Optical forces on two-level atoms by subcycle pulsed focused vector fields

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The optical forces on neutral two-level atoms by the subcycle pulsed focused vector field are investigated by numerically solving the density-matrix equations without the rotating-wave approximation. The phenomena of focusing or defocusing of the neutral atom are analyzed. For the transverse optical force, an acceleration of an atom that is 10 orders of magnitude higher than the Earth's gravitational acceleration g can be obtained. The optical force can change from a focusing force to a defocusing force depending on the initial state of the atom. The optical potential may be used to trap the neutral two-level atoms in the experiment of cold atoms.

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I. INTRODUCTION

In recent years, many theoretical and experimental studies on optical force have been carried out in various contexts [1–14]. The interaction of a laser with atoms forms the basis of atomic cooling and trapping, which enables us to get full control of the atomic motion and has paved the way for applications, such as atom optics and Bose-Einstein condensation [8–10]. Many authors have studied the radiation forces exerted on neutral atoms. In this context, the two-level system may be the one most studied. The two-level atomic system is the simplest scheme for calculating the radiation forces [15]. The forces can be calculated using the steady-state solutions of the optical Bloch equations within the rotating-wave approximation (RWA) [16]. However, the rotating-wave approximation and the slowly varying envelope approximation can no longer be used in the femtosecond and attosecond regimes. Recently, experiments on semiconductors have shown that, in the regime of extreme nonlinear optics, the description of the atomic system in the terms of two-level systems has been able to reproduce the experimental results amazingly well [17]. Some theoretical analyses in the context of optical dipole trapping and light force on a two-level atomic system in a few-cycle pulse laser field beyond RWA have been carried out [14,18]. In these researches, the few-cycle pulse is seen as the Gaussian pulse, and only the transverse component is considered. But, in fact, the ultrashort pulse is vectorial, and the longitudinal component cannot be ignored. With the great progress in the generation of ultrashort laser pulses, the electromagnetic pulses, which consist of a few or less than one optical cycle, have been realized in the laboratory [19–23]. For the subcycle pulse, the carrier-envelope model is found to cause nonphysical results and cannot be used [24,25]. Therefore, an accurate description of the pulse-matter interaction requires precise knowledge of all components of the electromagnetic field. Analytical expressions of few-cycle, single-cycle, and subcycle pulsed beams with arbitrary polarization that are exact solutions of Maxwell's equations have been presented by us [25,26]. With the expressions, we analyzed the optical force on the two-level atoms with the subcycle and few-cycle pulsed focused vector beams. The transverse focusing, defocusing force of the neutral atoms is found, and the optical potential is analyzed. The transverse focusing or defocusing force of the neutral atoms is dependent on the initial states of the atom. If the atom initially is at the ground state, the transverse trapping force is always inward, directed toward the optical axis. The ultrastrong deceleration of neutral atoms with a magnitude as high as 10 orders of magnitude times the Earth's gravitational acceleration g is gained if the velocity of the atom deviates from the optical axis. Focusing of the neutral atoms may be possible by the transverse trapping force and may lead to new applications in the experiment of cold atoms and atomic optics.

II. THEORY

In order to calculate the radiation force exerted on a two-level atom under the influence of a pulsed focused vector beam, we review the expressions of subcycle pulsed focused vector beams which are presented by us [25,26]. We consider two opposite charges oscillating against each other. They can be described by an oscillating dipole located at the origin of the coordinate system with the dipole moment,

$$p(r,t) = \frac{p_0(t)}{\sqrt{1+\xi^2}}(e_x + i\xi e_y)\delta(r), \tag{1}$$

where the parameter ξ determines the polarization state such that $\xi = 0$ corresponds to the linear polarization in the x direction, $\xi = \pm 1$ corresponds to the circular polarization, and $\xi = \infty$ corresponds to the linear polarization in the y direction. The function $p_0(t)$ cannot be arbitrary. For the model of the few-cycle pulse, the dc component in the carrier-envelope model with an envelope, which is multiply a carrier, is found to cause nonphysical results. Another problem is that an expression of subcycle and few-cycle pulses is only physically meaningful as long as the spectrum content remains invariant under the change in its carrier-envelope phase. In order to overcome the two problems, an analytic vector potential model has been proposed. Here, the $p_0(t)$ must be analytic. $p_0(t)$ can be written as a product of a carrier and a complex Lorentz analytical envelope,

$$p_0(t) = p_0 \frac{T}{T - it}\exp(i\omega t + i\phi_0). \tag{2}$$

The peak value p_0 of the dipole moment determines the peak power of the beam as will be shown below, and ϕ_0 denotes the carrier-envelope phase or absolute phase. ω is the carrier frequency. A focused pulse propagating in a certain direction, for which we take the z direction, can be obtained by moving the source dipole from the origin to a complex position along the z axis. We introduce the spatiotemporal translation,

$$z \to z' = z + iz_0, \qquad t \to t' = t - t_0 + i\frac{z_0}{c}.$$

We shift both the spatial coordinate z and the time ct by the same imaginary amount. This way, on axis (for $x = y = 0$), the shift cancels from the retarded time, and we will be able to retrieve the standard Lorentz beam in the paraxial approximation. In the latter, the parameter $z_0 = kw_0^2/2$ will become the Rayleigh range determined by the beam waist w_0. We introduce the complex distance $R' = \sqrt{x^2 + y^2 + (z + iz_0)^2}$. By the sink-source method [27,28], the optical field can be obtained by folding the source with

$$D\left(R',t\right) = \frac{\mu_0}{4\pi}\frac{\delta\left(t - R'/c\right) - \delta\left(t + R'/c\right)}{R'},$$

where μ_0 is the magnetic permeability of vacuum. The expressions of the field are

$$\vec{E}(x,y,z,t) = \frac{c^2\mu_0 k^2}{4\pi R'\sqrt{1+\xi^2}}\left\{[fp(\tau) + f'p(\tau')](\vec{e}_x + i\xi\vec{e}_y) + \frac{\vec{R}}{R'^2}[gp(\tau) + g'p(\tau')]\right\}, \tag{3}$$

$$\vec{H}(x,y,z,t) = \frac{ck^2}{4\pi R'^2\sqrt{1+\xi^2}}[hp(\tau) + h'p(\tau')][-i\xi z\vec{e}_x + z\vec{e}_y + (i\xi x - y)\vec{e}_z]. \tag{4}$$

The source $p(\tau)$ as well as the functions,

$$f = \left[1 + \frac{1}{\omega(T - i\tau)}\right]^2 + \frac{1}{\omega^2(T - i\tau)} - \frac{1}{k^2R'^2}\left[1 + ikR' + \frac{iR'}{c(T - i\tau)}\right],$$

$$g = -f + \frac{2}{k^2R'^2}\left[1 + ikR' + \frac{iR'}{c(T - i\tau)}\right], \qquad h = f + \frac{1}{k^2R'^2}$$

are evaluated at the complex retarded time$\tau = t' - R'/c$. Another source$p(\tau')$ and the functions,

$$f' = \left[1 + \frac{1}{\omega(T - i\tau')}\right]^2 + \frac{1}{\omega^2(T - i\tau')^2} - \frac{1}{k^2R'^2}\left[1 + ikR' + \frac{iR'}{c(T - i\tau')}\right],$$

$$g' = -f' + \frac{2}{k^2R'^2}\left[1 + ikR' + \frac{iR'}{c(T - i\tau')}\right], \qquad h' = -f' + \frac{1}{k^2R'^2}$$

are evaluated at the complex retarded time $\tau' = t' + R'/c$. The physical fields are obtained as the real parts of the fields (3) and (4). The peak value E_0 of the electromagnetic field is related to the (time-dependent) Poynting vector $S = \mathrm{Re}(E) \times \mathrm{Re}(H)$ of the beam through $E_0 = \sqrt{c\mu_0 S_{\max}}$. This yields the relation $p_0 = 4\pi z_0 A_0 E_0/(c^2k^2\mu_0)$, where

$$A_0^{-2} = \left(B_1 + B_2\frac{T}{T + 2z_0/c}e^{-2z_0/c}\right)\left(C_1 + C_2\frac{T}{T + 2z_0/c}e^{-2z_0/c}\right),$$

$$B_1 = \left(1 + \frac{1}{\omega T}\right)^2 + \frac{1}{\omega^2T^2} + \frac{1}{k^2z_0^2}\left(1 - kz_0 - \frac{z_0}{cT}\right), \qquad C_1 = B_1 - \frac{1}{k^2z_0^2},$$

$$B_2 = -\left(1 + \frac{1}{\omega(T + 2z_0/c)}\right)^2 - \frac{1}{\omega^2(T + 2z_0/c)^2} - \frac{1}{k^2z_0^2}\left(1 + kz_0 + \frac{z_0}{c(T + 2z_0/c)}\right),$$

$$C_2 = -B_2 - \frac{1}{k^2z_0^2}.$$

The components of the electric and magnetic fields are

$$E_x = \mathrm{Re}\left\{A\left[(fp + f'p') + \frac{x\bar{x}}{R^2}(gp + g'p')\right]\right\}, \qquad H_x = \mathrm{Re}\left[-A\frac{i\xi z}{Rc}(hp + h'p')\right],$$

$$E_y = \mathrm{Re}\left\{A\left[i\xi(fp + f'p') + \frac{y\bar{x}}{R^2}(gp + g'p')\right]\right\}, \qquad H_y = \mathrm{Re}\left[A\frac{z}{Rc}(hp + h'p')\right], \tag{5}$$

$$E_z = \mathrm{Re}\left[\frac{Az}{Rc}(hp + h'p')\right], \qquad H_z = \mathrm{Re}\left[A\frac{i\xi x - y}{Rc}(hp + h'p')\right],$$

where $\bar{x} = x + i\xi y$ and $A = E_0z_0A_0/(p_0R\sqrt{1+\xi^2})$.

The light force acting on the atomic center of mass can be written as follows:

$$\vec{F} = M\ddot{\vec{r}} = \left\langle\frac{d\vec{P}}{dt}\right\rangle = \langle\vec{\nabla}(\vec{\mu}\cdot\vec{E})\rangle,$$

where $\vec{\mu}$ is the atomic dipole moment operator. If α,β, and $\gamma$are the angles between $\vec{E}$and the x, y, and z axes, $\vec{\mu}\cdot\vec{E}$ can be written as follows:

$$\vec{\mu}\cdot\vec{E} = \mu(\rho_{12}+\rho_{21})[E_x \cos\ \alpha + E_y \cos\ \beta + E_z \cos\ \gamma],$$

where $\cos\ \alpha = \frac{E_x}{E}$, $\cos\ \beta = \frac{E_y}{E}$, and $\cos\ \gamma = \frac{E_z}{E}$. The components of the light force are

$$\begin{aligned}\vec{F}_x &= \mu(\rho_{12}+\rho_{21})\frac{\partial}{\partial x}[E_x \cos\ \alpha + E_y \cos\ \beta + E_z \cos\ \gamma]\vec{e}_x,\\ \vec{F}_y &= \mu(\rho_{12}+\rho_{21})\frac{\partial}{\partial y}[E_x \cos\ \alpha + E_y \cos\ \beta + E_z \cos\ \gamma]\vec{e}_y,\\ \vec{F}_z &= \mu(\rho_{12}+\rho_{21})\frac{\partial}{\partial z}[E_x \cos\ \alpha + E_y \cos\ \beta + E_z \cos\ \gamma]\vec{e}_z.\end{aligned} \tag{6}$$

The Bloch vector component $u=\rho_{12}+\rho_{21}$is described by the following optical Bloch equations:

$$\frac{du}{dt} = \omega_{12}v - \frac{u}{T_2}, \qquad \frac{dv}{dt} = -\omega_{12}u - 2\Omega w - \frac{v}{T_2}, \qquad \frac{dw}{dt} = 2\Omega v - \frac{w+1}{T_1}, \tag{7}$$

where u, v, and w are the three components of the Bloch vector. T_2 and T_1, respectively, are the dipole-dephasing and spontaneous decay times. The detuning parameter is defined as $\Delta = \omega_{12} - \omega$. Ω is the Rabi frequency and is defined as $\Omega = \vec{\mu}\cdot\vec{E}(\vec{r},t)/\hbar$.

III. RESULTS AND DISCUSSIONS

Using the expression of the Lorentz pulsed laser field described by Eqs. (5), we solve Eqs. (7) numerically without RWA. We compare the cases where the atom initially is at the ground state, the excitation state, and the coherent state. We find that the focusing, defocusing forces of the atomic beam are obtained depending on the initial state of the atom. We choose the following parameters for our numerical calculations: $\omega_{12} = 2.2758$ rad/fs, $T_2 = 200$ fs, $\mu = 2.65e$ Å, and $M = 200$ amu. The numerical parameters chosen for the atom are typical for the research of atomic physics, such as a similar transition frequency and a similar dipole moment can be found in rubidium atoms and other atoms. The dephasing time is much longer than the duration of the subcycle pulse. So, the effect of the dephasing time can be neglected. It is a heavy atom, and the numerical parameters do not correspond to a specific atom. But the conclusion is applicable for real atoms. The main results in the article are independent of the numerical parameters of the atom. Figures 1–3 depict the temporal evolution of the light force on the two-level atoms. The figures in the article show the oscillations of the optical force on extremely fast time scales. The period of oscillation of the optical force is equal to half the optical oscillation period, and the optical oscillation period is several femtoseconds. The time step in the numerical calculation is 10^{-17}s, so, the time step is small enough to see the exact optical force structure. The ground state is initially populated, and $T = 0.8T_0 = 0.8(2\pi/\omega)$ is chosen for Figs. 1–3.

Figures 1 and 2 show that the transverse force is inward, directed toward the z axis. The transverse force can produce an acceleration which points to the z axis, and it is a focusing force on the transverse direction. An acceleration of an atom that is 10 orders of magnitude higher than the Earth's gravitational acceleration g can be obtained by the force. Because of the ultrashort duration of the subcycle pulse, the speed change in the atom is small by a subcycle pulse, and we can use the focusing force to fine-control an atom in the experiment of

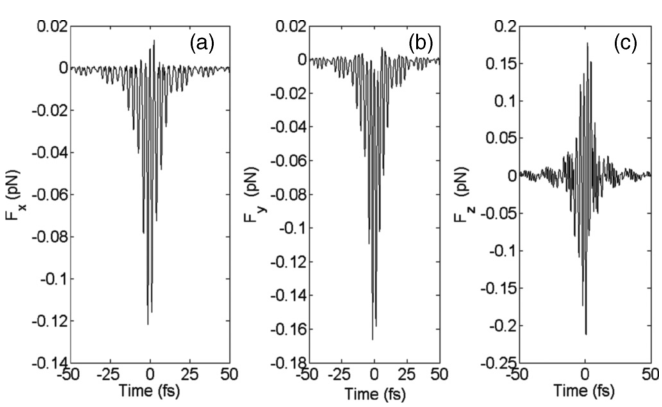

FIG. 1. Temporal evolution of the light force. The force is calculated at $x = 0.5,\ y = 0.5\ \mu$m,$z = 0$, and $\Delta = 0.75 \times 1.7758$ rad/fs. The Rabi frequency is $\Omega = 2.2758$ rad/fs.

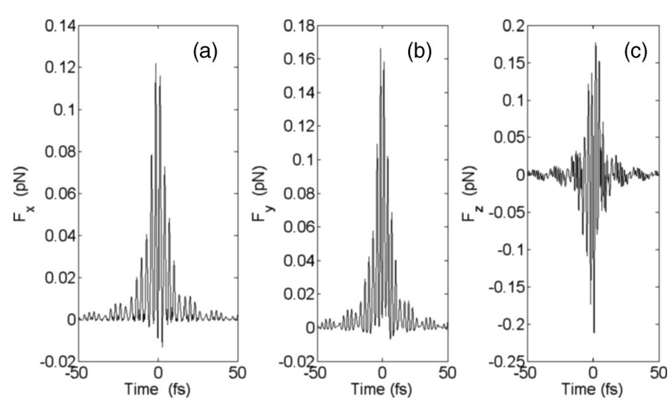

FIG. 2. Temporal evolution of the light force. The force is calculated at $x = -0.5,\ y = -0.5\ \mu$m,$z = 0$, and $\Delta = 0.75 \times 1.7758$ rad/fs. The Rabi frequency is $\Omega = 2.2758$ rad/fs.

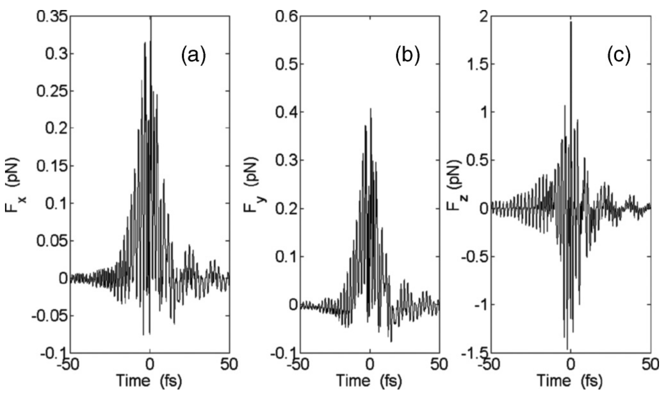

FIG. 3. Temporal evolution of the light force. The force is calculated at $x = 0.5$, $y = 0.5$ μm, and $z = 0$ with $\Delta = 0.5 \times 1.7758$ rad/fs.The Rabi frequency is $\Omega = 5 \times 2.2758$ rad/fs.

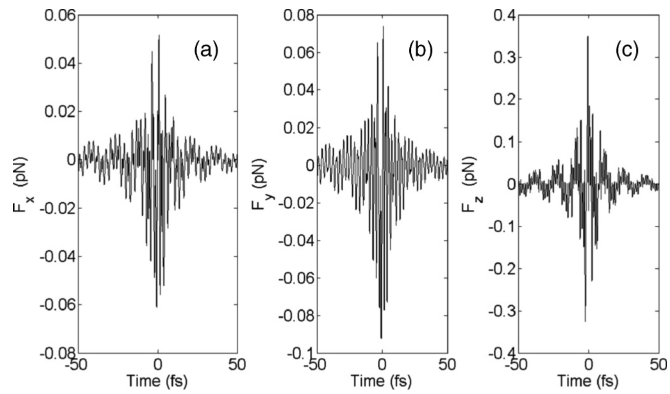

FIG. 5. Temporal evolution of the light force. The force is calculated at $x = 0.5$, $y = 0.5$ μm, and $z = 0$. The atom is initially prepared in the coherent state $\psi(0) = \sqrt{2}/2\,|1\rangle + \sqrt{2}/2\,|2\rangle$.

cold atoms. Figure 3 shows that the direction of the optical force will change with the detuning and Rabi frequency. The transverse optical can be a focusing force or a defocusing force by different detunings or Rabi frequencies, whereas, the optical force value increases with the Rabi frequency. The longitudinal component of the light force F_z oscillates positively and negatively, and the time average of the longitudinal force tends to zero.

Now, we consider the cases where the atom is initially prepared at the excited state and at the coherent state. Figures 4 and 5 show the evolution of the optical force. The parameters are chosen as $\Omega = 2.2758, T = 0.8T_0 = 0.8\,(2\pi/\omega)$, and $\Delta = 0.75 \times 1.7758$ rad/fs.

For the same condition as in Fig. 1, except for the initial state of the atom, Fig. 4 shows that the transverse force is directed away from the optical axis. For the atom initially being in the excited state, the optical force is a defocusing force on the transverse direction. Figure 5 shows that the time average of the longitudinal force tends to zero for the atom initially being in the maximum coherent state. So, there is no focusing or defocusing effect for the atom. The optical force on the transverse direction can change from a focusing force to a defocusing force when the initial state of the atom changes from the ground state to the coherent state to the excited state, and the focusing and defocusing controls of the atom can be obtained according to the initial state of the atom.

The spatiotemporal profile of the optical potential is plotted in Figs. 6 and 7. The atom is initially in the ground state. Figure 6 shows the potential energy function $U(x)$ at $z = 0$ and $y = 0.5$ μm, and Fig. 7 shows the potential-energy function $U(y)$ at $z = 0$ and $x = 0.5$ μm.

From Figs. 6 and 7, one can see that the optical potential is negative around the z axis. A two-level atom in an atomic beam may be trapped by the time-dependent optical potential. For the numerical parameters of the atom given in the article, a transverse acceleration of an atom that is 10 orders of magnitude higher than the Earth's gravitational acceleration g can be obtained. The transverse speed of the atom will alter a few millimeters per second by a subcycle pulse in several femtoseconds. For a light atom, the transverse speed will alter a few centimeters per second in several femtoseconds. In the subcycle pulse train, the speed of an atom can alter a few meters per second in a few picoseconds. So, the cold atom can be considered to be trapped for the laser pulse's short time duration. For a different initial state of the atom, the transverse optical force can change from a focusing force to a defocusing force. So, distinguishing of the different states of an atom may be possible by the transverse optical force in the

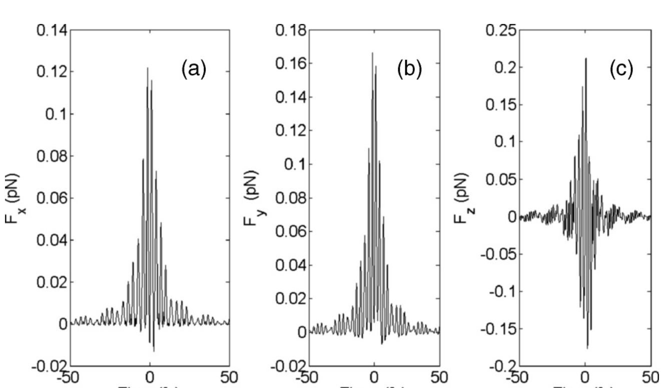

FIG. 4. Temporal evolution of the light force. The force is calculated at $x = 0.5$, $y = 0.5$ μm, and $z = 0$. The atom is initially prepared in the excited state.

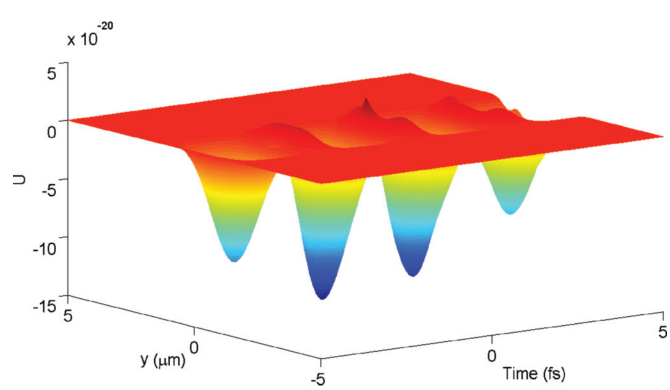

FIG. 6. (Color online) Spatiotemporal profile of the optical potential. The parameters are chosen as $\Omega = 2.2758$ rad/fs $T = 0.8T_0 = 0.8(2\pi/\omega)$, and $\Delta = 0.75 \times 1.7758$ rad/fs. The potential-energy function $U(x)$ is calculated at $z = 0$ and $y = 0.5$ μm.

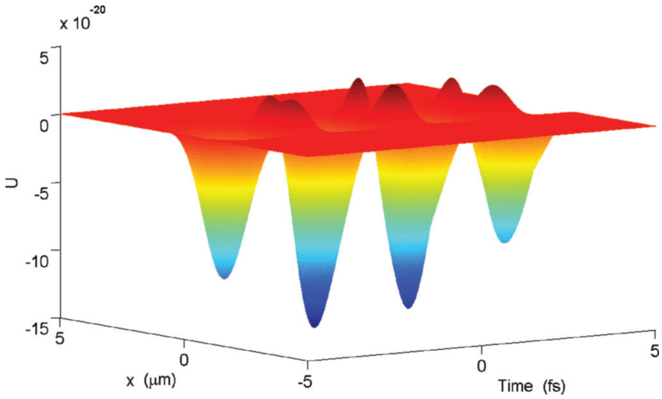

FIG. 7. (Color online) Spatiotemporal profile of the optical potential. The parameters are chosen as $\Omega = 2.2758$ rad/fs, $T = 0.8T_0 = 0.8\,(2\pi/\omega)$, and $\Delta = 0.75 \times 1.7758$ rad/fs. The potential-energy function $U(y)$ is calculated at $z = 0$ and $x = 0.5\ \mu$m.

femtosecond time regime. Therefore, the optical force makes the fine-control of atomic motion possible.

IV. CONCLUSION

In conclusion, we studied the optical force on a beam of neutral two-level atoms in a subcycle pulsed focused vector optical beam. The transverse component and the longitudinal component of the optical force are analyzed. The optical force can be a focusing or a defocusing force on the transverse direction by changing the detuning and Rabi frequency of the optical field. For the transverse optical force, the acceleration of an atom is 10 orders of magnitude higher than the Earth's gravitational acceleration g can obtain. On the same condition as the optical field, the optical force on the transverse direction can change from a focusing force to a defocusing force depending on the initial state of the atom. For the longitudinal force, the force oscillates with time, and the time average of the force tends to zero on most locations in the field. So, the longitudinal force cannot be used to steer an atom. The optical force on a neutral two-level atom by the subcycle pulse may be used to fine-control an atom in the experiment of cold atoms.

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