Mean-field optical bistability of two-level atoms in structured reservoirs

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We consider N driven two-level atoms interacting with a structured reservoir. By dressing the collective operators within a semiclassical approach, we derive a master equation and a mean-field single-particle effective Hamiltonian. This Hamiltonian describes the optical bistability phenomenon occurring in the relation between an input electromagnetic field and the effective output generated by the N atoms. The dissipative part of the master equation and the effective single-particle Hamiltonian contain new terms due the reservoir structure of modes. In plotting the output field amplitude and phase, for a structured reservoir, as a function of the input amplitude, one verifies the bistable behavior in both. We illustrate our results for two structured reservoirs: one having a Lorentzian shape for the distribution of modes, and the second is modeled as a photonic band-gap structure.

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I. INTRODUCTION

Optical and dynamic properties of atoms coupled to dissipative environments with a tailored density of modes have been a hot topic in current research [1–13]. Several interesting and potentially useful effects such as (i) suppression of spontaneous emission [1,2,13]; (ii) modifications in the resonance fluorescence and absorption spectra of strongly driven two-level atoms [3–10]; (iii) amplification without inversion [9]; and (iv) the possibility of effective control of atomic states [11], have been reported by considering the interaction of atoms with structured reservoirs. In general, theoretical studies of dissipative dynamics in structured reservoirs has been essentially devoted to systems of one or a few atoms. However, for a large number of atoms, one interesting phenomenon is the one related to the *optical bistability* (OB) [14–16]. It originates from a nonlinear relation between the intensity of an input field and an effective output field emerging from a collection of two-level atoms. Graphically, one sees the occurrence of an S-shaped curve, which corresponds to the existence of two stationary stable states for the atomic system, thus allowing the use of the system as an optical switch [16].

Since its prediction and observation in the 1970s the OB [17–19] has been the object of intense research due to its theoretical and experimental usefulness for studying nonlinear [20–24], together with far-from-equilibrium effects arising in complex systems [25–27]. The main motivation was the potential applications in the construction of optical devices ([16], and references therein). The basic standard successful description of the OB consists in considering a system of homogeneously broadened two-level atoms driven by a coherent resonant field [28]. As a matter of fact, the phenomenon arises from the interplay between (1) dissipation due to the presence of a reservoir responsible for the atomic decay, (2) feedback due to the mean-field many-atoms effects, and (3) pumping of the atomic sample by an external field, occurring simultaneously. In a more specific context, in Ref. [29] the authors study the change in the bistable S-shaped curve occurring in the population difference in impurity two-level atoms in a pseudophotonic band-gap background to an applied laser field. So, the structure of modes of the reservoir may significantly influence the behavior of the bistable system, as we are going to further explore in this paper.

We shall treat the problem of atomic bistability in a two-level N-atom system interacting with structured reservoirs. Formally we develop our calculations using the semiclassical dressed atom approach for the collective operators, extending the treatment considered in [7,8] for a single atom. We derive a master equation for K ($1 < K < N$) atoms and for a dilute system we obtain an effective single-particle nonlinear Hamiltonian for a representative particle. The effective Hamiltonian and the dissipative terms in the master equation contain additional terms that are absent when a structureless reservoir is considered. The relation between input and output fields is obtained and, in contrast with the case of a structureless reservoir, we found that besides the output amplitude, also the phase presents a bistable behavior. So, this feature allows probing the presence of a reservoir having a nonflat structure of modes. We illustrate our results for (1) a reservoir having a Lorentzian shape for the modes distribution, and (2) a reservoir displaying a photonic band-gap structure.

The article is organized as follows: In Sec. II we introduce the Hamiltonian describing the system of atoms plus fields. In Sec. III we obtain a master equation for the N atoms interacting with a structured reservoir. In Sec. IV, a mean-field approximation is developed and an effective single-particle Hamiltonian is obtained. In Sec. V the relation between input and output fields in the stationary state is obtained. In Sec. VI we illustrate the bistable behavior considering two illustrative models for the reservoir. Finally, in Sec. VII we present a summary and conclusions.

II. ATOM-FIELD SYSTEM

We consider N two-level atoms, with transition frequency ω_0, interacting within the rotating wave approximation (RWA)

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with a laser, assumed as a classical electromagnetic field of frequency ω_L and with the electric component $E_{in}e^{i\varphi}$, having an arbitrary phase φ. Additionally, the atoms interact with a reservoir at 0 K, made of a continuum of modes, which is responsible for the atomic decay. The Hamiltonian of the whole system is given by

$$H = H_A + H_R + V_{AR}, \tag{1}$$

where

$$H_A = \frac{\hbar\omega_0}{2} S_0 + \hbar F(e^{i\omega_L t} S_- + e^{-i\omega_L t} S_+), \tag{2}$$

$$H_R = \int d\omega (\hbar\omega) b^+(\omega) b(\omega), \tag{3}$$

$$V_{AR} = \hbar \int d\omega\, g(\omega)[b(\omega) S_+ e^{i\varphi} + b^+(\omega) S_- e^{-i\varphi}]. \tag{4}$$

The Hamiltonian (2) represents the N two-level atoms pumped by the laser field, with coupling constant $F = \mu E_{in}$ (we assume all atoms have the same atomic dipole moment μ). Admitting that the size of the atoms is much smaller than the laser wavelength, the two active levels of the atoms are described by the collective operators

$$S_0 = \sum_{i=1}^{N} \sigma_0(i); \qquad S_\pm = e^{\mp i\varphi} \sum_{i=1}^{N} \sigma_\pm(i), \tag{5}$$

and $\sigma_0(i)$ and $\sigma_\pm(i)$ are the Pauli operators for a single particle satisfying the commutation relations of the SU(2) algebra $[\sigma_0(i),\sigma_\pm(j)] = \pm 2\delta_{i,j}\sigma_\pm(i)$ and $[\sigma_+(i),\sigma_-(j)] = \delta_{i,j}\sigma_0(i)$. We remind one that $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, $\sigma_0 = [|e\rangle\langle e| - |g\rangle\langle g|]/2$, where $|e\rangle$ and $|g\rangle$ refer to the higher and lower energy levels, respectively, while $\hbar\omega_0$ is the energy difference between the levels. The Hamiltonian (3) represents the reservoir modes, where the operator $b(\omega)$ $[b^+(\omega)]$ annihilates (creates) a quantum of frequency ω, and both satisfy the bosonic commutation relations $[b(\omega),b^+(\omega')] = \delta(\omega - \omega')$. Finally, Hamiltonian (4) corresponds to the coupling between the reservoir modes and atoms, and $g(\omega)$ is the coupling parameter, assumed to be frequency dependent, that characterizes a structured reservoir. Furthermore, we assume that the atomic system is quite diluted such that we disregard the direct interaction between the atoms, so they will correlate and feel each other indirectly, as an effect of their coupling with the reservoir modes. We also consider an undepleted laser field so its dynamics is not taken into account.

In a referential frame rotating at frequency ω_L and in the interaction picture, with respect to the reservoir modes, the Hamiltonian (1) becomes

$$H = H_{0S} + V_{AR}(t), \tag{6}$$

where

$$H_{0S} = \delta S_0 + F(S_- + S_+), \tag{7}$$

and $\delta = (\omega_0 - \omega_L)/2$ is the detuning frequency between atoms and laser field, and

$$V_{AR}(t) = \int d\omega\, g(\omega)[b(\omega) S_+ e^{i(\omega_L - \omega)t + i\varphi} + \text{H.c.}] \tag{8}$$

is the interaction between atoms and the reservoir.

In order to derive a dynamical equation for the atomic system, usually a master equation, we follow an approach considered in Refs. [7,8]. Instead of considering the atomic decay process independently of the driving field, one assumes a coupled atom-driving field decay, i.e., a generic atom is dressed by the driving field and coupled to the reservoir modes. Within this approach we define the semiclassical dressed collective operators

$$\tilde{S}_0 = \frac{1}{\Delta}[\delta S_0 + F(S_- + S_+)], \tag{9a}$$

$$\tilde{S}_- = \frac{1}{2\Delta}[(\delta + \Delta) S_- - F S_0 + (\delta - \Delta) S_+)], \tag{9b}$$

$$\tilde{S}_+ = \frac{1}{2\Delta}[(\delta - \Delta) S_- - F S_0 + (\delta + \Delta) S_+)], \tag{9c}$$

that satisfy the very same commutation relation of the SU(2) algebra $[\tilde{S}_0,\tilde{S}_\pm] = \pm 2\tilde{S}_+$, $[\tilde{S}_+,\tilde{S}_-] = \tilde{S}_0$. For later convenience we write Eqs. (9a)–(9c) in short as

$$\tilde{S}_j = \sum_{l=-1}^{1} c_{jl} S_l, \quad j = -1,0,1, \tag{10}$$

where c_{jl} are the entries of the matrix

$$\mathbb{C}(F,\delta,\Delta) = \frac{1}{2\Delta}\begin{pmatrix} \delta+\Delta & -F & \delta-\Delta \\ 2F & 2\delta & 2F \\ \delta-\Delta & -F & \delta+\Delta \end{pmatrix}, \tag{11}$$

and $\Delta = \sqrt{\delta^2 + F^2}$. Definition (10) is invertible, so the operators $S_\pm$ are related to the dressed ones through the relation

$$S_k = \sum_{j=-1}^{1} \tilde{c}_{kj} \tilde{S}_j. \tag{12}$$

As $\mathbb{C}^{-1}(F) = \mathbb{C}(-F)$, the entries of the matrix $\tilde{\mathbb{C}}(F,\delta,\Delta)$ are $\tilde{c}_{kj}(F) = c_{kj}(-F)$, and the dependence on the other parameters remains the same.

Defining the single index coefficients $C_j = \tilde{c}_{1j} = \tilde{c}_{-1,-j}$ [where $C_{j=\pm1} = (\delta + j\Delta)/(2\Delta), C_0 = F/2\Delta$], the interaction Hamiltonian (8) can be written as

$$V_{AR}(t) = \sum_{j=-1}^{1} C_j \int d\omega\, g(\omega)[B_j(t,\omega)\tilde{S}_j + B_j^\dagger(t,\omega)\tilde{S}_{-j}], \tag{13}$$

where $B_j(t) = b(\omega) e^{-i(\omega - \omega_L - 2\Delta j t)t + i\varphi}$.

III. *N*-ATOM MASTER EQUATION

In order to describe the dynamical evolution of an N-atom system state we begin with the general non-Markovian master equation in the interaction picture [30],

$$\frac{d\rho_{I,N}(t)}{dt} = -\int_0^t d\tau\, \text{Tr}_{\mathcal{R}}[V_{AR}(t),[V_{AR}(t-\tau),\rho_{I,N}(t-\tau)\rho_R]], \tag{14}$$

for the evolution of the density operator $\rho_{I,N}(t)$ of N atoms coupled to an unperturbed reservoir in thermal equilibrium.

As usual, Eq. (14) is obtained by tracing over the reservoir degrees of freedom, and ρ_R is the reservoir density operator. The index I stands for interaction picture. Additionally, Eq. (14) was obtained in the weak-coupling approximation, by assuming that the atomic system plus reservoir density operator factorizes as $\rho_{S+R}(t) = \rho_S(t)\rho_R$, at any time.

By inserting the interaction term given by Eq. (13) in the master equation (14) we obtain

$$\frac{d\rho_{I,N}(t)}{dt} = -\sum_{j=-1}^{1}\sum_{j'=-1}^{1} C_j C_{j'} \int_0^\infty d\omega\, g(\omega) \int_0^\infty d\omega'\, g(\omega') \int_0^t d\tau \times \mathrm{Tr}_{\mathcal{R}}[B_j(t,\omega)\tilde{S}_j + B_j^\dagger(t,\omega)\tilde{S}_{-j},[B_{j'}(t-\tau,\omega)\tilde{S}_{j'} + B_{j'}^\dagger(t-\tau,\omega)\tilde{S}_{-j'},\rho_{I,N}(t-\tau)\rho_R]]. \tag{15}$$

We note that the density operator of the N-atom system at time t depends on the density operator at the previous time $t-\tau$. So, at this point, we invoke the Markov assumption by replacing $\rho_{I,N}(t-\tau)$ in Eq. (15) by $\rho_{I,N}(t)$ and extend the upper limit in the integral to infinity. As the reservoir is assumed in a vacuum state (0 K), within that approximation the trace operations result in

$$\mathrm{Tr}_{\mathcal{R}}\{b(\omega)b^\dagger(\omega')\rho_R\} = \delta(\omega-\omega'), \tag{16}$$

$$\mathrm{Tr}_{\mathcal{R}}\{b^\dagger(\omega)b(\omega')\rho_R\} = 0, \tag{17}$$

and Eq. (15) becomes

$$\frac{d\rho_{I,N}(t)}{dt} = -\sum_{j,j'=-1}^{1} C_j C_{j'} e^{2i\Delta(j+j')t}\xi_{j'}(\omega_L + 2j\Delta)\{[\tilde{S}_j,\tilde{S}_{-j'}\rho_{I,N}(t)] \times e^{2i\Delta(j-j')t} - [\tilde{S}_{-j},\rho_{I,N}(t)\tilde{S}_{j'}]e^{-2i\Delta(j-j')t}\}, \tag{18}$$

where

$$\xi_j(\omega_L+2j\Delta) = \int_0^\infty d\omega[g(\omega)]^2\left[\pi\delta(\omega-\omega_L-2j\Delta) \pm i\mathcal{P}\frac{1}{\omega-\omega_L-2j\Delta}\right], \tag{19}$$

and $\mathcal{P}$ stands for the principal part. The first term on the right-hand side of Eq. (19) is responsible for the atomic decay rate, while the second term gives a shift in the atomic frequencies. Usually, these shifts are incorporated in the atomic frequencies, but since our aim is to study the decay rates, in this work it will be neglected and we write

$$\xi_j \simeq \pi g^2(\omega_L + 2j\Delta) \equiv \frac{\Gamma_j}{2V} \tag{20}$$

($j = -1,0,+1$), where we introduced Γ_j as the decay rates that depend on the driving field and V is the volume of the atomic cell that contains the N atoms.

Back to the Schrödinger representation, the master equation for an N-atom system becomes

$$\frac{d\rho_N(t)}{dt} = -i[H_{0S},\rho_N(t)] - \frac{\tilde{\Gamma}_+}{2V}\{[S_+,S_-\rho_N] + \text{H.c.}\} - \frac{\tilde{\Gamma}_0}{2V}\{[S_+,S_0\rho_N] + \text{H.c.}\} - \frac{\tilde{\Gamma}_-}{2V}\{[S_+,S_+\rho_N] + \text{H.c.}\}, \tag{21}$$

where H.c. means Hermitian conjugate, and the effective decay rates are given by

$$\begin{pmatrix}\tilde{\Gamma}_-\\ \tilde{\Gamma}_0\\ \tilde{\Gamma}_+\end{pmatrix} = \frac{1}{4\Delta^2}\begin{pmatrix} -F^2 & 2F^2 & -F^2\\ -(\Delta+\delta)F & 2\delta F & (\Delta-\delta)F\\ (\Delta+\delta)^2 & 2F^2 & (\Delta-\delta)^2\end{pmatrix}\begin{pmatrix}\Gamma_-\\ \Gamma_0\\ \Gamma_+\end{pmatrix}. \tag{22}$$

So, in a structured reservoir the N-atom system contains decay rate parameters $\tilde{\Gamma}_j$ that have a dependence on the frequency and the intensity of the laser field. In contrast, in a nonstructured reservoir $g(\omega) = g_0$, the decay rates are either constant or zero, $\tilde{\Gamma}_+ = 2\pi g^2 = \Gamma$, $\tilde{\Gamma}_0 = \tilde{\Gamma}_- = 0$, and the master equation (21) reduces to the well-known form [30]

$$\frac{d\rho_N(t)}{dt} = -i[H_{0S},\rho_N(t)] - \frac{\Gamma}{2V}\{[S_+,S_-\rho_N] + \text{H.c.}\}. \tag{23}$$

The extra terms, third and fourth, appearing in Eq. (21) look like the dissipative terms that appear due to a squeezed vacuum reservoir [31] and were obtained in the context of a single atom in [8].

IV. MEAN-FIELD APPROXIMATION AND EFFECTIVE HAMILTONIAN

Now we will treat the N-atom system as a quantum Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy of equations similar to that of classical kinetic theory [32,33]. We consider a subsystem constituted of K atoms, with $K < N$. The equation of motion for the K atoms density operator ρ_K is obtained by calculating the trace over the remaining $K+1, K+2, \ldots, N$ atoms degrees of freedom in Eq. (21), so getting

$$\frac{d\rho_K}{dt} = \mathrm{Tr}_{K+1,\ldots,N}\left(\frac{d\rho_N}{dt}\right) = -i\,\mathrm{Tr}_{K+1,\ldots,N}\{\delta[S_0,\rho_N] + F[S_+,\rho_N] + F[S_-,\rho_N]\} - \frac{\tilde{\Gamma}_+}{2V}\{\mathrm{Tr}_{K+1,\ldots,N}([S_+,S_-\rho_N]) + \mathrm{Tr}_{K+1,\ldots,N}([\rho_N S_+,S_-])\} - \frac{\tilde{\Gamma}_0}{2V}\{\mathrm{Tr}_{K+1,\ldots,N}([S_+,S_0\rho_N]) + \mathrm{Tr}_{K+1,\ldots,N}([\rho_N S_0,S_-])\} - \frac{\tilde{\Gamma}_-}{2V}\{\mathrm{Tr}_{K+1,\ldots,N}([S_+,S_+\rho_N]) + \mathrm{Tr}_{K+1,\ldots,N}([\rho_N S_-,S_-])\}. \tag{24}$$

Using the sums (5) in terms of the microscopic operators, Eq. (24) becomes

$$\frac{d\rho_K}{dt} = -i\sum_{i=1}^{K}\{[\delta\sigma_0(i) + Fe^{-i\varphi}\sigma_+(i) + Fe^{i\varphi}\sigma_-(i),\rho_K]\} - \frac{(N-K)}{V}\sum_{i=1}^{K}\bigg\{\frac{\tilde{\Gamma}_+}{2}[\sigma_+(i),\mathrm{Tr}_{K+1}[\sigma_-(K+1)\rho_{K+1}]] + \frac{\tilde{\Gamma}_0}{2}e^{-i\varphi}[\sigma_+(i),\mathrm{Tr}_{K+1}[\sigma_0(K+1)\rho_{K+1}]] + \frac{\tilde{\Gamma}_-}{2}e^{-2i\varphi}[\sigma_+(i),\mathrm{Tr}_{K+1}[\sigma_+(K+1)\rho_{K+1}]] + \mathrm{H.c.}\bigg\} - \sum_{i,i'=1}^{K}\bigg\{\bigg(\frac{\tilde{\Gamma}_+}{2}[\sigma_+(i),\sigma_-(i')\rho_K] + \frac{\tilde{\Gamma}_0}{2}e^{-i\varphi}[\sigma_+(i),\sigma_0(i')\rho_K] + \frac{\tilde{\Gamma}_-}{2}e^{-2i\varphi}[\sigma_+(i),\sigma_+(i')\rho_K]\bigg) + \mathrm{H.c.}\bigg\}. \tag{25}$$

For a single representative atom of the system ($K = 1$), the equation of motion for ρ_1 will depend on the two atoms density operator ρ_2, and so on for the whole hierarchy, meaning that the equation of motion for ρ_K will depend on the state ρ_{K+1}. However, for a dilute system the higher-order atomic correlations may be disregarded, and this is achieved when one factorizes ρ_2 as $\rho_1 \otimes \rho_1$, so a generic atom is assumed to move in a mean field produced by all the other atoms, which is a kind of Hartree approximation. Implementing this approximation and dropping the subscript in ρ_1, Eq. (25) reduces to

$$\frac{d\rho}{dt} = -i[H_{ef}[\rho],\rho] - \frac{\tilde{\Gamma}_+}{2V}([\sigma_+,\sigma_-\rho] + \mathrm{H.c.}) - \frac{\tilde{\Gamma}_0}{2V}(e^{-i\varphi}[\sigma_+,\sigma_0\rho] + \mathrm{H.c.}) - \frac{\tilde{\Gamma}_-}{2V}(e^{-i2\varphi}[\sigma_+,\sigma_+\rho] + \mathrm{H.c.}), \tag{26}$$

where the single-particle effective Hamiltonian is

$$H_{\mathrm{ef}}[\rho] = \delta\sigma_0 + \bigg\{e^{-i\varphi}\bigg[\mu E_{\mathrm{in}} - i\frac{(N-1)}{V}\bigg(\frac{\tilde{\Gamma}_-}{2}\langle\sigma_+\rangle e^{-i\varphi} + \frac{\tilde{\Gamma}_0}{2}\langle\sigma_0\rangle + \frac{\tilde{\Gamma}_+}{2}\langle\sigma_-\rangle e^{i\varphi}\bigg)\bigg]\sigma_+ + \mathrm{H.c.}\bigg\}, \tag{27}$$

which contains nonlinear terms corresponding to a mean field that is due to the remaining $N-1$ atoms.

From the second term within brackets in the Hamiltonian (27) we see that effectively a single generic atom is excited by the input field amplitude E_{in} plus an extra polarization field density $\epsilon^*_{\mathrm{pol}}(t)$, where

$$\epsilon_{\mathrm{pol}}(t) = i\frac{(N-1)}{2\mu V}e^{i\varphi}(\tilde{\Gamma}_-\langle\sigma_-\rangle e^{i\varphi} + \tilde{\Gamma}_0\langle\sigma_0\rangle + \tilde{\Gamma}_+\langle\sigma_+\rangle e^{-i\varphi}) \tag{28}$$

originated from the other $(N-1)$ atoms that produce a mean-field effect, and is proportional to the uniform atomic density in the cell N/V, for $N \gg 1$. When the function $g^2(\omega)$ is assumed to be frequency independent (white noise limit) the polarization field reduces to the simple expression

$$\epsilon_{\mathrm{pol}}(t) = i\frac{(N-1)}{2\mu V}\Gamma\langle\sigma_+\rangle. \tag{29}$$

The equations of motion for the atomic operators mean values, derived from master equation (26), are

$$\frac{d}{dt}\langle\sigma_0\rangle = 2i\mu[\epsilon_{\mathrm{out}}(t)\langle\sigma_-\rangle - \epsilon_{\mathrm{out}}{}^*(t)\langle\sigma_-\rangle^*] + \tilde{\Gamma}_0(\langle\sigma_+\rangle e^{-i\varphi} + \langle\sigma_-\rangle e^{i\varphi}) - \tilde{\Gamma}_+(1 + \langle s_0\rangle), \tag{30}$$

$$\frac{d}{dt}\langle\sigma_-\rangle = -2i\delta\langle\sigma_-\rangle + i\mu\epsilon_{\mathrm{out}}{}^*(t)\langle\sigma_0\rangle + \frac{e^{-i\varphi}}{2}(\tilde{\Gamma}_-\langle\sigma_+\rangle e^{-i\varphi} - \tilde{\Gamma}_+\langle\sigma_-\rangle e^{i\varphi}) + \frac{\tilde{\Gamma}_0}{2}e^{-i\varphi}, \tag{31}$$

and $\langle\sigma_+\rangle = \langle\sigma_-\rangle^*$. The total effective output field transmitted from the sample is defined as

$$\epsilon_{\mathrm{out}}(t) = E_{\mathrm{in}}e^{i\varphi} + \epsilon_{\mathrm{pol}}(t) = E_{\mathrm{in}}e^{i\varphi} + i\frac{(N-1)}{2\mu V}\tilde{\Gamma}_+\langle\sigma_+\rangle + i\frac{(N-1)}{2\mu V}e^{i\varphi}(\tilde{\Gamma}_-\langle\sigma_-\rangle e^{i\varphi} + \tilde{\Gamma}_0\langle\sigma_0\rangle). \tag{32}$$

When $g(\omega) = g_0$, in Eq. (32), the only remaining term is $\tilde{\Gamma}_+ \to \Gamma$ (proportional to σ_+), a constant, while the other terms vanish. The mean-field extra terms, proportional to $\langle\sigma_-\rangle$ and $\langle\sigma_0\rangle$, are due to the structured reservoir. Equation (32), for the output field $\epsilon_{\mathrm{out}}(t)$, is more inclusive than the others deduced in Refs. [25,28,34], because the mean-field approximation contributes with additional terms that are sensible to the mode-structured reservoir.

V. STATIONARY SOLUTION AND THE INPUT-OUTPUT FIELDS RELATION

When the solutions of Eqs. (30) and (31) are inserted in Eq. (32) we get the output field (total field) as a function of the input one. Here we are interested in studying the influence of the structured reservoir on the bistable steady state output field amplitude as a function of input field E_{in}. The stationary solutions of Eqs. (30) and (31) are obtained by setting $d\langle\sigma_0\rangle/dt = d\langle\sigma_-\rangle/dt = 0$, resulting in

$$\langle\sigma_0\rangle_{\mathrm{ss}} = -\frac{\tilde{\Gamma}_+(16\delta^2 + \tilde{\Gamma}_+^2 - \tilde{\Gamma}_-^2) - 2\tilde{\Gamma}_0^2(\tilde{\Gamma}_+ + \tilde{\Gamma}_-) - 2\tilde{\Gamma}_0[i(-4i\delta + \tilde{\Gamma}_+ + \tilde{\Gamma}_-)\mu\epsilon_{\mathrm{ss}}e^{-i\varphi} + \mathrm{c.c.}]}{\tilde{\Gamma}_+(16\delta^2 + \tilde{\Gamma}_+^2 - \tilde{\Gamma}_-^2) + 2\mu\tilde{\Gamma}_0[i(4i\delta + \tilde{\Gamma}_+ + \tilde{\Gamma}_-)\epsilon_{\mathrm{ss}}e^{-i\varphi} + \mathrm{c.c.}] - 4\mu^2\tilde{\Gamma}_-\big(\epsilon_{\mathrm{ss}}^2e^{-2i\varphi} + \mathrm{c.c.}\big) + 8\mu^2\tilde{\Gamma}_+|\epsilon_{\mathrm{ss}}|^2} \tag{33}$$

and

$$e^{-i\varphi}\langle\sigma_+\rangle_{ss} = -\frac{\left\{-\tilde\Gamma_0\tilde\Gamma_+(4i\delta+\tilde\Gamma_++\tilde\Gamma_-)+2i\left[\tilde\Gamma_0^2-\tilde\Gamma_+(4i\delta+\tilde\Gamma_+)\right]\mu\epsilon_{ss}e^{-i\varphi} \atop +2i\left(\tilde\Gamma_0^2+\tilde\Gamma_+\tilde\Gamma_-\right)\mu\epsilon_{ss}^*e^{i\varphi}-4\mu^2\tilde\Gamma_0\left(\epsilon_{ss}^2e^{-2i\varphi}+|\epsilon_{ss}|^2\right)\right\}}{\tilde\Gamma_+(16\delta^2+\tilde\Gamma_+^2-\tilde\Gamma_-^2)+2\mu\tilde\Gamma_0[i(4i\delta+\tilde\Gamma_++\tilde\Gamma_-)\epsilon_{ss}e^{-i\varphi}+\text{c.c.}]-4\mu^2\tilde\Gamma_-\left(\epsilon_{ss}^2e^{-2i\varphi}+\text{c.c.}\right)+8\mu^2\tilde\Gamma_+|\epsilon_{ss}|^2}, \tag{34}$$

where ϵ_{ss} is the output field at the stationary state.

In particular, for a nonstructured (ns) reservoir $g(\omega)=g_0$, Eqs. (33) and (34) simplify to the well-known results [28]

$$\langle\sigma_0\rangle_{ss}^{(ns)} = -\frac{(16\delta^2+\Gamma^2)}{16\delta^2+\Gamma^2+8\mu^2|\epsilon_{ss}|^2},$$
$$e^{-i\varphi}\langle\sigma_+\rangle_{ss}^{(ns)} = \frac{2(-4\delta+i\Gamma)\mu\epsilon_{ss}e^{-i\varphi}}{16\delta^2+\Gamma^2+8\mu^2|\epsilon_{ss}|^2}, \tag{35}$$

for a single atom pumped by an external field $\epsilon_{ss}=E_{in}e^{i\varphi}+i[(N-1)/2\mu V]\Gamma\langle\sigma_+\rangle_{ss}$, which is due to a collective effect produced by the mean field. For a structured reservoir the stationary solution changes in an essential way: besides the terms proportional to $|\epsilon_{ss}|^2$, others terms, proportional to ϵ_{ss} and ϵ_{ss}^2, appear additionally. These terms show some similarity to those produced by the decay in a squeezed vacuum [31,34–36], without mean-field effects, as reported in [8] in the case of a single atom.

Writing $\epsilon_{ss}=E_{ss}e^{i\theta}$ (with the explicit introduction of a phase θ) in the stationary state of Eq. (32), we obtain the following relation between the input and output fields:

$$E_{in}^2 = \left[E_{ss}\cos\Phi+\frac{(N-1)}{2V\mu}(\tilde\Gamma_+-\tilde\Gamma_-)\text{Im}(e^{-i\varphi}\langle\sigma_+\rangle_{ss})\right]^2 + \left\{E_{ss}\sin\Phi-\frac{(N-1)}{2V\mu}(\tilde\Gamma_++\tilde\Gamma_-)[\text{Re}(e^{-i\varphi}\langle\sigma_+\rangle_{ss}) + \tilde\Gamma_0\langle\sigma_0\rangle_{ss}]\right\}^2, \tag{36}$$

where $\langle\sigma_+\rangle_{ss}$ and $\langle\sigma_0\rangle_{ss}$, from Eqs. (33) and (34), depend on the phase difference $\Phi=\theta-\varphi$. We note that the nonlinear dependence of θ on E_{in} is a manifestation of the intrinsic frequency distribution of the reservoir. Thus, in order to determine graphically the relations between the real and imaginary parts of ϵ_{ss} versus E_{in}, or E_{ss} and Φ versus E_{in}, for each input value E_{in}, the two output dependent variables must satisfy Eq. (36) for the unique independent variable E_{in}. In the white noise approximation we have $\tilde\Gamma_-=\tilde\Gamma_0=0$, $\tilde\Gamma_+=\Gamma$, so Eq. (36) simplifies to the already known phase independent form [16]

$$E_{in} = E_{ss}\left\{\left(1+\frac{(N-1)}{V\left[1+16(\delta^2/\Gamma^2)+8\mu^2E_{ss}^2/\Gamma^2\right]}\right)^2 + \left(\frac{4(N-1)\delta/\Gamma}{V\left[1+16(\delta^2/\Gamma^2)+8\mu^2E_{ss}^2/\Gamma^2\right]}\right)^2\right\}^{1/2}, \tag{37}$$

which displays the bistability phenomenon when output versus input fields are plotted. The first term within the braces of Eq. (37) corresponds to the absorptive regime, when the atoms are driven near resonance, while the second term stands for the dispersive regime, when the atoms are driven far from resonance ($\delta/\Gamma\gg 1$) and nonlinear refractive effects dominate [16]. Very characteristically, in this approximation (structureless reservoir), there is no phase dependence induced by the atoms. Therefore the presence of a phase shift indicates the existence of a structured reservoir (colored noise).

VI. STRUCTURED RESERVOIRS: OUTPUT FIELD AMPLITUDE AND PHASE BISTABILITY

In order to get a direct insight into the problem of bistability with a structured reservoir we consider the case when frequencies of the laser field and atomic transition are resonant ($\delta=0$). Furthermore, calling $\text{Re}(\epsilon_{out})\equiv\epsilon_x$ and $\text{Im}(\epsilon_{out})\equiv\epsilon_y$, Eq. (33) becomes

$$\langle\sigma_0\rangle_{ss} = \frac{-(\tilde\Gamma_++\tilde\Gamma_-)\left[\tilde\Gamma_+(\tilde\Gamma_+-\tilde\Gamma_-)-2\tilde\Gamma_0^2+4\mu\tilde\Gamma_0\epsilon_y\right]}{(\tilde\Gamma_++\tilde\Gamma_-)\left[\tilde\Gamma_+(\tilde\Gamma_+-\tilde\Gamma_-)-4\mu\tilde\Gamma_0\epsilon_y+8\mu^2\epsilon_y^2\right]+8\mu^2(\tilde\Gamma_+-\tilde\Gamma_-)\epsilon_x^2} \tag{38}$$

and the real and imaginary components of $\langle\sigma_+\rangle_{ss}$ are

$$\text{Re}\langle\sigma_+\rangle_{ss} = \frac{(\tilde\Gamma_0-2\mu\epsilon_y)\tilde\Gamma_+(\tilde\Gamma_++\tilde\Gamma_-)+8\mu^2\tilde\Gamma_0\epsilon_x^2}{(\tilde\Gamma_++\tilde\Gamma_-)\left[\tilde\Gamma_+(\tilde\Gamma_+-\tilde\Gamma_-)-4\mu\tilde\Gamma_0\epsilon_y+8\mu^2\epsilon_y^2\right]+8\mu^2(\tilde\Gamma_+-\tilde\Gamma_-)\epsilon_x^2}, \tag{39}$$

$$\text{Im}\langle\sigma_+\rangle_{ss} = \frac{2\left[\tilde\Gamma_+(\tilde\Gamma_+-\tilde\Gamma_-)-2\tilde\Gamma_0^2+4\mu\tilde\Gamma_0\epsilon_y\right]\mu\epsilon_x}{(\tilde\Gamma_++\tilde\Gamma_-)\left[\tilde\Gamma_+(\tilde\Gamma_+-\tilde\Gamma_-)-4\mu\tilde\Gamma_0\epsilon_y+8\mu^2\epsilon_y^2\right]+8\mu^2(\tilde\Gamma_+-\tilde\Gamma_-)\epsilon_x^2}. \tag{40}$$

Inserting Eqs. (38)–(40) into Eq. (32), and equating the real and imaginary parts, we get the following set of equations:

$$E_{in} = \epsilon_x+\frac{(N-1)}{2V\mu}(\tilde\Gamma_+-\tilde\Gamma_-)\text{Im}(\langle\sigma_+\rangle_{ss}), \tag{41}$$

$$0 = \epsilon_y-\frac{(N-1)}{2V\mu}(\tilde\Gamma_++\tilde\Gamma_-)[\text{Re}(\langle\sigma_+\rangle_{ss})+\tilde\Gamma_0\langle\sigma_0\rangle_{ss}], \tag{42}$$

for the components ϵ_x and ϵ_y, and whose solutions determine $E_{ss} = \sqrt{\epsilon_x^2 + \epsilon_y^2}$ and $\Phi = \arctan(\epsilon_y/\epsilon_x)$.

By assuming, in particular, that the function $g^2(\omega)$ is symmetric around the atomic transition frequency—an even function with respect to ω_0, $g^2(\omega_0 - \omega) = g^2(\omega_0 + \omega)$—the decay rates Γ_+ and Γ_- become equal, so the effective decay rates in Eq. (22) reduce to $\tilde{\Gamma}_\pm = \frac{1}{2}(\Gamma_0 \pm \Gamma_+)$ and $\tilde{\Gamma}_0 = 0$. In this case the output field does not depend on $\langle\sigma_0\rangle$, from Eq. (42) we have $\epsilon_y = 0$, and the unique physical solution occurs for $\Phi = 0$ as in the case of a structureless reservoir. Nevertheless, the decay rates depend on the input field, so the relation between input and output field amplitudes simplifies to

$$E_{\rm in} = E_{ss}\left[1 + \frac{N-1}{V}\frac{\Gamma_+(E_{\rm in})}{\Gamma_0 + \frac{16\mu^2 E_{ss}^2}{\Gamma_0 + \Gamma_+(E_{\rm in})}}\right], \qquad (43)$$

where we wrote explicitly the dependence on $E_{\rm in}$. In the case of a structureless reservoir (white noise), $\Gamma_0 = \Gamma_+$, Eq. (43) reduces to the well-known equation [16,28]

$$E_{\rm in} = E_{ss}\left[1 + \frac{(N-1)}{V}\frac{1}{\left(1 + 8\mu^2 E_{ss}^2/\Gamma^2\right)}\right], \qquad (44)$$

whose quite simple input-output amplitude nonlinearity is due to the term proportional to $N-1$. In order to compare with previous works [28], using the authors' notation, we write Eq. (43) as

$$y = x + \frac{2C(y)x}{D(y) + x^2}, \qquad (45)$$

where we have defined the input and output fields as $y = \sqrt{8}\mu E_{\rm in}/\Gamma_0$ and $x = \sqrt{8}\mu E_T/\Gamma_0$, respectively, $C(y) = [(N-1)/V](\Gamma_0 + \Gamma_+)\Gamma_+/4\Gamma_0^2$, and $D(y) = (\Gamma_0 + \Gamma_+)/2\Gamma_0$. For a structureless vacuum we have $C(y) = C = (N-1)/V$, $D(y) = 1$, so Eq. (45) reduces to

$$y = x + \frac{2Cx}{1 + x^2}, \qquad (46)$$

which is identical to the well-known result obtained in [28]. Equation (46) displays a bistable behavior for $C > 4$, and for $C \gg 1$, the range for input field, allowing three solutions, lies in the interval $\sqrt{8C} < y < C$. Now, regarding Eq. (45), the dependence of x on y is obtained by inverting the expression and solving the cubic equation, which, for a structured environment, has its coefficients depending nonlinearly on the input field amplitude. So, the range of values for a bistable solution is determined by the density of particles and by the proper input field.

For a more general physical situation, when $g(\omega)$ is not symmetric around the atomic frequency ω_0, all terms in Eq. (32) contribute to the output field. In this case $\tilde{\Gamma}_0 \neq 0$ and an imaginary component ϵ_y arises in the output field. Thus, in contrast with the case of a symmetric $g(\omega)$ (or just being flat), the asymmetric structure of modes induces a relative phase in the output field, and both amplitude and phase display a bistable behavior. We illustrate below the input-output field dependence for two reservoir models.

A. Example 1: Weighted Lorentzian shape

We assume a structured reservoir having a weighted Lorentzian shape for the frequency distribution [6,12],

$$g^2(\omega) = \frac{\Gamma_0}{2\pi}\left[\beta + (1-\beta)\frac{\gamma^2}{(\omega - \omega_r)^2 + \gamma^2}\right], \qquad (47)$$

with ω_r a characteristic frequency of the reservoir and the parameter $0 \leqslant \beta \leqslant 1$. In Eq. (47), the first term within the brackets represents a background vacuum (white noise), whereas the second term represents the structured vacuum (colored noise), assumed to have a Lorentzian shape of width γ; the parameter β interpolates between the two limiting cases respectively. The effective decay rates are

$$\tilde{\Gamma}_- = \Gamma_0\frac{(1-\beta)}{4}\left[-\frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar + \eta)^2} + \frac{2\gamma^2}{\gamma^2 + \eta^2} - \frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar - \eta)^2}\right],$$

$$\tilde{\Gamma}_0 = \Gamma_0\frac{(1-\beta)}{4}\left[-\frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar + \eta)^2} + \frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar - \eta)^2}\right],$$

$$\tilde{\Gamma}_+ = \Gamma_0\left\{\beta + \frac{(1-\beta)}{4}\left[\frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar + \eta)^2} + \frac{2\gamma^2}{\gamma^2 + \eta^2} + \frac{\gamma^2}{\gamma^2 + (2\mu E_{\rm in}/\hbar - \eta)^2}\right]\right\},$$

where $\eta = \omega_r - \omega_0$ is the detuning between atomic frequency and the reservoir characteristic frequency. It is worth noting that while $\tilde{\Gamma}_-$ and $\tilde{\Gamma}_+$ are even functions regarding the change $\eta \longrightarrow -\eta$, $\tilde{\Gamma}_0$ is an odd function. The existence of an input-output phase difference depends on a nonzero $\tilde{\Gamma}_0$, and when one changes the sign of η, also that phase changes sign.

In Fig. 1 we plotted the output versus input amplitudes for three values of β, a detuning $\eta = 0$, and $N = 50$. In this case $\epsilon_y = 0$ and $\epsilon_x = E_{ss}$, so the phase difference is $\Phi = 0$.

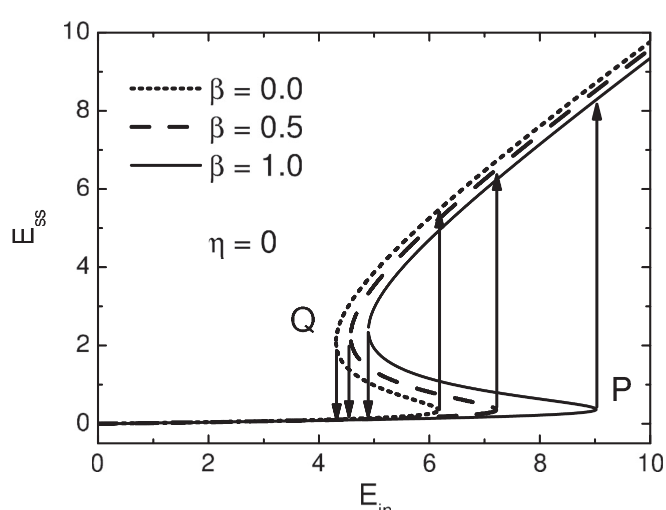

FIG. 1. Output versus input field amplitudes for $\eta = 0$, $N = 50$, $\gamma/\Gamma_0 = 20$ and different values of β. The solid line corresponds to a structureless reservoir, $\beta = 1$. The parameters are dimensionless.

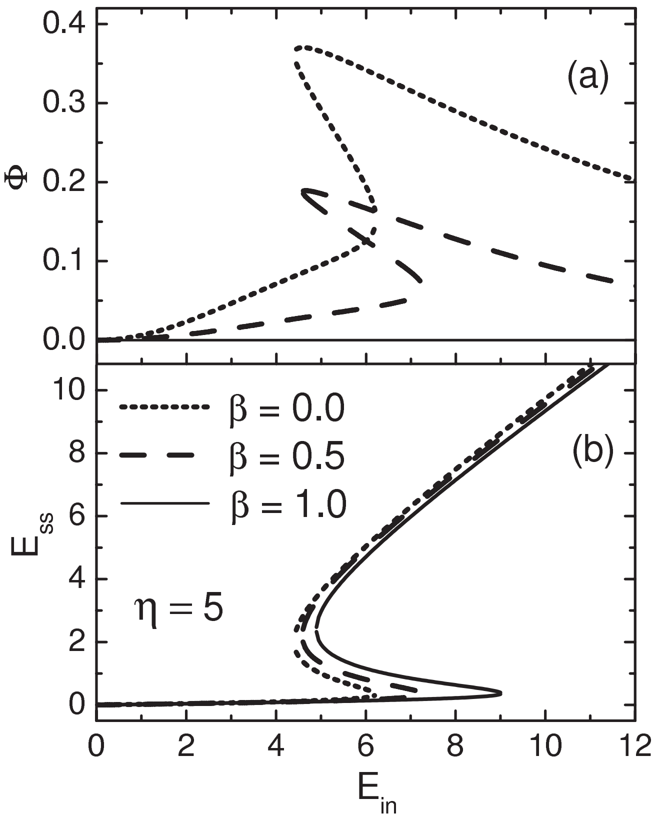

FIG. 2. Output (a) phase difference Φ, in radians, and (b) amplitude E_{ss}, versus input amplitude for $\eta = 5$, $N = 50$, $\gamma/\Gamma_0 = 20$ and different values of β. The solid line corresponds to a structureless reservoir, $\beta = 1$. The parameters are dimensionless.

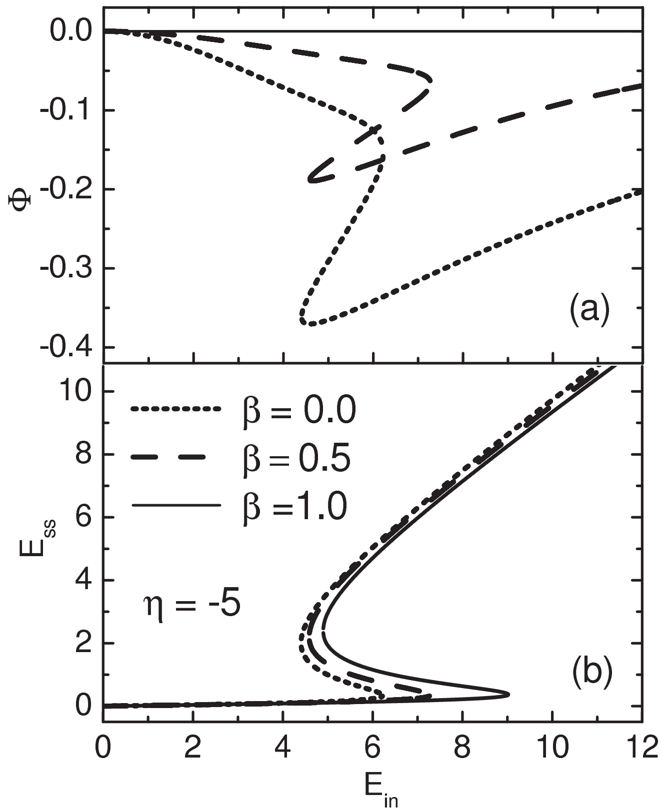

FIG. 3. The same as in Fig. 2 but for $\eta = -5$.

In both cases, $\beta = 0.5$ and $\beta = 0.0$, we observe a variation in the distances between the switching points, P and Q, that indicate the location of the lower and upper branches of the S-shaped curve. Comparing to the structureless reservoir, $\beta = 1.0$ (solid line), there is a reduction in the range of values of E_{in} where the bistability occurs. We also observe that the deviations from the white noise curve (solid line) is more pronounced at the switching points (P) from lower to upper branches. As the effective decay rate diminishes with the increase of the input field, less energy is transferred from the atomic sample to the reservoir, thus the energy goes through the sample carried by the output field. We now consider the detuning $\eta = 5.0$ and draw in Figs. 2(a) and 2(b) the output phase and amplitude, respectively, showing bistability in both. In Fig. 2(a) we observe that the phase difference is not null because of the emergence of the component ϵ_y. The variation of Φ is more pronounced the more the mode distribution ($\beta = 0.5$ and $\beta = 0.0$) deviates from the white noise ($\beta = 1.0$). The phase Φ goes to zero for large values of the input amplitudes because the effective decay rate $\tilde{\Gamma}_0$ goes to zero. Regarding the $\beta < 1$ cases, we note that amplitude and phase have the same switching points, and the P-Q distance is reduced. By admitting a negative detuning $\eta = -5.0$ the input-output amplitude relation does not change; however, as can be seen in Fig. 3, the bistable behavior of the phase changes by a sign inversion.

B. Example 2: Photonic band gap

Here we adopt a simple reservoir structure used to analyze the resonance fluorescence phenomenon in a photonic band gap [10], where it is assumed that there is a discontinuity at specific frequencies of the photonic density of modes $g^2(\omega)$, although it is constant over spectral regions in the dressed atomic frequencies; so

$$g^2(\omega) = \begin{cases} \frac{\Gamma_1}{2\pi}, & \omega < \omega_0 \\ \frac{\Gamma_2}{2\pi}, & \omega \geqslant \omega_0. \end{cases} \tag{48}$$

In this case, at each dressed frequency, i.e., $E_{\text{in}} \neq 0$, the decay rates are $\Gamma_-(\omega_0 - 2\mu E_{\text{in}}/\hbar) \equiv \Gamma_1$ and $\Gamma_0(\omega_0) = \Gamma_+(\omega_0 + 2\mu E_{\text{in}}/\hbar) \equiv \Gamma_2$, and the effective decay rates become $\tilde{\Gamma}_0 = \tilde{\Gamma}_- = (\Gamma_2 - \Gamma_1)/4$ and $\tilde{\Gamma}_+ = (\Gamma_1 + 3\Gamma_2)/4$.

In Figs. 4(a) and 4(b) we plotted the output field phase and amplitude, respectively, as a function of the input amplitude, for some ratios of Γ_2/Γ_1. We set $N = 50$, and all parameters are dimensionless. In Fig. 4(a), as in the previous example, we observe a change in the amplitude input range for bistability. Also, the switching point from the lower to the upper branch is more sensitive to the ratio Γ_2/Γ_1. Regarding the phase, from Fig. 4(b) we observe that, depending on the ratio Γ_2/Γ_1, at the region of bistability the phase difference may change sign and also can display a more complex behavior (a loop) than an S-shaped form. For large values of the input amplitude, from Eqs. (38)–(42), we get $\epsilon_x \cong E_{\text{in}}$, ϵ_y attains, asymptotically, the constant value $(N-1)\Gamma_2(\Gamma_2 - \Gamma_1)/4V\mu(\Gamma_1 + \Gamma_2)$, and the phase Φ goes to zero with a sign that depends on the difference $\Gamma_2 - \Gamma_1$. For larger values of E_{in} the phase

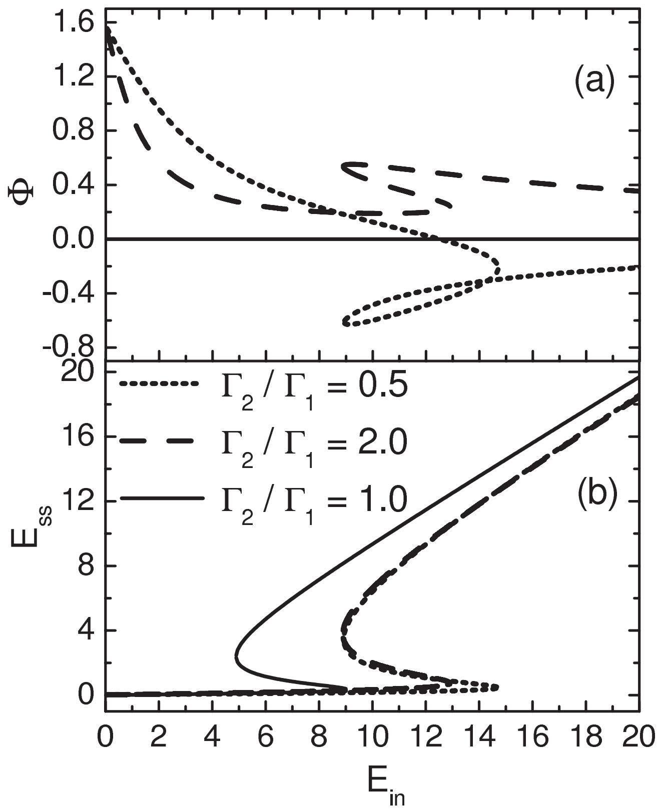

FIG. 4. Output (a) phase difference Φ, in radians, and (b) amplitude E_{ss}, versus input field amplitudes for $N = 50$, and different values of Γ_2/Γ_1. The solid line corresponds to a structureless reservoir, $\Gamma_2/\Gamma_1 = 1$. The parameters are dimensionless.

difference Φ becomes proportional to the jump (discontinuity) in the structure of modes.

VII. SUMMARY AND CONCLUSIONS

We presented a study of optical bistability using the mean-field approximation by considering a system of N two-level atoms interacting with structured reservoirs. Our approach consisted in dressing the atoms collective operators with the classical input field and coupling them to the structured reservoir. The dynamical system is described by a master equation which contains extra terms (compared to that obtained from a structureless reservoir) resembling those present in the master equation, derived under the influence of a squeezed reservoir. The master equation contains effective decay rates, associated to each dressed frequency, that depend on the frequency and intensity of the input field. Adopting the mean-field approximation, we deduced a single particle effective Hamiltonian containing extra nonlinear terms which are absent in the case of a structureless reservoir. In the stationary regime of the atomic and laser field system, we analyzed the relation between the input and output fields, observing that they are related in a nonlinear form and present typical bistable behavior. For a structured reservoir the S-shaped curve is present in the amplitude, but not necessarily in the phase. This theoretical result compares with a similar treatment given in [8] where the authors analyzed the resonance fluorescence and absorption spectra of a single atom and in which a similar phase dependence in the system dynamics occurs. However, both the resonance fluorescence and the absorption spectra have no phase dependence. Our results indicate that the presence of the induced phase shift in the output field of an N-atom system could be an interesting probe about the nature of the reservoir. We have considered in detail the case of resonance between atoms and the input field frequencies and verified that the output phase shift appears for reservoirs having an asymmetric structure of modes. We presented two illustrative examples of reservoirs: (1) a mixing of white noise and Lorentzian shaped frequency distribution and (2) a simplified photonic band-gap distribution of the reservoir modes. We noted that the output field bistable behavior is as sensible in the phase difference, or acquired phase, as for the amplitude. That characteristic can be explored by using a Mach-Zehnder interferometer, with a phase shifter in one of the arms, where the output field phase can be determined by measuring the difference of the pulse intensities at the interferometer output ports. By slightly changing the input field E_{in} at the transition values in the instability region, it should be noted, as a response, the occurrence of sudden and discontinuous changes in the difference between the photocurrents at the two exit ports of the interferometer. On the other hand, by engineering reservoirs and atomic samples one could use the output amplitude and phase to design optical control devices. In addition, we will address, in a future work, the influence of structured reservoirs on bistability, in the dispersive regime ($\delta \neq 0$).

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