Wigner rotations and an apparent paradox in relativistic quantum information

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It is shown that a general model for particle detection in combination with a linear application of the Wigner rotations, which correspond to momentum-dependent changes of the particle spin under Lorentz transformations, to the state of a massive relativistic particle in a superposition of two counterpropagating momentum states leads to a paradox. The paradox entails the instantaneous transmission of information between two spatially separated parties. A solution to the paradox is given when the physical construction of the corresponding state is taken into account, suggesting that we cannot in general linearly apply the Wigner rotations to a quantum state without considering the appropriate physical interpretation.

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I. INTRODUCTION

A Wigner rotation corresponds to a momentum-dependent change of the spin state of a relativistic particle with a change of reference frame [1,2]. It is a direct consequence of the imposition of the special relativistic space-time structure to quantum mechanics. It is thus deeply connected with the basic structure of the universe that, as far as we know, is both quantum and relativistic. The influence of the Wigner rotations on the field of quantum information has been intensively investigated in the past 10 years [3–28]. Since the seminal paper of Peres, Scudo, and Terno [3], who concluded that because of the momentum-dependent Wigner rotations the spin entropy of a relativistic particle is not a relativistic scalar, many works have appeared in the literature discussing how the entropy [3,9,11–13,16–18,22,26] and entanglement [5,8,19,20,23–25] of the reduced spin state of a relativistic system change under Lorentz transformations, as well as the influence of the Wigner rotations on the violation of Bell's inequalities with relativistic particles [6,7,10,14,21,28]. However, in our previous works [27,28], we showed that it is not possible to consistently define a reduced spin density matrix for a system with one or more relativistic particles, as is done in most of the cited papers [3,5,8,9,11–13,16–26].

Here we go further. We show that if, under a change of the reference frame, we simply linearly apply Wigner rotations to the quantum state of a massive particle that is in a superposition of two counterpropagating momentum states and consider a general model for particle detection, we obtain a paradox that could, among other things, permit an instantaneous transmission of information between two arbitrarily separated regions of space. This is, of course, highly undesirable given that the whole point of imposing special relativity is to avoid action at a distance. A solution of the paradox is given here based on the physical interpretation of the Wigner rotations recently given by us [27] and on a discussion about the preparation method of the quantum state of the particle. In particular, we show that the Wigner rotation depends on how the particle's quantum state is prepared, such that it is not possible to compute the rotation for each momentum component separately—a subtle consideration that removes the paradox. In other words, the solution we present for the paradox is based on the fact that the Wigner rotation operation cannot in general be linearly applied to an arbitrary superposition of different momentum states. Our conclusions affect much of the literature on relativistic quantum information which has to be reevaluated in order to avoid inconsistencies like the one to be presented here.

II. PHYSICAL SYSTEM OF THE PARADOX

Consider that Alice prepares a pair of relativistic massive spin-1/2 particles in the quantum state

$$|\Psi^{(2)}\rangle = \tfrac{1}{2}\{|p\hat{\mathbf{y}},+Z\rangle_1[|p\hat{\mathbf{y}},-Z\rangle_2 - |-p\hat{\mathbf{y}},-Z\rangle_2] + \\ -|p\hat{\mathbf{y}},-Z\rangle_1[|p\hat{\mathbf{y}},+Z\rangle_2 - |-p\hat{\mathbf{y}},+Z\rangle_2]\} \quad (1)$$

in reference frame $S^{(0)}$, where $|\mathbf{p},\pm Z\rangle_j$ represents a state for particle j with 4-momentum $(p^0,\mathbf{p})$, with $p^0 = \sqrt{m^2c^4 + c^2|\mathbf{p}|^2}$, and spin state pointing in the $\pm\hat{\mathbf{z}}$ direction, being the eigenvector of the Pauli matrix $\hat{\sigma}_z$ with eigenvalue ± 1. We are using Wigner's definition for spin [1], which refers to the particle angular momentum in the rest frame for each momentum component. From now on we will use a system of units in which the speed of light in vacuum is $c = 1$. In the above state particle 1 has momentum $p\hat{\mathbf{y}}$, particle 2 is in a superposition of two counterpropagating momenta $p\hat{\mathbf{y}}$ and $-p\hat{\mathbf{y}}$, and the particles are in an singlet state of spin, which is a maximally entangled state. This state can be constructed, for instance, if there is a decay of a particle without spin into two spin-1/2 particles inside a potential well that retains only particle 2 and particle 1 propagates in the $\hat{\mathbf{y}}$ direction. This decay naturally produces a singlet state of spin, and the potential well can be constructed such that particle 2 is stored with the above superposition of two counterpropagating momenta.

The apparent paradox emerges if Alice measures the spin of particle 1 in reference frame $S^{(0)}$ and Bob, in a reference frame

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$S^{(1)}$ that moves with velocity $\beta\hat{\mathbf{z}}$ in relation to $S^{(0)}$, measures the position of particle 2. When Alice measures the spin of particle 1, the spin of particle 2 "collapses" in the opposite direction. Since Bob is in another reference frame, with the change of reference frame Wigner rotations [1,2] act on the spin state of particle 2. A Wigner rotation depends on the momentum of the particle, so the two momentum components of particle 2, $p\hat{\mathbf{y}}$ and $-p\hat{\mathbf{y}}$, may generate different rotations for the particle spin, causing a correlation between spin and momentum in Bob's reference frame. As we show in the following, using the formalism which is generally used by the relativistic quantum information community, this correlation may affect the probability of finding particle 2 in different regions of space in a way that depends on the basis that Alice uses to measure the spin of particle 1. So, if Alice and Bob share a set of particles prepared in state (1), this would permit that Bob discovers the basis chosen by Alice to measure her particles in an arbitrary distant place by measuring the position of his particles, which would permit Alice to instantaneously transmit one bit of information to Bob. Of course this is impossible, so there must be a failure in the treatment. As we discuss below, we believe that the culprit is a careless linear application of the Wigner rotations.

III. THE PARADOX

Let us first consider that Alice, having a pair of particles in the state (1), makes a measurement of the $\hat{\mathbf{z}}$ component of the spin of particle 1 and obtains -1. Then the quantum state of particle 2 in reference frame $S^{(0)}$ "collapses" to

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|p\hat{\mathbf{y}},+Z\rangle - |-p\hat{\mathbf{y}},+Z\rangle]. \tag{2}$$

Using the Foldy-Wouthuysen transformation on the Dirac Hamiltonian for a spin-1/2 massive relativistic particle, the z component of the mean spin operator is a constant of motion of the free Hamiltonian and the mean position operator is independent of spin [29]. It is in the Foldy-Wouthuysen representation that the Pauli matrix $\hat{\sigma}_z$ is the z component of the mean spin operator of a relativistic particle as we consider in this work [29]. The components of the particle 2 wave function with mean spin state $|\pm Z\rangle$ in the mean position representation can be written as [30]

$$\Psi_{\pm Z}(\mathbf{r}) = \int d^3p K(p^0) e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}\langle \mathbf{p},\pm Z|\Psi_2\rangle, \tag{3}$$

where $K(p^0)$ is a factor that depends on the specific position operator that we use. Since the state of Eq. (2), as well as the other quantum states that we will consider here, has a superposition of two momenta with equal magnitudes, our results do not depend on the specific form of $K(p^0)$. For the state of Eq. (2) we have $\Psi_{+Z}(\mathbf{r}) \propto \sin(py/\hbar)$ and $\Psi_{-Z}(\mathbf{r}) = 0$.

If we make a change of reference frame to a frame $S^{(1)}$ that moves with velocity $\beta\hat{\mathbf{z}}$ in relation to $S^{(0)}$, each momentum component of the state (2) suffers a different spin transformation due to the dependence of the Wigner rotation with the particle momentum. The spin transformations are [31]

$$\hat{R}(\beta\hat{\mathbf{z}},\pm p\hat{\mathbf{y}}) = \cos\left(\frac{\varphi}{2}\right)\hat{\sigma}_0 \pm i\sin\left(\frac{\varphi}{2}\right)\hat{\sigma}_x, \tag{4}$$

where $\hat{\sigma}_0$ represents the identity and $\hat{\sigma}_x$ the x Pauli matrix, with

$$\sin\left(\frac{\varphi}{2}\right) = \sqrt{\frac{(\gamma_p-1)(\gamma_\beta-1)}{2(1+\gamma_p\gamma_\beta)}}, \tag{5}$$

where $\gamma_\beta \equiv 1/\sqrt{1-\beta^2}$ and $\gamma_p \equiv \sqrt{m^2+p^2}/m = 1/\sqrt{1-v^2}$ if $\mathbf{v}$ is the particle velocity corresponding to the momentum $\mathbf{p}$. The momentum state of the particle also changes with the change of reference frame, but the y component remains the same. Here we concentrate on the y dependence of the particle wave function, so we will not worry about the momentum in the x or z directions. Of course, the state of Eq. (2) must be seen as an approximation, since the wave function of Eq. (3) must decay to zero with large x and z, but we will simply consider that the wave function can be decomposed as $\Psi_{\pm Z}(\mathbf{r}) = \psi_{\pm Z}(y)\xi_{\pm Z}(x,z)$ and concentrate our discussion on $\psi_{\pm Z}(y)$. According to Eqs. (2), (3), and (4), in the new frame we have

$$\begin{aligned}\psi'_{+Z}(y) &\propto \cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{py}{\hbar}\right),\\ \psi'_{-Z}(y) &\propto \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{py}{\hbar}\right).\end{aligned} \tag{6}$$

Repeating the calculations for the case in which Alice measures the $\hat{\mathbf{z}}$ component of the spin of particle 1 and obtanis $+1$, the components of the wave function of particle 2 in reference frame $S^{(1)}$ are

$$\begin{aligned}\psi''_{+Z}(y) &\propto \sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{py}{\hbar}\right),\\ \psi''_{-Z}(y) &\propto \cos\left(\frac{\varphi}{2}\right)\sin\left(\frac{py}{\hbar}\right).\end{aligned} \tag{7}$$

If, on the other hand, Alice measures the $\hat{\mathbf{x}}$ component of the spin of particle 1 and obtains -1, the quantum state of particle 2 in reference frame $S^{(0)}$ "collapses" to

$$|\Phi\rangle = \frac{1}{\sqrt{2}}[|p\hat{\mathbf{y}},+X\rangle - |-p\hat{\mathbf{y}},+X\rangle], \tag{8}$$

with the particle spin state prepared in the eigenstate of $\hat{\sigma}_x$ with eigenvalue $+1$. Making the change of reference frame to $S^{(1)}$ and using the same treatment and a similar notation as before, we obtain the wave functions

$$\phi'_{+X}(y) \propto \sin\left(\frac{py}{\hbar}\right), \quad \phi'_{-X}(y) = 0 \tag{9}$$

in the new frame, since the axis of the Wigner rotation is in the direction of the particle spin in this case. For the case in which Alice measures the $\hat{\mathbf{x}}$ component of the spin of particle 1 and obtains $+1$, we have

$$\phi''_{+X}(y) = 0, \quad \phi''_{-X}(y) \propto \sin\left(\frac{py}{\hbar}\right) \tag{10}$$

in reference frame $S^{(1)}$.

Let us consider now that Bob, in a reference frame $S^{(1)}$ that moves with velocity $\beta\hat{\mathbf{z}}$ in relation to $S^{(0)}$, measures the position of particle 2 using a detector that, by definition, responds only to the charge or the mass of the particle but not to its spin. The probability of the particle detection with the central part of the detector placed at a position y_c is assumed

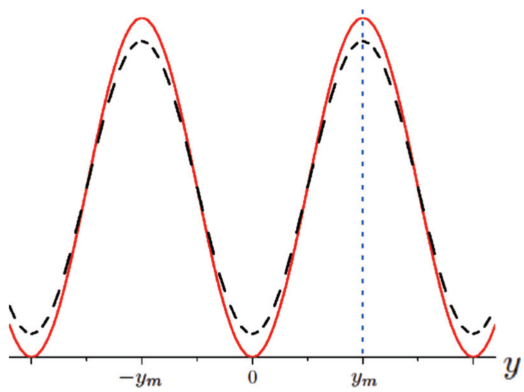

FIG. 1. (Color online) Modulus squared of particle 2 wave functions in reference frame $S^{(1)}$: $|\phi'(y)|^2$ from Eq. (9) and $|\phi''(y)|^2$ from Eq. (10) (continuous red curve) and $|\psi'(y)|^2$ from Eq. (6) and $|\psi''(y)|^2$ from Eq. (7) (dashed black curve) for $\gamma_\beta = 10$ and $\gamma_p = 1.2$ in Eq. (5).

to be

$$P(y_c) = \int dy \Gamma(y - y_c)|\psi(y)|^2, \tag{11}$$

with $|\psi(y)|^2 = |\psi_{+Z}(y)|^2 + |\psi_{-Z}(y)|^2$, and the same for wave functions $\phi_{\pm X}(y)$. However, despite the fact that $|\psi(y)|^2$ and $|\phi(y)|^2$ are the same in reference frame $S^{(0)}$, in reference frame $S^{(1)}$ $|\psi'(y)|^2$ and $|\phi'(y)|^2$, according to Eqs. (6) and (9), are not the same anymore. Note also that, according to Eqs. (6), (7), (9), and (10), we have $|\psi'(y)|^2 = |\psi''(y)|^2$ and $|\phi'(y)|^2 = |\phi''(y)|^2$ in reference frame $S^{(1)}$. This means that the modulus squared of the particle 2 wave function in reference frame $S^{(1)}$ depends on the basis that Alice uses to measure the spin of particle 1, not on the results of the measurements. In Fig. 1 we illustrate $|\psi'(y)|^2 = |\psi''(y)|^2$ and $|\phi'(y)|^2 = |\phi''(y)|^2$ for $\gamma_\beta = 10$ and $\gamma_p = 1.2$.

The exact form of $\Gamma(y)$ in (11) depends on details of the detection scheme, but for the sake of simplicity we will consider $\Gamma(y) \propto e^{-y^2/w^2}$. Since it is not possible to localize a particle in dimensions smaller than its Compton wavelength $\lambda = \hbar/(mc)$, we must have $w > \lambda$. We can define $R = P(0)/P(y_m)$ as the ratio between the probability of finding the particle around the minimum of the modulus of the wave function, at $y = 0$, and the probability of finding it around the maximum, at $y = y_m$ in Fig. 1. Considering that the superposition of momenta of the states (2) and (8) is not very relativistic, such that we can write $p \simeq mv$ and $\gamma_p \simeq 1 + v^2/2$ up to the second order in v, it is straightforward to show, using Eqs. (11), (5) and each of Eqs. (6), (7), (9), and (10), that

$$R_\phi \simeq \frac{m^2 w^2 v^2}{2\hbar^2}, \quad \frac{R_\psi}{R_\phi} \simeq 1 + \frac{(\gamma_\beta - 1)}{2(\gamma_\beta + 1)} \frac{\lambda^2}{w^2}, \tag{12}$$

where R_ϕ represents the ratio $R = P(0)/P(y_m)$ for the wave functions of Eqs. (9) and (10) and R_ψ the ratio for the wave functions of Eqs. (6) and (7) in reference frame $S^{(1)}$. For $\gamma_\beta \gg 1$ and $w \simeq \lambda$, we have $R_\psi/R_\phi \approx 1.5$.

According to the results of the previous paragraph, if Alice and Bob share a set of pairs of particles in the quantum state (1), Bob can measure the ratio R of the number of particles found around $y = 0$ and the number of particles found around $y = y_m$ in Fig. 1. In an ideal experiment, verifying if the ratio is closer to R_ψ or R_ϕ from Eq. (12), he could discover the basis chosen by Alice to measure her particles in an arbitrarily distant place, which would permit Alice to instantaneously transmit one bit of information to Bob. Of course, this is impossible, so there must be a failure in the treatment used so far. We believe that the culprit is a careless linear application of Wigner rotations, as we discuss in the next section.

It is worth discussing the relation of our calculations so far to the work of Peres *et al.* [3]. In Ref. [3], the authors consider a pure state for a relativistic particle separable in the spin-momentum partition, thus having pure reduced states for spin and momentum in the considered reference frame. But the system may not be separable in another reference frame due to the momentum dependence of the Wigner rotations. The entanglement between spin and momentum in the new frame results in a mixed reduced spin density matrix in the new frame, such that the spin entropy is not a relativistic scalar [3]. Here we are facing the same phenomenon for the state of Eq. (2) but considering the momentum reduced state. In the frame $S^{(1)}$ the reduced momentum state is not pure anymore due to the momentum-spin entanglement generated by the Wigner rotations, which causes the visibility reduction of the interference pattern of the position wave function represented in Fig. 1. It is important to reinforce that, as we showed in our previous work [27], it is not possible to consistently define a reduced density matrix for the spin of a relativistic particle as done in Ref. [3], since it is not possible to measure the particle spin independently of its momentum in a relativistic setting. However, in principle it is possible to measure the momentum of a relativistic particle independently of its spin, such that the definition of a reduced momentum state should be reasonable.

IV. SOLUTION OF THE PARADOX

The deduction of the Wigner rotations always assumes free-particle states [1,2]. The states of Eqs. (2) and (8) correspond to the superposition of free-particle solutions, consequently being also free-particle solutions. However, how can one physically construct states like the ones from Eq. (2) or Eq. (8), with a standing wave pattern? To obtain states like these, one must partially reflect the particle wave function, which is not possible with a uniform potential occupying the whole space. Although Eqs. (2) and (8) do represent free-particle solutions, the construction of such states needs the presence of a potential barrier, such that the simply application of the Wigner rotation to each momentum component is not a valid procedure (i.e., we are no longer in the domain of special relativity since the potential barrier accelerates the particle).

To understand why this is the case, we can make use of the physical interpretation of the Wigner rotations recently given by us [27] that says that these rotations are a consequence of the fact that different observers compute different quantization axes for a spin measurement. This interpretation is supported by a recent work from Palmer *et al.* that presents a detailed analysis of the Stern-Gerlach measurement process in a relativistic setting [32]. To compute the Wigner rotation for the state of Eq. (2), we must describe how the spin measurement is made and consider how the moving observer describes the quantization axis of this measurement. Let us first consider a situation in which the state of Eq. (2) is obtained trough measurements on particle 2 directly, without considering particle 1. To construct the state, one can make a spin measurement on a particle that propagates in the $+\hat{\mathbf{y}}$ direction with a Stern-Gerlach apparatus with magnetic field in the $+\hat{\mathbf{z}}$ direction, obtaining eigenvalue $+1$. After that, the particle is sent to a region that has a potential barrier that reflects its wave function and produces the standing wave pattern. In this case, the moving observer will describe the Wigner rotation for both momentum components of the wave function, with values $+p\hat{\mathbf{y}}$ and $-p\hat{\mathbf{y}}$, as the one for a free particle with momentum $+p\hat{\mathbf{y}}$, since the spin measurement is made when the particle has momentum $+p\hat{\mathbf{y}}$ and the particle spin should not change with the reflection on the potential barrier. Analogously, if the spin measurement is made while the particle propagates in the $-\hat{\mathbf{y}}$ direction, the Wigner rotation is the one of a free particle with momentum $-p\hat{\mathbf{y}}$ for both momentum components. If, on the other hand, the particle is confined in a potential well with both momentum components $+p\hat{\mathbf{y}}$ and $-p\hat{\mathbf{y}}$ while the spin measurement is made, the quantization axis is given by the average field seen by the particle such that no Wigner rotation occurs with the change of reference frame.

In the three examples of the particle state preparation described in the preceding paragraph, the Wigner rotation is the same for both momentum components of the state of Eq. (2), such that after tracing out spin the spatial wave function is pure, as in the case of the state of Eq. (8). So the probability of finding the particle in different regions is the same in the new reference frame, no matter its spin state.

Let us now come back to the physical situation described by the quantum state of Eq. (1). To compute the Wigner rotations acting on particle 2 with the change of reference frame, we must consider how this quantum state is prepared. If the physical state is the result of the decay of a particle without spin into two spin-1/2 particles in the rest frame of the original particle, then particle 2 is emitted with momentum $-p\hat{\mathbf{y}}$ and reflects back and forth on the potential well walls forming the interference pattern. According to the interpretation described in the previous paragraphs for the Wigner rotations, with the change of reference frame both momentum components of the particle wave function must suffer a Wigner rotation corresponding to the $-p\hat{\mathbf{y}}$ momentum, since the reflections on the potential barriers do not affect the particle spin. So the spatial wave function of particle 2 is pure no matter what measurement is made by Alice, such that the probabilities for Bob to find particle 2 in different regions of space are always the same, a fact that solves the paradox.

V. WIGNER ROTATIONS AND THE DETECTION OF GENUINE FREE PARTICLE STATES

Before concluding, we would like to briefly discuss the influence of the Wigner rotations on the detection of genuine free particle states in different reference frames. A particle in a superposition of momenta $\tilde{\psi}(p) \propto e^{-p^2W^2/2}$ in the y direction in the reference frame $S^{(0)}$, for instance, is what we call a genuine free particle state, since the state can be constructed without reflections of the wave function, with a uniform potential in the whole space. We will consider here that the spatial wave function $\psi(y)$ is given by Eq. (3) with $K(p^0) \propto \sqrt{m/p^0}$ [30]. In Fig. 2 we plot $|\psi'(y)|^2$ for the state in a reference frame $S^{(1)}$ that moves with velocity $\beta\hat{\mathbf{z}}$ in relation to $S^{(0)}$ when the particle spin in $S^{(0)}$ is prepared in eigenstates of $\hat{\sigma}_z$ and $\hat{\sigma}_x$ for $W = \lambda/\hbar = 1/(mc)$ and $\beta = 0.995c$. The difference between the two cases is very small, especially when it is considered that the detection probability corresponds to the integral of the square modulus of the wave function in regions greater than λ. This difference does not increase much by choosing other values for W and β. However, the probabilities of finding the particle in each region must be exactly the same in both situations, otherwise a paradox similar to the one previously discussed would appear. So, if in some cases the difference is found to be above the quantum fluctuations, this indicates that the definitions of the wave function and/or detection probability used are nonphysical.

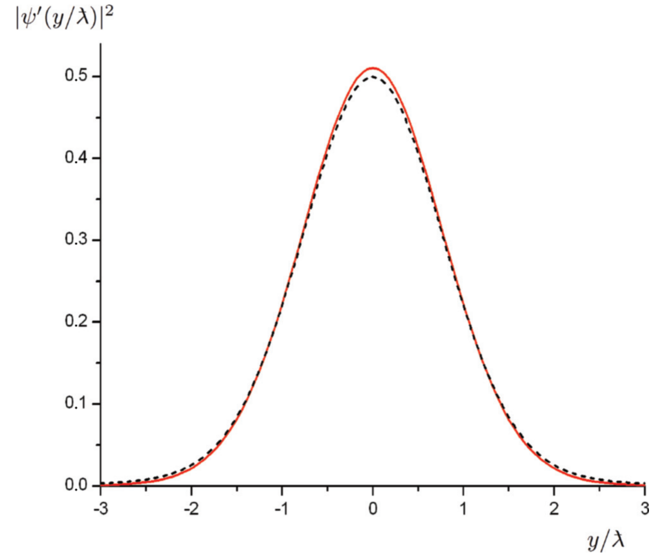

FIG. 2. (Color online) Modulus squared of the position wave function of a particle in a quantum state with a momentum wave function $\tilde{\psi}(p) \propto e^{-p^2/(2m^2c^2)}$ in the y direction seen from a observer moving with velocity $0.995c\hat{\mathbf{z}}$ when the spin state points in the x direction (continuous red curve) and the z direction (dashed black curve).

VI. CONCLUSION

To summarize, we have shown that the linear application of the momentum-dependent Wigner rotations to the quantum state of a massive relativistic particle in a superposition of counterpropagating momentum states in combination with a general model for particle detection leads to a paradox. The paradox can be stated as an apparent consequence of the imposition of the special relativity structure to quantum mechanics implying that the Wigner rotations could permit an instantaneous transmission of information between two spatially separated parties, thereby violating special relativity. Considering the physical implementation of the quantum state, we discussed that the Wigner rotation depends on the preparation method, such that, with a change of the reference frame, the spin transformation of a state in a superposition of different momenta is not necessarily equivalent to the linear application of the momentum-dependent Wigner rotation to each momentum component of the state, a conclusion that solves the paradox. The present work, together with our previous works on the subject [27,28], shows that relativistic quantum transformations cannot in general be computed only by following a mathematical procedure. The physical meaning of the transformations must always take precedence.

It is worth mentioning that it may be possible that by modeling the particle detection by some more complicated scheme, the paradox could be solved keeping the linearity of the Wigner rotations. But note that the position operator would have a very complicated dependence on the particle momenta and spin in this case. Although we do not rule out such a possibility, we believe that the solution we present for the paradox is more reasonable due to its simplicity and clear physical interpretation in the relativistic quantum information context.

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