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Destruction of shape-invariant solitons in nematic liquid crystals by noise

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We investigate the destructive influence of noise on the shape-invariant solitons in a three-dimensional model that includes the highly nonlocal nature of nematic liquid crystals. We first determine the fundamental shape-preserving solitons and then establish that any noise added to the medium or to the solitons induces them to breathe at short propagation distances and to disperse at long propagation distances. The characteristics of breathing solitons at short distances are well predicted by the variational calculation [Phys. Rev. A **85**, 033826 (2012)]. At longer propagation distances soliton beams suddenly spread, almost without radiation losses. Their power remains almost conserved until they reach the transverse boundaries of the sample. The increase in the amount of noise accelerates beam spreading and soliton destruction. The influence of the correlation length of noise is more complex. An initial increase in the correlation length causes solitons to disperse at shorter propagation distances. However, further increase in the correlation length leads to a reversal—to prolonged stability and dispersal at longer propagation distances. We give theoretical explanation for such behavior in terms of mean-field evolution equations.

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I. INTRODUCTION

The fundamental spatial optical soliton is a beam that propagates without changing its transverse profile in a nonlinear (NL) medium [1]. Such shape-invariant solutions are easily identified in $(1+1)$-dimensional $[(1+1)\mathrm{D}]$ NL systems, because the inverse scattering theory guaranties their existence [2]. The situation is less clear in the multidimensional and multicomponent systems. No complete inverse scattering theory is formulated in more than one dimension; in fact, wave instability and the collapse of solutions are overriding concerns in multidimensional NL systems [3]. Additional compounding difficulties arise in the vector models or in the scalar nonlocal models in which the medium response is driven by the optical field itself. Such are the models describing the generation of solitary waves—nematicons—in nematic liquid crystals (NLCs).

Nonlocality is an important characteristic of many NL media. A highly nonlocal situation arises in a nonlocal NL medium in which the characteristic size of the response is much wider than the size of the excitation itself. In NLCs both experiments [4,5] and theoretical calculations [6] demonstrated that the nonlinearity is highly nonlocal. Even a high degree of nonlocality does not guarantee the existence of stable higher-order solitary structures [7,8]. Orientational nonlinearity in NLCs is highly nonlocal, but the NL response is not perfectly quadratic, implying that if one launches a Gaussian beam into the cell, it is only possible to observe breathing solitons [9,10]. For the more general vectorial model, in which the order parameter in NLC is not constant, steady elliptical soliton profiles have been found numerically by including all three components of the optical field [11].

However, what is puzzling is that even though everybody agrees that shape-preserving solitons do exist in highly nonlocal NLCs, very few cared to present them explicitly. Experimental accounts frequently mention steady nematicons, but careful inspection of published figures reveals self-focusing oscillations. True, experimental results may not be of much help in this regard, because all experimental setups feature a few millimeter-long cells, which cannot capture slow (if any) convergence to a steady profile.

Noise is unavoidable in any realistic medium. Stochastic variation of the director field due to the liquid crystal temperature fluctuations is inherent to the nematic phase [12]. It is well known that disorder in an NL system is equivalent to the existence of an effective loss [13,14]. Nonlocality affects much the dynamics of self-trapped beams in the presence of randomness; soliton random walk can be very much suppressed in highly nonlocal media [15]. In addition, nonlocality effectively increases the correlation length of random perturbations, leading to the stabilization of solitons [16]. On the other hand, there exists abundant literature describing the demise of solitons in the presence of noise [17–19]. Interplay of nonlinearity, nonlocality, and randomness leads to very interesting novel physical phenomena.

In this paper we study numerically soliton propagation in a highly nonlocal noisy medium, utilizing a widely accepted scalar model of uniaxial NLCs. To find exact fundamental solitons in a $(2+1)\mathrm{D}$ model, we use an iterative numerical eigenvalue technique [20,21]. We analyze soliton and Gaussian beam propagation using two different propagation methods. We check their stability in propagation and demonstrate that any small change in the input shape, as well as in the medium, leads to the soliton breathing with the characteristics well predicted by the variational calculation [20]. We note that the same reference towards the end mentions the influence of noise, however, only as it relates to causing the breathing of nematicons. The more destructive influence of noise is covered in this paper.

An ideal soliton—determined with high accuracy—can propagate without change in a noiseless medium for arbitrary long. Even a nonideal soliton—still determined with sufficient numerical accuracy—can propagate in a noiseless medium

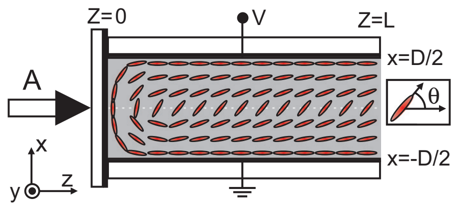

FIG. 1. (Color online) Liquid-crystal cell model adopted.

arbitrary long, but in an oscillatory fashion. On the other hand, an ideal soliton propagating in a noisy medium will oscillate initially but will suffer radiation losses and spread after some propagation distance. The larger the noise, the faster the soliton destruction. To explain such behavior, we introduce a system of mean-field evolution equations, which account well for the observed dynamical phenomena.

The text is structured as follows. In Sec. II the model is introduced. Section III describes the numerical methods utilized. Section IV discusses the influence of noise and Sec. V compares our results with the results reached by others. Section VI concludes the paper.

II. THE MODEL

We adopt the well-known three-dimensional (3D) scalar model of NLCs, which provides good agreement with experimental data [5]. The liquid-crystal cell of interest is sketched in Fig. 1. The optical beam polarized along the x axis propagates in the z direction, while the NLC molecules can rotate in the x-z plane. The total orientation of molecules with respect to the z axis is denoted as θ, whereas the orientation induced by the static electric field only is denoted by θ_0 (the pre-tilt angle). The bias field points in the x direction and is uniform in the z direction; hence the pre-tilt angle is uniform along the z axis as well. The quantity $\hat{\theta} = \theta - \theta_0$ corresponds to the optically induced molecular reorientation.

The equations of interest consist of the scaled NL Schrödinger-like equation for the propagation of the optical field A and the diffusionlike equation for the relaxation of the molecular orientation angle θ. After the rescaling $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to z$, where x_0 is the transverse scaling length and $L_D = kx_0^2$ is the diffraction length, the following model equations in the computational domain are obtained [5,6,9]:

$$2i\frac{\partial A}{\partial z} + \triangle A + \alpha[\sin^2\theta - \sin^2\theta_0]A = 0, \quad (1)$$

$$\tau\frac{\partial \theta}{\partial t} = 2\triangle\theta + [\beta + \alpha|A|^2]\sin(2\theta), \quad (2)$$

where $\triangle$ is the transverse Laplacian. Usually, the material response equations contain the full spatial Laplacian; here, however, in the spirit of the paraxial approximation made in Eq. (1), it is presumed that the second derivative of the θ field in the z direction in Eq. (2) can be neglected. The coefficients α and β are proportional to the optical and static permittivity anisotropies of the NLC molecules ($\alpha = k_0^2x_0^2\Delta\varepsilon^{\rm OPT}$ and $\beta = \varepsilon_0 x_0^2\Delta\varepsilon^{\rm dc}|E^{\rm dc}|^2/K$), and τ is the relaxation time. We also scaled the optical field intensity $\frac{\varepsilon_0}{2Kk_0^2}|A|^2 \to |A|^2$. The wave numbers in the medium and vacuum are k and k_0, respectively. The amplitude of the static bias electric field is $E^{\rm dc} = V/D$, where V is the applied bias voltage and D is the cell thickness. $\Delta\varepsilon^{\rm OPT}$ and $\Delta\varepsilon^{\rm dc}$ are the optical and static permittivity anisotropies of the NLC molecules, respectively. K is Frank's elastic constant.

Thus, the localized paraxial field A propagates along the z axis, adjusting at all times to the slowly varying material field θ, which in turn is influenced by A through the optically induced change in the index of refraction. Hard boundary conditions (BCs) on the molecular orientation at the NLC cell faces in the x direction are assumed: $\theta(x = -D/2, y) = \theta(x = D/2, y) = 2^\circ$ [22], while in the y direction different BCs are assumed, depending on the situation. Different BCs affect the solutions differently.

In all calculations the following data are kept constant: $L_D = 78.6\ \mu$m, $x_0 = 2\ \mu$m, $\lambda = 514$ nm, $n_0 = 1.53$, $D = 75\mu$m, $V = 1$V, $\Delta\varepsilon^{\rm OPT} = 0.4$, $\Delta\varepsilon^{\rm dc} = 20$, $K = 12 \times 10^{-12}$ N. These data correspond to typical experimental conditions (optical power in the milliwatt range, NLC parameters of commercially available liquid crystals). The propagation distance in simulations varied between $20L_D$ and $135L_D$. The first distance corresponds to a typical experimental length of about 1.6 mm over which nematicons have been observed; the second is as an estimate of length—about 10 mm—over which the nematicons are surely destroyed by any reasonable amount of noise.

III. NUMERICS

In our calculations we use data corresponding to typical experimental conditions [6,9,22]. In the case of fundamental beam propagation, with single-peak on-axis intensity, we observe slow convergence of beam amplitude A and θ to their steady-state values. Therefore, we confine our attention to the steady state only. The steady-state pre-tilt angle θ_0, which figures in Eq. (1), is found from Eq. (2) in the absence of optical field:

$$2\triangle\theta_0 + \beta\sin(2\theta_0) = 0. \quad (3)$$

This calculation has to be performed before the actual integration of Eqs. (1) and (2) can commence. It is a simple elliptic boundary-value problem that can be solved by any of the numerous methods available. But, the determination of the solitary eigensolutions of Eqs. (1) and (2) is not so simple.

The solitary wave solutions of the full system of Eqs. (1) and (2) are determined using the modified Petviashvili's iteration method [21,23,24]. The system of two PDEs possesses two eigenfunctions—for A and θ; the shape-preserving soliton solutions are presented in Ref. [20], for two different types of BCs: zero and periodic. An example with zero BCs is shown in Fig. 2. The shape and the power of the fundamental shape-invariant solutions depend on the BCs applied. In this paper we will be concerned only with the solutions with zero BCs.

Spatial solitons in highly nonlocal media with quadratic response possess Gaussian profiles [25]. However, the fundamental soliton profile is not Gaussian. The soliton intensity profile compared to a Gaussian is shown in Fig. 2(a). To check the stability of fundamental solitons, we propagate them numerically; peak intensities as functions of the propagation distance are presented in Fig. 2(b). Also included in Fig. 2(b)

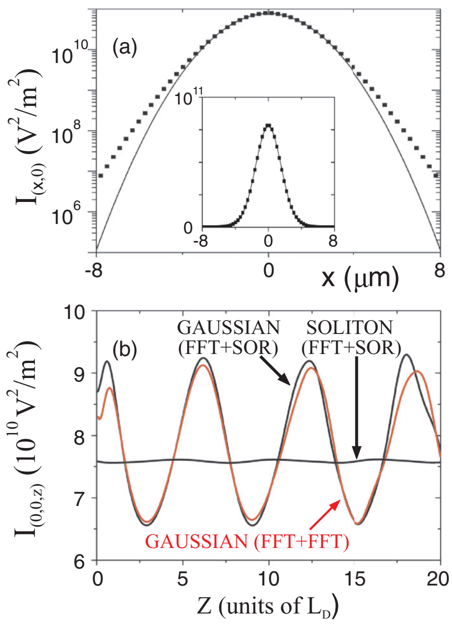

FIG. 2. (Color online) (a) Fundamental soliton intensity profile obtained by the eigenvalue method (black dots), fitted with a Gaussian. (Inset) The same profile but on a linear scale. Parameters are as follows: $P = 10.6$ mW, $\mu = 3.84 L_D^{-1}$; zero BCs. (b) Soliton and Gaussian propagation using two different propagation methods, FFT + SOR and FFT + FFT. The soliton power $P = 10.6$ mW; Gaussian power $P = 10.6$ mW for FFT + SOR, $P = 10.1$ mW for FFT + FFT.

is a case presenting the propagation of a Gaussian with similar parameters, but obtained using two different numerical methods. In both methods a split-step beam propagation procedure based on the fast Fourier transform (FFT) is used for the propagation of the optical field. In the first method the diffusion equation for the optically induced molecular reorientation is treated using the successive overrelaxation (SOR) method, until convergence is achieved; this procedure is referred to as the FFT + SOR. In the second method the diffusion equation is treated using the split-step procedure again—this is the FFT + FFT procedure. One can see that the methods provide similar results; however, the first method is more accurate.

The problem with the FFT + FFT procedure is that it treats an array of transversely periodic cells. Since the molecular reorientation is wide, it tends to slightly spill over into the adjacent cells, i.e., back onto itself, adding to the optical field. This is not an overriding problem in the propagation of a Gaussian, as it only leads to a slightly amplified oscillation of the breathing solution. Nonetheless, it makes a huge difference in the propagation of the fundamental soliton—it makes it impossible for the field to keep its shape-invariant input profile.

Indeed, any small change in the orientation angle θ pushes the system from the self-organized equilibrium and forces it to oscillate about the shape-invariant soliton. In a highly nonlocal system, the potential well is broad, making it difficult for the narrow localized solution to radiate and relax to the fundamental soliton. It just keeps oscillating, forming a steady breathing soliton. Therefore, the FFT procedure for θ should be discarded. Even the SOR soliton solution slightly oscillates at lower accuracy; this, however, becomes imperceptible as the accuracy is improved. In Fig. 2(b) we show a case where the oscillation of the amplitude is still perceptible. When one considers the propagation of a Gaussian beam using the two propagation methods, the results are close. The propagation of a Gaussian invariably leads to breathing beams, regardless of the method of integration. Breathers are closely related to the perturbed fundamental solitons in highly nonlocal systems [26].

IV. THE INFLUENCE OF NOISE

However, when the fundamental soliton is propagated through the medium in which a small white noise is added to the pre-tilt angle θ_0, a breathing solution is also obtained. In earlier studies [14] only longitudinal random perturbations were considered. In [16] randomness was a function of both the propagation variable and the transverse coordinate. Here, we investigate a more general case: Randomness is chosen as a function of all spatial coordinates. To display the influence of the correlation length of noise in our simulations, we divide the computational space into equal blocks and introduce noise by adding randomly distributed white noise to θ_0 so that in each block θ_0 is perturbed by the same relative amount of noise. Thus, in each block we have delta-correlated white noise of certain strength. The size of blocks varies and is denoted by two numbers, representing its dimensions in the transverse and longitudinal directions, measured in the numbers of basic computational cells. For example, "4 × 1" means that the block size is $4\Delta x \times 4\Delta y \times \Delta z$. Hence, 1 × 1 block size represents a fine-grained noise, while 4 × 4 represents a coarse-grained noise.

In this manner, the noise is transversely and longitudinally spatially dependent, with the correlation length in a specific direction equal to the size of the block in that direction. This manner of introducing the noise and its correlation length seems natural, because the size of grains of noise—i.e., the fluctuations in NLCs—affects naturally the behavior of solutions. It also allows an easy control of the influence of noise. Introducing noise in a more traditional way, by introducing a stochastic term in the material response equation, as, for example, done in [16], leads to qualitatively similar behavior. In any case, the important finding—the destruction of solitons by noise—is there regardless of how the noise or the correlation length is introduced.

The maximum beam intensity of the fundamental soliton in a noisy medium for propagation distances up to 20 L_D is presented in Fig. 3, for four different noise block sizes. Here, L_D denotes the diffraction length. We pick the relatively short propagation distance of 20 L_D because it corresponds well to the experimental distance of about 1.6 mm over which the nematicons have been observed and because the same distance (or the "evolution coordinate") has been used in Refs. [15,16], with which we later compare our results.

It is seen in Fig. 3 that the greater block sizes lead to more coarse irregular oscillations, although for a while one can discern a simple sinusoidal breather in the background with the same period $T \approx 10.6 L_D$. Characteristics of the perturbed breathing fundamental solitons are well predicted by the variational calculation, and according to Ref. [26]

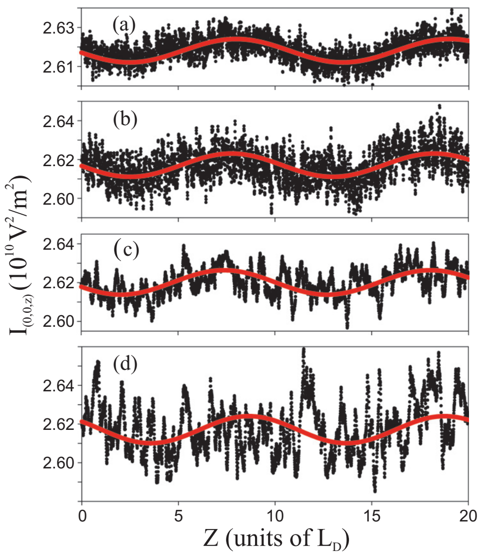

FIG. 3. (Color online) Peak intensity of the fundamental soliton propagating in a noisy medium. An amount of 0.1% randomly changing noise is added to the pre-tilt angle. Four cases with different noise block sizes are considered: (a) 1×1; (b) 1×4; (c) 4×1; (d) 4×4. The red sinusoidal fit is to guide the eye. Parameters are as follows: $\mu = 2L_D^{-1}$, zero BCs.

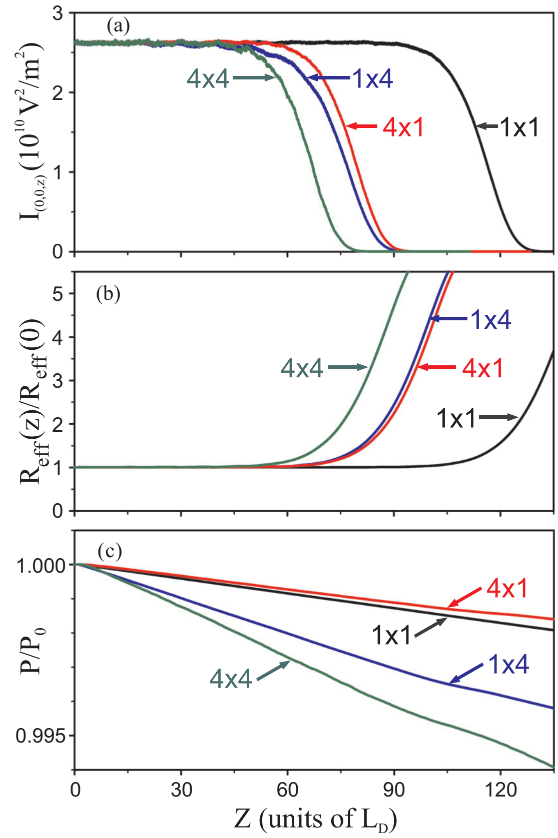

FIG. 4. (Color online) Propagation of the fundamental soliton in a noisy medium for longer propagation distances: (a) peak intensity; (b) effective radius; (c) power. Noise block dimensions are indicated by arrows. Parameters are as in Fig. 3. Note a very small loss of power.

$T = 9.7L_D$ for small propagation distances. Adding more noise leads to larger and more irregular oscillations (not shown here). In addition, increasing the correlation length of the noise leads—at least initially—to the speedier destruction of solitons. However, after the correlation length becomes comparable to the transverse size of the beam, the influence is reversed and the destruction is delayed.

Such a situation is physically plausible: The existence of noise or random fluctuations in the director field of NLCs is a well-established fact [12]. The same induced oscillation phenomenon happens as well when a small intensity noise is added to the fundamental profile, but θ_0 kept unchanged. In all three possible scenarios of the propagation (nonideal soliton in an ideal medium, ideal soliton in a noisy medium, and nonideal soliton in a noisy medium) the period of oscillation is the same, suggesting the existence of a robust breathing soliton in the background. Nevertheless, that breather is not stable, as is easily established when longer propagation distance is considered.

Propagation of the fundamental soliton in a noisy medium at longer propagation distances is depicted in Fig. 4. To characterize the spreading of the beam, we introduce the effective beam width $R_{\text{eff}} = \sqrt{\int |A(x,y)|^2(x^2+y^2)dxdy/P}$, where P is the beam power. It is seen that the noise effectively destroys breathers at large propagation distances, as solitons start to spread rapidly. The radiation loss remains very small, until the beam reaches the absorbing transverse boundaries of the sample. The beam spreads in the medium and gets absorbed at the cell boundaries. Our results suggest that the beam spreading affects more the destruction of solitons than the beam radiation. Small power radiation cannot stabilize solitons in highly nonlocal noisy NLCs. To confirm power conservation, we put zero noise in our simulations and find the radiation loss smaller than 10^{-6}. In addition, the destruction of solitons is absent then. Similar behavior was reported in [5].

The influence of the correlation length of noise on the destruction of solitons is more subtle, as already mentioned. Starting from the smallest noise block size, i.e., the fine-grain noise, the increase in the correlation length, in both transverse and longitudinal directions, causes the beam to spread sooner. This trend is visible in Fig. 4; however, it is not uniform. As the transverse block size becomes comparable to the width of the beam, the trend gets reversed: Further increase in the correlation length leads to an increased stability of the soliton. This is understandable: As the correlation length becomes larger than the beam width, the medium becomes more uniform to the soliton and therefore the noise exerts less influence on its behavior. Still, the destruction of solitons by noise is always there.

To explain such behavior, we derive from Eqs. (1) and (2) a system of mean-field evolution equations for the ensemble-averaged fields in the lowest order of approximation [27]:

$$2i\frac{\partial\langle A\rangle}{\partial z} + \triangle\langle A\rangle + \bar{\alpha}\langle\hat{\theta}\rangle\langle A\rangle = -i\delta_1\langle A\rangle + i\delta_2\triangle\langle A\rangle, \qquad (4)$$

$$\tau\frac{\partial\langle\hat{\theta}\rangle}{\partial t} = 2[\triangle\langle\hat{\theta}\rangle + \bar{\beta}\langle\hat{\theta}\rangle] + \bar{\alpha}|\langle A\rangle|^2, \qquad (5)$$

where $\bar{\alpha} = \alpha \sin(2\theta_0)$, $\bar{\beta} = \beta \cos(2\theta_0)$, and $\delta_1(z)$ and $\delta_2(z)$ are monotonically increasing but limited functions. The δ_1 term represents the linear losses in the system, commonly originating from the scattering losses on fluctuations and defects in the crystal; it is reassuring that such a term appears in our model of noisy medium. It is an obvious universally present loss mechanism that accounts for the effective linear losses introduced by the noise. Nevertheless, it is the presence of the δ_2 term that is new and unexpected.

It should perhaps be mentioned that in addition to the scattering coming from fluctuations, other scattering mechanisms are present in NLCs as well. These come from the physical properties of molecules, such as rotovibrational levels and resonance occurrences when the molecules are illuminated by laser light. Most notably they include Rayleigh scattering and to a lesser degree Brillouin and Raman scattering [12]. Rayleigh scattering is also due to orientational fluctuations of elongated molecules, however, it is caused by individual molecular movements and takes place on a much shorter time scale than the typical noisy temperature fluctuations of interest here. The main scattering mechanism in liquid crystals is due to collective orientational fluctuations that take place on a slower time scale and as such are imbedded in the Rayleigh wing scattering [12].

The propagation equation [Eq. (4)] should be compared with an analogous equation in [16]. The second term on the right side of Eq. (4), introduced here for the first time, is crucial; it is responsible for the diffusion of $\langle A \rangle$ and naturally accounts for the beam spreading. This points to an important difference in the models used here and in [16], best evident from Eq. (23) in [16] for the mean-field amplitude and our Eq. (4). While the equations are similar in appearance, Eq. (4) introduces a new and essential term which further destabilizes the disturbed propagating solitons and leads to the behavior more complex than that in [16]. The details of this theory will be presented elsewhere.

V. COMPARISON WITH OTHERS

The influence of noise on solitons has been considered in a number of papers [13–19]. The influence has been dubbed in different terms, such as the evolution under fluctuating nonlinearity, with stochastic contribution, or in random potentials, but the essence is the same: The influence of noise introduced in various forms on the propagation of solitons in noisy media. Consequently, widely differing conclusions have been drawn.

We concentrate on the conclusions that are relevant to our work. We should point first that all the references mentioned above treat one-dimensional (1D) NLSE-like models, most often perturbed with white noise. None treat multidimensional models and, except for [15,16], none treat nonlocal models. We could not locate in the literature any publication treating multidimensional multicomponent nonlocal noisy models. Even as such, Refs. [13,17–19] report similar findings to ours—that the fundamental solitons decay while propagating and easily get destroyed by noise. Thus, the abstract of [13] states that the fluctuations reduce the phase correlation and act as an effective loss to solitary waves. In the abstract of [17] it is similarly stated that the fluctuations of the nonlinearity induce effective nonlinear losses and that in quadratic media the amplitude of the fundamental wave decreases faster than the amplitude of the second harmonic.

Reference [18] deals with the Gross-Pitaevskii equation of Bose-Einstein condensates; it is a variant of NLSE that still displays the dispersal of solitons in random potentials. This is dully reported in [18]; however, it is also reported that under specific conditions the solitons might get stabilized; see Fig. 5 in [18]. Reference [19] considers stochastic contributions to 1D NLSE in the form of both multiplicative and additive white noise and finds that in the subcritical case of nonlinearity—which corresponds to our model—the averaged propagating profile behaves like a solitary wave localized in space, which is damped and whose maximum amplitude decays (see Fig. 6 in [19], which looks much like our Fig. 4). The situation with Refs. [15,16] is somewhat different.

References [15,16] consider a similar physical problem to ours—the influence of noise on the solitary waves in a nonlocal medium. Among other things, the authors have established that the stability of solitons increases with the degree of nonlocality and nonlocality-induced correlation length of noise in the transverse direction. On the other hand, one of the main results of our work is that the greater correlation length of the noise in the transverse plane leads to the less stable solitons during propagation. Even though this influence is reversed at large correlation lengths, it seems as if the authors of Refs. [15,16] have reached the opposite conclusions to ours: In the limit of strong nonlocality and large correlation length of the noise, the solitons in their case should be stable, whereas in our case they always decay. Although seemingly in contradiction, these results are actually consistent with each other, as it will be demonstrated below. Hence, in the reminder we point to the differences in approaches as well as to the similarities in conclusions between theirs and our paper.

First, the models considered in Refs. [15,16] and here are different. They utilize a nonlocal model with the nonlinearity of Kerr-type that can be represented by exponential response functions. We utilize a model that describes the generation of solitons in NLCs through the coupling with the reorientation angle of the director field; this model is represented by a system of two PDEs that cannot be cast in terms of simple exponential response functions.

Secondly, noise is introduced differently. In Refs. [15,16] it is an additive white noise, added to the solitary wave intensity in the medium equation. This feature causes the appearance of colored noise in the propagation equation, owing to the presence of the response function. In our case, the white noise is added to the director pre-tilt angle, which figures in the propagation equation. This—because of the form of model equations—makes it essentially the multiplicative noise. (Note, however, that the latter part of [16] also deals with the multiplicative noise). We believe that adding noise directly to the medium and not to the optical field is physically more appropriate, because the medium possesses defects, fluctuations, and irregularities, which actually present the source of noise. Nonetheless, either way of introducing noise leads to similar results, because the model equations are coupled.

We should mention that we have also explored a more traditional way of introducing noise, by adding a stochastic term to the material equation (2). This term contains random noise with well-defined correlation lengths in all spatial directions. The model leads to rich dynamical behavior that will be reported in detail elsewhere. Here, it suffices to mention that it also leads to the behavior very similar to the one reported in Fig. 4.

Finally, the models used are of different dimensionality. In Refs. [15,16] the models are $(1+1)$D; ours is $(2+1)$D. They consider an evolution problem in which the response function depends on the spatial coordinate. The evolution variable can be interpreted as time t or the propagation distance z. Ours is a steady-state spatial problem in which the nonlocality and noise are spatially distributed. Noisy nonlocal models in 1D and 2D may offer substantially different behavior, because the response functions—essentially Green's functions—possess different forms in different dimensions.

In Refs. [15,16] nonlocality is introduced through an exponential response function; its width σ measures the degree of nonlocality and determines the behavior of the propagating solitons. It is also related to the correlation length of the noise. Much of the discussion in Refs. [15,16] is concerned with what happens as σ is varied. In [15] the authors showed that in the presence of beam randomness the nonlocality suppresses random walk of the self-trapped beam, which ideally vanishes for an infinite degree of nonlocality. In [16] the authors demonstrated that the radiative losses can adiabatically transform a soliton into another member of the soliton family with lower power. A general conclusion was that the increase in σ leads to a smaller radiation and a reduced random walk of the disturbed solitons. However, even at high values of σ, as in Fig. 2(a) of [15] or Fig. 1(a) in [16], the solitons are unstable in propagation and slowly decay, due to small but finite radiation. It is precisely this behavior at high σ that we are interested in.

In our case σ—determined by the ratio of widths of $\hat{\theta}$ and the propagating soliton—is constant. We remain at all times in the highly nonlocal regime, in which σ is fixed to a large value (of the order of 10). Small extra energy added to the system by noise moves the soliton away from the self-organized equilibrium and causes it to oscillate. This extra energy stays for long with the soliton and is not radiated away easily. Noise introduced in the director's pre-tilt angle makes the system effectively lossy and causes the soliton not to randomly walk and decay but to oscillate and decay. We consider how this process of oscillation and decay is influenced by the coarse graining and the increasing correlation length of the noise, introduced by the variable computational block size. This behavior is depicted in Fig. 4.

Thus, in our case the correlation length is not connected with the degree of nonlocality but with the size of the noise grains or the computational blocks. Still, our results are in qualitative agreement with those mentioned in [15,16] for high but fixed σ and for propagation intervals considered in Refs. [15,16]. The longer propagation lengths, as displayed in Fig. 4 offer different behavior. It would have been a more interesting comparison with [15,16], had the propagation distance there—or the time interval—been longer than 20.

VI. CONCLUSION

Concluding, we have studied numerically soliton propagation in a highly nonlocal noisy medium. White noise added to the medium in specific blocks induces the fundamental solitons to breathe at small propagation distances, with the characteristics well predicted by the variational calculation. For larger propagation distances, at a certain distance the soliton beams start to spread rapidly with small radiation loss. This leads to the destruction of solitons after a prolonged propagation. The destruction weakly depends on the correlation length of noise, with the larger correlation length initially leading to the speedier demise of solitons. However, this trend is reversed when the correlation length becomes comparable to the transverse width of the beam. Naturally, the speedier demise is also induced by increasing the level of noise. We introduced a system of mean-field evolution equations that explains the observed dynamical behavior.

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[1] Yu. S. Kivshar and G. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego, 2003).
[2] G. L. Lamb, *Elements of Soliton Theory* (Wiley, New York, 1980).
[3] C. Sulem and P. Sulem, *The Nonlinear Schrödinger Equation: Self-focusing and Wave Collapse* (Springer, Berlin, 2000).
[4] J. F. Henninot, J. F. Blach, and M. Warenghem, J. Opt. A **9**, 20 (2007).
[5] X. Hutsebaut, C. Cambournac, M. Haelterman, J. Beeckman, and K. Neyts, J. Opt. Soc. Am. B **22**, 1424 (2005).
[6] J. Beeckman, K. Neyts, X. Hutsebaut, C. Cambournac, and M. Haelterman, Opt. Express **12**, 1011 (2004).
[7] S. Skupin, O. Bang, D. Edmundson, and W. Krolikowski, Phys. Rev. E **73**, 066603 (2006).
[8] A. I. Yakimenko, Y. A. Zaliznyak, and Yu. S. Kivshar, Phys. Rev. E **71**, 065603(R) (2005).
[9] A. Strinić, M. Petrović, D. Timotijević, N. Aleksić, and M. Belić, Opt. Express **17**, 11698 (2009).
[10] M. Petrović, A. Strinić, and M. Belić, Phys. Scr. **85**, 015403 (2012).
[11] J. Beeckman, K. Neyts, P. J. M. Vanbrabant, R. James, and F. A. Fernandez, Opt. Express **18**, 3311 (2010).
[12] I. C. Khoo, *Liquid Crystals* (Wiley, New York, 2007).
[13] C. B. Clausen, O. Bang, Yu. S. Kivshar, and P. L. Christiansen, Opt. Lett. **22**, 271 (1997).
[14] F. Abdullaev and J. Garnier, Prog. Opt. **48**, 35 (2005).
[15] V. Folli and C. Conti, Phys. Rev. Lett. **104**, 193901 (2010).

[16] F. Maucher, W. Krolikowski, and S. Skupin, Phys. Rev. A **85**, 063803 (2012).
[17] F. Abdullaev, S. A. Darmanyan, A. Kobyakov, E. Schmidt, and F. Lederer, Opt. Commun. **168**, 213 (1999).
[18] F. K. Abdullaev and J. Garnier, Phys. Rev. A **72**, 061605(R) (2005).
[19] A. Debussche and L. Di Menza, Physica D **162**, 131 (2002).
[20] N. B. Aleksić, M. S. Petrović, A. I. Strinić, and M. Belić, Phys. Rev. A **85**, 033826 (2012).
[21] V. I. Petviashvili, Fiz. Plazmy **2**, 469 (1976) [Sov. J. Plasma Phys. **2**, 257 (1976)].
[22] X. Hutsebaut, C. Cambournac, M. Haelterman, A. Adamski, and K. Neyts, Opt. Commun. **233**, 211 (2004).
[23] J. Yang, I. Makasyuk, A. Bezryadina, and Z. Chen, Stud. Appl. Math. **113**, 389 (2004).
[24] D. Jović, S. Prvanović, R. Jovanović, and M. Petrović, Opt. Lett. **32**, 1857 (2007).
[25] A. W. Snyder and D. J. Mitchell, Science **276**, 1538 (1997).
[26] A. Strinić, M. Petrović, N. Aleksić, and M. Belić, Opt. Commun. **286**, 309 (2013).
[27] I. V. Klyatzkin, *Stohastic Equation and Waves in Random Media* (Nauka, Moscow, 1980) (in Russian).