Dissipation-driven two-mode mechanical squeezed states in optomechanical systems

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In this paper, we propose two quantum optomechanical arrangements that permit the dissipation-enabled generation of steady two-mode mechanical squeezed states. In the first setup, the mechanical oscillators are placed in a two-mode optical resonator while in the second setup the mechanical oscillators are located in two coupled single-mode cavities. We show analytically that for an appropriate choice of the pump parameters, the two mechanical oscillators can be driven by cavity dissipation into a stationary two-mode squeezed vacuum, provided that mechanical damping is negligible. The effect of thermal fluctuations is also investigated in detail and shows that ground-state precooling of the oscillators is not necessary for the two-mode squeezing. These proposals can be realized in a number of optomechanical systems with current state-of-the-art experimental techniques.

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I. INTRODUCTION

Quantum squeezing and entanglement have been observed in a number of atomic and photonic systems and are expected to play an increasing role in applications ranging from the measurement of feeble forces and fields to quantum information science [1–4]. For example, it has been know for over three decades that squeezed vibrational states are of importance for the measurement beyond the standard quantum limit of the weak signals expected to be produced, e.g., in gravitational wave antennas [5]. Although achieving such effects in macroscopic systems remains a major challenge due in particular to the increasing rate of environment-induced decoherence [6], recent progress toward the ground-state cooling of micromechanical systems [7–11] may change the situation significantly in the near future. In particular, the characterization of quantum ground-state mechanical motion [12], the quantum control of a mechanical resonator deep in the quantum regime by coupling it to a qubit [13], and the demonstration of optomechanical ponderomotive squeezing [14] are important steps toward the broad exploration of quantum effects in truly macroscopic systems [15–23]. Several proposals have been put forward to generate mechanical squeezing in an optomechanical oscillator, including the injection of nonclassical light [24], conditional quantum measurements [25], and parametric amplification [26–29]. In all cases, decoherence and losses are a dominant limiting factor in the amount of squeezing that can be achieved.

A new paradigm in quantum state preparation and control has recently received increased attention. Its key aspect is that it exploits quantum dissipation in the generation of specific quantum states. Quantum reservoir engineering has been proposed to prepare desirable quantum states [30–35] and perform quantum operations [36], and the creation of steady-state entanglement between two atomic ensembles by quantum reservoir engineering has been experimentally demonstrated [37]. This dissipative approach to quantum state preparation presents the double advantage of being independent of specific initial states and of leading to steady states robust to decoherence.

In this paper we propose to exploit the combined effects of optomechanical coupling and cavity dissipation to generate the steady-state, two-mode squeezing of two spatially separated mechanical oscillators. These states are also entangled states of continuous variables, a basic resource in quantum information precessing. We consider specifically two different setups. In the first one, analyzed in Sec. II, two mechanical oscillators are placed inside a two-mode optical resonator, while in the second one, discussed in Sec. III, the mechanical oscillators are located in two separate single-mode cavities coupled by photon tunneling. The cavities are driven in both cases by amplitude-modulated lasers. In the first setup we show analytically that for appropriate mechanical oscillator positions and pump laser parameters the mechanics can be driven into a stationary two-mode squeezed vacuum by cavity dissipation, provided that mechanical damping is negligible. In the second case a two-step driving sequence can likewise give rise to a steady two-mode mechanical squeezed vacuum state. In both cases the effect of thermal fluctuations on the resulting squeezed states is also investigated in detail. Finally, Sec. IV is a conclusion and outlook.

II. MECHANICAL OSCILLATORS IN A SINGLE TWO-MODE CAVITY

A. Model and equations

Consider an extension of the "membrane-in-the-middle" arrangement of cavity optomechanics [38] where two mechanical oscillators, modeled as vibrating dielectric membranes of identical frequencies ω_m, are located inside a driven two-mode optical resonator, as illustrated in Fig. 1. The mechanical modes are characterized by the bosonic annihilation operators $\hat{C}_j$ and the cavity modes, at frequencies $\omega_{cj} (j = 1,2)$, by annihilation operators $\hat{A}_j$. These modes are driven by amplitude-modulated lasers of frequencies ω_{lj}. The Hamiltonian of the

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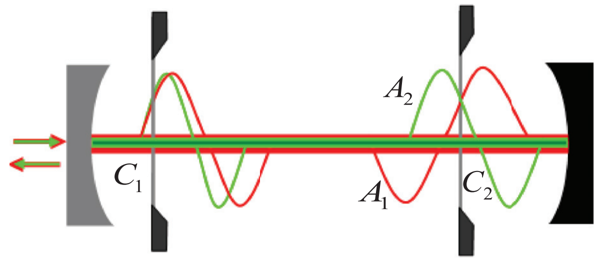

FIG. 1. (Color online) Schematic plot of two vibrating membranes (C_j) placed at appropriately chosen positions in a driven cavity with two frequency-nondegenerate resonant modes (A_j).

driven cavity-oscillators system reads

$$H = \sum_{j,k=1,2} \{\omega_{cj}\hat{A}_j^\dagger \hat{A}_j + \omega_m \hat{C}_j^\dagger \hat{C}_j + g_{jk}\hat{A}_j^\dagger \hat{A}_j(\hat{C}_k + \hat{C}_k^\dagger) + i\mathcal{E}_j(t)e^{-i\omega_{lj}t}\hat{A}_j^\dagger - i\mathcal{E}_j^*(t)e^{i\omega_{lj}t}\hat{A}_j\}, \tag{1}$$

where $\mathcal{E}_j(t)$ are the time-dependent amplitudes of the pump lasers. Their specific forms will be given later. The single-photon optomechanical coupling constants are [38]

$$g_{jk} = \frac{\omega_{cj} f_{jk}(\bar{x}_k)}{L}\sqrt{\frac{\hbar}{m\omega_m}}, \tag{2}$$

where L is the length of the cavity, m are the identical masses of the membranes, and

$$f_{jk}(\bar{x}_k) = \frac{2\mathcal{R}_k \sin(2k_{cj}\bar{x}_k)}{\sqrt{1-\mathcal{R}_k^2\cos^2(2k_{cj}\bar{x}_k)}}. \tag{3}$$

Here $\mathcal{R}_k$ and $\bar{x}_k$ are the reflection coefficients and equilibrium positions of the two membranes and k_{cj} the wave numbers of the cavity modes.

For an appropriate combination of cavity length and membrane positions of the membranes (see Fig. 1), it is possible to find a situation such that the "symmetrical" and "antisymmetric" optomechanical coupling strengths satisfy the equalities

$$g_{11} = g_{12} = g_1, \quad g_{21} = -g_{22} = g_2, \tag{4}$$

as discussed in Ref. [39]. Introducing then the new bosonic annihilation operators

$$\hat{B}_1 = (\hat{C}_1 + \hat{C}_2)/\sqrt{2}, \tag{5a}$$

$$\hat{B}_2 = (\hat{C}_1 - \hat{C}_2)/\sqrt{2}, \tag{5b}$$

the Hamiltonian H becomes $H = \sum_j \tilde{H}_j$, where

$$\tilde{H}_j = \omega_{cj}\hat{A}_j^\dagger \hat{A}_j + \omega_m B_j^\dagger \hat{B}_j + g_j \hat{A}_j^\dagger \hat{A}_j(\hat{B}_j + \hat{B}_j^\dagger) + i\mathcal{E}_j(t)e^{-i\omega_{lj}t}\hat{A}_j^\dagger - i\mathcal{E}_j^*(t)e^{i\omega_{lj}t}\hat{A}_j. \tag{6}$$

In terms of the normal operators $\hat{B}_j$, the system is therefore decoupled and reduces to two independent single-membrane optomechanical systems.

Further decomposing the operators $\hat{A}_j$ and $\hat{B}_j$ as

$$\begin{aligned}\hat{A}_j &= \langle \hat{A}_j(t)\rangle + \hat{a}_j \equiv \alpha_j(t) + \hat{a}_j,\\ \hat{B}_j &= \langle \hat{B}_j(t)\rangle + \hat{b}_j \equiv \beta_j(t) + \hat{b}_j.\end{aligned} \tag{7}$$

With $|\alpha_j(t)|^2 \gg \langle \hat{a}_j^\dagger \hat{a}_j\rangle$ and $|\beta_j(t)|^2 \gg \langle \hat{b}_j^\dagger \hat{b}_j\rangle$ one can linearize the Hamiltonians $\tilde{H}_j$ to get

$$\tilde{H}_j^{\text{lin}} = \Delta_j \hat{a}_j^\dagger \hat{a}_j + \omega_m \hat{b}_j^\dagger \hat{b}_j + [\chi_j^*(t)\hat{a}_j + \chi_j(t)\hat{a}_j^\dagger](\hat{b}_j + \hat{b}_j^\dagger), \tag{8}$$

where

$$\Delta_j = \delta_j + g_j(\beta_j + \beta_j^*),$$

with $\delta_j = \omega_{cj} - \omega_{lj}$, and the effective optomechanical coupling strengths are given by

$$\chi_j(t) = g_j \alpha_j(t). \tag{9}$$

The density matrix $\tilde{\rho}_j$ of the subsystem composed of the cavity mode a_j and the normal mode b_j satisfies the master equation

$$\begin{aligned}\dot{\tilde{\rho}}_j(t) = {}& -i[\tilde{H}_j^{\text{lin}},\tilde{\rho}_j] + \frac{\kappa_j}{2}(2\hat{a}_j\tilde{\rho}_j\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\tilde{\rho}_j - \tilde{\rho}_j\hat{a}_j^\dagger\hat{a}_j)\\ &+ \frac{\gamma_m}{2}(\bar{n}_{\text{th}}+1)(2\hat{b}_j\tilde{\rho}_j\hat{b}_j^\dagger - \hat{b}_j^\dagger\hat{b}_j\tilde{\rho}_j - \tilde{\rho}_j\hat{b}_j^\dagger\hat{b}_j)\\ &+ \frac{\gamma_m}{2}\bar{n}_{\text{th}}(2\hat{b}_j^\dagger\tilde{\rho}_j\hat{b}_j - \hat{b}_j\hat{b}_j^\dagger\tilde{\rho}_j - \tilde{\rho}_j\hat{b}_j\hat{b}_j^\dagger),\end{aligned} \tag{10}$$

where κ_j is the cavity dissipation rate, γ_m the mechanical damping rate, taken to be the same for both oscillators, and the mean thermal phonon number is

$$\bar{n}_{\text{th}} = (e^{\hbar\omega_m/k_B T} - 1)^{-1}, \tag{11}$$

with k_B the Boltzmann constant and T the temperature.

B. Stationary two-mode mechanical squeezed vacuum via cavity dissipation

We now show how cavity dissipation can be exploited to prepare the stationary two-mode mechanical squeezed vacuum of the vibrating membranes. To this end we consider an effective optomechanical coupling strength $\chi_j(t)$ of the form

$$\chi_j(t) = \chi_{j1}e^{-i(\Omega_j t - \phi_j)} + \chi_{j2}, \tag{12}$$

where χ_{j1} and χ_{j2} are constants. This situation can be realized by a pump laser of the form discussed in Sec. II C.

For weak optomechanical coupling we have $\Delta_j \approx \delta_j$, so that if the cavity-laser detuning δ_j and the modulation frequency Ω_j are

$$\delta_j = \omega_m, \quad \Omega_j = 2\omega_m, \tag{13}$$

and the transformations $\hat{a}_j \to \hat{a}_j e^{-i\Delta_j t}$ and $\hat{b}_j \to \hat{b}_j e^{-i\omega_m t}$ reduce the Hamiltonian $\tilde{H}_j^{\text{lin}}$ to

$$\tilde{H}_j^{\text{lin}} = (\chi_{j1}e^{-i\phi_j}\hat{b}_j + \chi_{j2}\hat{b}_j^\dagger)a_j + (\chi_{j1}e^{2i\omega_m t - i\phi_j}\hat{b}_j + \chi_{j2}e^{-2i\omega_m t}\hat{b}_j^\dagger)\hat{a}_j + \text{H.c.} \tag{14}$$

This form can be further simplified for a mechanical frequency $\omega_m \gg \chi_{jk}$, in which case the rapid oscillating terms $e^{\pm 2i\omega_m t}$ can be neglected and

$$\tilde{H}_j^{\text{lin}} \simeq (\chi_{j1}e^{-i\phi_j}\hat{b}_j + \chi_{j2}\hat{b}_j^\dagger)\hat{a}_j + \text{H.c.} \tag{15}$$

This Hamiltonian describes the coupling of the cavity field a_j to the normal mode b_j simultaneously via parametric amplification—through the term proportional to χ_{j1}– and via a beam-splitter-type coupling—through the constant contribution χ_{j2}. It is known that the first term in the Hamiltonian (15) leads to photon-phonon entanglement and phononic heating of the normal mode b_j, while the second term results in quantum state transfer between the cavity mode a_j and the mechanical mode b_j as well as to mechanical cooling (cold damping). To ensure the stability of the system, the coupling strengths must satisfy the inequality $\chi_{j2} > \chi_{j1}$, that is, cooling should dominate over antidamping.

When absorbing a laser photon, the parametric amplification term results in the simultaneous emission of a photon into the cavity mode a_j and a phonon in the normal mode b_j, while the beam-splitter interaction corresponds to the annihilation of a photon and the emission of a phonon. We now show that the combined effect of these processes is the generation of steady-state, single-mode squeezing of the normal mechanical mode b_j, or equivalently, of two-mode squeezing of the original modes c_1 and c_2.

We proceed by first performing the unitary transformation

$$\varrho_j(t) = \hat{S}_j^\dagger(\xi_j)\tilde{\rho}_j(t)\hat{S}_j(\xi_j), \tag{16}$$

where the squeezing operator

$$\hat{S}(\xi_j) = \exp\left[-\xi_j^*\hat{b}_j^{\dagger 2}/2 + \xi_j\hat{b}_j^2/2\right] \tag{17}$$

with

$$\xi_j = r_j e^{-i\phi_j}, \quad r_j = \tanh^{-1}(\chi_{j1}/\chi_{j2}). \tag{18}$$

For $\gamma_m = 0$ the master equation (10) becomes then

$$\dot{\varrho}_j(t) = -i\left[\mathcal{H}_j^{\rm lin},\varrho_j\right] + \frac{\kappa_j}{2}(2\hat{a}_j\varrho_j\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\varrho_j - \varrho_j\hat{a}_j^\dagger\hat{a}_j), \tag{19}$$

where

$$\mathcal{H}_j^{\rm lin} = G_j(\hat{a}_j^\dagger\hat{b}_j + \hat{b}_j^\dagger\hat{a}_j) \tag{20}$$

with

$$G_j = \chi_{j2}\sqrt{1-(\chi_{j1}/\chi_{j2})^2} \tag{21}$$

describes quantum-state transfer between the cavity mode a_j and the mechanical mode b_j in the transformed picture. It can be inferred from that master equation that in the transformed picture and for $\gamma_m = 0$ the cavity mode a_j and the mechanical mode b_j asymptotically decay to the ground state

$$\varrho_j(\infty) = \left|0_{a_j}0_{b_j}\right\rangle\!\left\langle 0_{a_j}0_{b_j}\right|. \tag{22}$$

Simply reversing the unitary transformation (16) we have then that in the steady-state regime the normal mode b_j is indeed in the squeezed vacuum state

$$\tilde{\rho}_{b_j}(\infty) = \hat{S}(\xi_j)\left|0_{b_j}\right\rangle\!\left\langle 0_{b_j}\right|\hat{S}^\dagger(\xi_j). \tag{23}$$

It is possible to adjust the amplitude χ_{jk} and phase ϕ_j of the optomechanical coupling coefficient (12) in such a way that the squeezing parameters satisfy the conditions

$$r \equiv r_j, \tag{24a}$$
$$\phi \equiv \phi_1 = \phi_2 - \pi, \tag{24b}$$

in which case the two normal modes b_1 and b_2 exhibit the same amount of steady-state squeezing, but in perpendicular directions. With Eqs. (5) and (23) we then have

$$\rho_{c_1c_2}(\infty) = \hat{S}_{12}(\xi_{12})\left|0_{c_1},0_{c_2}\right\rangle\!\left\langle 0_{c_1},0_{c_2}\right|\hat{S}_{12}^\dagger(\xi_{12}), \tag{25}$$

where we have introduced the two-mode squeezing operator

$$\hat{S}_{12}(\xi_{12}) = \exp(-\xi_{12}^*\hat{c}_1^\dagger\hat{c}_2^\dagger + \xi_{12}\hat{c}_1\hat{c}_2), \tag{26}$$

and $\xi_{12} = re^{-i\phi}$. This two-mode squeezed vacuum of the mechanical oscillators can be thought of as the output from a 50:50 beam splitter characterized by the unitary transformation (5), the two inputs being the normal modes b_j squeezed in perpendicular directions. This shows that for $\gamma_m \to 0$ the dissipation of the intracavity field can be exploited to prepare pure two-mode mechanical squeezed vacuum state. The minimum time $t_{\rm min}$ required for preparing such states can be evaluated from the eigenvalues of Eq. (19),

$$\eta_{j\pm} = -\frac{\kappa_j}{2} \pm \sqrt{\frac{\kappa_j^2}{4} - G_j^2}. \tag{27}$$

For symmetric parameters $\kappa = \kappa_j$, $\chi_j = \chi_{jk}$, and $G_j = G$ one finds $t_{\rm min} = 4/\kappa$ for $G \geqslant \kappa/2$.

C. The choice of pump lasers

Next we consider the choice of pump laser amplitudes $\mathcal{E}_j(t)$ that result in the time-dependent optomechanical coupling (12), and the specific form (24) that results in the pure two-mode mechanical squeezing (25). From Eq. (9) an obvious starting ansatz is

$$\mathcal{E}_j(t) = \mathcal{E}_{j1}e^{-i(\Omega_j t - \varphi_{j1})} + \mathcal{E}_{j2}e^{i\varphi_{j2}}, \tag{28}$$

where φ_{jk} are the initial phases of two components, with amplitudes $\mathcal{E}_{jk}$ and Ω_j the modulation frequencies. The corresponding amplitudes $\alpha_j(t)$ and $\beta_j(t)$ of the cavity and mechanical modes are

$$\begin{aligned}\frac{d}{dt}\alpha_j(t) &= -[\kappa/2 + i\delta_j + ig_j(\beta_j + \beta_j^*)]\alpha_j \\ &\quad + \mathcal{E}_{j1}e^{-i(\Omega_j t - \varphi_{1j})} + \mathcal{E}_{j2}e^{i\varphi_{j2}}, \\ \frac{d}{dt}\beta_j(t) &= -(\gamma_m/2 + i\omega_m)\beta_j - ig_j|\alpha_j|^2.\end{aligned} \tag{29}$$

It is difficult to find exact solutions of these equations in general. For the case of weak optomechanical coupling strengths g_j, however, approximate analytical solutions can be found by expanding the amplitudes α_j and β_j in powers of g_j as $\alpha_j = \alpha_j^{(0)} + \alpha_j^{(1)} + \alpha_j^{(2)} + \cdots$ and $\beta_j = \beta_j^{(0)} + \beta_j^{(1)} + \beta_j^{(2)} + \cdots$. Substituting these into Eqs. (29) and for times $t \gg 1/\kappa$, $\omega_m \gg \gamma_m$, $\delta_j = \omega_m$, and $\Omega_j = 2\omega_m$ one then finds (higher-order corrections can be derived straightforwardly)

$$\alpha_j^{(0)} = \frac{\mathcal{E}_{j1}}{\sqrt{\kappa^2/4 + \omega_m^2}}e^{-i(\Omega_j t - \phi_j)} + \frac{\mathcal{E}_{j2}}{\sqrt{\kappa^2/4 + \omega_m^2}}, \tag{30a}$$

$$\alpha_j^{(1)} = 0, \quad \beta_j^{(0)} = 0, \quad \beta_j^{(2)} = 0, \tag{30b}$$

$$\begin{aligned}\alpha_j^{(2)} &= \frac{2ig_j^2\mathcal{E}_{j2}\left(2\mathcal{E}_{j1}^2 + 3\mathcal{E}_{j2}^2\right)e^{-i\varphi_{j2}}}{3\omega_m\left(\kappa^2/4 + \omega_m^2\right)^2} \\ &\quad + \frac{2ig_j^2\mathcal{E}_{j1}\left(3\mathcal{E}_{j1}^2 + 2\mathcal{E}_{j2}^2\right)e^{-2i\omega_m t - i\varphi_{j1}}}{3\omega_m\left(\kappa^2/4 + \omega_m^2\right)^2}\end{aligned}$$

$$- \frac{2ig_j^2\mathcal{E}_{j1}\mathcal{E}_{j2}^2 e^{2i\omega_m t - i\varphi_{j1}}}{3\omega_m(\kappa^2/4+\omega_m^2)^2} - \frac{2ig_j^2\mathcal{E}_{j1}^2\mathcal{E}_{j2} e^{-4i\omega_m t + 2i\varphi_{j1}}}{3\omega_m(\kappa^2/4+\omega_m^2)^{3/2}(\kappa/2 - 3i\omega_m)}, \tag{30c}$$

$$\beta_j^{(1)} = -\frac{g_j(\mathcal{E}_{j1}^2+\mathcal{E}_{j2}^2)}{\omega_m(\kappa^2/4+\omega_m^2)} - \frac{g_j\mathcal{E}_{j1}\mathcal{E}_{j2}e^{2i\omega_m t - i\varphi_{j2}}}{3\omega_m(\kappa^2/4+\omega_m^2)} + \frac{g_j\mathcal{E}_{j1}\mathcal{E}_{j2}e^{-2i\omega_m t + i\varphi_{j2}}}{\omega_m(\kappa^2/4+\omega_m^2)}, \tag{30d}$$

with the phases

$$\phi_j = \varphi_{j1} - \varphi_{j2}, \quad \varphi_{j2} = \arctan(2\omega_m/\kappa). \tag{31}$$

For coupling strengths $g_j \ll \{\kappa, \varepsilon_{jk}, \omega_m\}$, for example, for $g_j \sim 10^{-6}\omega_m$, $\kappa \sim 0.05\omega_m$, and $\mathcal{E}_{jk} \sim 10^4\omega_m$, we find $|\alpha_j^{(2)}| \sim 1 \ll |\alpha_j^{(0)}| \sim 10^4$ and $\delta_j \sim \omega_m \gg g_j(\beta_j + \beta_j^*) \sim 10^{-4}\omega_m$. We can therefore set $\Delta_j = \delta_j + g_j(\beta_j+\beta_j^*) \simeq \delta_j$, and the effective optomechanical coupling strength $\chi_j(t) \simeq g_j\alpha_j^{(0)}$ takes the required form (12) for

$$\chi_{jk} = \frac{g_j\mathcal{E}_{jk}}{\sqrt{\kappa^2/4+\omega_m^2}}. \tag{32}$$

Finally, Eqs. (24b) and (31) give

$$\varphi_{21} - \varphi_{11} = \pi, \quad \varphi_{j2} = \arctan(2\omega_m/\kappa). \tag{33}$$

D. Thermal fluctuations

For finite mechanical damping, $\gamma_m \neq 0$, the mechanical modes c_1 and c_2 are no longer in a pure squeezed state but rather in a two-mode squeezed thermal state. In that case we find from Eqs. (5) and (10)

$$\langle \hat{c}_j^\dagger \hat{c}_j\rangle_\infty = d_0 d_1 \cosh 2r - d_0 d_2 \sinh 2r + \sinh^2 r, \tag{34a}$$

$$\langle \hat{c}_1 \hat{c}_2\rangle_\infty = [-(d_0 d_1 + 1/2)\sinh 2r + d_0 d_2 \cosh 2r]e^{i\phi}, \tag{34b}$$

where

$$d_0 = 1 - \frac{4\kappa G^2}{(\kappa+\gamma_m)(\kappa\gamma_m + 4G^2)}, \tag{35a}$$

$$d_1 = \bar{n}_{\rm th}\cosh 2r + \sinh^2 r, \tag{35b}$$

$$d_2 = (\bar{n}_{\rm th} + 1/2)\sinh 2r. \tag{35c}$$

The quantum correlations between the mechanical oscillators can be quantified by the sum of variances [3,40]

$$\Delta_{\rm EPR} = \langle(\hat{X}_1^{\theta_1} + \hat{X}_2^{\theta_2})^2\rangle + \langle(\hat{X}_1^{\theta_1+\frac{\pi}{2}} - \hat{X}_2^{\theta_2+\frac{\pi}{2}})^2\rangle, \tag{36}$$

where $\hat{X}_j^{\theta_j} = (\hat{c}_j e^{-i\theta_j} + \hat{c}_j^\dagger e^{i\theta_j})/\sqrt{2}$ are quadrature operators with local phase θ_j. A value of $\Delta_{\rm EPR} < 2$ is a signature of Einstein-Podolsky-Rosen (EPR)-type correlations between the two mechanical modes, with $\Delta_{\rm EPR} = 0$ corresponding to the ideal quantum mechanical limit [3]. In our case we find that $\Delta_{\rm EPR,min}$ is minimum for the choice of local phases $\theta_1 + \theta_2 = \phi$. In the long-time limit it is equal to

$$\Delta_{\rm EPR,min} = 2e^{-2r}(1-d_0) + 2(2\bar{n}_{\rm th}+1)d_0. \tag{37}$$

We observe that for $\kappa = 0$ we have $d_0 = 1$ and hence $\Delta_{\rm EPR,min} = 4\bar{n}_{\rm th} + 2$, showing that in the absence of cavity dissipation the oscillators are just prepared in the thermal state imposed by their mechanical coupling to the environment. This confirms that cavity dissipation is an essential component of this realization of steady-state, two-mode mechanical squeezing. From Eq. (37), steady-state squeezing requires that

$$\bar{n}_{\rm th} < \bar{n}_{\rm th,max} = \frac{1-d_0}{2d_0}(1-e^{-2r}). \tag{38}$$

In practice, it is important to be able to operate at as high a number of mean thermal phonons as possible. This can be achieved by decreasing d_0 via keeping the "cooperative parameter" $4G^2/(\kappa\gamma_m) \gg 1$. From Eq. (35a) and for the realistic case $\kappa \gg \gamma_m$, d_0 reduces approximately to

$$d_0 \simeq \frac{\gamma_m}{\kappa} + \frac{\kappa\gamma_m}{4G^2}, \tag{39}$$

indicating that by increasing the coupling frequency G while keeping the ratio χ_1/χ_2 fixed it is possible to increase the value of $\bar{n}_{\rm th,max}$, which is approximately given by

$$\bar{n}_{\rm th,max} \simeq \frac{4\kappa\chi_1(\chi_2-\chi_1)}{\gamma_m[\kappa^2 + 4(\chi_2^2 - \chi_1^2)]}. \tag{40}$$

As a concrete example [38,41], for a mechanical frequency $\omega_m/2\pi \approx 2$ MHz, a cavity dissipation rate $\kappa/2\pi \approx 1$ MHz, a mechanical damping $\gamma_m/2\pi \approx 1$ Hz, and effective optomechanical coupling strengths $\chi_2/2\pi \approx 20$ kHz and $\chi_1 = 0.5\chi_2$, we find $\bar{n}_{\rm th,max} \approx 400$. Note also that strong coupling $\chi_2 \approx \kappa$ has recently been reported in Ref. [41]. This indicates that two-mode mechanical squeezing is robust against thermal fluctuations and ground-state precooling of the mechanical modes may not be necessary.

III. MECHANICAL OSCILLATORS IN SEPARATE SINGLE-MODE CAVITIES

In this section we show that the same combined effects of optomechanical coupling and dissipation can also be utilized to generate a two-mode squeezed state of two mechanical oscillators in separate single-mode cavities. The specific example that we consider consists of two identical single-mode cavities of frequency ω_c, optically coupled via a fiber (a) or evanescent-wave coupling (b) (see Fig. 2). Each cavity field is driven by a modulated laser and optomechanically coupled to a mechanical oscillator of frequency ω_m.

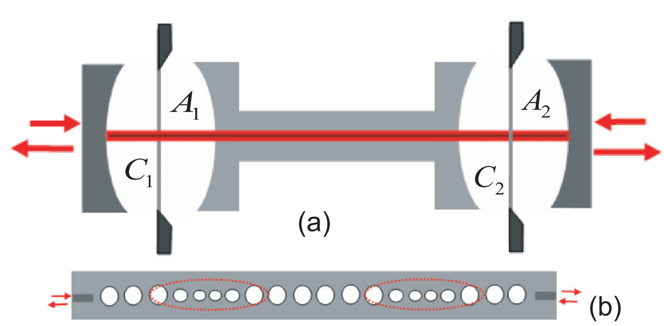

FIG. 2. (Color online) (a) Two vibrating membranes (C_j) in separate single-mode cavities (A_j) are optically coupled by an optical fiber. (b) Two evanescent-wave-coupled optomechanical crystal nanocavities.

Adopting the same symbols as before, the Hamiltonian of that system reads

$$H = \sum_{j=1,2}\{\omega_c \hat{A}_j^\dagger \hat{A}_j + \omega_m \hat{C}_j^\dagger \hat{C}_j + g\hat{A}_j^\dagger \hat{A}_j(\hat{C}_j + \hat{C}_j^\dagger) + i\mathcal{E}(t)e^{-i\omega_l t}\hat{A}_j^\dagger - i\mathcal{E}^*(t)e^{i\omega_l t}\hat{A}_j\} + \mathcal{J}_{12}(\hat{A}_1\hat{A}_2^\dagger + \hat{A}_1^\dagger \hat{A}_2), \tag{41}$$

where g accounts for the optomechanical coupling strengths, assumed to be identical, and $\mathcal{J}_{12}$ describes the coupling between the two cavities. Here we consider two pump fields of equal frequency ω_l and time-dependent amplitude

$$\mathcal{E}(t) = \mathcal{E}_1 e^{-i\Omega t + i\varphi_1} + \mathcal{E}_2 e^{i\varphi_2}. \tag{42}$$

For that symmetrical situation, both cavity fields have the same amplitude $\alpha_j(t) = \alpha(t)$ and the linearized Hamiltonian is

$$H_{\rm lin} = \sum_j \Delta \hat{a}_j^\dagger \hat{a}_j + \omega_m \hat{c}_j^\dagger \hat{c}_j + [\chi^*(t)\hat{a}_j + \chi(t)\hat{a}_j^\dagger](\hat{c}_j + \hat{c}_j^\dagger) + \mathcal{J}_{12}(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger \hat{a}_2), \tag{43}$$

where

$$\chi(t) = g\alpha(t) \tag{44}$$

and $\Delta \simeq (\omega_c - \omega_l)$ for weak optomechanical coupling. By introducing the new bosonic operators

$$\hat{b}_{1,2} = (\hat{c}_1 \pm \hat{c}_2)/\sqrt{2}, \tag{45a}$$

$$\hat{d}_{1,2} = (\hat{a}_1 \pm \hat{a}_2)/\sqrt{2}, \tag{45b}$$

the Hamiltonian separates as before into the sum of two uncoupled Hamiltonians, $H_{\rm lin} = \sum_{j=1,2} \tilde{H}_j^{\rm lin}$, where

$$\tilde{H}_j^{\rm lin} = \Delta_j \hat{d}_j^\dagger \hat{d}_j + \omega_m \hat{b}_j^\dagger \hat{b}_j + [\chi^*(t)\hat{d}_j + \chi(t)\hat{d}_j^\dagger](\hat{b}_j + \hat{b}_j^\dagger), \tag{46}$$

and the effective detunings are $\Delta_1 = \Delta + \mathcal{J}_{12}$ and $\Delta_2 = \Delta - \mathcal{J}_{12}$. The dynamics of the two independent optomechanical subsystems are governed by the same master equations as before [see Eq. (10)].

In contrast to the preceding case, though, in the two-cavity setup it is not possible to simultaneously achieve single-mode squeezing of the normal modes b_1 and b_2, since there is only one driving laser field and one set of control parameters $(\omega_l,\Omega,\varphi_j)$. However, for a sufficiently weak mechanical decoherence it is possible in principle to implement a two-step process that can still achieve that goal.

For the first step we choose the frequencies ω_l and Ω such that

$$\Delta_1 = \omega_c - \omega_l + \mathcal{J}_{12} = \omega_m, \quad \Omega = 2\omega_m, \tag{47}$$

and the phases

$$\varphi_1 = \phi_1 + \varphi_2, \quad \varphi_2 = \arctan(2\omega_m/\kappa), \tag{48}$$

where ϕ_1 is arbitrary. With the transformation $\hat{a}_j \to \hat{a}_j e^{-i\Delta_j t}$ and $\hat{b}_j \to \hat{b}_j e^{-i\omega_m t}$, the Hamiltonians $\tilde{H}_j^{\rm lin}$ reduce to

$$\tilde{H}_1^{\rm lin} = (\chi_1 e^{-i\phi_1}\hat{b}_1 + \chi_2 \hat{b}_1^\dagger)\hat{d}_1 + (\chi_2 e^{-2i\omega_m t}\hat{b}_1 + \chi_1 e^{i(2\omega_m t - \phi_1)}\hat{b}_1^\dagger)\hat{d}_1 + \text{H.c.}, \tag{49a}$$

$$\tilde{H}_2^{\rm lin} = (\chi_1 e^{i(2\mathcal{J}_{12}t - \phi_1)}\hat{b}_1 + \chi_2 e^{-2i\mathcal{J}_{12}t}\hat{b}_1^\dagger)\hat{d}_1 + (\chi_1 e^{2i(\mathcal{J}_{12}-\omega_m)t}\hat{b}_1 + \chi_2 e^{i[2(\mathcal{J}_{12}+\omega_m)t - i\phi_1]}\hat{b}_1^\dagger)\hat{d}_1 + \text{H.c.} \tag{49b}$$

For $\chi_j \ll \{\omega_m,\mathcal{J}_{12},|\mathcal{J}_{12}-\omega_m|\}$, the nonresonant terms in these Hamiltonians can be neglected and they reduce to

$$\tilde{H}_1^{\rm lin} \simeq (\chi_1 e^{-i\phi_1}\hat{b}_1 + \chi_2 \hat{b}_1^\dagger)\hat{d}_1 + \text{H.c.}, \tag{50a}$$

$$\tilde{H}_2^{\rm lin} \simeq 0, \tag{50b}$$

respectively. For the mode b_1 this is formally the same situation as encountered in Sec. II B. Neglecting then as before the mechanical damping, $\gamma_m = 0$, cavity dissipation brings likewise that normal mode into a steady-state, single-mode squeezed vacuum for long enough time, and at the same time mode b_2 simply decays into the vacuum, i.e.,

$$\tilde{\rho}_{b_1}(t \geqslant t_{\rm min}) = \hat{S}(\xi_1)|0_{b_1}\rangle\langle 0_{b_1}|\hat{S}^\dagger(\xi_1), \tag{51a}$$

$$\tilde{\rho}_{b_2}(t \geqslant t_{\rm min}) = |0_{b_2}\rangle\langle 0_{b_2}|, \tag{51b}$$

where $t_{\rm min}$, as defined before, is a time long enough that steady state has been reached. After the first step the mechanical oscillators c_1 and c_2 are therefore prepared in a pure two-mode squeezed state, although not a standard two-mode squeezed *vacuum*. That latter goal can be achieved in a second step by changing the frequency and phase of the pump laser at time $t_{\rm min}$ so that

$$\Delta_2 = \omega_c - \omega_l - \mathcal{J}_{12} = \omega_m, \quad \Omega = 2\omega_m, \tag{52}$$

and the phases

$$\varphi_1 = \phi_1 + \arctan(2\omega_m/\kappa) + \pi, \quad \varphi_2 = \arctan(2\omega_m/\kappa). \tag{53}$$

For weak optomechanical coupling, we then have

$$\tilde{H}_1^{\rm lin} \simeq 0, \tag{54a}$$

$$\tilde{H}_2^{\rm lin} \simeq (\chi_1 e^{-i\phi_2}\hat{b}_2 + \chi_2 \hat{b}_2^\dagger)\hat{d}_2 + \text{H.c.} \tag{54b}$$

That is, the mode b_1 evolves freely to the state of Eq. (51a), and after a time $t \geqslant 2t_{\rm min}$ both the normal modes b_1 and b_2 are in steady-state, single-mode squeezed vacua,

$$\tilde{\rho}_{b_1}(t \geqslant 2t_{\rm min}) = \hat{S}(\xi_1)|0_{b_1}\rangle\langle 0_{b_1}|\hat{S}^\dagger(\xi_1), \tag{55a}$$

$$\tilde{\rho}_{b_2}(t \geqslant 2t_{\rm min}) = \hat{S}(\xi_2)|0_{b_2}\rangle\langle 0_{b_2}|\hat{S}^\dagger(\xi_2). \tag{55b}$$

Hence the mechanical modes c_1 and c_2 evolve to the two-mode squeezed vacuum state

$$\rho_{c_1 c_2}(t \geqslant 2t_{\rm min}) = \hat{S}_{12}(\xi_{12})|0_{c_1},0_{c_2}\rangle\langle 0_{c_1},0_{c_2}|\hat{S}_{12}^\dagger(\xi_{12}).$$

When accounting for mechanical damping the two-step preparation scheme remains efficient for mean thermal phonon numbers such that $\gamma_m \bar{n}_{\rm th} \ll \kappa$ so that $[\gamma_m \bar{n}_{\rm th}]^{-1} \gg t_{\rm min} \gg \kappa^{-1}$ and thermal effects can be neglected during the state preparation. In case this condition is not satisfied, $\gamma_m \bar{n}_{\rm th} \geqslant \kappa$, following the second step the mode b_1 is thermalized while the mode b_2 is in a squeezed thermal state,

$$\tilde{\rho}_{b_1}(t \geqslant 2t_{\rm min}) = \tilde{\rho}_{{\rm th},b_1}, \tag{56a}$$

$$\tilde{\rho}_{b_2}(t \geqslant 2t_{\rm min}) = \hat{S}(\tilde{\xi}_2)\tilde{\rho}_{{\rm th},b_2}\hat{S}^\dagger(\tilde{\xi}_2), \tag{56b}$$

with

$$\varrho_{\mathrm{th},b_i} = \sum_{n_{b_i}=0}^{\infty} \frac{\bar{n}_i^{n_{b_i}}}{(\bar{n}_i+1)^{n_{b_i}+1}} |n_{b_i}\rangle\langle n_{b_i}|, \quad i=1,2,$$
$$\tilde{\xi}_2 = (r-\tilde{r})e^{-i\phi_1}, \tag{57}$$
$$\tilde{r} = \frac{1}{4}\ln\frac{2d_0(d_1+d_2)+1}{2d_0(d_1-d_2)+1},$$

and $\bar{n}_1 = \bar{n}_{\mathrm{th}}$ and

$$\bar{n}_2 = \sqrt{(d_0 d_1 + 1/2)^2 - d_0^2 d_2^2} - 1/2. \tag{58}$$

Note, however, that a two-step procedure is not required in that situation since a single step with laser parameters satisfying either Eq. (47) [or (52)] will result in the preparation of mode b_1 [or b_2] in a squeezed thermal state while the other mode in the thermal state (56a), with the degree of EPR correlations between the mechanical modes c_1 and c_2:

$$\Delta_{\mathrm{EPR,min}}(\infty) = e^{-2r}(1-d_0) + (2\bar{n}_{\mathrm{th}}+1)(1+d_0). \tag{59}$$

Hence, the maximum mean number of thermal phonons $\bar{n}_{\mathrm{th,max}}$ for which squeezing can be achieved is

$$\bar{n}_{\mathrm{th,max}} = \frac{1-d_0}{2(1+d_0)}(1-e^{-2r}), \tag{60}$$

which is obviously smaller than that in Eq. (38) because just one normal mode is now in a squeezed state.

IV. DISCUSSION AND CONCLUSION

To conclude this paper we comment briefly on possible ways to verify the successful generation of two-mode mechanical squeezing and to fully characterize that state. One method that is straightforward in principle would involve using two weak probe lasers to excite two additional cavity modes that are linearly coupled to the mechanical oscillators. In the weak-coupling regime, the two-mode mechanical squeezing can be mapped onto these two cavity modes via coherent quantum state transfer between the cavity and mechanical modes, similarly to the approach proposed by Vitali *et al.* [42] to quantify the optomechanical entanglement between a movable mirror and a cavity field. The two-mode mechanical squeezing can then be fully characterized via homodyne detection of the cavity field outputs.

In summary, we have proposed two possible quantum optomechanical setups to generate two-mode mechanical squeezed states of mechanical oscillators in optical cavities driven by modulated lasers. We showed analytically that for appropriate laser pump parameters the two spatially separated oscillators can be prepared into a stationary two-mode mechanical squeezed vacuum with the aid of the cavity dissipation. The effect of thermal fluctuations on the two-mode mechanical squeezing was also investigated in detail, and we showed that mechanical squeezing is achievable without precooling the mechanical oscillators to their quantum ground states. The present schemes are deterministic and can be implemented in a variety of optomechanical systems with current state-of-the-art experimental techniques.

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