Single-cycle-pulse passively-mode-locked laser with inhomogeneously broadened active medium

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(Received 29 January 2013; published 25 April 2013)

Generation of single-cycle pulses from a laser passively mode locked by the technique of coherent mode locking is theoretically demonstrated for the case when resonant lines of both the gain and absorbing media are inhomogeneously broadened. In contrast to conventional mode-locked lasers, for which the inhomogeneous nature of line broadening sets a severe limitation on their performance, here, the stable single-cycle pulse operation is shown to persist even when the spectral widths of the inhomogeneously broadened lines of the amplifier and the absorber become nearly as wide as the central resonance frequency of their two-level transitions. This study is aimed at showing the potential for application of the coherent-mode-locking technique for quantum-dot lasers, which are typically characterized by a great amount of inhomogeneous broadening.

DOI: 10.1103/PhysRevA.87.043836 PACS number(s): 42.60.Fc, 42.65.Re

I. INTRODUCTION

It is well-known textbook material that the gain spectral width of a laser medium sets the upper limit to the spectral width of pulses generated by means of a mode-locking mechanism. This explains the continuous quest for laser materials with gain spectral widths as wide as possible. The titanium-sapphire laser has the broadest gain spectrum. However, even this laser is not capable of supporting pulses shorter than two carrier oscillation periods; see Refs. [1–3]. In a recent paper [4] we theoretically demonstrated the possibility of generation of single-cycle pulses directly from a laser oscillator with a narrowband homogeneously broadened gain medium. This counterintuitive demonstration became possible due to the use of the technique of coherent mode locking (CML) [5], which is conceptually different from the widespread method of Kerr-lens mode locking [6].

All existing mode-locking methods, including the Kerr-lens mode-locking mechanism, assume that the interaction of a pulse circulating in the cavity with the gain medium is *linear*, or at most, only weakly nonlinear. In contrast, the CML technique requires that the pulse is strong enough to induce substantial nonlinear effects in the amplifier. The field-induced broadening of the gain spectrum is the key ingredient of the CML technique. In other words, the pulse itself broadens the medium. Such a nonlinearly broadened gain spectrum can accommodate shorter pulses. The formulation and the proof-of-principle demonstration of the CML technique can be found in Ref. [5], its application to the mode-locking of quantum-cascade lasers was theoretically tested in Ref. [7], while the ultimate limit in terms of generation of pulses with the shortest possible durations has been probed in Ref. [4]; see also the precursors of this study, where few-cycle dissipative solitons in the free-propagation regime were found [8–10]. In particular, it was shown in Ref. [4] that the CML technique was capable of delivering single-cycle pulses. Motivated by the expectation that a single-cycle mode-locked laser shows strong promise for diverse applications throughout science and technology, in particular in extreme nonlinear optics, attosecond science, and few-cycle physics [11–14], our goal is to proceed with further research on the CML technique.

In order to bring the CML idea close to practice, we need to work with a medium with a value of the dipole moment which is of the order of at least 1 D (a characteristic value for dipole-allowed transitions in alkali-metal atoms) or greater. The appropriate candidates for the realization of such a single-cycle-pulse coherently-mode-locked laser are semiconductor (bulk, quantum-well, quantum-dot) lasers. Among these semiconductor structures, quantum-dot-based lasers seem to be most promising, as quantum dots can be engineered with dipole moments which are an order of magnitude greater than those of the bulk semiconductors.

The geometric sizes of quantum dots embedded in a host matrix are poorly controlled during the manufacturing process, and as a result, the resonance frequency (of a two-level transition of interest) varies greatly from one quantum dot to another. Effectively, the resonance line of the entire ensemble becomes inhomogeneously broadened. The width of this inhomogeneously broadened line can sometimes reach a value close to the central resonance frequency of the two-level transition, and therefore becomes comparable to the bandwidth of a single-cycle pulse. Such a substantial modification of the properties of the gain (and also the absorbing) medium with respect to our previous models based on the assumption of a homogeneously broadened gain spectrum, requires a check as to whether the CML method still retains its attractive features and the capability of generating single-cycle pulses. This check is the major goal of the present study.

Note also that lasers with gain media with predominantly inhomogeneously broadened spectra are believed to be not well suited for mode-locking operation. This problem is particularly important for quantum-dot lasers; see, for instance, Ref. [15]. The inhomogeneous nature of the gain spectrum suppresses the mechanism of mode competition for the gain, and as a result, modes are no longer superposed coherently, thus preventing stable mode-locked operation. Indeed, a laser mode locked

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in a conventional manner tends to lock only modes within a bandwidth equal to the bandwidth of the homogeneously broadened contour, γ_{21}. Thus, the inhomogeneously broadened spectrum of width $(T^*)^{-1}$ appears to be split into approximately $(\gamma_{21}T^*)^{-1}$ portions. Modes within each of these spectral portions are locked independently of each other. Such a regime of generation of an incoherent superposition of multicolor pulses cannot be characterized as a mode-locked operation, because a mode-locked laser is required to deliver pulses that are fully coherent over their entire spectral width.

The single-cycle-pulse CML laser is immune to the nature of the broadening, as we show below. In particular, the limitations set by the inhomogeneous character of the gain spectral line, and inherent to conventional mode-locked lasers, simply does not show up for a CML laser, thanks to the ultrabroadband spectrum of single-cycle pulses. Indeed, even if the spectrum of the gain medium is extremely wide, as is the case for quantum-dot lasers, but still it is not as wide as the central resonance frequency of the two-level transition. Thus, virtually any gain medium appears to be narrowband for a single-cycle pulse CML laser.

II. PHYSICAL CONSIDERATIONS

In its essential properties, the model exploited here reproduces the theory developed in our previous study of the homogeneously broadened CML laser in Ref. [4]. Such continuity in modeling is also important from the viewpoint of ease of a direct comparison of pulse characteristics (energy, duration) of the two lasers, which are different only in the type of line broadening, while the other parameters of the gain and absorbing media are kept the same.

The details of the model can be found in Ref. [4], while here we reproduce only its main features, which are necessary for understanding the problem. The laser is a passively-mode-locked laser, which requires an intracavity absorber. Typically, absorbers are divided into two main categories: "fast" and "slow." Our absorber belongs to neither of these two classes. Instead, it is a "coherent" absorber. The term "coherent" means that the spectral width of the pulse circulating inside the cavity is much greater than the homogeneous width of the absorption line. Putting this differently, the pulse interacts with the absorber in a coherent manner. A similar requirement, to be coherent with respect to the intracavity pulse, is imposed on the gain medium: the amplifier medium must have a relaxation time of polarization much longer than the duration of the pulse. The coherent nature of the matter-field interactions explains the name of the technique—coherent mode locking.

The important observation here is that the coherence of the matter-field interaction is defined by the relation between the relaxation times of a single two-level atom (or another two-level system), and not by the optical response of the entire ensemble. That is, if the ensemble itself is characterized by a broad inhomogeneous line, the coherence of the matter-field interaction will be still preserved, requiring only that the homogeneously broadened spectrum of the individual two-level system remains sufficiently narrowband. The survival of this microscopic coherence even in the presence of the fast decorrelation of the collective optical response on the macroscopic (ensemble-averaged) level, lies at the heart of well-known effects such as photon echo and self-induced transparency (SIT); see Refs. [16,17].

This reasoning shows that the main feature of the CML technique—the coherent character of the matter-field interaction—extends also to the inhomogeneously broadened systems, provided the homogeneous broadening of an individual two-level system remains narrowband. However, the question of whether the performance of the CML technique will remain on an acceptable level when we use inhomogeneously broadened systems instead of homogeneously broadened ones is still awaiting resolution and is the main question that motivated this study. Along with the coherent gain medium, the CML method assumes the presence of a coherent absorber. It is reasonable to argue that if the interaction of the intracavity pulse with the absorber was of the SIT type for the case of the homogeneously broadened ensemble, as was shown to be the case in Ref. [4], then the SIT type of behavior will persist for the inhomogeneously broadened ensemble, no matter how broad is its spectrum. In other words, if the pulse propagated through the absorber with virtually no losses in the homogeneously broadened ensemble, then the amount of loss will not increase when the medium turns into an inhomogeneously broadened one. From the viewpoint of our laser system, this argument gives us encouragement to expect that the use of the inhomogeneously instead of homogeneously broadened absorbers will bring no additional losses to the system. This argument indeed appears to be true, as supported by the numerical results discussed below.

The consideration of the gain medium is less straightforward, and to our understanding, does not lead us to conclusions as promising as for the absorber. In the CML laser, the pulse circulating inside the cavity behaves as a π pulse with respect to the gain medium. That is, the medium is left maximally deexcited after the pulse has passed through it. However, if the inhomogeneously broadened spectrum of the gain medium is substantially wider than the pulse spectral width, then we cannot expect the spectral wings of the pulse to be powerful enough to fully deexcite the atoms having resonance frequencies distant from the center of the inhomogeneously broadened gain spectrum. This regime is not advantageous energetically, because the efficiency of extraction of the energy from the inhomogeneously broadened gain medium of width $(T^*)^{-1}$ by a pulse with spectral width $\Delta\omega$ is $\Delta\omega T^*$ times smaller than in the case of the homogeneously broadened medium. In order to bring the laser back to the same point of the gain-loss equilibrium as with the homogeneously broadened gain medium, we need to enlarge the linear gain accordingly, for example, by increasing the number of active centers.

Fortunately, such a supernormal amount of broadening is not typical for the quantum-dot single-pulse laser under our consideration, simply because no (known to us) semiconductor samples are characterized by inhomogeneous spectra as wide as the central resonance frequency itself. Even if such samples can be engineered, they will be not suitable for our purposes. Thus, we shall study here two cases, which we label for convenience as "narrowband" [with $(T^*) = 7.1\omega_{21}^{-1}$, where ω_{12} is the central frequency of the inhomogeneously broadened gain spectrum] and "broadband" (with $T^* = 1.77\omega_{21}^{-1}$). Basing on the reasoning given above, we anticipate that the characteristics of the narrowband inhomogeneously broadened

CML laser should not appear too different from those of its homogeneously broadened partner, while for the broadband inhomogeneously broadened laser the characteristics may differ more substantially. This expectation is correct, as we show below.

III. MODEL

The main requirement associated with the method of CML is sufficient power of the intracavity field. More precisely, it is not a requirement on the absolute power, but rather on the strength of the matter-field interaction, expressed in terms of the Rabi frequency (here the Rabi frequency Ω_R is the product of the atomic transition dipole moment d_p and the strength of the electric field E, divided by the Planck constant $\hbar$: $\Omega_R = d_p E/\hbar$). The condition for generating single-cycle pulses amounts to getting the Rabi frequency equal to the central resonance frequency of the two-level transition.

Another critical parameter of the coherently-mode-locked laser is the level of linear intracavity losses. We assume that all such losses are associated with the transmission of the pulses through the outcoupling mirror, as shown in Fig. 1(a). It is clear that the lower limit should appear on the amplitude reflection coefficient of the outcoupling mirror. Indeed, if too much energy leaks from the laser, then the pulse becomes less intense and therefore longer (because of smaller nonlinearity-induced broadening of the gain medium); the longer pulse loses too much energy in the incoherent losses inside the absorber and also cannot any longer efficiently deplete the amplifier. Therefore the losses increase in an avalanche manner, the feature which determines the sharp threshold in the reflection coefficient R of the outcoupling mirror. In support of this reasoning, we indeed observed an abrupt termination of the mode-locked operation when R dropped below 0.9 for the homogeneously broadened gain medium, and 0.92 for the narrowband, and 0.97 for the broadband inhomogeneously broadened gain media. These values of R are specific to our choice of the linear gain coefficient of the amplifier and the linear absorption coefficient of the intracavity absorber, detailed below.

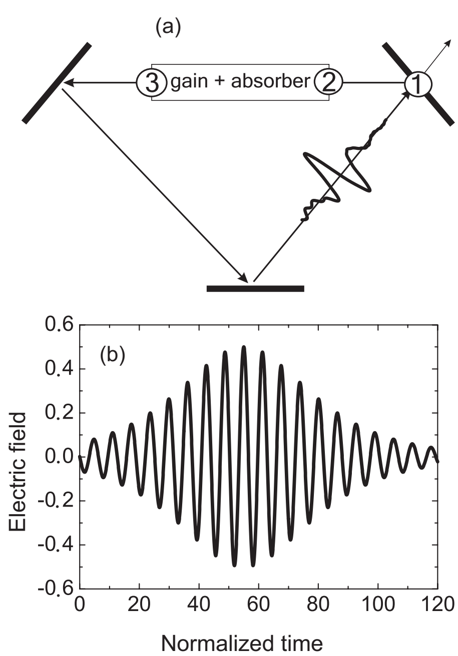

FIG. 1. (a) Schematic of the coherently-mode-locked laser. The amplifier and absorber are implemented within the same sample, as in the numerical model. If necessary, the gain and absorbing media can be placed in different locations inside the cavity. (b) The seed (input) pulse used to trigger the operation of the laser. The electric field is the normalized Rabi frequency Ω, and time is normalized to ω_{21}^{-1}.

In the extreme situation of generation of single-cycle pulses, the approximations commonly applied to conventional mode-locked lasers break down, so that we have to abandon the slowly varying, rotating-wave, and unidirectional-propagation approximations altogether. The two Maxwell equations

$$\frac{\partial D}{c\partial t} - \frac{\partial H}{\partial z} = 0 \quad \text{and} \quad \frac{\partial H}{c\partial t} - \frac{\partial E}{\partial z} = 0 \tag{1}$$

for the electric E and magnetic $H = B$ fields are coupled to two systems of nonreduced Bloch equations. The first of such systems describes the inhomogeneously broadened amplifier:

$$\frac{\partial}{\partial t}\rho_{21}^{(a)} = -\left[i\omega_{21}^{(a)} + \Delta\omega_a(z) + \gamma_{21}^{(a)}\right]\rho_{21}^{(a)} - i\frac{d_a E}{\hbar}\left(\rho_{22}^{(a)} - \rho_{11}^{(a)}\right), \tag{2}$$

$$\frac{\partial}{\partial t}\rho_{22}^{(a)} = -\gamma_2^{(a)}\rho_{22}^{(a)} - i\frac{d_a E}{\hbar}\left(\rho_{21}^{(a)} - \rho_{12}^{(a)}\right) + p, \tag{3}$$

$$\frac{\partial}{\partial t}\rho_{11}^{(a)} = \gamma_2^{(a)}\rho_{22}^{(a)} - \gamma_1^{(a)}\rho_{11}^{(a)} + i\frac{d_a E}{\hbar}\left(\rho_{21}^{(a)} - \rho_{12}^{(a)}\right), \tag{4}$$

and the second system is associated with the inhomogeneously broadened absorber:

$$\frac{\partial}{\partial t}\rho_{21}^{(p)} = -\left[i\omega_{21}^{(p)} + \Delta\omega_p(z) + \gamma_{21}^{(p)}\right]\rho_{21}^{(p)} - i\frac{d_p E}{\hbar}\left(\rho_{22}^{(p)} - \rho_{11}^{(p)}\right), \tag{5}$$

$$\frac{\partial}{\partial t}\rho_{22}^{(p)} = -\gamma_2^{(p)}\rho_{22}^{(p)} - i\frac{d_p E}{\hbar}\left(\rho_{21}^{(p)} - \rho_{12}^{(p)}\right), \tag{6}$$

$$\frac{\partial}{\partial t}\rho_{11}^{(p)} = \gamma_2^{(p)}\rho_{22}^{(p)} + i\frac{d_p E}{\hbar}\left(\rho_{21}^{(p)} - \rho_{12}^{(p)}\right). \tag{7}$$

Here $D = E + 4\pi P$ with the polarization $P = N_p d_p \rho_{12}^{(p)} + N_a d_a \rho_{12}^{(a)} + \text{c.c.}$; $N_{p,a}$ are the concentrations of passive (absorber) and active (amplifier) two-level systems; $d_{p,a}$ are the (real) dipole matrix elements of the transitions between the upper (2) and lower (1) states; $\omega_{21}^{(a,p)}$ are the transition frequencies, which are taken to be equal to each other, as well as to the carrier frequency ω_0 of the seed pulse which triggers the mode-locking operation; $\gamma_{21}^{(a,p)}$, $\gamma_2^{(a,p)}$, and $\gamma_1^{(a)}$ are the relaxation rates with obvious meanings; and finally p is the pump rate. Note that the absorber is modeled by a closed system, which implies a conservation law in the form $\rho_{11}^{(p)} + \rho_{22}^{(p)} = 1$, whereas the laser amplifier is modeled as usual by an open system, for which a similar conservation law does not hold.

In our numerical simulations we used the following set of parameters: The ratio of dipole moments $\sqrt{\mu} = d_p/d_a$ was 1.5. This value was intentionally made different from the ideal case of $\sqrt{\mu} = 2$, in order to test the relative insensitivity of the CML operation to the precise value of this ratio, which is important for the practical implementation of the CML technique. The relaxation rates were $\gamma_1^{(a)} = 0.025$, $\gamma_2^{(a)} = 0.005$, $\gamma_2^{(p)} = 0.006$, $\gamma_{21}^{(p)} = 0.0025$, and $\gamma_{21}^{(a)} = 0.015$, and the pump rate was $p = 0.004$. All of these quantities are expressed in units of the frequency $\omega_{21}^{(a)} = \omega_{21}^{(p)}$. Other parameters are chosen in such a way that the coupling constants between the field (which is expressed in terms of the dimensionless Rabi frequency $\Omega = \Omega_R/\omega_{21}^{(p)}$) and the polarizations induced by the passive and active systems read as $\beta = 4\pi N_p d_p^2/\hbar\omega_{21}^{(p)} = 0.1$ and $\kappa = 4\pi N_a d_p d_a/\hbar\omega_{21}^{(a)} = 0.02$. As an estimate for the intracavity intensity I we can write $I = (104/x^2)$ GW/cm^2 for $\Omega_R = \omega_{12}^{(a)} = 10^{15}$ s^{-1}, where $d_p = 1.6x \times 10^{-28}$ C m; here the spatial extension of the dipole x is expressed in nanometers. Thus, for $x = 10$ the intracavity intensity becomes as small as 1 GW/cm^2. This estimate shows the possibility of generation of intracavity single-cycle pulses with duration $\sim$6 fs and energy $\sim$0.4 pJ, provided that the active area of the gain medium is $\sim$6 μm^2.

The model equations (1)–(7) need to be supplemented by appropriate boundary conditions. We considered a ring-cavity configuration supporting predominantly unidirectional propagation, and consisting of a number of mirrors, one of which is partially transparent with the amplitude reflection coefficient R; see Fig. 1(a). In this case, the relation $\Omega_{\rm ref} = R\Omega_{\rm in}$ establishes the connection between the wave which is incident on the mirror $\Omega_{\rm in}$ and the reflected wave $\Omega_{\rm ref}$. The balance between the gain κ and the mirror outcoupling losses $(1 - R)$ provides a key relation in our model, as it ultimately determines the intracavity pulse power. In order to generate pulses of one carrier period in duration, i.e., with $\tau_p \approx \omega_{12}^{-1}$, we need to ensure a large gain-to-loss ratio which permits the condition $\Omega \approx 1$ to be reached. The next important parameter is the strength of the absorber β: its large value is necessary for the stability of the mode-locked operation. On the other hand, the precise values of the various relaxation constants are not very important (as long as they do not become too large). The only requirement here (dictated solely by the intention of easing the numerical modeling and having virtually no impact on the physics of operation of a generic CML laser) is that the total length L of the cavity is long enough in order to guarantee full recovery of the equilibrium population differences of the amplifier and the absorber before the pulse comes back after completing the round trip along the cavity.

The way of modeling the effect of inhomogeneous broadening of two-level ensembles implemented here is different from the conventional approach. The conventional method requires the introduction of a detuning term (which is the frequency shift between the carrier frequency of the electric field and the resonance frequency of the two-level transition) into the equation for the nondiagonal density matrix element of the two-level transition. Then the macroscopic polarization of the entire ensemble (which is the driving term in the wave equation) is obtained by averaging over all possible detunings with an appropriate weighting function, which has the shape of the inhomogeneously broadened spectrum. This approach becomes resource consuming when implemented numerically, because at each spatial step it requires spanning over a representative number of detunings, typically 50–100, and thus slows down numerical simulations by about two orders of magnitude in comparison with simulations of similar homogeneously broadened systems. In terms of computational resources, it is nearly equivalent to adding a new dimension to the problem.

An alternative approach to modeling the inhomogeneous broadening was suggested in Ref. [9], and recently theoretically verified in Ref. [18]. Its implementation amounts to the introduction of terms

$$\Delta\omega_a(z) = (T^*\sqrt{2})^{-1} F_a(z), \tag{8}$$

$$\Delta\omega_p(z) = (T^*\sqrt{2})^{-1} F_p(z) \tag{9}$$

into Eqs. (2) and (5), respectively, with $F_{a,p}(z)$ being spatial stochastic Gaussian processes with zero mean and unit standard deviation (that is, distributed according to the normal distribution). In other words, $\Delta\omega_{a,p}$ now becomes a function of the spatial coordinate z, and at each subsequent z we take a random Gaussian-distributed number (positive or negative) and multiply it by a factor of $(T^*\sqrt{2})^{-1}$. The stochastic process is a Markovian one, since at each subsequent spatial step $\Delta\omega_{a,p}$ are selected truly randomly and independently of the value of $\Delta\omega_{a,p}$ in the preceding step. As shown in Ref. [18], $(T^*)^{-1}$ is the characteristic width of the inhomogeneously broadened spectrum. This way of modeling has been proved to be statistically identical to the standard procedure, provided that the shapes of the inhomogeneously broadened lines are approximated by a Gaussian function, as is the case in the present study. From the viewpoint of the computational load, the stochastic approach to modeling the inhomogeneous broadening adds neither complexity nor addditional resources, and takes essentially similar running times, when compared with simulations of homogeneously broadened systems. Here, we choose to work with similar inhomogeneous contours for both gain and absorbing media [note the same parameter T^* in the expressions for $\Delta\omega_a(z)$ and $\Delta\omega_p(z)$] in Eqs. (8) and (9).

IV. RESULTS AND DISCUSSION

We performed numerical simulations of the approximation-free Maxwell-Bloch equations based on the finite-difference time-domain integration method. Our numerics mimics the intracavity dynamics of the pulse circulating between mirrors: starting from point 1 in Fig. 1(a), the seed many-cycle pulse, which is shown in Fig. 1(b), experiences free propagation in vacuum until point 2. Next the pulse experiences gain and shaping in the amplifier as well as absorption and shaping in the absorber until it reaches the point 3, where the pulse is again subjected to free propagation towards point 1, where the circulating pulse experiences partial transmission at the outcoupling mirror. The round trip is thus completed. Typically, fewer than 100 round trips are needed to reach the stationary mode-locked regime. Additional losses appear as a result of reflection of the pulse from the boundary between vacuum and the resonant media. The back-propagating wave which is generated in each round trip rapidly vanishes (numerically, it is artificially absorbed), as we assume that the cavity contains elements supporting only unidirectional beam propagation.

The CML laser oscillation does not start from spontaneous noise since the cavity is lossy for low-intensity cw radiation. Therefore a relatively long and powerful many-cycle femtosecond pulse [shown in Fig. 1(b)] was used for initiating the mode-locked operation. Note that for $R < 0.96$ and $T^* = 7.1\omega_{21}^{-1}$, as well as for $R = 0.97$ and $R = 0.98$ for $T^* = 1.77\omega_{21}^{-1}$, the amplitude of the seed pulse was doubled, to ensure that the enhancement of losses of the radiation (particularly significant in the transient regime) caused by the effect of line broadening would not quench the laser operation in its initial stage.

We allowed the reflection coefficient R to vary between 0.89 and 0.98. We found that for the narrowband line with $T^* = 7.1\omega_{21}^{-1}$ and for $R < 0.92$, the total intracavity losses were too large, and the laser could not reach a stable mode-locked operation, while for $R \geqslant 0.92$ it was possible to initiate the desirable mode-locked regime. For the broadband line with $T^* = 1.77\omega_{21}^{-1}$, the minimal value of R, necessary for sustaining the stable mode-locked operation became as high as 0.97. Note that for the case of the homogeneously broadened line the minimal value of R was as small as 0.9; see Ref. [4]. The differences and similarities between all these three regimes can be analyzed by looking at the energy plots shown in Figs. 2(a) and 2(b). As long as the width of the inhomogeneously broadened line remains substantially narrower than the pulse spectral width in the stationary mode-locked regime, the difference from the case of homogeneous broadening is not essential; compare the black squares and red circles in Figs. 2(a) and 2(b). However, when the linewidth $(T^*)^{-1}$ becomes comparable to the pulse spectral width, the total intracavity energy is noticeably reduced; see the green triangles in Fig. 2(a). At the same time, the fraction of pulse energy contained in the "fundamental" spectral band (between $\omega = 0$ and $\omega = 1.77\omega_{21}$) is approximately the same as in the other two cases; see the green triangles in Fig. 2(b).

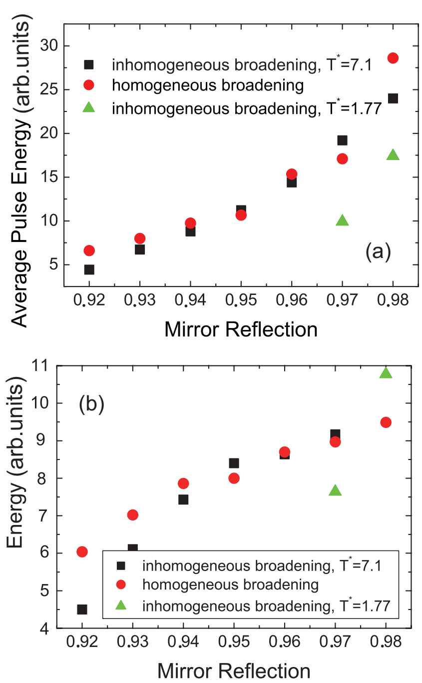

FIG. 2. (Color online) (a) Total intracavity energy and (b) fraction of energy contained in the fundamental spectral band (between $\omega = 0$ and $\omega = 1.77\omega_{21}$) of a single pulse in a stationary mode-locked regime as a function of reflectivity R of the outcoupling mirror for three cases: (i) narrowband inhomogeneously broadened line with $T^* = 7.1\omega_{21}^{-1}$; (ii) homogeneously broadened line; (iii) broadband inhomogeneously broadened line with $T^* = 1.77\omega_{21}^{-1}$. Note that in (a) we plot the energy averaged over the round trip, while (b) shows the energy at the location of the outcoupling mirror. Here $T' = T^*\omega_{21}$.

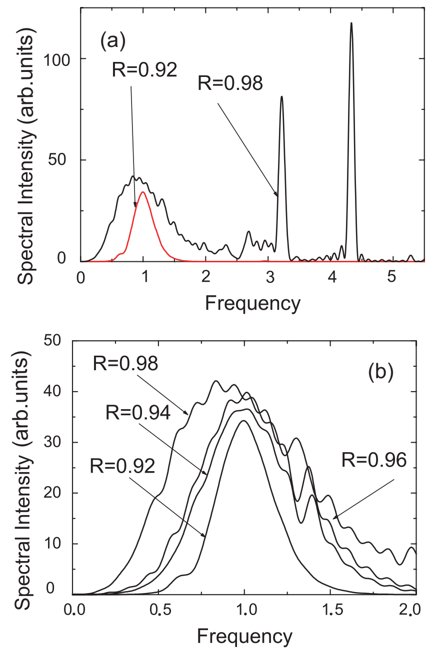

FIG. 3. (Color online) (a) Spectra of the pulse in the stationary mode-locked regime for two values of the reflection coefficient R of the outcoupling mirror; (b) spectra of the pulses for different values of the reflection coefficient R in the fundamental spectral band. Frequency is measured in units of ω_{21}. The gain and absorbing media are both inhomogeneously broadened with $T^* = 7.1\omega_{21}^{-1}$.

With increasing R, more and more energy is deposited into higher harmonics of the fundamental frequency ω_{21}. This tendency can be clearly seen from Fig. 3(a), where two spectra, for $R = 0.92$ and $R = 0.98$, are compared. The efficient generation of the third and higher harmonics arises from the fact that our model implies that the cavity contains no dispersive elements in addition to the intrinsic dispersion of the resonant media. In practice, extra sources of dispersion are always present (if not in the band of interest below $2\omega_{21}$ due to a careful design of dispersion-compensating schemes, then at least for frequencies greater than $2\omega_{21}$), and therefore the generation of the higher harmonics will be totally suppressed (unless phase-matching conditions are provided intentionally), and only the generation in the fundamental spectral band will survive. Based on this reasoning, we believe that the fraction of energy contained in the fundamental spectral band is more important than the total amount of intracavity energy. Figure 3(b) shows the spectra of pulses in the spectral band of interest for different values of R. In order to complete the picture, we plot in Fig. 4 the temporal shapes of pulses for different values of R. Clearly, for all shown values of R we obtain genuine single-cycle pulses.

Note the peculiarity of the spectrum shown in Fig. 3(a) for $R = 0.98$. Two strong peaks appearing at $3.2\omega_0$ and $4.4\omega_0$

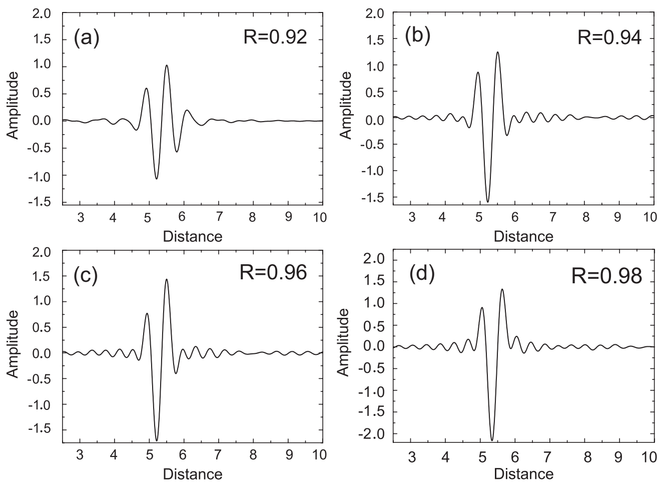

FIG. 4. Temporal shapes of intracavity pulses in the stationary mode-locked regime after they passed through a broadband low-pass filter with the transmission band between $\omega = 0$ and $\omega = 1.77\omega_{21}$, for different values of the reflection coefficient R (shown on each plot). The gain and absorbing media are both inhomogeneously broadened with $T^* = 7.1\omega_{21}^{-1}$. Time is measured in units of ω_{21}^{-1}.

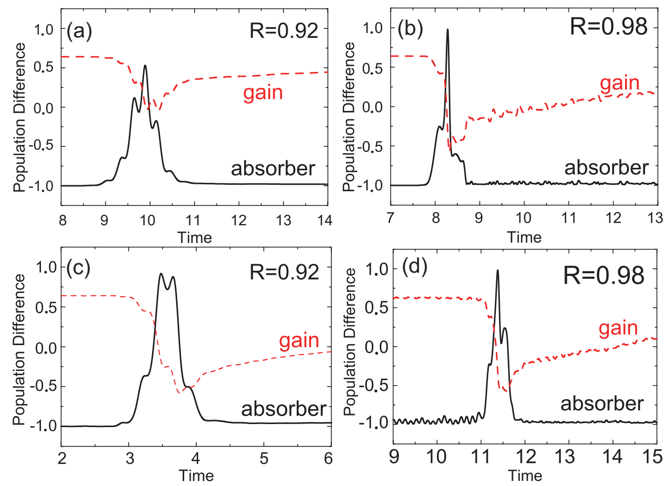

FIG. 5. (Color online) Population differences for the absorbing and gain media for the case when both media are (a) and (b) inhomogeneously broadened with $T^* = 7.1\omega_{21}^{-1}$; (c) and (d) homogeneously broadened. The profiles shown are obtained after the laser reaches the stationary mode-locked regime. The temporal profiles are taken at the point exactly at the center of the combined absorbing-gain medium. Values of the reflection coefficients are shown on the plots.

are attributed to multiple-wave frequency-mixing processes (in our model the central frequency ω_0 of the spectrum of the intracavity pulse in the fundamental band coincides with ω_{21}). If the intracavity pulse had a narrowband spectrum, then these peaks would appear exactly at the third and fifth harmonics of the fundamental frequency ω_0. In our case, the pulse has an ultrabroadband spectrum, for which generation of the third and the fifth harmonics is suppressed in favor of generation of peaks at $3.2\omega_0$ and $4.4\omega_0$, appearing due to a broadband continuum of four-wave mixing processes (and higher-order mixing processes, as well) like $\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$, where the frequencies ω_1, ω_2, and ω_3 all lie within the ultrabroadband spectrum of the single-cycle pulse. Each of these frequencies has its own nonlinear phase shift. The coherent superposition of all possible sets of triples $E(\omega_1)$, $E(\omega_2)$, and $E(\omega_3)$ with their individual phase shifts results in the appearance of the spectral pattern shown in Fig. 3(a).

The (theoretical) observation of the "abnormal" shift of the sum- and difference-frequency components from their canonical positions (at $3\omega_0$ and $5\omega_0$), expected on the basis of the standard nonlinear optics intuition developed predominantly for quasimonochromatic fields, has been reported recently in Ref. [19] in connection with propagation of single-cycle pulses through a Kerr medium. In our case, the physics of the nonlinear frequency conversion has a similar origin, but with two important differences. First, for the case of the Kerr medium only four-wave mixing processes are active in the system, while in our model the resonant nature of the nonlinearity initiates not only four-wave but also all higher-order mixing processes as well. Second, and most important, is that in contrast to the conservative nature of the free-propagation problem of an intense single-cycle pulse through a Kerr medium, studied in Ref. [19], here we are dealing with a dissipative system with feedback. Thus, on each round trip the pulse circulating inside the cavity gets energy from the amplifier and deposits a part of this energy into higher harmonics through the process of nonlinear frequency conversion. The constant flow of energy into the pulse from the amplifier can result in generation of very energetic higher harmonics, in extreme cases even exceeding the energy in the fundamental frequency band. Note also that similar abnormal spectral shifts of higher harmonics have been reported recently in our study of the CML laser with homogeneously broadened gain and absorbing media in Ref. [20].

The physics of the CML laser shows up most distinctly from analysis of the plots in Fig. 5, where we show graphs of population differences at an arbitrarily chosen spatial location inside the gain and absorbing media. Surprisingly, in a wide range of parameters, and independently of the intracavity power, the pulse always behaves as an ideal 2π pulse with respect to the absorber, and simultaneously as a nearly ideal π pulse with respect to the amplifier. The complete excitation-deexcitation cycle (an analog of the 2π rotation of the Bloch vector) of the absorber initiated by the pulse proves the conjecture put forward in Sec. II that the inhomogeneous broadening indeed does not bring additional losses with respect to homogeneously broadened systems.

A long tail following the passage of the pulse through the chosen point inside the amplifier appears as a result of the pumping process [the term p in the right-hand side of Eq. (3)]. The pumping process is required to restore the initial level of the population inversion fast enough to ensure that, until the pulse returns to this point after completing the round trip, the upper state will again appear as maximally populated.

V. CONCLUSION

We have reached the main goal of this study by way of successful demonstration of the possibility of generation of single-cycle pulses from the CML laser even in the case when the inhomogeneous lines of the gain medium and the intracavity absorber were extremely wide (with widths nearly equal to the central resonance frequencies). Essentially, the laser keeps delivering single-cycle pulses under a rather broad range of parameters. Moreover, the properties of these pulses are only weakly dependent on the type of line broadening and its width, provided the broadening is not wider than the spectral width of the pulse. We did not explore the case of wider spectral lines, because such a supernormal amount of broadening is not characteristic of known semiconductor systems.

ACKNOWLEDGMENTS

This study is partially supported by a grant from the University of ITMO, by the Russian Foundation for Basic Research (Grants No. 11-02-12250-ofi-m and No. 12-02-90020-Bel-a), by the Ministry of Education and Science of Russia, Project No. 14.B37.21.0907, and by the St.-Petersburg University.

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