

Homodyne detection as a near-optimum receiver for phase-shift-keyed binary communication in the presence of phase diffusion

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We address binary optical communication channels based on phase-shift-keyed coherent signals in the presence of phase diffusion. We prove theoretically and demonstrate experimentally that a discrimination strategy based on homodyne detection is robust against this kind of noise for any value of the channel energy. Moreover, we find that a homodyne receiver beats the performance of a Kennedy receiver as the signal energy increases and achieves the Helstrom bound in the limit of large noise.

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I. INTRODUCTION

In a binary quantum communication channel the sender encodes the logical symbols 1 and 0 on two states of a physical system described by the density operators $\hat{\rho}_1$ and $\hat{\rho}_0$, respectively. In order to retrieve the logical information, the receiver should discriminate between the two signals, which, in general, may be not orthogonal, due to the encoding process itself ($\hat{\rho}_1$ and $\hat{\rho}_0$ may refer to nonorthogonal states) or because of the noise during the propagation stage [1]. In this case, an unavoidable probability of error $P_e = \frac{1}{2}[P(1|0) + P(0|1)]$ appears, where $P(j|k)$ is the probability of inferring the symbol j when the signal sent over the channel was meant for k and we assume that the two states are sent with equal probability. The minimum error probability allowed by quantum mechanics is given by the Helstrom bound [1] $P_Q = \frac{1}{2}(1 - \frac{1}{2} \text{Tr}|\hat{\rho}_0 - \hat{\rho}_1|)$, where $|A| = \sqrt{A^\dagger A}$. Optimal discrimination of nonorthogonal states is a crucial topic for the effective implementation of quantum communication channels and, as a consequence, different strategies have been employed to attack the problem in different situations [2–4].

In the following, we address binary communication channels with phase-shift-keyed (PSK) signals [5,6], i.e., channels where the information is encoded on two coherent states $|\psi_1\rangle = |\alpha\rangle$ and $|\psi_0\rangle = |-\alpha\rangle$ (without lack of generality we can assume $\alpha \in \mathbb{R}$, $\alpha > 0$). In this case, the Helstrom bound is rewritten as $P_Q = \frac{1}{2}[1 - \sqrt{1 - |\langle\psi_0|\psi_1\rangle|^2}]$, i.e.,

$$P_Q = \frac{1}{2}[1 - \sqrt{1 - \exp(-4N)}], \quad (1)$$

where $N = |\alpha|^2$ is the average number of photons contained in each signal; it will be referred to as the *signal energy* throughout the paper. A detection strategy achieving this level of error probability is said to be an *optimum receiver*.

Quantum state discrimination strategies for PSK signals is an active field of research since it encompasses the read-out problem, and it is an important tool to assess the performances of quantum limited measurements in optical communication, e.g., for deep-space missions [7]. On the one hand, in the high-energy regime, a near-optimum receiver based on photodetection, the so-called Kennedy receiver, was proposed long ago for coherent signals propagating in an ideal channel [8,9]. The Kennedy receiver has been also extended [10–12]

and, in particular, an optimum receiver based on an adaptive scheme has been recently experimentally realized [13]. On the other hand, strategies based on homodyne detection have great practical advantages [14] and for this reason their performances have been analyzed in realistic situations. In particular, receivers based on homodyne detection have been theoretically [15–17] and experimentally investigated [18] in the presence of losses and phase-insensitive thermal noise. More generally, these receivers have been proved to represent the optimum Gaussian strategy for the discrimination of the PSK coherent signals [19] and it has been also demonstrated that it possible to emulate adaptive processes by means of postprocessing and Bayesian analyses [20]. Hybrid [21] and displacement-based [22] detectors also have been implemented to decrease the error probability, with application to M -ary communication channels [23].

Communication schemes based on coherent signals may be useful in scenarios in which quantum resources (as single photons and entanglement) cannot be fully exploited, as in free-space communication. In this paper we analyze a source of noise, namely, phase diffusion, which is detrimental for coherent-signal-based channels. At the same time, this provides a relevant example of non-Gaussian noise. We demonstrate experimentally the robustness of the homodyne receiver against phase noise. Moreover, we find that in the presence of phase noise, the homodyne receiver beats the performances of the Kennedy receiver as the signal energy increases and/or the noise is larger than an intensity-dependent threshold value. Finally, we show that homodyne detection achieves the Helstrom bound in the limit of large noise.

II. PHASE DIFFUSION

Any phase diffusion process coming from a nondissipative interaction with a bosonic environment may be described by a suitable phase-diffusion master equation [24]. The overall effect of phase diffusion on a coherent state $\hat{\rho}_\alpha = |\alpha\rangle\langle\alpha|$ is described by the map [25]

$$\hat{\rho}_\alpha \rightarrow \hat{\rho}'_\alpha = \int_{\mathbb{R}} d\phi g(\phi, \delta) |\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}|, \quad (2)$$

where $g(\phi, \delta)$ is a normal distribution of the variable ϕ with zero mean and standard deviation δ . At the end of a channel affected by phase diffusion the receiver is faced with the problem of discriminating between \hat{Q}'_{α} and $\hat{Q}'_{-\alpha}$. In order to calculate the corresponding Helstrom bound, one has to diagonalize the traceless operator $\Lambda = \hat{Q}'_{\alpha} - \hat{Q}'_{-\alpha}$ whose expansion in the photon-number basis reads

$$\Lambda = \sum_{n,m=0}^{\infty} \frac{\alpha^{n+m}}{\sqrt{n!m!}} e^{-\alpha^2 - [(n-m)^2/2]\delta^2} [1 - (-1)^{n-m}] |m\rangle \langle n|.$$

This can be done numerically after a suitable truncation of the Hilbert space, which is determined by the average number of photons of the two signals. The Helstrom bound for different values of the signal energy N and the diffusion coefficient δ is reported in Fig. 3 together with the experimental data for the homodyne receiver. For small values of the coherent amplitude we may truncate Λ at low dimension, achieving the analytic expression

$$P_Q(\delta) \simeq \frac{1}{2}(1 - \alpha e^{-\delta^2/2}), \quad (3)$$

whereas in the limit $\delta \gg 1$ of large noise we have

$$P_Q(\delta) \simeq \frac{1}{2}[1 - g_Q(\alpha)e^{-\delta^2/2}], \quad (4)$$

where $g_Q(\alpha)$ is a decreasing function of the amplitude.

III. KENNEDY RECEIVER IN THE PRESENCE OF PHASE NOISE

In the ideal case, a near-optimum strategy is provided by the Kennedy receiver [8] based on photodetection: The signal mode \hat{a} excited in either $|\alpha\rangle$ or $|\alpha\rangle$ interferes with a reference mode \hat{b} excited in the coherent state $|\beta\rangle$ at a beam splitter (BS) with transmissivity $\tau = \cos^2 \varphi$ and a photodetector detects the light at one output. The overall evolved state reads $U_{\tau} |\pm\alpha\rangle \otimes |\beta\rangle = |\pm\sqrt{\tau}\alpha + \sqrt{1-\tau}\beta\rangle \otimes |\sqrt{\tau}\beta \mp \sqrt{1-\tau}\alpha\rangle$, where U_{τ} is the evolution operator associated with the BS. If we choose $\beta = \alpha\sqrt{\tau}/\sqrt{1-\tau}$, take $\tau \rightarrow 1$, and consider only the first output port, then we have the input-output relation $|\pm\alpha\rangle \rightarrow |\pm\alpha + \alpha\rangle$. Therefore, a natural discrimination strategy is to associate the absence of light (the vacuum) with the symbol 0, i.e., $|\alpha\rangle$, and the detection of any number of photons with the symbol 1, i.e., $|\alpha\rangle$. This strategy also implies a nonzero probability of inferring the wrong symbol, which is determined by the conditional probabilities [16]

$$P(0|1) = \lim_{\tau \rightarrow 1} \text{Tr}[\hat{U}_{\tau} \hat{Q}_{\alpha} \otimes \hat{Q}_{\beta} \hat{U}_{\tau}^{\dagger} \hat{\Pi}_0 \otimes \hat{\Pi}] = e^{-4N}, \quad (5a)$$

$$P(1|0) = \lim_{\tau \rightarrow 1} \text{Tr}[\hat{U}_{\tau} \hat{Q}_{-\alpha} \otimes \hat{Q}_{\beta} \hat{U}_{\tau}^{\dagger} \hat{\Pi}_1 \otimes \hat{\Pi}] = 0, \quad (5b)$$

where we introduced the projectors $\hat{\Pi}_0 = |0\rangle\langle 0|$ and $\hat{\Pi}_1 = \hat{\mathbb{1}} - \hat{\Pi}_0$, describing an on-off detector detecting the absence or the presence of photons, respectively. The overall probability of error (still assuming the two signals sent with the same probability) is

$$P_K = \frac{P(0|1) + P(1|0)}{2} = \frac{\exp(-4N)}{2}. \quad (6)$$

It is worth noting that, in the limit $N \gg 1$, $P_K \approx 2P_Q$, i.e., the Kennedy receiver based on photodetection is nearly optimal.

In the presence of phase diffusion, Eqs. (5) become

$$P_{\delta}(0|1) = \int_{\mathbb{R}} d\phi g(\phi, \delta) \exp[-4\alpha^2 \cos^2(\phi/2)], \quad (7a)$$

$$P_{\delta}(1|0) = 1 - \int_{\mathbb{R}} d\phi g(\phi, \delta) \exp[-4\alpha^2 \sin^2(\phi/2)], \quad (7b)$$

and

$$P_K(\delta) = \frac{1}{2}[P_{\delta}(0|1) + P_{\delta}(1|0)],$$

which can be easily evaluated numerically. For small values of the coherent amplitude we have the analytic expression

$$P_K(\delta) \simeq \frac{1}{2}(1 - 4\alpha^2 e^{-2\delta^2}), \quad (8)$$

whereas in the limit $\delta \gg 1$ of large noise we have

$$P_K(\delta) \simeq \frac{1}{2}[1 - g_K(\alpha)e^{-2\delta^2}], \quad (9)$$

where $g_K(\alpha)$ is a decreasing function of the amplitude.

IV. HOMODYNE RECEIVER IN THE PRESENCE OF PHASE NOISE

Let us now focus on a different strategy based on homodyne detection: The receiver measures the quadrature $\hat{x}_{\psi} = 2^{-1/2}(\hat{a}^{\dagger} e^{i\psi} + \hat{a} e^{-i\psi})$, where $\psi = \arg[\alpha]$ (in our case $\psi = 0$ or π) and associates the symbol 1 (0) with a positive (negative) outcome. In the presence of losses and thermal noise but without phase noise, this strategy has been proven to beat the photodetection strategy in either the low- or the high-energy regime [16]. In the presence of phase diffusion we have the following conditional probabilities (the noiseless case is recovered in the limit $\delta \rightarrow 0$):

$$Q_{\delta}(0|1) = \int_{-\infty}^0 dx p_{\delta}(x; \alpha), \quad (10)$$

$$Q_{\delta}(1|0) = \int_0^{+\infty} dx p_{\delta}(x; -\alpha),$$

where

$$p_{\delta}(x; \pm\alpha) = \int_{\mathbb{R}} \frac{d\phi}{\sqrt{\pi}} g(\phi, \delta) e^{-(x \mp \sqrt{2}\alpha \cos \phi)^2} \quad (11)$$

is the homodyne probability, namely, the probability of obtaining as outcome x addressing the quadrature \hat{x}_0 given the input $|\pm\alpha\rangle$. The overall probability of error thus is written

$$P_H(\delta) = \frac{1}{2}[Q_{\delta}(0|1) + Q_{\delta}(1|0)] \quad (12)$$

and can be easily evaluated numerically. For small values of the coherent amplitude we have the analytic expression

$$P_H(\delta) \simeq \frac{1}{2} \left(1 - \alpha \sqrt{\frac{2}{\pi}} e^{-\delta^2/2} \right), \quad (13)$$

whereas in the limit $\delta \gg 1$ of large noise we have

$$P_H(\delta) \simeq \frac{1}{2}[1 - g_H(\alpha)e^{-\delta^2/2}], \quad (14)$$

where $g_H(\alpha)$ is a decreasing function of the amplitude with $g_H(\alpha) < g_Q(\alpha)$ and $g_H(\alpha)$ approaching $g_Q(\alpha)$ for increasing amplitude.

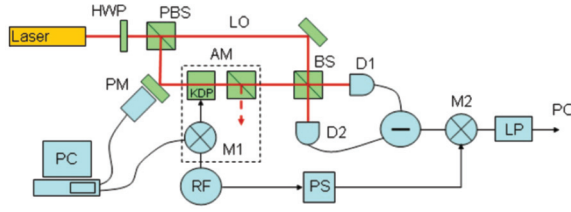


FIG. 1. (Color online) Schematic diagram of the experimental apparatus. The primary source is a He:Ne laser, then split into two paths, the local oscillator (LO) and the signal, by means of a half-wave plate (HWP) and a polarizing beam splitter (PBS). The phase modulation (PM) is obtained by a piezo connected to a computer (PC). The amplitude modulation (AM) of the coherent signals is controlled by a suitable combination of a potassium dihydrogen phosphate (KDP) crystal and a PBS [26]. The actual homodyne receiver is composed of a 50:50 BS and a balanced amplifier detector based on silicon photodiodes (D1 and D2). M1 is a mixer used to set the amplitude modulation by a computer, whereas M2 is a mixer used to demodulate the signal @4MHz.

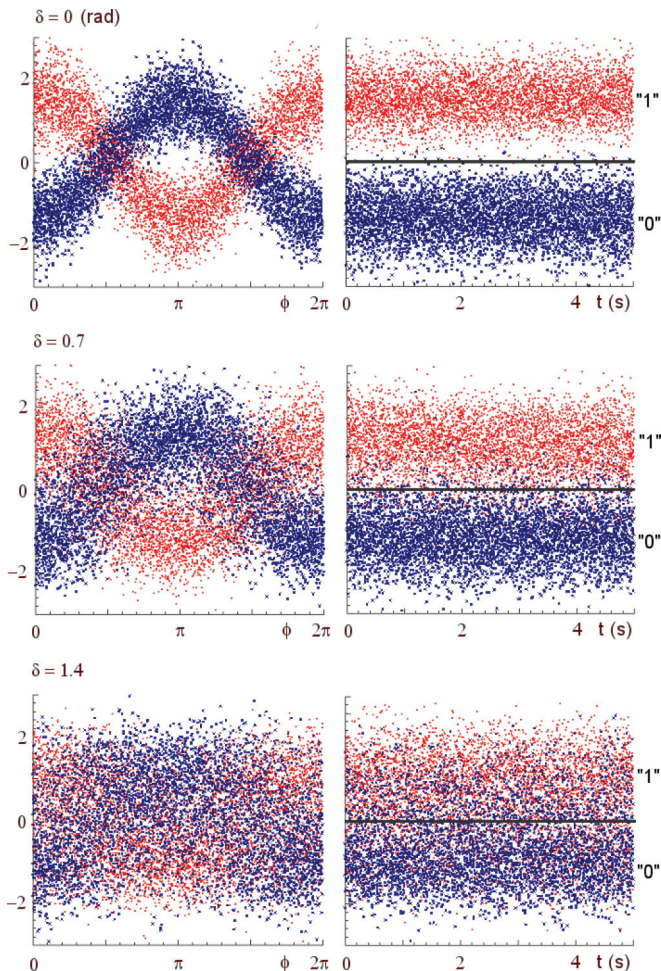


FIG. 2. (Color online) Panels on the left show homodyne traces for the quadrature \hat{x}_ϕ , of the coherent signals $|\alpha\rangle$ [red (light gray)] and $|\!-\alpha\rangle$ [blue (dark gray)] for increasing noise (from top to bottom, $\delta = 0, 0.7$, and 1.4 rad) in the case of $N = \alpha^2 = 1.0$. Panels on the right show homodyne traces for the quadrature \hat{x}_0 versus the detection time. The horizontal line refers to the threshold for the discrimination strategy: If the dots fall above (below) that line, the signal is chosen to be 1 (0).

V. EXPERIMENT

The experimental apparatus we used to investigate the performance of the homodyne receiver in the presence of phase diffusion is sketched in Fig. 1. The output beam of a He:Ne laser is split into two paths, the local oscillator (LO in Fig. 1) and the signal, by means of a suitable combination of a half wave plate (HWP) and a polarizing beam splitter (PBS). The phase modulation (PM) is obtained by a piezo connected to the computer (PC), which sends a voltage signal with a frequency of 1 kHz corresponding to the phase diffusion. The amplitude modulation (AM) of the coherent signals is controlled by a suitable combination of a potassium dihydrogen phosphate (KDP) crystal and a PBS [26]. The actual homodyne receiver is composed of a 50:50 BS and a balanced amplifier detector based on silicon photodiodes (D1 and D2). In order to experimentally determine the probability of error, for a fixed value of the signal energy $N = \alpha^2$, we performed ten runs with 5×10^3 shots each, randomly chosen between $|\alpha\rangle$ and $|\!-\alpha\rangle$.

VI. DATA ANALYSIS AND DISCUSSION

In Fig. 2 we report the data of our experiments for different values of the noise parameter. In the left panels we show the homodyne traces for the quadrature \hat{x}_ϕ , of the coherent signals $|\alpha\rangle$ [red (light gray)] and $|\!-\alpha\rangle$ [blue (dark gray)] as a function of ϕ and for increasing level of noise. In the right panels we report homodyne data for the optimal quadrature \hat{x}_0 versus the detection time. The horizontal line refers to the threshold for the discrimination strategy: If the dots (i.e., the homodyne outcomes) fall above (below) that line, the signal is inferred to be 1 (0).

In Fig. 3 we report the behavior of the probabilities of error $P_Q(\delta)$, $P_K(\delta)$, and $P_H(\delta)$ as functions of the phase diffusion

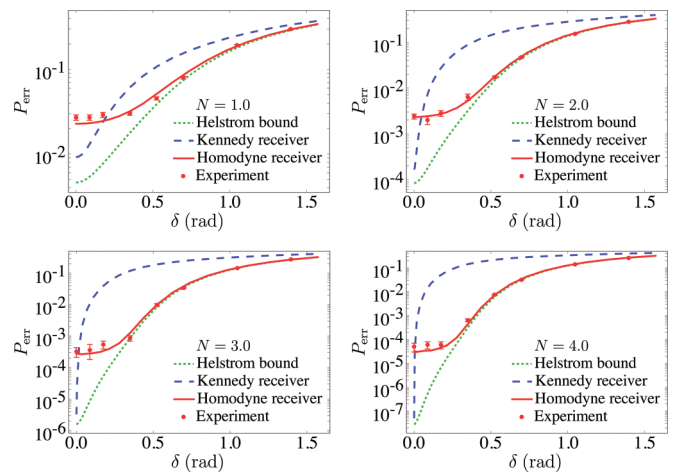


FIG. 3. (Color online) A log-linear plot of the homodyne probability of error (red circles) as a function of the diffusion parameter δ for different values of the signal energy N . Each point corresponds to the average of ten acquisitions of 5×10^3 values with a phase diffusion frequency of 1 kHz. The errors bars correspond to the standard deviation of the mean. The theoretical predictions $P_H(\delta)$ (solid red line) is reported for comparison. We also show the error probability $P_K(\delta)$ of a Kennedy receiver (dashed blue line) in the same experimental conditions and the Helstrom bound $P_Q(\delta)$ (dotted green line).

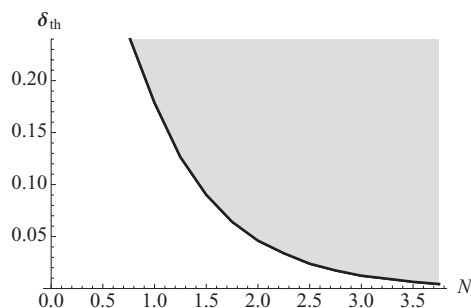


FIG. 4. Threshold value δ_{th} of the diffusion coefficient δ as a function of the signal energy N . The meaning of the threshold value is the following: If $\delta \geq \delta_{\text{th}}$, then the homodyne receiver shows a smaller error probability than does the Kennedy receiver (gray region); if $\delta < \delta_{\text{th}}$, then the opposite holds (white region).

coefficient δ and for different values of the input energy N . As one may expect, the presence of the phase noise dramatically affects the performance of the Kennedy receiver: It is worth noting that as the input energy increases, a small amount of phase noise is enough to increase by several orders of magnitude the probability of error P_K . In contrast, the strategy based on the homodyne detection is quite robust with respect to the phase noise. As is apparent from the plots, for $\delta \lesssim 0.2$ the value of P_H is almost constant. In addition, P_H approaches the quantum mechanical limit given by the Helstrom bound P_Q as δ increases, as anticipated by the asymptotic expansions in Eqs. (4) and (14). In general, as the signal energy and the

noise increase, the homodyne receiver becomes more effective than the Kennedy receiver. This behavior is illustrated in Fig. 4, where we plot the threshold $\delta_{\text{th}} \equiv \delta_{\text{th}}(N)$ on the diffusion coefficient δ as a function of the signal energy N : At any fixed value of N , if $\delta \geq \delta_{\text{th}}$ then the homodyne receiver exhibits a smaller error probability than does the Kennedy receiver in the same experimental conditions, in agreement with Eqs. (9) and (14).

VII. CONCLUSION

We have addressed PSK binary optical communication in the presence of phase diffusion, i.e., a detrimental noise for schemes based on coherent signals. We demonstrated experimentally that a discrimination strategy based on homodyne detection is robust against this kind of noise. In addition, we have also demonstrated that homodyne receivers beat the performances of Kennedy receivers as long as the noise is larger than an energy-dependent threshold. Finally, we have shown that homodyne receivers achieve the Helstrom bound on the error probability in the limit of large noise and for any value of the signal energy. Our results help clarify the fundamental limits of quantum communications and show that receivers that perform near the quantum limit in noisy conditions are realizable with current technology [27].

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