

# Spin model in the effective staggered magnetic field in optical lattices

Huirong Zhang,<sup>1</sup> Qiuyi Guo,<sup>1</sup> Zhaoyuan Ma,<sup>1,2</sup> and Xuzong Chen<sup>1,3</sup>

<sup>1</sup>*School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China*

<sup>2</sup>*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, People's Republic of China*

<sup>3</sup>*State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China*

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Ultracold atoms with two hyperfine internal states in optical lattices are proposed to demonstrate a two-dimensional spin-1/2 model under a transverse effective staggered magnetic field. The transverse magnetic field is produced by a Raman process and can be modulated by the intensities of the additional light fields. This optical lattice protocol can investigate magnetic quantum phenomena induced by the effective staggered magnetic field in a wider parameter range with controllable interaction between the spins of the nearest-neighbor atoms.

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## I. INTRODUCTION

The theoretical and experimental work [1–4] on the superfluid–Mott-insulator phase transition opens the new way to simulate complex many-body physics in an ultracold atom system under controllable conditions. In certain limits, spin models [5–10] can be obtained with two-state bosonic or fermionic atoms in an optical lattice. A great deal of research has been done to simulate this pseudospin behavior in optical lattices, such as magnetically ordered states [11–13], bosons and fermions in Kagomé optical lattices [14,15], antiferromagnetic spin chains [16,17], superexchange in superlattices [18,19], and so on.

The phenomena induced by magnetic fields have been considerably studied theoretically and experimentally for a long time in condensed-matter physics. Recently, there is increasing interest in the effects of the staggered magnetic field motivated by the experimental work on a number of materials. An effective staggered  $g$  tensor and a staggered Dzyaloshinskii-Moriya interaction give rise to the effective staggered magnetic field [20–25]. Generally, the magnitude of the staggered field depends on the relative direction and magnitude of the applied uniform magnetic field on the sample. In the quasi-one-dimensional case, for the spin-1/2 system, the magnetic phenomena under the staggered magnetic field have been explored [20,26–29]. The staggered magnetic field induces an excitation energy gap in the spin systems, which has been observed in crystal materials [21–23]. In the two-dimensional case, considering the interchain interaction [30–32], there are different physical phenomena [33–36] from the one-dimensional (1D) chain case. Quantum phase transition occurs due to the competition between the interchain coupling and the staggered magnetic field. The power-law dependence of the excitation energy gap is different also.

In this work we shall study a controllable spin-1/2 model in a tunable transverse effective staggered magnetic field for the two-dimensional (2D) system with two component ultracold atoms trapped in optical lattices. We consider a protocol that the additional  $\sigma^-$ -polarized light fields along the  $z$  direction are applied to the ultracold atoms of two hyperfine internal states (denoting the effective spin states) trapped in  $xy$  plane by a standing-wave configuration formed by  $\pi$ -polarized laser beams. The frequency of the additional lights is two-photon resonant with  $x$ -direction trapping lights

and is two-photon large detuned with the other trapping lights. Under the two-photon resonance condition, the additional lights and the  $x$ -direction trapping lights drive the Raman transition and induce an effective coupling between the two hyperfine internal states. The distribution of the effective coupling varies periodically along the  $x$  direction with the period equal to the wavelength of the  $x$ -direction trapping lights. The coupling is positive and negative staggered for the atoms trapped in different optical potential wells, which produces the transverse effective staggered magnetic field for the spin system.

This protocol allows us to explore the fundamental physics of the spin model in a wider range of parameter with the controllable spin coupling intensity and the adjusting transverse staggered magnetic field. The spin model can be realized using this protocol which is difficult for the experimental realization in condensed-matter physics.

The paper is organized as follows. In Sec. II we describe the optical lattice setup and obtain the effective staggered coupling between the two internal states of the trapped ultracold atoms. In Sec. III, we give out the Hamiltonian of the spin model under the transverse staggered field for the ultracold atomic system and demonstrate the advantages of the model with the independent and easily controlled parameters. In Sec. IV, we summarize the work.

## II. SETUP AND MODEL

In this section, we demonstrate the scheme in details that the ultracold atoms of two hyperfine internal states are trapped in  $xy$  plane with the additional light fields along the  $z$  direction. We write out the Hamiltonian of the system and obtain the effective spatially periodic coupling between the two hyperfine internal states which induces the effective staggered transverse magnetic field.

As shown in Fig. 1(a) the standing-wave configuration is formed by counterpropagating  $\pi$ -polarized laser beams along  $x$ ,  $y$ , and  $z$  directions with the frequencies  $\omega_1$ ,  $\omega_2$ , and  $\omega_2$ , and the wave-vector values  $k_1$ ,  $k_2$ , and  $k_2$ . The polarizations of the propagating fields along  $x$ ,  $y$ , and  $z$  directions are orthogonal to each other. The intensity of the standing wave along the  $z$  direction is large enough so that there is no hopping in this direction and the atoms are confined in the  $xy$  plane. The spatial light fields are  $\mathbf{E}_\pi^+(\mathbf{x}, t) = \mathbf{E}_x(x)e^{-i\omega_1 t} + \mathbf{E}_y(y)e^{-i\omega_2 t} + \mathbf{E}_z(z)e^{-i\omega_2 t}$

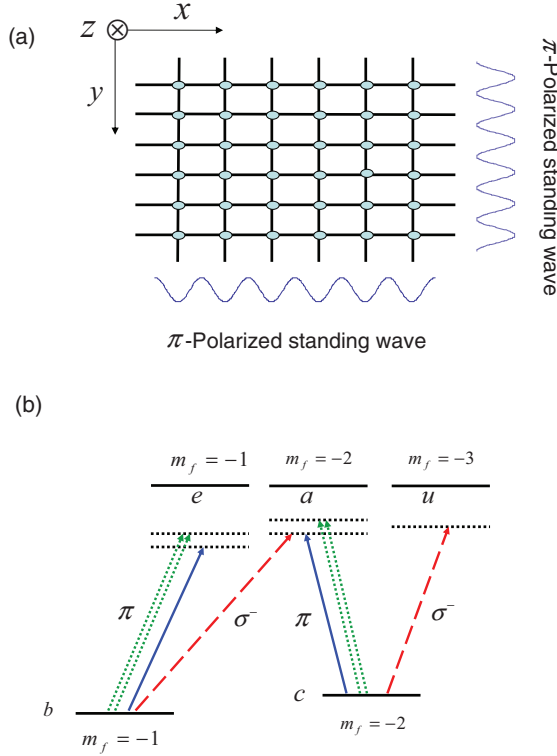


FIG. 1. (Color online) (a) 2D optical lattices with two-component ultracold atoms. The atoms are confined in the  $xy$  plane by the  $\pi$ -polarized standing wave along  $z$  direction with frequency  $\omega_2$ . The standing wave along  $x$  and  $y$  directions are formed by the  $\pi$ -polarized fields with different frequencies  $\omega_1$  and  $\omega_2$ . The additional  $\sigma^-$ -polarized fields are applied to the system counterpropagating along  $z$  direction with frequency  $\omega_3$ . (b) Energy-level scheme of atoms interacting with  $\pi$ -polarized trapping fields (the solid blue arrows with  $x$ -direction fields; the dotted green arrows with  $y$ - and  $z$ -direction fields) and interacting with  $\sigma^-$ -polarized additionally applied fields (the dashed red arrows). The frequencies  $\omega_3$ ,  $\omega_1$  of the additional fields and the trapped fields along  $x$  direction satisfy the condition of two-photon resonance  $\omega_c = \omega_3 - \omega_1$ .

with  $\mathbf{E}_x(x) = \mathbf{E}_x(e^{ik_1x} + e^{-ik_1x})$ ,  $\mathbf{E}_y(y) = \mathbf{E}_y(e^{ik_2y} + e^{-ik_2y})$ , and  $\mathbf{E}_z(z) = \mathbf{E}_z(e^{ik_3z} + e^{-ik_3z})$ . An ensemble of ultracold atoms with two hyperfine internal states  $|b\rangle$  and  $|c\rangle$  is trapped in the  $xy$  plane standing-wave configuration. The additionally counterpropagating  $\sigma^-$ -polarized lasers are applied on the systems along the  $z$  direction. The light field is  $\mathbf{E}_\sigma^+(z, t) = \mathbf{E}_\sigma(z)e^{-i\omega_3 t}$  and  $\mathbf{E}_\sigma(z) = \mathbf{E}_0(e^{ik_3z} + e^{-ik_3z})$  with the frequency  $\omega_3$  and the wave-vector value  $k_3$ . The frequencies of the additional lights and  $x$ -direction trapping lights satisfy the two-photon resonance condition  $\omega_c = \omega_3 - \omega_1$  with  $\omega_c$  the energy spacing of the internal states  $|c\rangle$  to  $|b\rangle$ . The frequency of the trapping lasers along  $y$  and  $z$  directions is two-photon large detuned, i.e.,  $|\Delta| \gg 0$  with  $\Delta = \omega_3 - \omega_2 - \omega_c$ . An effective coupling between the two internal states  $|b\rangle$  and  $|c\rangle$  can be induced by the additional lights and the  $x$ -direction trapping lights under the two-photon resonance condition.

We consider the case of bosonic atoms of the hyperfine internal states  $|b\rangle$  and  $|c\rangle$  with the magnetic quantum numbers  $m_f = -1$  and  $m_f = -2$ , respectively, in details. This is the same as the condition of fermionic atoms with the

two appropriate hyperfine internal states. The energy-level configuration of the bosonic atom is shown in Fig. 1(b). By the selection rules, the transition from the state  $|b\rangle$  to the excited state  $|a\rangle$  and the transition from state  $|c\rangle$  to the excited state  $|u\rangle$  are driven by the additionally applied  $\sigma^-$ -polarized lights. The  $\pi$ -polarized trapping lights couple the ground state  $|b\rangle$  to the excited states  $|e\rangle$  and couple the state  $|c\rangle$  to the excited states  $|a\rangle$  with the position-dependent Rabi frequencies, respectively. Under the case of two-photon resonance  $\omega_c = \omega_3 - \omega_1$ , an effective coupling between the internal states  $|b\rangle$  and  $|c\rangle$  can be induced by the Raman process of the additional fields and  $x$ -direction trapping fields. Due to the large two-photon detuning  $\Delta$ , no effective coupling forms from the additional lights and the  $y$ - and  $z$ -direction trapping lights. The effects of the trapping lasers along  $y$  and  $z$  directions on the atom are the periodical potentials with the period of  $\lambda_2/2$  ( $\lambda_2$  is the optical wavelength). The Hamiltonian in terms of interaction of an atom with the additional fields and the  $x$ -direction trapping fields reads

$$\begin{aligned}
 H = & \sum_{\mu=u,a,e,c} \omega_\mu \sigma_{\mu\mu} \\
 & + \sum_{\mu\nu=ac,eb} [\Omega_{\mu\nu}(x)e^{-i\omega_1 t} \sigma_{\mu\nu} + \text{H.c.}] \\
 & + \sum_{\mu\nu=ab,uc} [\Omega_{\mu\nu}(z)e^{-i\omega_3 t} \sigma_{\mu\nu} + \text{H.c.}], \quad (1)
 \end{aligned}$$

where  $\omega_\mu$  are the atomic energy-level spacings with respect to the internal states  $|\mu\rangle$  to the ground state  $|b\rangle$ , respectively,  $\sigma_{\mu\nu} = |\mu\rangle\langle\nu|$ . The position-dependent Rabi frequencies  $\Omega_{\mu\nu}(x) = -\mathbf{d}_{\mu\nu} \cdot \mathbf{E}_x(x)$  (with  $\mathbf{d}_{\mu\nu} = e\langle\mu|\mathbf{r}|\nu\rangle$ ,  $\mu\nu = ac, eb$ ) are the couplings of  $x$ -direction trapping lights to the atom between the internal states  $\mu$  and  $\nu$ . The Rabi frequencies  $\Omega_{\mu\nu}(z) = -\mathbf{d}_{\mu\nu} \cdot \mathbf{E}_\sigma(z)$  (with  $\mathbf{d}_{\mu\nu} = e\langle\mu|\mathbf{r}|\nu\rangle$ ,  $\mu\nu = ab, uc$ ) correspond to the  $\sigma^-$ -polarized lights coupling the internal states  $\mu$  and  $\nu$ . For the system the Raman process by the  $x$ -direction trapping lights and additional lights will induce the effective coupling between the states  $|b\rangle$  and  $|c\rangle$ . The atoms in the internal states  $|b\rangle$  and  $|c\rangle$  are subjected to the spatially periodical potentials induced by the  $\pi$ -polarized standing-wave fields.

Using Fröhlich's transformation [37], in the large detuning case, the effective Hamiltonian  $H_{\text{eff}}$  can be written as

$$H_{\text{eff}} = V_{\pi b}(x)\sigma_{bb} + V_{\pi c}(x)\sigma_{cc} + [\Omega_{\text{eff}}(x)\sigma_{bc} + \text{H.c.}], \quad (2)$$

where  $V_{\pi b}(x) \approx V_{\pi c}(x)$  is the trapping potential of the atoms in states  $|b\rangle$  and  $|c\rangle$  by  $\pi$ -polarized trapping lights along the  $x$  direction, which is independent on the internal states of the atoms. The trapping potentials are  $V_{\pi b}(x) = V_{bx} \cos^2(k_1 x)$  with  $V_{bx} = |\mathbf{d}_{eb} \cdot \mathbf{E}_x|^2 / \Delta_{xeb}$  and  $V_{\pi c}(x) = V_{cx} \cos^2(k_1 x)$  with  $V_{cx} = |\mathbf{d}_{ac} \cdot \mathbf{E}_x|^2 / \Delta_{xac}$ , which are periodically varied with the period  $\lambda_1/2$  ( $\lambda_1$  being the wavelength of the  $x$ -direction trapping lights). The detuning is  $\Delta_{x\mu\nu} = \omega_1 - (\omega_\mu - \omega_\nu)$  ( $\mu\nu = eb, ac$ ). Here we ignore the potentials  $V_{\sigma c}(z) = |\Omega_{uc}(z)|^2 / \Delta_{uc}$  and  $V_{\sigma b}(z) = |\Omega_{ab}(z)|^2 / \Delta_{ab}$  with  $\Delta_{\mu\nu} = \omega_3 - (\omega_\mu - \omega_\nu)$  ( $\mu\nu = ab, uc$ ) for the internal states  $|b\rangle$  and  $|c\rangle$  induced by the  $\sigma^-$ -polarized optical fields, which are much less than the trapping potential along the  $z$  direction due to the weak additional fields. The effective coupling intensity

between the internal states  $|b\rangle$  and  $|c\rangle$  is

$$\Omega_{\text{eff}}(x) = \Omega_0 \cos(k_1 x), \quad (3)$$

where  $\Omega_0 = 2[(\mathbf{d}_{ac} \cdot \mathbf{E}_x)(\mathbf{d}_{ab} \cdot \mathbf{E}_0)^* / \Delta_{xac} + (\mathbf{d}_{ac} \cdot \mathbf{E}_x)(\mathbf{d}_{ab} \cdot \mathbf{E}_0)^* / \Delta_{ab}]$  with  $z = 0$  is adopted.

In the case of red detuning the effective coupling  $\Omega_{\text{eff}}(x)$  is positive and negative staggered for the atoms trapped in different optical potential wells and varies with the spatial period  $\lambda_1$  along the  $x$  direction.

### III. MAGNETIC PHENOMENA

In this section, we write out the spin model for the ultracold atom system under the effective transverse staggered magnetic field which is produced by the effective couple  $\Omega_{\text{eff}}(x)$  of the two internal states  $|b\rangle$  and  $|c\rangle$ . We demonstrate the advantages of this model to study the magnetic phenomena induced by the staggered field.

The atoms are confined in  $xy$  plane with enough intense standing-wave fields along  $z$  direction and there is no hopping in this direction. Considering the 2D case, the trapping potentials for the atom in internal states  $|b\rangle$  and  $|c\rangle$  are  $V_b(\mathbf{x}) = V_{bx} \cos^2(k_1 x) + V_{by} \cos^2(k_2 y)$  with  $V_{bm} = |\mathbf{d}_{eb} \cdot \mathbf{E}_m|^2 / \Delta_{meb}$  and  $V_c(\mathbf{x}) = V_{cx} \cos^2(k_1 x) + V_{cy} \cos^2(k_2 y)$  with  $V_{cm} = |\mathbf{d}_{ac} \cdot \mathbf{E}_m|^2 / \Delta_{mac}$  ( $m = x, y$ ). The detuning is  $\Delta_{y\mu\nu} = \omega_2 - (\omega_\mu - \omega_\nu)$  (with  $\mu\nu = eb, ac$ ). Here the coordinates  $\mathbf{x} = \{x, y\}$ . Considering  $N$  atoms trapped, from the representation (2) the second quantized Hamiltonian reads

$$\begin{aligned} H = & \sum_{\mu=b,c} \int d\mathbf{x} \hat{\Psi}_\mu^\dagger(\mathbf{x}) \left[ -\frac{1}{2m} \nabla^2 + V(\mathbf{x}) \right] \hat{\Psi}_\mu(\mathbf{x}) \\ & + \frac{1}{2} \sum_{\substack{\mu\nu=cc, \\ bb, bc}} \frac{4\pi a_{\mu\nu}}{m} \int d\mathbf{x} \hat{\Psi}_\mu^\dagger(\mathbf{x}) \hat{\Psi}_\nu^\dagger(\mathbf{x}) \hat{\Psi}_\nu(\mathbf{x}) \hat{\Psi}_\mu(\mathbf{x}) \\ & + \left[ \int d\mathbf{x} \Omega_{\text{eff}}(x) \hat{\Psi}_b^\dagger(\mathbf{x}) \hat{\Psi}_c(\mathbf{x}) + \text{H.c.} \right], \quad (4) \end{aligned}$$

where  $V(\mathbf{x})$  is the trapping potential. For  $\pi$ -polarization trapping lights there is  $V(\mathbf{x}) = V_b(\mathbf{x}) \approx V_c(\mathbf{x})$ . The Schrödinger field operators  $\hat{\Psi}_{\mu(v)}(\mathbf{x})$  describe atoms with the internal states  $|\mu\rangle$  ( $|v\rangle$ );  $a_{\mu\nu}$  are the corresponding scattering lengths between the atoms with the internal states  $|\mu\rangle$  and  $|v\rangle$ .

Using the single band and tight-binding approximation, we can write the Hamiltonian in the Wannier presentations [2] as follows:

$$\begin{aligned} H = & - \sum_{i,j} (t_1 c_{i,j}^\dagger c_{i+1,j} + t_1 b_{i,j}^\dagger b_{i+1,j}) \\ & + t_2 c_{i,j}^\dagger c_{i,j+1} + t_2 b_{i,j}^\dagger b_{i,j+1}) \\ & + \frac{1}{2} \sum_{i,j;\mu=b,c} U_{\mu\mu} n_{i,j\mu} (n_{i,j\mu} - 1) + U_{bc} \sum_{i,j} n_{i,jb} n_{i,jc} \\ & + \sum_{i,j} (-1)^i (\Omega b_{i,j}^\dagger c_{i,j} + \text{H.c.}), \quad (5) \end{aligned}$$

where  $i$  and  $j$  denote the indexes of the atom site along  $x$  and  $y$  -directions,  $c_{i,j}$ ,  $b_{i,j}$  are annihilation operators for the atom of internal states  $|c\rangle$  and  $|b\rangle$  localized on site  $(i, j)$ , and  $n_{i,jb} = b_{i,j}^\dagger b_{i,j}$ ,  $n_{i,jc} = c_{i,j}^\dagger c_{i,j}$ ,  $t_1$ , and  $t_2$  are

the tunneling between adjacent sites for particles along  $x$  direction and  $y$  direction.  $U_{\mu\mu}$  is the on-site interaction between the two atoms of the same internal states  $|\mu\rangle$ ;  $U_{bc}$  is on-site interaction between the different internal states  $|b\rangle$  and  $|c\rangle$ . The definitions of these parameters are given  $U_{\mu\nu} = 4\pi a_{\mu\nu} / m \int d^2x |w(\mathbf{x})|^4$  (with  $\mu\nu = bb, cc, bc$ ),  $t_1 = \int d^2x w^*(\mathbf{x} - \mathbf{x}_{i,j}) [-1/(2m)\nabla^2 + V(\mathbf{x})] w(\mathbf{x} - \mathbf{x}_{i+1,j})$ , and  $t_2 = \int d^2x w^*(\mathbf{x} - \mathbf{x}_{i,j}) [-1/(2m)\nabla^2 + V(\mathbf{x})] w^*(\mathbf{x} - \mathbf{x}_{i,j+1})$ .

In the red detuned case, the induced tunneling between the states  $|b\rangle$  and  $|c\rangle$  at nearest-neighbor site is omitted. The coupling  $\Omega$  is

$$\Omega = e^{i\phi} \int d^2x w^*(\mathbf{x}) |\Omega_{\text{eff}}(x)| w(\mathbf{x}), \quad (6)$$

where  $w(\mathbf{x}) = w(x)w(y)$  are localized Wannier functions and  $\phi$  is the phase of  $\Omega_{\text{eff}}(x)$ . Here we consider the case  $\phi = 0$ .

Quantum spin systems can be simulated by the ultracold atoms in optical lattices. Considering the case  $\langle n_{i\uparrow} \rangle + \langle n_{i\downarrow} \rangle \simeq 1$ , corresponding to one atom per site, in the Mott-insulator phase and the regime in which  $t, \Omega \ll U_{bb}, U_{cc}, U_{bc}$ , from Eq. (5) the effective Hamiltonian describing the spin can be derived as [10]

$$\begin{aligned} H = & -J_{1\perp} \sum_{i,j} (\mathbf{S}_{i,j}^x \mathbf{S}_{i+1,j}^x + \mathbf{S}_{i,j}^y \mathbf{S}_{i+1,j}^y) - J_{1z} \sum_{i,j} S_{i,j}^z S_{i+1,j}^z \\ & - J_{2\perp} \sum_{i,j} (\mathbf{S}_{i,j}^x \mathbf{S}_{i,j+1}^x + \mathbf{S}_{i,j}^y \mathbf{S}_{i,j+1}^y) - J_{2z} \sum_{i,j} S_{i,j}^z S_{i,j+1}^z \\ & + \sum_{i,j} (-1)^i h_x \mathbf{S}_{i,j}^x + \sum_{i,j} h_z S_{i,j}^z, \quad (7) \end{aligned}$$

where the spin operators are  $S_{i,j}^z = 1/2(n_{i,jc} - n_{i,jb})$ ,  $S_{i,j}^x = 1/2(c_{i,j}^\dagger b_{i,j} + b_{i,j}^\dagger c_{i,j})$ , and  $S_{i,j}^y = -i/2(c_{i,j}^\dagger b_{i,j} - b_{i,j}^\dagger c_{i,j})$ . The parameters are

$$J_{\gamma\perp} = \pm \frac{4t_\gamma^2}{U_{bc}}, \quad (8)$$

$$J_{\gamma z} = 2 \left( \frac{2t_\gamma^2}{U_{bb}} + \frac{2t_\gamma^2}{U_{cc}} - \frac{2t_\gamma^2}{U_{bc}} \right), \quad (9)$$

$$h_x = 2\Omega, \quad h_z = \sum_{\gamma} 8t_\gamma^2 / U_{cc} - 8t_\gamma^2 / U_{bb}, \quad (10)$$

where  $\gamma = 1, 2$ . The  $\pm$  signs in the expression of  $J_{\gamma\perp}$  correspond to the cases of bosonic and fermionic atoms, respectively. This Hamiltonian describes the spin coupling in a transverse staggered magnetic field and a  $z$ -direction homogeneous magnetic field. We shall use this model to investigate the effects induced by the staggered magnetic field in an ultracold atom system.

Now we demonstrate the advantages of this model. In Hamiltonian (7) all the parameters can be adjusted [6,10,38,39] in a wider parameter range than those in the crystal materials in condensed-matter physics. The transverse staggered magnetic field is independent of the  $z$ -direction uniform magnetic field, while in the crystal materials these two fields are both proportional to the uniform magnetic field applied to the sample. The value of  $h_z$  can be modulated by an additionally applied magnetic field and the values of  $h_x$  can be controlled by adjusting the intensity of the additional

light fields. The magnitude of spin coupling intensity  $J_{y\perp}$  and  $J_{yz}$  is controlled [6] by modulating the intensities of trapping light fields and by adjusting the scattering lengths with the Feshbach resonance. Thus this protocol can explore fundamental physical phenomena in a new physical regime which is difficult to reach in condensed-matter physics and simulate the new spin models in the staggered magnetic field which have been only theoretically studied. The system can also reduce to a 1D spin chain along  $x$  direction with  $y$  direction confined by the strong enough standing-wave fields.

We consider a specific example. For the 2D fermionic atoms, the couplings satisfy  $J_{y\perp} = J_{yz} = J_y$  since  $U_{bb}, U_{cc} \gg U_{bc}$ . The spin model (7) reduces to the Hamiltonian (4) of Ref. [36] by selecting the magnetic field  $h_z = 0$ . This model has been theoretically studied with a continuous-time Monte Carlo method; however, there is not yet an appropriate material to investigate it. According to the results, for the antiferromagnetic interchain interaction case, i.e., the  $y$ -direction spin coupling  $J_2$  being negative ( $J_2 < 0$ ), the competition between the interchain coupling and the staggered magnetic field results in quantum phase transition. For a finite transverse staggered magnetic field, there are two different phases; a symmetric phase and a spontaneous symmetry-breaking (SSB) phase. The symmetric phase is that the spins point in the directions of the transverse staggered magnetic field and the SSB phase means the ordered phase with spontaneous symmetry-breaking in the plane normal to the staggered magnetic field. A phase-transition diagram was given. We show it in Fig. 2 with the parameters in this ultracold atoms system. A transverse staggered magnetic field lowers symmetries and induces an excitation energy gap. Here the dependent law of excitation energy gap is different from the 1D case for which the gap is  $\Delta \propto h_x^{2/3}$ . For the competition case, i.e., the interchain coupling  $J_2$  being antiferromagnetic, the gap opens at a finite value  $h_c$  of  $h_x$  and the dependence law is shown in Ref. [36] for the region  $h_x - h_c < J_2/2$ . Here we show it in Fig. 3 by the energy-gap variation with the effective coupling. Using this model, these phenomena can be investigated in ultracold atom systems of two internal states

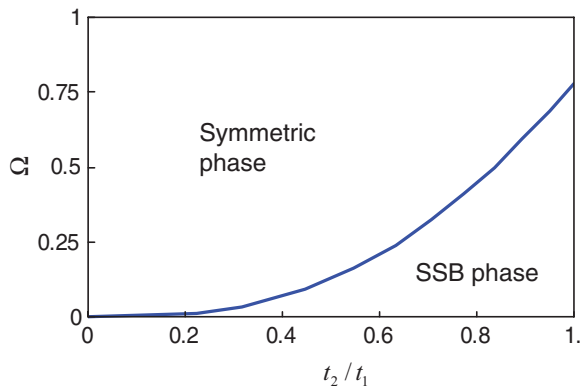


FIG. 2. (Color online) Phase-transition diagram for competition case, i.e., the interchain coupling is antiferromagnetic. Here we set the coupling constant  $J_1$  as the energy unit. SSB denotes spontaneous symmetry breaking in the plane normal to the staggered field. Symmetric phase denotes that the spins are oriented in the staggered field directions.

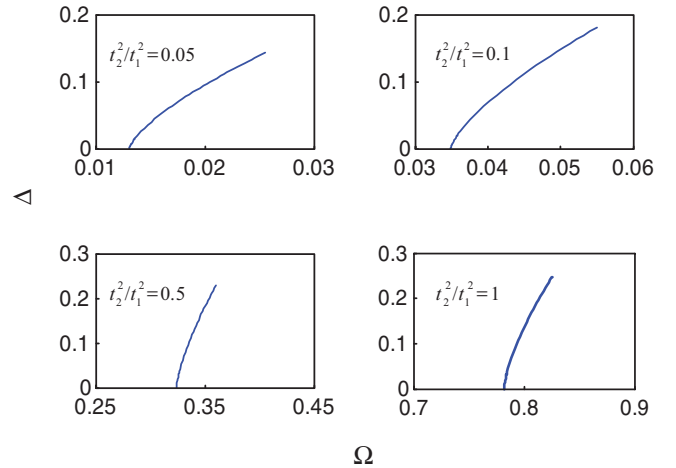


FIG. 3. (Color online) Energy-gap dependence on the value of effective coupling  $\Omega$  for the competition case for different  $t_2^2/t_1^2$  values. The coupling constant  $J_1$  is the energy unit.

trapped in optical lattices. The ratio  $J_2/J_1 = t_2^2/t_1^2$  can be easily controlled by adjusting the intensities of the trapping fields along  $y$  and  $x$  directions.

Although spin models under the staggered magnetic field have been considerably studied in condensed-matter physics, the great advantage of the scheme for the ultracold atom system is that one can investigate the physical phenomena with the variation of the coupled strength between the spins and with the variation of the independent transverse staggered and longitudinal uniform magnetic fields, which allows one to study the model in a wider parameter range experimentally.

#### IV. CONCLUSIONS

In summary, we propose a scheme to simulate the 2D Heisenberg spin model in a transverse effective staggered magnetic field with two component ultracold atomic system trapped in optical lattices. The staggered magnetic field is induced by the Raman process by controlling frequencies and polarizations of the  $x$  direction trapped lights and the additionally applied light fields. All the parameters including the coupling intensity and the effective magnetic fields are adjustable with the state-of-the-art experimental techniques. The model also can be reduced to the 1D spin chain by enough intense trapping fields in  $y$  direction. This scheme can provide a clean and controllable environment to explore the fundamental magnetic phenomena induced by the effective transverse staggered magnetic field in a new physical parameter regime.

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