Localized detection of quantum entanglement through the event horizon

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(Received 18 July 2012; published 28 May 2013)

We present a localized solution to the problem of entanglement degradation in noninertial frames. A two-mode squeezed state is considered from the viewpoint of two observers, Alice (inertial) and Rob (accelerated), each observing a single localized mode of the field. We study the state of these modes to determine how much entanglement the observers can extract from the initial state. The dominant source of degradation is an inevitable mode mismatch between the mode of the squeezed state Rob is given and the mode he is able to observe from his accelerated frame. Leakage of the initial mode through Rob's horizon places a limit on his ability to fully measure the state, leading to an inevitable degradation of entanglement that even in principle cannot be fully retrieved by any measurement device.

DOI: 10.1103/PhysRevA.87.052326 PACS number(s): 03.67.Mn, 03.65.Ud, 03.30.+p, 04.62.+v

I. INTRODUCTION

The semiclassical combination of gravity with quantum theory that leads to the apparent paradox of information loss in black holes [1] naturally raises questions about the robustness of basic resources in quantum information theory in the presence of strong gravitational fields. Perhaps the most fundamental of these questions can be stated as follows: "How is the entanglement in the state of a quantum field affected by noninertial motion of the observer?"

The pioneering works that attempted to answer this question involved nonlocalized states, usually plane waves [2,3] or Unruh modes [4] that spanned through the whole space-time (for a list of other works using Unruh modes, see Ref. [5]). However, the observed entanglement degradation [4] was mostly due to a particular acceleration-dependent parametrization of the initial state in terms of Unruh modes that masked the true effects of the acceleration itself. The origin of this issue is traced back to the inability to control the size of the global mode and the location of its observation, thus leaving the physical interpretation of the setting unclear.

In this work we present a localized solution to the question of entanglement degradation due to uniformly accelerated motion. Since the observations of quantum fields are localized in space, our setup is in principle implementable. Our approach leads to interesting new insights into the nature of the degradation process.

For example, we find perhaps counterintuitively that for low accelerations the thermal noise of the vacuum state is not the dominant cause of the degradation of entanglement. Rather, the entanglement loss is traced back to an inevitable mode mismatch that cannot be corrected. This limitation arises because the accelerated observer cannot observe field modes outside his event horizon.

II. SETUP

For simplicity we work with a massless two-dimensional (2D) scalar field. Two inertial observers Alice and Bob, at rest with respect to each other, prepare an entangled two-mode squeezed state,

$$\hat{S}_{\mathrm{AB}}|0\rangle_{\mathrm{M}} = \exp[s(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b})]|0\rangle_{\mathrm{M}}, \tag{1}$$

where the annihilation operators $\hat{a}$ and $\hat{b}$ are associated with two localized, orthogonal, and spatially separated field modes $\phi_{\mathrm{A}}(x,t)$ and $\phi_{\mathrm{B}}(x,t)$ respectively. In order that these modes form well-defined annihilation operators we demand that the wave packets be superpositions of positive Minkowski frequencies only. Such states well approximate entangled states commonly obtained via parametric down conversion in nonlinear crystals [6] that have been used, for example, in violations of Bell's inequality experiments [7].

In an inertial frame the entanglement in these states can be detected by projective measurements of quadratures carried out independently by Alice and Bob. The entanglement can then be determined by calculating the entanglement logarithmic negativity [8]. The question that we study in the present work is the following: What happens when one of the modes is actually observed by a relativistically accelerated observer, Rob instead of inertial Bob. In this case one expects that the entanglement accessible to the two parties changes due to a relativistic transformation acting on the accelerated subsystem, effectively squeezing the Minkowski vacuum state [9].

In order to investigate this noninertial effect we implicitly have in mind a scheme introduced in Ref. [10] that focuses on a single, localized mode of the field studied by Rob. However, it is sufficient in what follows to work at the abstract level of wave packets, where we invariably use ϕ to denote the mode that the state is prepared in and ψ to denote the mode that Rob is observing. In what follows we use the conformal Rindler coordinates $(c\tau,\xi)$ parameterized by a:

$$ct = \frac{c^2}{a}e^{a\xi/c^2}\sinh\frac{a\tau}{c}, \quad x = \frac{c^2}{a}e^{a\xi/c^2}\cosh\frac{a\tau}{c}, \tag{2}$$

to cover the region $x > c|t|$. Rob is assumed to travel along the world line $\xi = 0$.

We suppose that Alice (Rob) can carry measurements only on a spatially localized single mode of the field $\psi_{\mathrm{A}}(x,t)$

$[\psi_B(\xi,\tau)]$ with a corresponding annihilation operator $\hat{d}_A$ ($\hat{d}_B$), which is assumed to be a superposition of positive Minkowski (Rindler) frequencies. We assume Rob only makes measurements on Bob's part of the state. At $t = 0$ some part of Bob's mode lies beyond Rob's event horizon; therefore we immediately notice that Rob's accelerated motion limits his ability to measure this part of the initial state.

The effective state accessible to Alice and Rob is the mixed Gaussian state formed by taking the original entangled state and tracing out all the modes orthogonal to ψ_A and ψ_B. However, we find in practice that this trace does not need to be done.

Any Gaussian state is fully characterized by its covariance matrix σ. For simplicity we study the state at $t = 0$ when Rob's velocity is zero and Doppler-shift effects [11] do not arise. Since the state is Gaussian, it is enough to measure two orthogonal quadratures of each mode, $\hat{x}_l = \frac{1}{\sqrt{2}}(\hat{d}_l + \hat{d}_l^\dagger)$ and $\hat{p}_l = \frac{1}{\sqrt{2}i}(\hat{d}_l - \hat{d}_l^\dagger)$, where $l \in \{A,B\}$, and construct the correlations between the measurement outcomes:

$$\sigma_{ij} \equiv \langle \hat{X}_i\hat{X}_j + \hat{X}_j\hat{X}_i\rangle, \tag{3}$$

where $\hat{\boldsymbol{X}} = (\hat{x}_A,\hat{p}_A,\hat{x}_B,\hat{p}_B)$.

Details of the calculation of the covariance matrix of the state (1) as viewed by Alice and Rob are provided in the Appendix. We find that this covariance matrix only depends on three scalar products, $\alpha \equiv (\psi_A,\phi_A)$, $\beta \equiv (\psi_B,\phi_B)$, $\beta' \equiv (\psi_B,\phi_B^\star)$, and the average number of particles seen by an accelerated detector in the vacuum, $\langle\hat{n}\rangle_U$ (see Eq. (A7) and also Ref. [10]). We find that

$$\sigma = \mathbb{1} + 2\langle\hat{n}\rangle_U \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} + 2\sinh^2 s \begin{pmatrix} |\alpha|^2 & 0 & 0 & 0\\ 0 & |\alpha|^2 & 0 & 0\\ 0 & 0 & |\beta+\beta'^\star|^2 & 2\,\mathrm{Im}(\beta\beta')\\ 0 & 0 & 2\,\mathrm{Im}(\beta\beta') & |\beta-\beta'^\star|^2 \end{pmatrix}$$
$$+ \sinh 2s \begin{pmatrix} 0 & 0 & -\mathrm{Re}[\alpha(\beta+\beta'^\star)] & -\mathrm{Im}[\alpha(\beta-\beta'^\star)]\\ 0 & 0 & -\mathrm{Im}[\alpha(\beta+\beta'^\star)] & \mathrm{Re}[\alpha(\beta-\beta'^\star)]\\ -\mathrm{Re}[\alpha(\beta+\beta'^\star)] & -\mathrm{Im}[\alpha(\beta+\beta'^\star)] & 0 & 0\\ -\mathrm{Im}[\alpha(\beta-\beta'^\star)] & \mathrm{Re}[\alpha(\beta-\beta'^\star)] & 0 & 0 \end{pmatrix}. \tag{4}$$

When the squeezing parameter $s \to 0$, the state becomes the usual Minkowski vacuum $|0\rangle_M$ and, unlike with Unruh-DeWitt detectors [12], we find that no correlations are present between Alice and Rob. This is because our detector is assumed to operate on a particle absorption principle [10]. This situation should also be distinguished from the case when Alice and Rob are counteraccelerating with equal magnitude; in that case correlations do exist [10].

III. ENTANGLEMENT DEGRADATION

In order to quantify the nonlocal correlations between Alice's and Rob's modes we use the logarithmic negativity, which constitutes an upper bound to the distillable entanglement. It can be completely calculated from the elements of the covariance matrix σ [8]:

$$E_{\mathcal{N}} = \mathrm{Max}\left[0, -\ln\sqrt{\frac{\Delta - \sqrt{\Delta^2 - 4\det\sigma}}{2}}\right], \tag{5}$$

where $\Delta = \sigma_{11}\sigma_{22} - \sigma_{12}^2 + \sigma_{33}\sigma_{44} - \sigma_{34}^2 - 2\sigma_{13}\sigma_{24} + 2\sigma_{14}\sigma_{23}$.

When Rob's proper acceleration vanishes and the modes ϕ_l and ψ_l (for $l \in \{A,B\}$) are chosen to match perfectly, we have $\alpha = \beta = 1$, $\langle\hat{n}\rangle_U = \beta' = 0$ and then $E_{\mathcal{N}} = 2s$. For nonzero proper accelerations Eq. (4) can be substituted into Eq. (5) and studied numerically. In order to perform this calculation, specific forms of $\phi_A(x,t)$, $\phi_B(x,t)$, $\psi_A(x,t)$, and $\psi_B(\xi,\tau)$ need to be chosen. Since we consider measurements carried out at $t = \tau = 0$, it is sufficient to specify the modes by their wave packets and their first derivatives at this time only. Since Alice is inertial we assume the idealized scenario when she observes the mode $\hat{a}$ of the entangled state, i.e., $\alpha = 1$. Any degradation of entanglement therefore will be a consequence of the specification of Rob's mode.

We model the functional shape of Bob's initial state mode with the Gaussian

$$\phi_B(x,0) = \frac{1}{\sqrt{N\sqrt{2\pi}}}\exp\left[-\frac{x^2}{L^2} + i\frac{N}{L}x\right],$$
$$\partial_t\phi_B(x,0) = -i\frac{Nc}{L}\phi_B(x,0), \tag{6}$$

but modify it with a low-frequency cutoff, eliminating all frequencies characterized by $k < \frac{1}{2L}$. This removes an infrared divergence in the spectrum at zero wave number but does not appreciably change the shape or localization of the mode near $t = 0$. Here, N is the characteristic frequency about which the mode is centered. For convenience we take it as a large natural number (>3), which ensures the component of negative Minkowski frequency plane waves present in ψ_B is negligible. L is the spacial width of the localized mode and in combination with Rob's acceleration the dimensionless quantity, $\frac{aL}{c^2}$, will set the scale at which entanglement degradation effects become important.

For each acceleration Rob is given an identical initial state. However, his position, $x(0) = \frac{c^2}{a}$, is acceleration dependent and therefore the initial mode must be translated according to $\phi_B(x,0) \to \phi_B(x - \frac{1}{a},0)$. It is important to realize that such repositioning does not change the initial state; it is merely a computational convenience allowing us to describe each acceleration using a single Rindler coordinate chart. Alternatively, the initial state could be kept fixed and the origin of the Rindler coordinate chart could be adjusted such that Rob passes through the center of the mode for each acceleration.

Having chosen a localized initial state, there remains the question of choosing Rob's mode. As mentioned, inertial observers such as Alice can always make the idealized assumption effectively setting $\alpha = 1$, but in Rob's case such a setting is immediately ruled out by the existence of negative Rindler frequencies in ϕ_B. Thus some alternative choice must be made. Let us first try a mode that (apart from the spacial translation) is obtained by replacing the spacial coordinate x with the conformal Rindler coordinate ξ and replacing L with the appropriate length in the conformal coordinates, $\widetilde{L} = \frac{2c^2}{a}\operatorname{asinh}(\frac{aL}{2c^2})$. As a result,

$$\psi_B(\xi,0) = \frac{1}{\sqrt{N\sqrt{2\pi}}}\exp\left[-\frac{\xi^2}{\widetilde{L}^2} + i\frac{N}{\widetilde{L}}\xi\right],$$
$$\partial_\tau\psi_B(\xi,0) = -i\frac{Nc}{\widetilde{L}}\psi_B(\xi,0). \tag{7}$$

Again we assume a low frequency cutoff and take N large, producing an annihilation mode with the correct properties in the Rindler frame. This exemplary choice of Rob's mode has been physically motivated in Ref. [10]. To ensure that the acceleration is approximately uniform over the support of the mode, we also assume that $\frac{a\widetilde{L}}{c^2} \ll 1$, or equivalently that $\widetilde{L} \approx L$. In this limit, using Eqs. (6) and (7) we obtain analytic estimates for the scalar products appearing in the covariance matrix:

$$|\beta| \approx \left(1 + \left(\frac{NaL}{4c^2}\right)^2\right)^{-1/4}, \tag{8}$$

and $\beta' \approx 0$. Since $|\beta| < 1$ for nonzero accelerations, a component of the degradation of entanglement comes from a mode mismatch between Rob's mode and the mode ϕ_B that he observes. Indeed, in the large N limit $\beta \to 0$ and this mismatch leads to the complete degradation of the entanglement even for small accelerations.

One may wonder how this source of disentanglement compares with that coming from the ambient particles (A7) that will exist even when the mode is unpopulated, $s = 0$. The expected number of such particles per Rindler frequency satisfies a Bose-Einstein distribution at the Unruh temperature,

$$\langle \hat{n}_k \rangle = \frac{1}{e^{2\pi|k|c^2/a} - 1}. \tag{9}$$

These particles are mostly populated in frequencies below a critical value $k_c = \frac{a}{2\pi c^2}$. However, the assumed low-frequency cutoff in the spectrum of Eq. (7) and the assumption of a small acceleration spread over the range of the mode together imply that the frequency spectrum of the mode Rob measures is greater than the critical value, $k > k_c$. Therefore virtually all the thermal particles are undetectable to him, $\langle n_U \rangle \sim 0$. Thus, the mode mismatch is truly the dominant source of entanglement degradation in these localized models. In Fig. 1 we plot the numerically calculated entanglement as a function of the dimensionless parameter $\frac{aL}{c^2}$ for several values of initial squeezing. For all values of the initial squeezing, the entanglement approaches zero as $\frac{aL}{c^2}$ is increased.

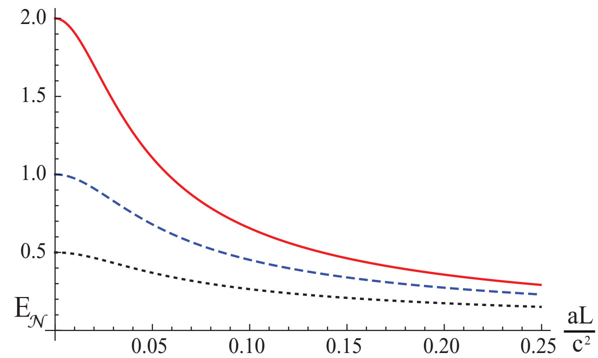

FIG. 1. (Color online) Logarithmic negativity $E_{\mathcal{N}}$ (adimensional) as a function of the dimensionless parameter $\frac{aL}{c^2}$ for $N = 100$ and the squeezing parameter $s = 1$ (solid line), $s = 0.5$ (dashed line), and $s = 0.25$ (dotted line).

This simple example illustrates the key features of the degradation of entanglement that are present generally for other mode shapes. Yet one may be left wondering if the degradation effects are truly fundamental to the acceleration or are merely a consequence of looking in a mode which is poorly matched with the source. In particular, could Rob completely eliminate all of the mode mismatch by cleverly redefining the mode that he observes at each acceleration? In what follows we consider an alternative definition of Rob's mode, defining it at each acceleration to be the one which minimizes the mode mismatch.

IV. OPTIMIZED MODE OF THE ACCELERATING OBSERVERS

Consider the decomposition of Bob's mode in terms of the positive-frequency Minkowski plane waves, $u_k \equiv (4\pi c|k|)^{-1/2}\exp\{i(kx - |k|ct)\}$, and the region I ($x > c|t|$) and region II ($x < -c|t|$) positive-frequency Rindler plane waves, $w_{\mathrm{I}k} \equiv (4\pi c|k|)^{-1/2}\exp\{i(k\xi - c|k|\tau)\}$ and $w_{\mathrm{II}k} \equiv (4\pi c|k|)^{-1/2}\exp\{-i(k\xi' + c|k|\tau')\}$ [13] respectively:

$$\phi_B = \int dk (u_k,\phi_B)u_k \tag{10}$$
$$= \int dk\{(w_{\mathrm{I}k},\phi_B)w_{\mathrm{I}k} + (w_{\mathrm{II}k},\phi_B)w_{\mathrm{II}k} - (w^\star_{\mathrm{I}k},\phi_B)w^\star_{\mathrm{I}k} - (w^\star_{\mathrm{II}k},\phi_B)w^\star_{\mathrm{II}k}\}. \tag{11}$$

From the second line we see that this mode contains contributions from region II Rindler modes, $w_{\mathrm{II}k}, w^\star_{\mathrm{II}k}$, and negative-frequency region I Rindler modes, $w^\star_{\mathrm{I}k}$. Due to the event horizon, the region II frequencies are completely inaccessible to Rob. Likewise, the negative Rindler frequency modes in region I are also unusable in the construction of the annihilation operator associated with his mode. Therefore, we first try defining the optimum mode by simply removing these

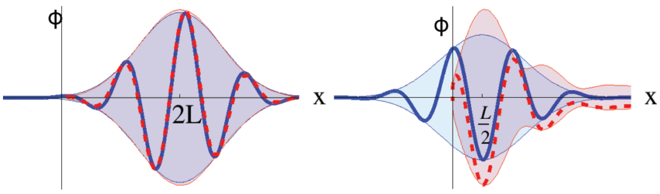

FIG. 2. (Color online) Visualization of the optimized mode (dashed line, red online) overlaid with ϕ_B (solid line, blue online) as a function of the position for low, $a = 1/2L$ (left), and large, $a = 2/L$ (right), accelerations. $N = 6$. The interior of the positive and negative absolute value envelopes have been shaded, and the real part is drawn inside, indicating the oscillation of the wave. The black vertical line shows the position of Rob's horizon, illustrating that as acceleration increases more of ϕ_B is inaccessible to Rob.

components from ϕ_B and normalizing:

$$\psi_{\text{opt}} \equiv |N| \int dk (w_{\text{I}k}, \phi_{\text{B}}) w_{\text{I}k}; \quad |N| = \frac{1}{\sqrt{\int dk |(\phi_{\text{B}}, w_{\text{I}k})|^2}}. \tag{12}$$

A small calculation shows that $(\psi_{\text{opt}}, \phi_{\text{B}}) = |N|^{-1}$. Indeed, this mode is optimized in the sense that for any other mode in region I, $\psi' = \int dk (w_{\text{I}k}, \psi') w_{\text{I}k}$, the magnitude of $|\beta| = |(\psi', \phi_{\text{B}})|$ is bounded from above by $(\psi_{\text{opt}}, \phi_{\text{B}})$:

$$\begin{aligned} |(\psi', \phi_{\text{B}})|^2 &= \left| \int dk (w_{\text{I}k}, \psi')(w_{\text{I}k}, \phi_{\text{B}}) \right|^2 \\ &\leqslant \int dk |(w_{\text{I}k}, \psi')|^2 \int dj |(w_{\text{I}j}, \phi_{\text{B}})|^2 \\ &= \int dk |(w_{\text{I}j}, \phi_{\text{B}})|^2 = (\psi_{\text{opt}}, \phi_{\text{B}})^2, \end{aligned} \tag{13}$$

where the Cauchy-Schwarz inequality has been used on the second line. While ψ_{opt} tries as much as possible to fit to ϕ_B it never completely succeeds because of penetration of part of ϕ_B beyond Rob's horizon [14]. At low acceleration, the tail of the inertial mode penetrating the horizon is very small and so the mode shape becomes approximately Gaussian, approximating the mode it is observing; see the left figure in Fig. 2. However, at large acceleration the tail of Bob's mode penetrates the horizon by a larger extent (see the right of the same figure), and so Rob is never able to completely reconstruct all of the state, thereby leading to an inevitable loss of entanglement.

In Fig. 3 we plot the maximal amount of entanglement available to Rob and Alice, calculated using the optimized mode. We compare it with the simple Gaussian model that we used in the previous section. For the range of accelerations where the spread in acceleration over the effective size of the mode is small, Rob can reconstruct nearly all of the entanglement. It is only once the acceleration becomes large compared with $\frac{c^2}{L}$ that degradation effects begin to appear, precisely when a modest amount of the inertial mode is out of Rob's view.

We have therefore found that at large accelerations entanglement degradation can never be completely avoided. However, it can be made negligible at least for low accelerations, which is a previously unknown result.

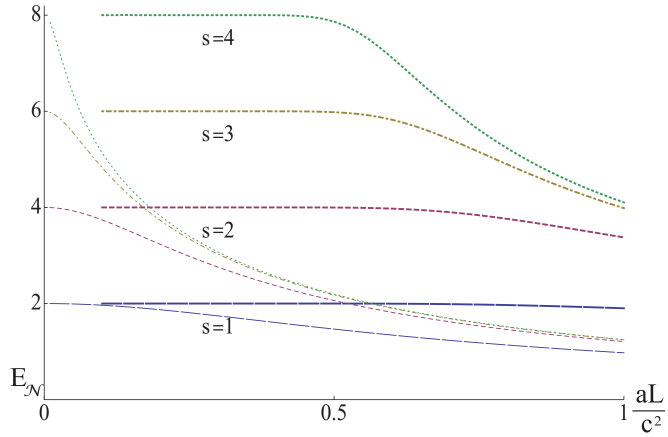

FIG. 3. (Color online) Maximal amount of entanglement, $E_{\mathcal{N}}$ (adimensional), available to Alice and Rob for different values of the initial squeezing, $s = 1,2,3,4$ (thick lines), as a function of the dimensionless acceleration parameter $\frac{aL}{c^2}$. Also shown is a comparison with the entanglement that would have been extracted for the choice of Gaussian modes (thin lines) described in the previous section of the text. In all plots, $N = 6$ and the low Rindler frequency cutoff is assumed to be $\frac{c}{2L}$.

V. CONCLUSION

We have revisited the problem of the degradation of entanglement in noninertial frames, giving a localized discussion of the problem. Our setup closely parallels the design of existing experiments and because we only consider local observables of physically realisable states we expect that our results will be more suitable to experimental enquiry.

We have answered the question of how entanglement is affected by accelerated motion, which was originally posed several years ago. We have traced the source of entanglement degradation to a mode mismatch that will be ubiquitous in any choice of the observed mode, although some choices can be made better than others.

We have found at low accelerations that the noise coming from the particles in the vacuum state does not lead to a significant amount of degradation. It would seem in practice that for realistic observations of single modes this noise is a sub-leading effect to the mode-loss effect we have described. This suggests that entanglement degradation would be actually much easier to observe than the Unruh effect itself, which could be of importance for the future experimental studies of noninertial effects on quantum phenomena.

ACKNOWLEDGMENT

J. Doukas thanks B. Munro, K. Nemoto, and J. Louko for useful discussions.

APPENDIX: CALCULATION OF THE COVARIANCE MATRIX

In this Appendix we provide the details of the calculation of the covariance matrix (4). We assume that initially the field is prepared in the pure Gaussian state (1). Since we are describing the outcome of measurements made by the two-party system (Alice and Rob), the covariance matrix is 4×4 dimensional and its elements are found by calculating Eq. (3). To do so one needs to evaluate the operators $\hat{S}^{\dagger}_{AB}\hat{d}_l\hat{S}_{AB}$, where $l \in \{A,B\}$. This can be done by trivially rewriting $\hat{d}_A = (\psi_A,\phi_A)\hat{a} + \hat{d}_A - (\psi_A,\phi_A)\hat{a}$, where $(\cdot,\cdot)$ is the Klein-Gordon inner product and noting that the part, $\hat{d}_A - (\psi_A,\phi_A)\hat{a}$, does not contain $\hat{a}$ operators. A similar decomposition can be introduced for the $\hat{d}_B$ operator (taking out $\hat{b}$ and $\hat{b}^{\dagger}$), leading to the following commutation relations:

$$\hat{S}^{\dagger}_{AB}\hat{d}_A\hat{S}_{AB} = \alpha(\cosh s\,\hat{a} - \sinh s\,\hat{b}^{\dagger}) + \hat{d}_A - \alpha\hat{a}, \quad (A1)$$

$$\hat{S}^{\dagger}_{AB}\hat{d}_B\hat{S}_{AB} = \beta(\cosh s\,\hat{b} - \sinh s\,\hat{a}^{\dagger}) + \hat{d}_B - \beta\hat{b} - \beta'\hat{b}^{\dagger} + \beta'(\cosh s\,\hat{b}^{\dagger} - \sinh s\,\hat{a}). \quad (A2)$$

With these relations and the fact that the operators $\hat{a}$, $\hat{b}$, and $\hat{d}_A$ annihilate the vacuum $|0\rangle_M$ each element of the covariance matrix can be calculated. For concreteness we present here the calculation of the σ_{33} element:

$$\sigma_{33} = {}_M\langle 0|(\hat{S}^{\dagger}_{AB}(\hat{d}_B + \hat{d}^{\dagger}_B)\hat{S}_{AB})^2|0\rangle_M. \quad (A3)$$

When we apply Eq. (A2) and remove terms which annihilate the vacuum, it results in

$$\sigma_{33} = |\beta + \beta'^{\star}|^2(\cosh s - 1)^2 + \sinh^2 s|\beta + \beta'^{\star}|^2 + 2\mathrm{Re}((\beta + \beta'^{*})(\cosh s - 1)_M\langle 0|\hat{b}(\hat{d}_B + \hat{d}^{\dagger}_B)|0\rangle_M) + {}_M\langle 0|(\hat{d}_B + \hat{d}^{\dagger}_B)^2|0\rangle_M. \quad (A4)$$

The last term can be calculated by expressing the Minkowski vacuum in terms of the Rindler vacuum, $|0\rangle_M = \hat{S}_{I,II}|0\rangle_R$, where the squeezing operator, $\hat{S}_{I,II}$, is characterized by the squeezing parameter, $r_k = \mathrm{arctanh}(e^{-\pi|k|c^2/a})$, and fulfills the following relations:

$$\hat{S}^{\dagger}_{I,II}\hat{b}_{l,I}\hat{S}_{I,II} = \cosh r_l\hat{b}_{l,I} + \sinh r_l\hat{b}^{\dagger}_{l,II}, \quad (A5)$$

where $\hat{b}_{l,I}$ and $\hat{b}_{l,II}$ are the annihilation operators associated with the Rindler modes w_{Ik} and w_{IIk} respectively. We find that

$${}_M\langle 0|(\hat{d}_B + \hat{d}^{\dagger}_B)^2|0\rangle_M = 1 + 2\langle\hat{n}\rangle_U, \quad (A6)$$

where

$$\langle\hat{n}\rangle_U \equiv \int dk\frac{|(\psi_B,w_{Ik})|^2}{e^{\frac{2\pi|k|c^2}{a}} - 1}. \quad (A7)$$

Finally, the term ${}_M\langle 0|\hat{b}(\hat{d}_B + \hat{d}^{\dagger}_B)|0\rangle_M$ can be evaluated by decomposing the operators into Minkowski plane waves:

$${}_M\langle 0|\hat{b}(\hat{d}_B + \hat{d}^{\dagger}_B)|0\rangle_M = \int dk(\phi_B,u_k)((\psi_B,u_k) + (\psi_B,u_k)^{\star}), = \beta' + \beta^{\star}. \quad (A8)$$

By putting Eqs. (A6) and (A8) into Eq. (A4) and simplifying, we obtain

$$\sigma_{33} = 2|\beta + \beta'^{\star}|^2\sinh^2 s + 1 + 2\langle\hat{n}\rangle_M. \quad (A9)$$

Calculation of the remaining covariance elements follows in a similar fashion, resulting in Eq. (4).

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[13] (τ',ξ') are defined from Eq. (2) by adding primes to τ and ξ and replacing $x \to -x$.

[14] It is also possible to relate the negative region frequency contribution directly to this horizon penetration. The combination $w^{\star}_{Ik} + e^{\frac{\pi|k|c^2}{a}}w_{IIk}$ is a superposition of pure negative-frequency Minkowski waves; therefore $(w^{\star}_{Ik},\phi_B) = -e^{\frac{\pi|k|c^2}{a}}(w_{IIk},\phi_B)$.