j(f;\gamma_j) = \sum_{j=1}^{N_f}\frac{\gamma_j}{\gamma_j^2+4\pi^2 f} \propto \frac{1}{f^\alpha}. \tag{17}$$

In fact, the sum $\sum_{j=1}^{N_f}S_j(f;\gamma_j)$, with the γ_j belonging to the distribution $p_\alpha(\gamma_j)$, can be viewed as the Monte Carlo sampling of the integral $N_f\int_{\gamma_1}^{\gamma_2}S(f)p_\alpha(\gamma)\,d\gamma$, which is, but for a constant, the one in Eq. (6). Therefore, in order to obtain a $1/f^\alpha$ spectrum, it is necessary to consider a sufficiently large number of fluctuators, and that the selected γ_j are a representative sample of the distribution $p_\alpha(\gamma_j)$ in the range $[\gamma_1,\gamma_2]$. This means that the minimum number of fluctuators we can sum up depends on the range of integration. In fact, a large number of fluctuators is required to sample the distribution of the switching rates over a large range, while few fluctuators are sufficient in the case of a narrow range. Note that we assume that all the fluctuators have the same coupling constant with the environment, that is, $\nu_j = \nu$ for $j = 1,\ldots,N_f$.

The global evolution operator $U(t) \propto e^{-i\sum_j\varphi_j(t)}$ for fixed values of the parameters associated with each fluctuator permits us to compute the density matrix of the global system as a function of a total noise phase $\varphi(t) = \sum_j\varphi_j(t)$. Following the approach used in [76] to evaluate the dynamics of two qubits subject to a single RTN, the time-evolved density matrix of the system at time t can be expressed as

$$\overline{\rho(t)} = \int \rho(\varphi,t)p_T(\varphi,t)d\varphi, \tag{18}$$

where $p_T(\varphi,t) = \prod_j p(\varphi_j,t)$ is the global noise phase distribution. The density matrix in Eq. (18) depends on the average of the phase factor $e^{im\varphi(t)}$, which can be computed in terms of the RTN coefficient $D(t)$ of Eq. (12). Note that for the sake of simplicity, we omitted the subscripts $m\nu$ in writing the $D(t)$ coefficient.

We have

$$\langle e^{im\varphi(t)}\rangle = \left\langle\prod_j e^{im\varphi_j}\right\rangle = \prod_j D_j(t), \tag{19}$$

where the last equality holds since the RTN phase coefficients are independent. By inserting Eq. (19) into Eq. (18), we evaluate the two-qubit density matrix:

$$\overline{\rho}_{de(ce)}(t,\{\gamma_j\}) = \frac{1}{2}[(1+\Gamma_{de(ce)})|\phi^+\rangle\langle\phi^+|$$
$$+(1-\Gamma_{de(ce)})|\psi^+\rangle\langle\psi^+|]. \tag{20}$$

The coefficients appearing in the density matrix are

$$\Gamma_{de} = \prod_j D_{jA}D_{jB}(t) \quad\text{with}\quad D(t) = D_{2\nu}(t),$$
$$\Gamma_{ce} = \prod_j D_j(t) \quad\text{with}\quad D(t) = D_{4\nu}(t). \tag{21}$$

Note that the fluctuators have fixed switching rates $\{\gamma_j\}$, $j = 1, \ldots, N_f$.

C. $1/f^\alpha$ noise from a collection of fluctuators with random switching rates

For the sake of completeness, here we report also a third scenario which describes an environment with a $1/f^\alpha$ spectrum. This is the case of two qubits subject to a collection of fluctuators whose switching rates are known with an uncertainty determined by the $1/\gamma^\alpha$ distribution in the finite range $[\gamma_1, \gamma_2]$. The sources of RTN do not have a fixed γ, so they are described by an ensemble of many fluctuators. The dynamics is evaluated by averaging the two-qubit density matrix for specific values of the switching rates (20) over the γ in a range $[\gamma_1, \gamma_2]$:

$$\overline{\rho}_{de(ce)}(t) = \int_{\gamma_1}^{\gamma_2} \overline{\rho}_{de(ce)}(t, \{\gamma_j\}) p_\alpha(\{\gamma_j\}) \, d\{\gamma_j\}, \qquad (22)$$

where $p_\alpha(\{\gamma_j\}) = \prod_j p_\alpha(\gamma_j)$ and $d\{\gamma_j\} = \prod_j d\gamma_j$. The time evolution also in this case preserves the X shape of the density matrix and the evolved state has the same functional expression of Eq. (20), but with coefficients

$$\Gamma_{de(ce)} \rightarrow \Gamma'_{de(ce)} = [\Lambda_{de(ce)}]^{N_f}, \qquad (23)$$

where $\Lambda_{de(ce)}$ are given in Eq. (16).

IV. RESULTS

In this section, we present analytical expressions for the negativity and the quantum discord for qubits interacting with a single random fluctuator and a collection of N_f bistable fluctuators, with either fixed or random switching rates. As we will see in detail in the following, negativity and discord show the same qualitative behavior in all the analyzed regimes. The reason for this behavior lies in two features of the physical systems we are addressing: (a) the choice of a maximally entangled state as the initial state of the two qubits, and (b) the sheer dephasing nature of the interaction. Overall, these two facts imply that the evolved state is a mixture of Bell states. For this class of states, discord is a function of negativity only, as follows straightforwardly from Eqs. (1) and (3).

A. Single fluctuator

In the case of two qubits interacting with a single random fluctuator, the dynamics depends upon the selected range $[\gamma_1, \gamma_2]$ of the switching rate. Following the definition of negativity in Eq. (1), the negativity reads

$$N_{de}(t) = \Lambda_{de}(t), \qquad (24)$$

$$N_{ce}(t) = |\Lambda_{ce}(t)|, \qquad (25)$$

where the Λ's are given in Eq. (16). Since the density matrix preserves its X shape during the time evolution, the discord can be evaluated using Eq. (3):

$$Q_{de(ce)}(t) = h(\Lambda_{de(ce)}), \qquad (26)$$

where

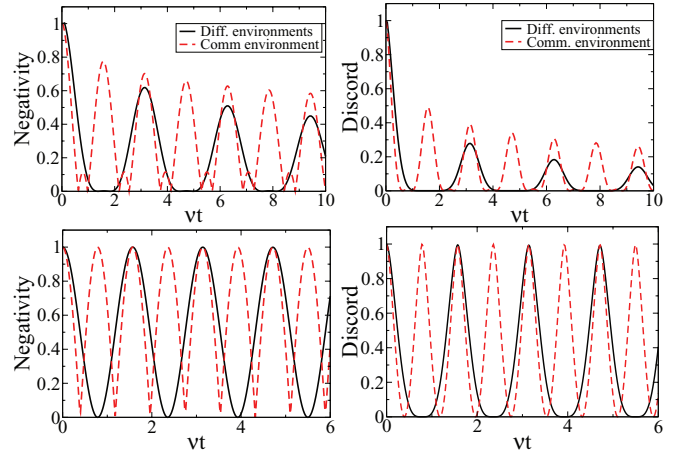$$h(x) = \tfrac{1}{2}[(1 + x) \log_2 (1 + x) + (1 - x) \log_2 (1 - x)].$$

FIG. 1. (Color online) Upper panels: Time evolution of negativity (left) and discord (right) for two qubits interacting with a single random bistable fluctuator with spectrum $1/f$ for different (solid line) and common (dashed line) environments when $[\gamma_1, \gamma_2]/\nu = [10^{-4}, 10^4]$. Bottom panels: Time evolution of negativity (left) and discord (right) for two qubits interacting with a single random fluctuator with spectrum $1/f^2$ for different (solid line) and common (dashed line) environments when $[\gamma_1, \gamma_2]/\nu = [10^{-4}, 10^4]$.

The dynamics of N and D is shown in Fig. 1 for the cases of pink and brown noise. The range of integration is $[10^{-4}, 10^4]/\nu$. Negativity and discord have been obtained analytically, except for the integral $\Lambda_{de(ce)}$, which must be computed numerically. Quantum correlations decay with damped oscillations. In the case of a qubit subject to different environments with a $1/f$ spectrum, the oscillations have a periodicity of π. This periodicity can be explained by analyzing the analytical expressions of the quantum correlations. In particular, we note that in the integral of Eq. (16), the $D_{2\nu}$ functions exhibit damped oscillations for $\gamma < 2\nu$ with periodicity $2\pi/\kappa_{m\nu}$, and for $\gamma > 2\nu$ they decay monotonically. Their weighted superposition leads to an interference effect that can be summarized as follows: the oscillating components result in the formation of alternatively positive and negative peaks spaced by $\pi/2$. On the other hand, the monotonic decaying components combine to cancel the negative peaks and to preserve the positive ones. Finally, we are left with an oscillating function with a periodicity of π. The same concept applies in the case of a common environment, but now the $D_{4\nu}$ sum up in an oscillating function with periodicity of $\pi/2$.

The $1/f^2$ noise spectrum leads to oscillating functions of time with periodicity $\pi/2$ and $\pi/4$ for different and common environments, respectively. Again, this periodicity is related to the fact that with such a distribution, the selected values of γ accumulate near the lower value of the frequency range, thus leading to a beat phenomenon with constructive interference with the above-mentioned periodicity. If different ranges of integration are considered, a different time behavior for the quantum correlations can arise, but we will not analyze this effect in this paper.

B. Collection of fluctuators

In the case of two qubits interacting with a collection of fluctuators with fixed switching rates, the dynamics is
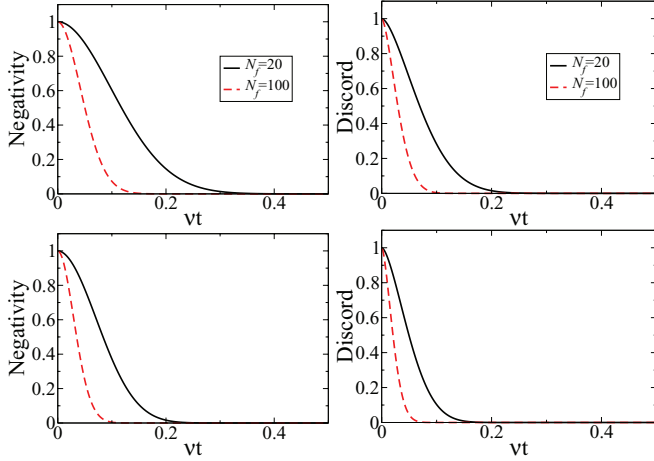
FIG. 2. (Color online) Upper panels: Time evolution of negativity (left) and discord (right) for two qubits interacting with two independent environments, consisting in a collection of N_f bistable fluctuators with spectrum $1/f^2$ when $[\gamma_1, \gamma_2]/\nu = [10^{-4}, 10^4]$. Lower panels: same as before, but with qubits subject to a common environment.
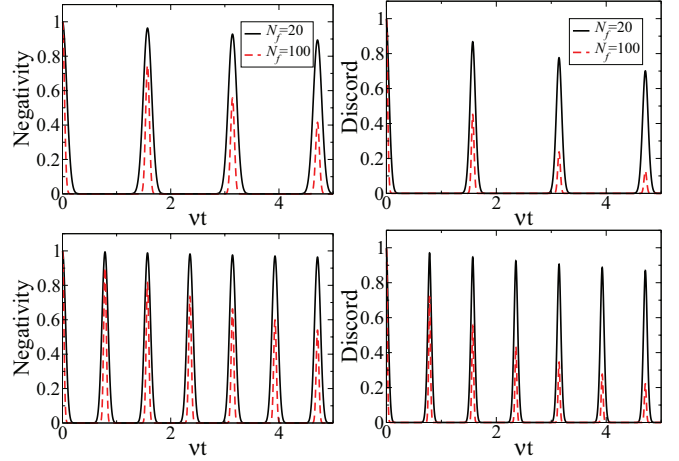

FIG. 3. (Color online) Upper panels: Time evolution of negativity (left) and discord (right) for two qubits interacting with two independent environments, consisting in a collection of N_f bistable fluctuators with spectrum $1/f^2$ when $[\gamma_1, \gamma_2]/\nu = [10^{-4}, 10^4]$. Lower panels: same as before, but with qubits subject to a common environment.

very different, depending on the spectrum of the noise. The analytical expression for the negativity and the discord is computed starting from the density matrix in Eq. (20). The negativity reads

$$N_{de}(t) = |\Gamma_{de}(t)|, \quad N_{ce}(t) = |\Gamma_{ce}(t)|, \tag{27}$$

where the Γ's are given in Eq. (21). Also in this case the density matrix preserves the X form during time evolution, such that the discord is calculated by applying the Luo formula (3):

$$Q_{de(ce)}(t) = h(\Gamma_{de(ce)}). \tag{28}$$

The negativity is the product of many oscillating coefficients, with various periodicities. In the case of switching rates taken from a $1/\gamma$ distribution, the product of these terms results in a monotonic decay for both entanglement and discord. In Fig. 2, we report the behavior of such quantities in the case of 20 and 100 fluctuators. As the number of fluctuators is increased, the quantum correlations decay faster. We consider 20 sources of RTN as the minimum number of fluctuators needed to obtain both a reliable profile of the frequency spectrum and a representative sample of the $p(\gamma)$ distribution. Although it is possible to obtain a pink noise spectrum even with a smaller number of fluctuators, this approximation does not describe a sample of $1/\gamma$-distributed switching rates. A very different behavior arises when the γ's are selected from a $1/\gamma^2$ distribution; see Fig. 3. Phenomena of sudden death and revivals appear for both entanglement and discord. As the number of fluctuators is increased, the heights of the peaks decrease. The peaks have a periodicity of $\pi/2$ and $\pi/4$ for different and common environments, respectively. As in the case of the single random fluctuator, this is explained by considering that the selected switching rates have small and very close values. The product of functions with almost the same periodicity gives a periodic behavior with narrow peaks.

Our results clearly show that mere knowledge of the spectrum is not sufficient to determine the dynamical evolution of correlations. Indeed, the number of decoherence channels also

plays a key role. Different physical models of environments can lead to the same spectrum. But, if the two-qubit system interacts with only one decoherence channel, then revivals appear because the system is affected only by one source of classical noise and the information can flow back. If many sources of decoherence are present, then the information can be completely lost depending on the channel characteristics, that is, the distribution of the switching rates.

C. Collection of fluctuators with random switching rates

For the sake of completeness, we report also the expressions for negativity and discord in the case of two qubits subject to a collection of bistable fluctuators with stochastic switching rates. From Eq. (23), we can write the negativity and discord as

$$N_{de}(t) = \Gamma'_{de}(t) = [\Lambda_{de(ce)}]^{N_f}, \tag{29}$$

$$N_{ce}(t) = |\Gamma'_{ce}(t)| = |[\Lambda_{de(ce)}]^{N_f}|, \tag{30}$$

$$Q_{de(ce)}(t) = h(\Gamma'_{de(ce)}). \tag{31}$$

The quantum correlations decay exponentially in the case of $1/f$ noise and with smooth damped oscillations in the case of $1/f^2$ noise. The latter is the most general case, in which the collection of fluctuators is described by a statistical ensemble. Also in this scenario, brown noise leads to nonmonotonic decay of negativity and discord, since all the selected $D_{m\nu}(t)$ terms have similar periodicity and beat effects arise, with constructive interference.

V. CONCLUSIONS

We have analyzed in detail the dynamics of quantum correlations, entanglement, and discord for a two-qubit system initially prepared in a maximally entangled state and then subjected to classical noise, i.e., coupled with an external environment consisting in a collection of classical bistable fluctuators. In particular, due to its relevance for solid-state

devices, we have addressed environments characterized by a $1/f^\alpha$ noise spectrum. We analyzed both the case of independent environments associated with each qubit and the case of a common environment, and we have taken into account explicitly the structure of the environment. More specifically, we have analyzed distinct configurations of the environment, characterized by the same $1/f^\alpha$ spectrum, but with a different number of decoherence channels. Analytical expressions have been found for the negativity and the discord, showing the same qualitative behavior in all the analyzed regimes. Indeed, the choice of a maximally entangled initial state together with the sheer dephasing nature of the interaction with the environment make discord a function of negativity only. In these systems, in fact, the evolved state is a mixture of Bell states.

In the first configuration we have addressed, the two qubits interact with a single bistable fluctuator, which has a random switching rate leading to an overall $1/f^\alpha$ spectrum. An oscillating behavior of quantum correlations for both pink and brown noise has been found. The effect of a common environment is that of better preserving the correlations and to double the number of revivals compared to the case of independent baths. In the presence of pink noise, the peaks of negativity and discord decay faster than in the presence of brown noise. In fact, the switching rates are more evenly distributed over the range $[\gamma_1, \gamma_2]$ than in the case of the $1/f^2$ noise, thus leading to massive destructive interference. In the second configuration, the two qubits interact with a collection of bistable fluctuators, each one with fixed switching rate chosen from a distribution $1/\gamma^\alpha$; also in this case, the overall noise spectrum is $1/f^\alpha$. In the case of pink noise, entanglement and discord show a monotonic decay, while for brown noise, sudden death and revivals occur. Quantum correlations can be written as a product of oscillating and exponential functions. Since the switching rates of the fluctuators are selected from

a distribution $1/f^\alpha$, for the pink noise they lead to destructive interference, while brown noise enhances constructive interference. The action of independent or common environments has different effects on the robustness of correlations, and this agrees with previous results obtained with different noise models [29,34].

Our results clearly show that the behavior of quantum correlations is influenced not only by the spectrum of the environment but also by the number of decoherence channels, i.e., by the very structure of the environment. With a single decoherence channel, revivals of correlations appear, indicating that a backflow of information from the environment to the system is possible. When the number of decoherence channels is increased, the information is quickly lost, and no revivals can occur.

The characterization and the control of quantum correlations are fundamental for the development of quantum technology. Not only do quantum correlations constitute a resource to process quantum information, but a deep understanding of their nature will provide better insight into the nature of quantum states themselves and the transition between the quantum and the classical description of physical systems. Our results indicate that the microscopic structure of the environment, in addition to its noise spectrum, is relevant for the dynamics of quantum correlations and may be a valid starting point for the engineering of colored non-Gaussian environments.

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