Vacuum Rabi oscillation induced by virtual photons in the ultrastrong-coupling regime

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(Received 1 January 2013; published 19 April 2013)

We present an interaction scheme that exhibits a dynamical effect of virtual photons carried by a vacuum-field dressed two-level atom in the ultrastrong-coupling regime. We show that, with the aid of an external driving field, virtual photons provide a transition matrix element that enables the atom to evolve coherently and reversibly to an auxiliary level accompanied by the emission of a real photon. The process corresponds to a type of vacuum Rabi oscillation, and we show that the effective vacuum Rabi frequency is proportional to the amplitude of a single virtual photon in the ground state. Therefore, the interaction scheme could serve as a probe of ground-state structures in the ultrastrong-coupling regime.

DOI: 10.1103/PhysRevA.87.045804 PACS number(s): 42.50.Pq, 42.50.Ct, 42.50.Lc

A single-mode electromagnetic field interacting with a two-level atom has been a fundamental model in quantum optics capturing the physics of resonant light-matter interaction. In particular, the Jaynes-Cummings (JC) model [1,2], which describes the regime where the interaction energy $\hbar\lambda$ is much smaller than the energy scale of an atom $\hbar\omega_A$ and a photon $\hbar\omega_c$, has tremendous applications in cavity QED [3,4] and trapped ion systems [5]. Recently, there has been considerable research interest in the ultrastrong-coupling regime, where λ is an appreciable fraction of ω_c or ω_A. In particular, the possibility of realizing an ultrastrong coupling has been discussed in circuit QED architectures [6], and some experiments have explored the regime in various systems with artificial atoms and cavity photon resonators, including a superconducting qubit in a coplanar waveguide [7] or LC resonator [8], microcavities embedding doped quantum wells [9,10], and two-dimensional electron gas coupled to metamaterial resonators [11]. In addition, theoretical investigations have also found novel phenomena in the ultrastrong-coupling regime, such as the asymmetry of vacuum Rabi splitting [12], photon blockade [13], nonclassical state generation [14], superradiance transition [15], and collapse and revival dynamics [16].

A key feature in the ultrastrong-coupling regime is an appreciable number of virtual photons in the ground state of the system. The generation of such virtual photons is governed by counter-rotating terms in the Hamiltonian, and these photons do not correspond to radiation as they are bound to the atom. However, by modulating the atom-field coupling strength, virtual photons can be released as a form of quantum vacuum radiation [17]. In this paper, we indicate a different effect of the vacuum-field dressed atom, namely a type of vacuum Rabi oscillations that would not occur if virtual photons are absent.

Specifically, we investigate the quantum dynamics of a driven quantum Rabi model. The configuration of our system is shown in Fig. 1 in which a Ξ-type three-level atom is confined in a single-mode cavity. The atomic levels $|g\rangle$ and $|e\rangle$ are coupled to a cavity field of frequency ω_c. These two atomic levels and the cavity field mode constitute a Rabi model [18]. In addition, there is an external classical field driving the transition between $|e\rangle$ and the third atomic level $|f\rangle$. We note that some theoretical aspects of three-level artificial atoms in circuit QED were discussed in Refs. [19,20], and Ξ-type superconducting atoms have been demonstrated in experiments [21–23]. Recently, Carusotto *et al.* have studied the dynamics of a related system in a different driving configuration [24].

The Hamiltonian of our system is given by ($\hbar = 1$)

$$H = H_R + \omega_f |f\rangle\langle f| + \Omega \cos\omega_p t(|f\rangle\langle e| + |e\rangle\langle f|), \tag{1}$$

where H_R is the Hamiltonian of the Rabi model,

$$\begin{aligned} H_R = {} & \frac{\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \omega_c a^\dagger a \\ & + \lambda(a + a^\dagger)(|g\rangle\langle e| + |e\rangle\langle g|). \end{aligned} \tag{2}$$

Here $a^\dagger$ and a are creation and annihilation operators associated with the field mode, ω_0 is the (bare) transition frequency between $|e\rangle$ and $|g\rangle$, and $\omega_f - \omega_0/2$ is the transition frequency between $|f\rangle$ and $|e\rangle$. The parameter λ denotes the atom-cavity coupling strength, and the classical driving field has a frequency ω_p and an interaction strength Ω. In writing H_R, we have kept counter-rotating terms because λ can be comparable to ω_c in the ultrastrong-coupling regime. Note that the coupling between the cavity field and the atom involving $|f\rangle$ is assumed to be far off resonance and weak, so it is neglected in the Hamiltonian.

Initially, the system is prepared in the ground state of H_R in the absence of the driving field. Our task is to determine the dynamics after the driving field is turned on. To analyze the problem, we apply a unitary transformation to simplify the Hamiltonian. It is known that H_R can be transformed into a form of the Jaynes-Cummings Hamiltonian approximately by a unitary operator e^{-S} [25]. Here the operator S and its parameters are defined by

$$S = \frac{\lambda\xi}{\omega_c}(|g\rangle\langle e| + |e\rangle\langle g|)(a^\dagger - a), \tag{3}$$

$$\xi = \frac{\omega_c}{\omega_c + \eta\omega_0}, \tag{4}$$

$$\eta = \exp\left(-\frac{2\lambda^2\xi^2}{\omega_c^2}\right). \tag{5}$$

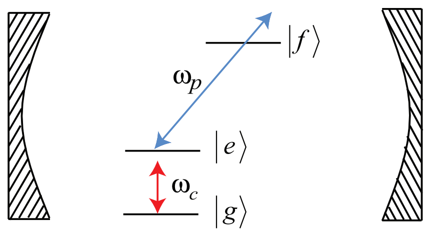

FIG. 1. (Color online) Interaction scheme of a Ξ-type three-level atom in a cavity. The atomic states $|g\rangle$ and $|e\rangle$ and a cavity field mode of frequency ω_c form a quantum Rabi model described by H_R, and an external classical field of frequency ω_p drives the transition between $|e\rangle$ and $|f\rangle$.

Then it can be shown that $H_R' = e^S H_R e^{-S}$ is approximately given by Refs. [25–28]

$$H_R' \approx \frac{\omega_0'}{2}(|e\rangle\langle e| - |g\rangle\langle g|) + \omega_c a^\dagger a + \lambda'(a|e\rangle\langle g| + \text{H.c.}) + \frac{\lambda^2\xi}{\omega_c}(\xi - 2)(|e\rangle\langle e| + |g\rangle\langle g|) \equiv H_{\text{JC}}, \tag{6}$$

where H_{JC} describes a JC model in which the atomic frequency and cavity-atom interaction strength are renormalized as $\omega_0' = \eta\omega_0$ and $\lambda' = 2\eta\omega_0\xi\lambda/\omega_c$, respectively. One can solve Eqs. (4) and (5) for ξ and η to obtain the renormalized parameters. In the regime where $\lambda < \omega_c$, ξ can be expanded as

$$\xi = \frac{\omega_c}{\omega_c + \omega_0} + \frac{2\omega_0\omega_c^3}{(\omega_c + \omega_0)^4}\left(\frac{\lambda}{\omega_c}\right)^2 + O\left(\frac{\lambda}{\omega_c}\right)^4. \tag{7}$$

In the resonance case $\omega_c = \omega_0$, ξ is roughly about 0.5 in the range $|\lambda|/\omega_c < 0.5$, and the corresponding η is slightly less than 1 obtained by Eq. (5).

Note that H_{JC} in Eq. (6) is an approximation to H_R', and the difference $H_R' - H_{\text{JC}}$ describes multiphoton processes that correspond to higher-order corrections [25–28]. Since $|g,0\rangle$ is the ground state of H_{JC}, $e^{-S}|g,0\rangle$ is an approximated ground state of H_R in the original frame. The accuracy of such an approximation has been tested in Ref. [25]. Specifically, if λ is comparable to but smaller than ω_c, the ground-state energy of H_{JC} is in good agreement with that of H_R obtained by exact numerical calculations over a range of parameters. For example, in the case $\omega_c = \omega_0 = 2\lambda$, the approximated ground-state energy obtained by H_{JC} has a percentage error of about 0.65%.

Now we apply the transformation to our system Hamiltonian H, which becomes

$$H' = e^S H e^{-S} \approx H_{\text{JC}} + \omega_f|f\rangle\langle f| + \Omega\cos\omega_p t(e^S|e\rangle\langle f| + |f\rangle\langle e|e^{-S}). \tag{8}$$

Since $e^S|e\rangle = \cosh[\frac{\lambda\xi}{\omega_c}(a^\dagger - a)]|e\rangle + \sinh[\frac{\lambda\xi}{\omega_c}(a^\dagger - a)]|g\rangle$, we expand the hyperbolic sine and cosine operator functions in normal order up to first order in $\lambda\xi/\omega_c$,

$$\cosh\left[\frac{\lambda\xi}{\omega_c}(a^\dagger - a)\right] \approx \eta^{1/4}, \tag{9}$$

$$\sinh\left[\frac{\lambda\xi}{\omega_c}(a^\dagger - a)\right] \approx \eta^{1/4}\frac{\lambda\xi}{\omega_c}(a^\dagger - a). \tag{10}$$

Therefore, the transformed Hamiltonian becomes

$$H' \approx H_{\text{JC}} + \omega_f|f\rangle\langle f| + \Omega'\cos\omega_p t(|f\rangle\langle e| + |e\rangle\langle f|) + \frac{\lambda\xi}{\omega_c}\Omega'\cos\omega_p t(|g\rangle\langle f| - |f\rangle\langle g|)(a^\dagger - a), \tag{11}$$

where $\Omega' = \eta^{1/4}\Omega$ is a renormalized driving field strength, and the last term indicates a new coupling between $|g\rangle$ and $|f\rangle$ through the cavity field mode.

A further simplification can be made by exploiting resonance when ω_p is tuned to a certain resonance frequency defined by the undriven system. In this paper, we consider the resonance at

$$\omega_p = \omega_f + \omega_c - \left[\frac{\lambda^2\xi}{\omega_c}(\xi - 2) - \frac{\omega_0'}{2}\right], \tag{12}$$

which corresponds to the transition between $|g,0\rangle$ to $|f,1\rangle$, since the square bracket term is the approximate ground-state energy of H_R obtained by the transformation method. By the condition (12), $|g,0\rangle$ and $|f,1\rangle$ are resonantly coupled, but the transition between $|f,1\rangle$ and $|e,1\rangle$ is far away from resonance (the corresponding detuning is of order ω_c). Therefore, if Ω' is not too strong, the system is confined to the two resonantly coupled states, i.e., all off-resonant transitions may be ignored. In this way, H' in the interaction picture is reduced to

$$H_I' \approx -\frac{\lambda\xi}{2\omega_c}\Omega'(|g,0\rangle\langle f,1| + |f,1\rangle\langle g,0|). \tag{13}$$

Equation (13) indicates that the system would execute a form of vacuum Rabi oscillations, in which $|g,0\rangle$ behaves like an excited atom in the vacuum field and $|f,1\rangle$ behaves like a ground atom with a single photon. In cavity QED, such oscillations lead to vacuum Rabi splitting [29–33]. Note that the effective vacuum Rabi frequency here is $\lambda\xi\Omega'/\omega_c$, which is significant in the ultrastrong-coupling regime where λ is comparable to ω_c.

It is useful to go back to the original frame in which the Rabi oscillations occur between the states $e^{-S}|f,1\rangle$ and $e^{-S}|g,0\rangle$. Since $e^{-S}|f,1\rangle = |f,1\rangle$, an initial ground state will evolve to $|f,1\rangle$ after half of a Rabi period. If we switch off the external field at this moment, the single photon described by $|f,1\rangle$ will be free to escape the cavity because the atom in the state $|f\rangle$ does not couple to the cavity field when $\Omega = 0$, i.e., the photon cannot be reabsorbed by the atom. In this way, a π pulse of the driving field can generate a real photon deterministically while the atom is excited to the $|f\rangle$ state.

To gain a better insight into the physical process without relying on the approximation made in Eqs. (6) and (11), we express the Hamiltonian by the eigenbasis of H_R. Let $|\psi_n\rangle$ be an eigenvector of H_R with the eigenvalue λ_n, i.e., $H_R|\psi_n\rangle = \lambda_n|\psi_n\rangle$ (the ground state is denoted by $|\psi_0\rangle$), and consider the expansion $|e,n\rangle = \sum_m c_{nm}|\psi_m\rangle$ with the coefficients $c_{nm} = \langle\psi_m|e,n\rangle$. Therefore,

$$|f\rangle\langle e| = \sum_n |f,n\rangle\langle e,n| = \sum_{nm} c_{nm}^*|f,n\rangle\langle\psi_m|. \tag{14}$$

Then the Hamiltonian (1) in the interaction picture is

$$H_I = \Omega\cos\omega_p t\sum_{nm} e^{i(\omega_f + n\omega_c - \lambda_m)t}c_{nm}^*|f,n\rangle\langle\psi_m| + \text{H.c.} \tag{15}$$

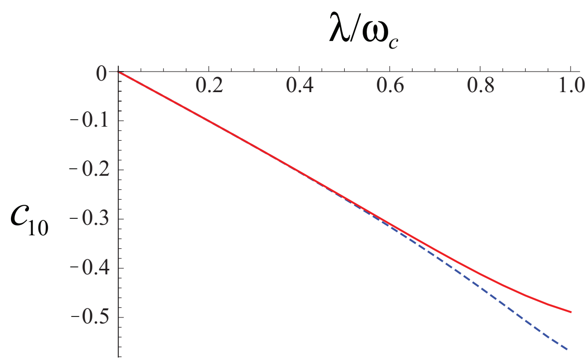

FIG. 2. (Color online) Probability amplitude of $|e,1\rangle$ in the ground state of H_R as a function of the coupling strength λ for the $\omega_0 = \omega_c$ case. The solid red line corresponds to exact numerical values, and the dashed blue line is obtained from the approximated ground state $e^{-S}|g,0\rangle$ according to Eq. (6).

At the resonant frequency $\omega_p = \omega_f + \omega_c - \lambda_0$, $|\psi_0\rangle$ and $|f,1\rangle$ are resonantly coupled. By keeping only the resonant terms, we have

$$H_I \approx \frac{\Omega c_{10}^*}{2}|f,1\rangle\langle\psi_0| + \text{H.c.} \tag{16}$$

Comparing with H_I' in Eq. (13) and noting that $|\psi_0\rangle \approx e^{-S}|g,0\rangle$, H_I describes the same type of resonant interaction as H_I'. However, we emphasize that H_I in Eq. (16) is a more accurate interaction Hamiltonian than H_I' because H_I is derived directly from the eigenbasis of H_R without making use of the approximation in Eq. (6). In this sense, the resonant condition (12) can be improved by replacing the square bracket term by λ_0.

The role of virtual photons is now explicitly seen in Eq. (16) through the effective vacuum Rabi frequency $\Omega|c_{10}|$. This is because c_{10} is precisely the probability amplitude of a single virtual photon state in $|\psi_0\rangle$. In other words, we may interpret that the interaction described in Eq. (16) is induced or mediated by a virtual photon. In Fig. 2, we plot c_{10} (solid line) as a function of λ/ω_c for the case $\omega_c = \omega_0$, and the figure shows that the magnitude of c_{10} is appreciable in the ultrastrong-coupling regime. As a comparison, we also plot the approximate amplitude $c_{10} \approx -\eta^{1/4}\xi\lambda/\omega_c$ (dashed line) obtained from $e^{-S}|g,0\rangle$. For the parameters used in Fig. 2, we see that the approximation agrees well with the exact numerical calculation up to $\lambda/\omega_c < 0.6$.

We have tested our prediction of the virtual-photon-induced Rabi oscillations by solving numerically the Schrödinger equation defined by the Hamiltonian (1) with the initial state $|\psi_0\rangle$. In Fig. 3, we plot the exact numerical probability P_{1f} of the system in the state $|f,1\rangle$ as a function of time. The parameter $\lambda = \omega_c/2$ used in the figure serves as an example of ultrastrong coupling. We see the Rabi cycles as predicted by the Hamiltonians (13) or (16) for relatively weak driving fields with $\Omega \leqslant 0.4\omega_c$. At a stronger driving field with $\Omega = 0.8\omega_c$ (red solid line), there is a high-frequency pattern due to counter-rotating terms of the classical driving field, and the Rabi oscillations are less perfect in the sense that the maximum $P_{1f} \approx 0.9$ is smaller than 1. Such a behavior is understood because the off-resonance transitions neglected in Eq. (13) or (16) would generate energy shifts which in turn could bring the driven system out of resonance. As a result, the amplitude of oscillations in P_{1f} is reduced. Since these energy shifts are generally proportional to Ω^2, as long as Ω is small compared with detunings associated with off-resonance transitions, it would be safe to use Eq. (16), and this is demonstrated in Fig. 3 for Ω up to $0.4\omega_c$.

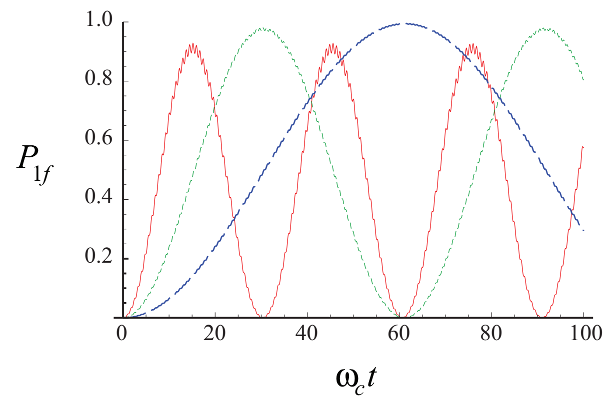

FIG. 3. (Color online) Probability of $|f,1\rangle$ as a function of time for $\Omega = 0.2\omega_c$ (blue long-dashed line), $0.4\omega_c$ (green short-dashed line), and $0.8\omega_c$ (red solid line). The parameters used are $\lambda = 0.5\omega_c$, $\omega_c = \omega_0 = \omega_f/3$, $\omega_p = \omega_f + \omega_c - \lambda_0$, and the numerical ground-state energy $\lambda_0 = -0.633\omega_c$. The figure is essentially the same if ω_p in Eq. (11) is used.

Finally, it is worth noting that the Hamiltonian in Eq. (15) has higher resonances at $\omega_p = \omega_f + n\omega_c - \lambda_0$ for odd positive integers n. The requirement of an odd n is because $|\psi_0\rangle$ has a definite parity in which the atomic state $|e\rangle$ and odd photon numbers are connected. In the case $n = 3$, the driving field at the corresponding ω_p would resonantly excite the atom to $|f\rangle$ with the emission of three real photons. The effective Hamiltonian would be of the same form as (16), but with $|f,1\rangle$ and c_{10}^* replaced by $|f,3\rangle$ and c_{30}^*, i.e., the effective Rabi frequency is proportional to $|c_{30}|$. Such a three-photon resonance was also observed in our numerical calculations.

To conclude, we have shown that virtual photons in the ultrastrong-coupling regime can play a role in quantum dynamics by providing transition matrix elements that allow the system to access relevant quantum states of interest. In our scheme, the system can exhibit a form of vacuum Rabi oscillations, and this can be regarded as a signature of virtual photons. Since our main focus in this paper is on the interaction induced by virtual photons, decoherence effects have not been included in the discussion. However, as long as the decoherence time is sufficiently short, coherent dynamics predicted by the Hamiltonian (13) or (16) would be justified. Specifically, given a vacuum Rabi period $T \approx 2\pi\omega_c/\lambda\xi\Omega'$, the cavity field damping rate γ_c, and the atomic decay rate γ_A, the condition $\gamma_j T \ll 1$ ($j = c,A$) ensures that the system can execute a Rabi cycle without being affected by the damping, and this is achievable in the ultrastrong-coupling regime with moderate small γ's. For the parameters used in Fig. 3, for example, $\gamma_j < 10^{-2}\omega_c$ would be sufficient. We emphasize that a finite interaction time within T is of practical importance, since the interaction (13) or (16) is switchable via the driving field. This feature could be a tool for performing quantum operations on qubits formed by the atom or the field, as well as for deterministic single-photon generation [34,35]. In addition, since the effective vacuum Rabi frequency is proportional to the corresponding virtual photon amplitude, our scheme can be used to probe the ground-state structure of the quantum Rabi model.

The author thanks H. T. Ng for discussions. This work is partially supported by a grant from the Research Grants Council of Hong Kong, Special Administrative Region of China (Project No. CUHK401812).

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