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Generation of long-time maximum entanglement between two dipole emitters via a hybrid photonic-plasmonic resonator

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We explore the entanglement generation between two dipole emitters via a hybrid photonic-plasmonic resonant structure which consists of two metal nanoparticles (MNPs) and a high-Q whispering-gallery-mode (WGM) microcavity. Because the coupling strength between the dipole emitter and the cavity can be substantially enhanced with the help of MNPs and maintained by the WGM cavity, the maximum entanglement plateau between two dipole emitters can be deterministically created even in the presence of the spontaneous emission of the dipole emitters and the decay induced by the microcavity and MNPs. The width of the plateau depends on the coupling strengths (or the radii of the MNPs) and the initial state, but is insensitive to the dipole-cavity detuning. These results are useful in real experiments.

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I. INTRODUCTION

Quantum entanglement and quantum entangled states are of crucial importance in many quantum information processes, such as quantum cryptography [1], teleportation [2,3], computation [4], and dense coding [5]. Numerous different systems have been proposed to prepare entangled states of matter, among which the cavity quantum electrodynamics (CQED) system provides one of the most promising and qualified candidates for quantum information processing [6–9]. In order to generate entanglement, it is important to create a light-matter strong interaction. Cavity QED offers an almost ideal platform to realize this strong-coupling regime even on experiments [10]. In particular, whispering-gallery mode (WGM) mirocavities [11] are promising due to their ultrahigh-quality factor (Q) and allowing for mass production on a chip. Unfortunately, the relatively large cavity mode volume of WGM mirocavities makes it difficult to realize strong coupling [12]. On the other hand, when a metal nanoparticle (MNP) is driven by an external electromagnetic field, the MNP can magnify the local optical field, which can significantly enhance interactions with atomic media [13] due to localized surface plasmon resonance (LSPR) [14]. This enhancement has been studied for increasing atomic radiative efficiency [15]. Recently, the interaction between a dipole emitter and a metal nanoparticle investigated by several works led to interesting interference effects [16–18].

Based on this achievement, we investigate the entanglement generation between two dipole emitters via a hybrid photonic-plasmonic resonant structure which consists of two MNPs and a WGM-type microcavity. Such a composite system possesses the advantages of both high-Q and ultralow-loss WGMs and highly localized plasmons. Two identical dipole emitters (quantum-mechanical two-level atoms) are placed in the vicinity of two MNPs, respectively. The interaction between dipole emitters and WGMs can be enhanced with the help of MNPs. We use entanglement concurrence to demonstrate the entanglement degree of two dipole emitters. There is a relative long-time maximum entanglement plateau whose width closely associates with the coupling strength between the dipole emitters and WGMs. We also demonstrate the influence of the dipole-cavity detuning, and the initial state of the coupled system on the degree of entanglement. Our main results are as follows: (i) The maximum entanglement of two dipole emitters can be deterministically generated in a very short time and the width of the entanglement plateau is $\sim$2 ns. (ii) The entanglement plateau strongly depends on the radii of the MNPs and the initial state of the system but is insensitive to the dipole-cavity detuning. (iii) Over enough time, the entanglement concurrence trends toward a steady value which can reach about 0.5 for appropriate system parameters.

II. PHYSICAL MODEL AND BASIC FORMULA

Figure 1 shows a schematic of the coupled system. Two spherical MNPs are placed close to the surface of a microtoroidal cavity which supports two counterpropagating WGMs with degenerate frequency ω_c, denoted as $a_{\rm cw}$ and $a_{\rm ccw}$. Two identical diploe emitters are located in the vicinity of two MNPs, respectively. The distance between two MNPs is far enough, so the interaction of two MNPs can be neglected. Experimentally we can use atomic force microscope manipulation to controllably position the MNPs and dipole emitters. The dipole moment is $\mu\hat{x}$ ($\hat{x}$ is a unit vector pointing in the direction of dipole moment). When the MNP is driven by an external monochromatic field $\mathbf{E}_0(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}_0 e^{-i\omega t}]$ (Re[$\cdot$] indicates the real part of the bracketed quantity), the total field which includes the response of the MNP is given by [13]: $\mathbf{E}(\mathbf{r}) = (1-\beta)E_{0x}\hat{x}$ $(r \leqslant r_m)$, $\mathbf{E}(\mathbf{r}) = (1+2\beta r_m^3/r^3)E_{0x}\hat{x} (r > r_m)$. E_{0x} is the xth component of $\mathbf{E}_0$ (here we consider the transverse coupling case). r_m is the radius of the MNP (r_m is much smaller than the light wavelength) and r is the radial coordinate of the position vector $\mathbf{r}$ (with respect to the center of the MNP). So

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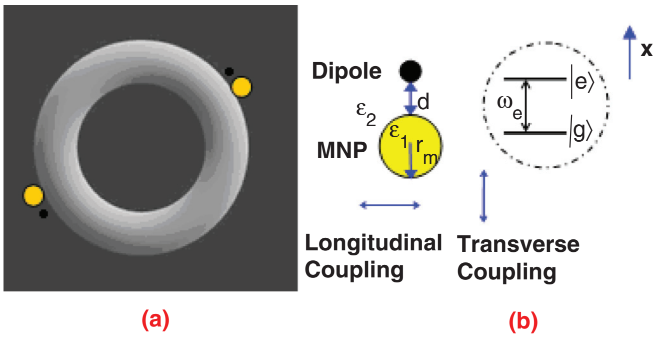

FIG. 1. (Color online) (a) Schematic illustration of the composite system composed of a microtoroidal cavity and two MNPs (not to scale). Two diploe emitters (such as two-level atoms) are located in the vicinity of two MNPs, respectively. The distance between two MNPs is far enough, so the interaction of two MNPs can be neglected. (b) The dipole-MNP system and two different incident field orientations, longitudinal and transverse. $\varepsilon_1(\varepsilon_2)$ denotes the relative permittivity of the MNP (the surrounding medium). r_m is the radius of the MNP and d is the distance between the dipole emitter and the surface of the MNP. The inset is the energy diagram of the dipole.

we have $r = r_m + d$ and d is the distance between the dipole emitter and the surface of the MNP. The complex coefficient β can be calculated to be $\beta = (\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + 2\varepsilon_2)$; here $\varepsilon_1(\varepsilon_2)$ denotes the relative permittivity of the MNP (the surrounding medium). The dielectric constant ε_1 can be well modeled by a Drude dispersion relation owing to the metallic of the MNPs, i.e., $\varepsilon_1(\omega) = 1 - \omega_p^2/[\omega(\omega + i\gamma_m)]$ [12,13], where ω_p is the plasma frequency of the metal and γ_m accounts for energy dissipation due to Ohmic losses in the medium.

Under the rotating-wave approximation, the Hamiltonian of this composite system can be written as [12]

$$H = H_1 + H_2, \tag{1a}$$

$$\begin{aligned} H_1 &= \hbar\left(\omega_e - i\frac{\gamma_s}{2}\right)\sum_{j=1,2}|e\rangle_j\langle e| \\ &\quad + \hbar\left(\omega_c - i\frac{\kappa_0}{2}\right)\sum_{n=\mathrm{cw,ccw}} a_n^\dagger a_n \\ &\quad + \sum_{j=1,2}\sum_{n=\mathrm{cw,ccw}} \hbar G_{c,j}(a_n^\dagger\sigma_j^- + a_n\sigma_j^+), \end{aligned} \tag{1b}$$

$$H_2 = \hbar\sum_{j=1,2}(h_j - i\kappa_{Rj} - i\kappa_{mj})\sum_{n,n'=\mathrm{cw,ccw}} a_n^\dagger a_{n'}, \tag{1c}$$

where ω_e is the transition frequency of the dipole emitters (ground state $|g\rangle$ and excited state $|e\rangle$). γ_s denotes the spontaneous emission rate of the dipole emitters (we assume that two dipole emitters have the same spontaneous emission rate). ω_c is the cavity frequency of a microtoroidal cavity which supports twin counterpropagating WGMs (a_{cw} and a_{ccw}). κ_0 is the decay rate of the cavity modes. The third part of H_1 [see Eq. (1b)] describes the dipole interaction between the dipole emitters and WGMs with the single-photon coupling strength $G_{c,1}$ and $G_{c,2}$. As mentioned before, in the presence of the MNP, the coupling strength G_c can be enhanced and calculated to be $|1 + 2\beta r_m^3/r^3|G$. Here $G = \mu f_c(\mathbf{R})[\omega_c/(2\hbar\varepsilon_0\varepsilon_c V_c)]^{1/2}$ is the coupling strength without the MNP. ε_0 and ε_c are the permittivity of vacuum and relative permittivity of the microcavity, respectively. V_c and $f_c(\mathbf{R})$ ($\mathbf{R}$ is the position of the spherical MNP) denote the mode volume and normalized field distribution of the WGMs. $\sigma_j^- = |g\rangle_j\langle e|$ and $\sigma_j^+ = |e\rangle_j\langle g|$ ($j = 1,2$) stand for the descending and ascending operators of the jth dipole emitter. The coupling strength $h_j = 2\pi r_{mj}^3\varepsilon_2\omega_c|\beta|^2 f_c^2(\mathbf{R})/(\varepsilon_c V_c)$ [see Eq. (1c)] describes the scattering induced by the jth MNPs into the same ($n = n'$) or the counterpropagating ($n \neq n'$) quantized WGM fields. On the other hand, the scattering can also result in the decay from WGMs to reservoir modes with the damping rate $\kappa_{Rj} = \varepsilon_2^{5/2}(4\pi r_{mj}^3)^2|\beta|^4\omega_c^4 f_c^2(\mathbf{R})/(6\pi c^3\varepsilon_c V_c)$. The last term of H_2 describes the absorption of the jth MNPs which results in Ohmic losses with the decay rate $\kappa_{mj} = 4\pi r_{mj}^3|1 - \beta|^2\omega_p^2\gamma_m f_c^2(\mathbf{R})/(3\varepsilon_c\omega_c^2 V_c)$ [12].

In the single excitation manifold together with the ground state, the bases of the whole coupled system are as follows: $\{|1\rangle = |e_1,g_2,0_{\mathrm{cw}},0_{\mathrm{ccw}}\rangle$, $|2\rangle = |g_1,e_2,0_{\mathrm{cw}},0_{\mathrm{ccw}}\rangle$, $|3\rangle = |g_1,g_2,1_{\mathrm{cw}},0_{\mathrm{ccw}}\rangle$, $|4\rangle = |g_1,g_2,0_{\mathrm{cw}},1_{\mathrm{ccw}}\rangle$, and $|5\rangle = |g_1,g_2,0_{\mathrm{cw}},0_{\mathrm{ccw}}\rangle\}$, where $|e_{1(2)}\rangle,|g_{1(2)}\rangle$ denote the state of the first (second) dipole emitter, and $|0_{\mathrm{cw(ccw)}}\rangle,|1_{\mathrm{cw(ccw)}}\rangle$ denote the number of photons in the cw and ccw WGMs. For simplicity, we ignore subscripts in the following.

If the initial state is $|\Psi(0)\rangle$, after time t, the state of the system becomes $|\Psi(t)\rangle = \sum_{i=1}^5 C_i(t)|i\rangle$. In order to investigate entanglement of the two dipole emitters, we need to trace out the cavity modes and obtain the reduced density matrix of two dipole emitters, i.e., $\rho_a(t)$. To quantify the degree of entanglement of the two dipole emitters, we use $\rho_a(t)$ to calculate the concurrence [19]

$$C(t) = C(\rho_a) = 2\ \max\{0,\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{2}$$

where λ_i ($i = 1$–5) is the square roots of the eigenvalues of $\rho_a\tilde{\rho}_a$ with $\tilde{\rho}_a = (\sigma_y \otimes \sigma_y)\rho_a^*(\sigma_y \otimes \sigma_y)$ in decreasing order, and σ_y being the Pauli operator. In our case, the concurrence can be written as [20]

$$C(t) = \frac{2|C_1C_2^*|}{|C_1|^2 + |C_2|^2 + |C_3|^2 + |C_4|^2}. \tag{3}$$

We consider a silica microtoroidal cavity in air whose intrinsic quality factor can achieve $Q_0 = 10^7$ [21]. We obtain other practical parameters: $\varepsilon_c = 1.45^2$, $\varepsilon_2 = 1$, $V_c \sim 200\ \mu\mathrm{m}^3$, $f_c(\mathbf{R}) \sim 0.3$, and $\kappa_0/2\pi = (\omega_c/2\pi)/Q_0 = 55$ MHz. On the other hand, for a gold MNP and the dipole emitter [such as chemically synthesized cadmium selenide (CdSe) quantum dot], we extracted the experimental parameters $\omega_p \sim 6 \times 10^6$ GHz, $\gamma_m \sim 3 \times 10^5$ GHz, $\mu = 2.4 \times 10^{-28}$ C m, and $\gamma_s = 2\pi \times 1.6$ GHz [12,22,23]. Using these experimental data and considering the LSPR [14], we can obtain the following relationship: $\varepsilon_1(\omega) = \varepsilon_1(\omega_c)$, $G_c/2\pi \simeq |1 + 2\beta\frac{r_m^3}{(r_m+d)^3}| \times 0.76$ GHz, $h/2\pi \simeq 9.88 \times 10^{22} r_m^3$ GHz, $\kappa_R/2\pi \simeq 2.69 \times 10^{46} r_m^6$ GHz, and $\kappa_m/2\pi \simeq 1.72 \times 10^{22} r_m^3$, respectively.

III. RESULTS AND DISCUSSION

First of all, we analyze the influence of the coupling strength $G_{c,j}$ ($j = 1,2$) on the entanglement degree of the two dipole emitters. In Fig. 2, we plot the concurrence of the two dipole emitters versus the time t for four different radii under the initial state $|\Psi(0)\rangle = |1\rangle = |eg00\rangle$. Here we

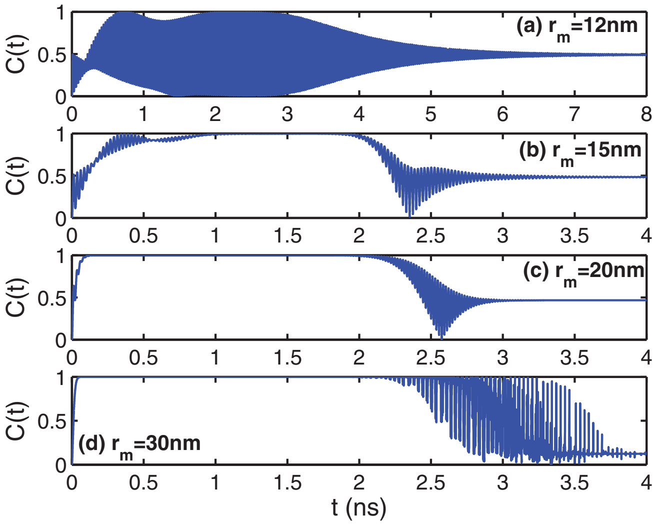

FIG. 2. (Color online) Concurrence of two dipole emitters versus the time t for four different radii [i.e., (a) $r_m = 12$ nm, (b) 15 nm, (c) 20 nm, and (d) 30 nm] under the initial state $|\Psi(0)\rangle = |1\rangle = |eg00\rangle$. Two MNPs have the same radius $r_{m1} = r_{m2} = r_m$ and no dipole-cavity detuning $\Delta = 0$. The distance between the dipole emitter and the surface of the MNP is $d = 3$ nm.

set $G_{c,1} = G_{c,2} = G_c$, $h_1 = h_2 = h$, $\kappa_{R1} = \kappa_{R2} = \kappa_R$, and $\kappa_{m1} = \kappa_{m2} = \kappa_m$ (or $r_{m,1} = r_{m,2} = r_m$), i.e., two MNPs are identical. The dipole-cavity detuning Δ ($\Delta = \omega_e - \omega_c$) is taken as zero and the distance d between the dipole emitter and the surface of the MNP is 3 nm. From Fig. 2, we can observe that, when the radius of the MNPs is small, i.e., $r_m = 12$ nm in Fig. 2(a), the concurrence curve oscillates rapidly and finally arrives at a steady value ($C \sim 0.5$) which needs a relative long time. With the radius increasing, the concurrence achieves the maximum ($C = 1$) more rapidly and holds it for some time, then the curve oscillates again and reaches one steady value at last, as shown in Figs. 2(b)–2(d). In other words, there is a plateau whose width becomes larger with increasing r_m. A similar phenomenon has been discussed by Montenegro *et al.* [24]. We also find that the last steady value decreases with increasing r_m. We can understand this phenomenon as follows. The concurrence reaches the maximum quickly, at first, and it is maintained for some time because there is a strong interaction between each dipole emitter and the WGMs in the presence of MNPs which can be maintained by the WGM cavity. As time goes on, the influence of decay terms such as γ_s, κ_0, κ_R, and κ_m is gradually enhanced, so the concurrence begins to decrease and finally reaches a stable equilibrium value.

It is worth mentioning that, when the initial state is chosen as $|\Psi(0)\rangle = |2\rangle = |ge00\rangle$, the concurrence is the same as in the initial state $|\Psi(0)\rangle = |1\rangle = |eg00\rangle$ due to the equivalence of two dipole emitters.

Secondly, we consider the off-resonance interaction of the cavity field with the dipole emitters, i.e., $\Delta \neq 0$. The concurrence of two dipole emitters as a function of time t is plotted in Fig. 3 with three different detunings Δ (i.e., $\Delta/2\pi = 0$, 10, and 50 GHz). The time which is spent to reach the maximum $C = 1$ becomes longer with Δ increasing but the moment of the obtained steady value is almost independent of

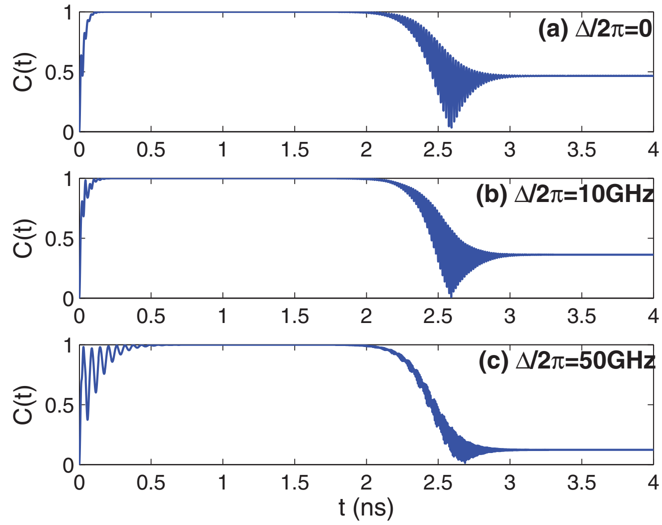

FIG. 3. (Color online) Concurrence of two dipole emitters versus the time t with three different emitter-cavity detunings Δ. The initial state is set to $|\Psi(0)\rangle = |1\rangle = |eg00\rangle$. The other system parameters are chosen as $r_m = 20$ nm and $d = 3$ nm.

the detuning Δ. That is to say, the width of the plateau becomes narrower. We also find that this narrowness is very small, i.e., there is a wide plateau even for the large dipole-cavity detuning Δ [see Fig. 3(c)]. On the other hand, the last steady value decreases rapidly with increasing Δ.

In what follows, we illustrate explicitly the above results from the perspective of normal modes of the resonator, which are just linear combinations of the cw and ccw modes. In this case, the two dipole emitters actually couple to a single normal mode of the resonator, i.e., $(a_{\mathrm{cw}} + a_{\mathrm{ccw}})/\sqrt{2}$, with frequency $\omega_c + 4h$. The system is therefore equivalent to a pair of two-level dipole emitters coupled off resonantly to a single cavity mode. Any initial excitation is coherently exchanged between the field mode and the dipole emitters and during

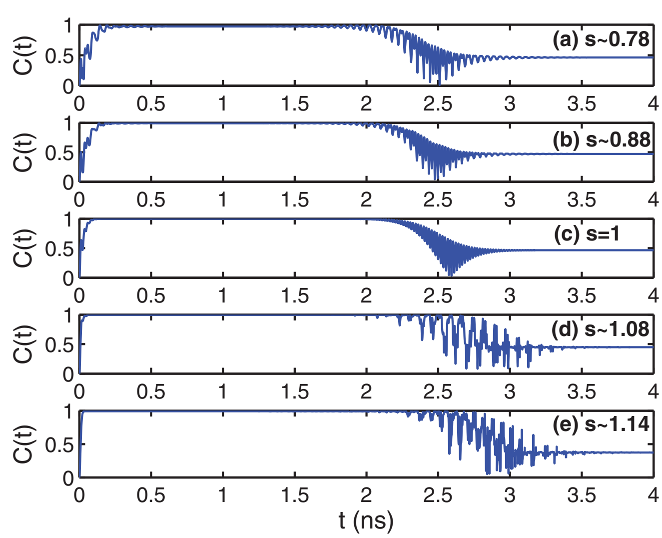

FIG. 4. (Color online) Concurrence of two dipole emitters versus the time t with five different scale factors s ($s = G_{c,2}/G_{c,1}$). The initial state is set to $|\Psi(0)\rangle = |1\rangle = |eg00\rangle$. The other system parameters are chosen as $r_{m,1} = 20$ nm and $\Delta = 0$.

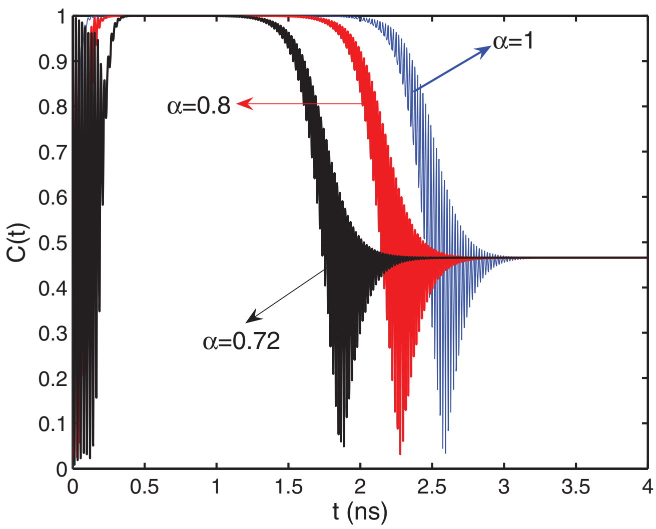

FIG. 5. (Color online) Concurrence of two dipole emitters versus the time t under the initial state $|\Psi(0)\rangle = \alpha|1\rangle + \sqrt{(1-\alpha^2)}|2\rangle$. The other system parameters are chosen as $r_m = 20$ nm, $d = 3$ nm, and $\Delta = 0$.

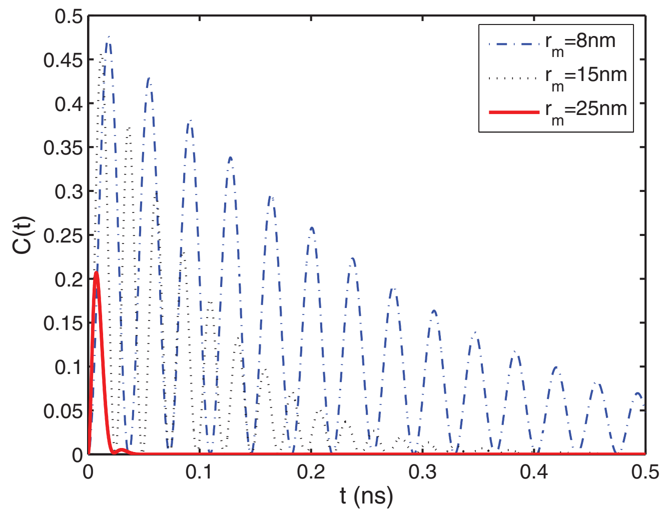

FIG. 6. (Color online) Concurrence of two dipole emitters versus the time t under the initial state $|\Psi(0)\rangle = |3\rangle$ or $|4\rangle$. The other system parameters are chosen as $d = 3$ nm and $\Delta = 0$.

certain intervals of time the state of the system is entangled, as one can expect. For large h, the cavity normal mode is far off resonant from the transition frequency of the dipole emitter and mediates an effective dipole-dipole interaction between the dipole emitters, which periodically produces maximally entangled states of the dipole emitters [3].

In order to show the influence of the coupling strengths $G_{c,j}$ $(j = 1,2)$ more clearly, we consider that two MNPs hold different coupling strengths, i.e., $G_{c,1} \neq G_{c,2}$ (or $r_{m,1} \neq r_{m,2}$). We define a parameter s to be the ratio of the coupling strength between each dipole emitter and the WGMs, i.e., $G_{c,2} = sG_{c1} = sG_c$. It can be found from Fig. 4 that concurrence can reach the maximum ($C = 1$) more rapidly and the width of the plateau becomes longer with the increase of the scale factor s, but the last steady value becomes smaller (see $s \sim 1.14$).

Finally, we investigate the influence of the initial state of the system. We consider the situation in that the initial state is a superposition state $|\Psi(0)\rangle = \alpha|1\rangle + \sqrt{(1-\alpha^2)}|2\rangle$. Figure 5 shows clearly that the width of the plateau becomes narrower and narrower with decreasing α. For example, when $\alpha = 1$, the width $\Delta t \simeq 1.5$ ns, and when $\alpha = 0.72$, the width $\Delta t \simeq 0.6$ ns. It is easy to see that, when $\alpha = 1/\sqrt{2}$, the width $\Delta t \to 0$, i.e., the concurrence will maintain the oscillation state until it reaches the steady value. Moreover, different α cannot change the last steady value.

If we consider that the initial state is $|\Psi(0)\rangle = |3\rangle$ or $|4\rangle$, there is no plateau as shown in Fig. 6. The concurrence curve is oscillatory between zero and the maximum (not to exceed 0.5). The maximum greatly decreases when the radius of the MNPs increase and the oscillation aggravates (see $r_m = 15$ nm). When the radius is increased to 25 nm, there is no oscillation and the concurrence obtains the maximum ($C \sim 0.21$) and soon decays to zero. From Figs. 5 and 6, it is clearly shown that the initial state can greatly influence the entanglement generation except for the single-photon coupling strength G_c.

IV. CONCLUSION

In summary, we have studied a composite of two MNPs and dipole emitters localized near the microtoroidal cavity. The coupling strengths between the dipole emitters and the microcavity have been substantially enhanced owing to the help of the MNPs. The coupling strengths increase with the radius r_m of the MNPs increasing. This effective strong interaction leads to the generation of the maximum entanglement between the two dipole emitters in a much shorter time. The corresponding maximum entanglement plateau can be held for a relative long time. The width of the plateau strongly depends on the coupling strength G_c (or the radius r_m of the MNPs) and the initial state but is insensitive to the dipole-cavity detuning. Our result is useful in real experiments.

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