Quantum correlating power of local quantum channels

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We define quantum-correlating power (QCP) of a local quantum channel acting on the left part of a bipartite quantum system as the maximum amount of left quantum correlation that can be created by this channel. We prove that for any local channel, the optimal input state, which corresponds to the maximum quantum correlation in the output state, must be a classical-classical state. Further, the single-qubit channels with maximum QCP can be found in the class of channels which take their optimal input states to rank-two quantum-classical states. A superactivation property of QCP, that is, two zero-QCP channels can constitute a positive-QCP channel, is observed and discussed for single-qubit phase damping channels. The analytic expression for QCP of single-qubit amplitude damping channel is obtained.

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I. INTRODUCTION

The quantum nature of correlation goes beyond quantum entanglement. There are separable states that contain correlation with no classical counterpart. Researches show that such separable states can still be useful for quantum computation [1], quantum state discrimination [2], and quantum communication [3–6]. The importance of quantum correlation also lies in its close connection to quantum entanglement [7–10]. Since the fundamental role of quantum correlation in quantum mechanics and its potential key role in quantum information processing, various ways for detecting and measuring the quantum correlation have been proposed [11–15]. The dynamics of quantum correlation under noise are also studied theoretically [16] and experimentally [17].

Creation of entanglement requires coherent operations on two parties. In order to quantify the ability of an operation to generate entanglement, quantum entangling power was defined [18] and attracted much attention. Contrary to entanglement, local operations alone can turn some classically correlated states into states with positive quantum correlation [19–24]. In particular, any separable state with positive quantum discord can be produced by local positive operator-valued measure (POVM) on a classical state in a larger Hilbert space [25]. The criteria for checking whether a local trace-preserving operation is able to generate quantum correlation have been obtained very recently for both single-qubit channels [19] and quantum channels of arbitrary finite dimension [20,21]. The fact of local creation of quantum correlation provides an opportunity to prepare a quantum-correlated state at no communication cost by modifying the local environment or actively performing local operations. Since no entanglement can be generated by local channels, this is a good regime for studying the fundamental properties of quantum correlation beyond entanglement. Considering the above problems, we ask the following question: How much quantum correlation can be built by local operation?

In this paper, we provide an answer to this question by introducing the concept of quantum-correlating power (QCP) for a local quantum channel, which is defined as the maximum quantum correlation that can be generated by the channel. QCP is an intrinsic attribute of a quantum channel, which quantifies the channel's ability to create quantum correlation. Some basic properties of QCP are explicitly studied. For any local channel, the input state which corresponds to the maximum quantum correlation in the output state is proved to be a classical-classical state. Further, the quantum state with maximum quantum correlation obtained by local operation on a two-qubit classical-quantum state can be found in the class of rank-two quantum-classical states. An interesting effect that two zero-QCP channels can constitute a positive-QCP channel is observed, which is named the superactivation of QCP. It implies that using two channels together is more efficient in creating quantum correlations than using them separately. As a by-product, we find a class of four-qubit states, where any two of the four qubits are not correlated at all but the quantum correlation between the bipartition $AA':BB'$ is not zero. These states are potentially useful for protocols where two parties have to cooperate to complete the task.

II. QUANTUM-CORRELATING POWER AND OPTIMAL INPUT STATE

Generally, a state is said to have zero quantum correlation on A if and only if there is a measurement on A that does not affect the total state. Such states are called classical-quantum states. We label $\mathcal{C}_0$ as the set of all classical-quantum states. Then $\mathcal{C}_0$ can be written as [26]

$$\mathcal{C}_0=\left\{\rho|\rho=\sum_i q_i\Pi^A_{\alpha_i}\otimes\rho^B_i\right\}, \tag{1}$$

where $\{\Pi^A_{\alpha_i}=|\alpha_i\rangle\langle\alpha_i|\}$ are a set of orthogonal basis of part A.

Various measures for quantifying quantum correlation have been proposed. For example, quantum discord [11] is defined as the minimum part of the mutual information shared between A and B that cannot be obtained by the measurement on A : $\delta_{B|A}(\rho)=\min_{\{F_i^A\}}S_{B|A}(\rho_{F_i^AB})-S_{B|A}(\rho)$, where $S_{A|B}(\rho)=S(\rho)-S(\rho_B)$ with $S(\rho)=-\mathrm{Tr}(\rho\log_2\rho)$ is conditional entropy, $\{F_i^A\}$ is a POVM on qudit A, and $\rho_{F_i^AB}=\sum_i F_i^A\rho F_i^{A\dagger}$ is the state of qudits A and B after the POVM. Another example is the distance-based measure of quantum correlation [19] $Q_D(\rho)=\min_{\sigma\in\mathcal{C}_0}D(\rho,\sigma)$, where the state distance satisfies the property that D does not increase under any quantum operation. Trace-norm distance $D_1=\mathrm{Tr}|\rho-\sigma|/2$ with

$|\hat{O}| = \sqrt{\hat{O}^\dagger \hat{O}}$ and relative entropy $S(\rho \parallel \sigma) = \mathrm{Tr}[\rho(\log_2 \rho - \log_2 \sigma)]$ are examples satisfying this property [27]. One-way quantum deficit $\Delta^{\leftarrow}_{B|A} = \min_{\{\Pi_A\}} S(\rho_{\Pi_i^A B}) - S(\rho)$ is in fact the minimum relative entropy to classical-quantum states [28] and thus belongs to this class of quantum correlation measure. Notice that the measures of quantum correlation are asymmetric for A and B. Here and hereafter, we discuss only the quantum correlation defined on A.

The measure of quantum correlation Q we discuss here satisfies the following three conditions: (a) $Q(\rho) = 0$ iff $\rho \in \mathcal{C}_0$; (b) $Q(U_A \otimes U_B \rho U_A^\dagger \otimes U_B^\dagger) = Q(\rho)$ where U_A and U_B are arbitrary local unitary operators on A and B; (c) $Q(I \otimes \Lambda_B(\rho)) \leqslant Q(\rho)$. Conditions (a) and (b) are satisfied by most of the quantum correlation measures. It has been proved that quantum discord satisfies condition (c) [8]. Here we briefly prove that Q_D satisfies condition (c). Suppose the closest classical-quantum state to ρ is labeled as σ; then we have $Q_D(\rho) = D(\rho,\sigma) \geqslant D(\Lambda_B(\rho),\Lambda_B(\sigma)) \geqslant Q_D(\Lambda_B(\rho))$. The last inequation holds because $\Lambda_B(\sigma)$ is still a quantum-classical state, but may not be the closest one to $\Lambda_B(\rho)$. It should be noticed that geometric quantum discord does not satisfy condition (c), and the counterexample will be shown later in this paper. Now we are ready to define quantum-correlating power.

III. DEFINITION (QUANTUM-CORRELATING POWER)

The quantum-correlating power of a quantum channel is defined as

$$\mathcal{Q}(\Lambda) = \max_{\rho \in \mathcal{C}_0} Q(\Lambda \otimes I(\rho)), \tag{2}$$

where Q is a measure of quantum correlation which satisfies conditions (a)–(c).

The input state $\rho \in \mathcal{C}_0$ that corresponds to the maximization in Eq. (2) is called the optimal input state. Here we give a general form of the optimal input state.

Theorem 1. For any d-dimensional local channel acting on A, The optimal input classical state with the maximum amount of quantum correlation in the output state is a classical-classical state of form

$$\varrho = \sum_{j=0}^{d-1} q_j \Pi^A_{\alpha_j} \otimes \Pi^B_{\beta_j}, \tag{3}$$

where $\{\Pi^B_{\beta_j} = |\beta_j\rangle\langle\beta_j|\}$ is the orthogonal basis for the Hilbert space of qudit B.

Proof. Consider a classical-quantum state $\varrho' \in \mathcal{C}_0$ as input state. After a local channel on A, the state becomes

$$\rho' = \sum_i q_i \Lambda\left(\Pi^A_{\alpha_i}\right) \otimes \rho_i^B. \tag{4}$$

For input state ϱ as in Eq. (3), the corresponding output state is

$$\rho = \sum_i q_i \Lambda\left(\Pi^A_{\alpha_i}\right) \otimes \Pi^B_{\beta_j}. \tag{5}$$

We first prove that ρ' can be prepared from ρ by a local operation on B. Writing the d states of qudit B in Eq. (4) as $\rho_k^B = \sum_{i=0}^{d-1} \lambda_i^{(k)} |\phi_i^{(k)}\rangle_B \langle\phi_i^{(k)}|$, $k = 1,\ldots,d$, we find a channel $\Lambda_B(\cdot) = \sum_{i=0}^{d-1}\sum_{k=1}^{d} \lambda_i^{(k)} E_i^{(k)}(\cdot) E_i^{(k)\dagger}$ with $E_i^{(k)} = |\phi_i^{(k)}\rangle\langle\beta_k|$, such that $\Lambda_B(\Pi^B_{\beta_j}) = \rho_k^B$. It means that $\rho = I \otimes \Lambda_B(\rho')$. Noting that local operation on B can never increase the quantum correlation on A, we have $Q(\rho) \geqslant Q(\rho')$. It means that for any state ρ' in the form of Eq. (4), we can always find a state ρ in the form of Eq. (5) whose quantum correlation is larger than ρ'. Therefore, the optimal input state must be in the form of Eq. (3). This completes the proof of Theorem 1.

IV. CHANNELS WITH MAXIMUM QCP

We have investigated the maximum quantum correlation that can be created by a specific local channel. It is also interesting to ask the following question: How much quantum correlation can be generated from a classical state when all the local quantum operation is allowed? In this section we focus on finding the single-qubit channels with maximum QCP.

Lemma 1. For any two states of a qubit ρ_j, $j = 0,1$, there exist two pure states $|\psi\rangle$ and $|\phi\rangle$, such that $\rho_j = p_j|\phi\rangle\langle\phi| + (1-p_j)|\psi\rangle\langle\psi|$, where $0 \leqslant p_j \leqslant 1$, $j = 1,2$.

Proof. We discuss this problem in the Bloch presentation: $\rho_j = (I + \vec{c}_j \cdot \vec{\sigma})/2$, $j = 1,2$, $|\psi\rangle\langle\psi| = (I + \vec{a}\cdot\vec{\sigma})/2$, and $|\phi\rangle\langle\phi| = (I + \vec{b}\cdot\vec{\sigma})/2$, where $\vec{\sigma} = \{\sigma_x,\sigma_y,\sigma_z\}$ are Pauli matrices, $\vec{c}_j = \mathrm{Tr}(\rho_j\vec{\sigma})$, $\vec{a} = \langle\psi|\vec{\sigma}|\psi\rangle$, and $\vec{b} \equiv \langle\phi|\vec{\sigma}|\phi\rangle$. Lemma 1 is proved by noticing the fact that $\vec{a}$ and $\vec{b}$ are just the two intersections of the Bloch sphere surface and the line $\overline{c_1c_2}$, where $\overline{c_1c_2}$ is the line fixed by the two points $\vec{c}_1$ and $\vec{c}_2$. Now we are ready to prove the second central result of this paper.

Theorem 2. The local single-qubit channel with maximum QCP can be found in the set of channels

$$\mathcal{D}_0 = \left\{\Lambda|\Lambda(\cdot) = \sum_{i=0}^{1} E_i(\cdot)E_i^\dagger, E_i = |\psi_i\rangle\langle\alpha_i|\right\}, \tag{6}$$

where $|\psi_0\rangle$ and $|\psi_1\rangle$ are two nonorthogonal pure states.

Proof. In order to find out the maximum-QCP qubit channel, we first prove that the optimal output two-qubit state, which contains the maximum quantum correlation created by local operations on a classical-classical state, can be found in the subset of the rank-2 quantum-classical state

$$\tilde{\mathcal{C}}_0 \equiv \left\{\tilde{\rho}|\tilde{\rho} = \sum_{i=0}^{1} p_i|\psi_i i\rangle\langle\psi_i i|\right\}. \tag{7}$$

Consider the optimal input state as in Eq. (3) and the corresponding output state as in Eq. (5) with $d = 2$. According to Lemma 1, each $\rho_j \equiv \Lambda(\Pi^A_{\alpha_j})$ $(j = 0,1)$ can be decomposed as $\rho_j = \sum_{i=0}^{1} p_i^{(j)}|\psi_i\rangle\langle\psi_i|$ with $|\psi_0\rangle$ and $|\psi_1\rangle$ two nonorthogonal pure states, and consequently, Eq. (5) can be written as

$$\rho = \sum_{i=0}^{1} p_i|\psi_i\rangle\langle\psi_i| \otimes \xi_i, \tag{8}$$

where $p_i = \sum_{j=0}^{1} q_j p_i^{(j)}$ and $\xi_i = (\sum_{j=0}^{1} q_j p_i^{(j)} \Pi^B_{\beta_j})/p_i$ for $i = 0,1$. From the proof of Theorem 1, any state ρ in form of Eq. (8) can be obtained from $\tilde{\rho}$ in Eq. (7) by some local operations on B. Therefore, the optimal output state can be found in $\tilde{\mathcal{C}}_0$. Further, for any output state $\tilde{\rho} \in \tilde{\mathcal{C}}_0$, we can find a channel $\Lambda \in \mathcal{D}_0$ which takes a classical input state to $\tilde{\rho}$. This completes the proof of Theorem 2.

Based on Theorem 2, we derive the local single-qubit channel with the maximum QCP based on quantum discord. We first need to find $\tilde{\rho} = p_0|00\rangle\langle 00| + p_1|\phi 1\rangle\langle\phi 1|$ in $\tilde{\mathcal{C}}_0$ which contains the maximum quantum discord. By using the Koashi-Winter relation [29]

$$\delta_{B|A} = \mathcal{E}_{BC} + S_{B|C}, \tag{9}$$

where $\mathcal{E}_{BC}$ is the entanglement of formation between qubits B and C, and qubit C is the purification of state $\tilde{\rho}$: $|\Psi\rangle_{ABC} = \sqrt{p_0}|000\rangle + \sqrt{p_1}|\phi 11\rangle$, we have $\delta_{B|A}(\tilde{\rho}) = h(\sqrt{\sin^2\phi - t^2\cos^2\phi}) + h(\sqrt{\cos^2\phi - t^2\sin^2\phi}) - h(t)$, where $h(x) = -\frac{1+x}{2}\log_2\frac{1+x}{2} - \frac{1-x}{2}\log_2\frac{1-x}{2}$, and $t = p_0 - p_1$. $\delta_{B|A}(\tilde{\rho})$ reaches its maximum at $\phi = \pi/4$ and $t = 0$. Therefore, the channels with maximum QCP should satisfy $\Lambda^{\max}(|\phi\rangle\langle\phi|) = |\phi\rangle\langle\phi|$ and $\Lambda^{\max}(|\phi+\pi/2\rangle\langle\phi+\pi/2|) = |\phi+3\pi/4\rangle\langle\phi+3\pi/4|$. It is direct to write a class of maximum-QCP channels, which are unitarily equivalent to $\tilde{\Lambda}(\cdot) = \sum_{i=0}^{1}\tilde{E}_i(\cdot)\tilde{E}_i^{\dagger}$, where

$$\tilde{E}_0 = |0\rangle\langle 0|, \tilde{E}_1 = |+\rangle\langle 1|, \tag{10}$$

and the corresponding QCP is

$$\mathcal{Q}_\delta(\Lambda^{\max}) = 2h\left(\frac{1}{\sqrt{2}}\right) - 1 \approx 0.2017. \tag{11}$$

It is worth mentioning that there are separable states containing larger quantum discord. For example, for separable state $\rho = (|\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|)/4 + |\Psi^+\rangle\langle\Psi^+|/2$ with $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ three Bell states, the quantum discord is $\delta(\rho) = 3(2 - \log_2 3)/4 \approx 0.311$, according to the result of Ref. [30]. Such states cannot be prepared by local operations from a single copy of a two-qubit classical state. It has been shown in Ref. [31] that the states that can be created from a classical-quantum state by local operations have measure zero. Our result shows that there is a threshold on the amount of quantum correlation that can be created locally from a classical-quantum state.

An interesting question is that whether the states created from classical states by local channel are useful in concrete tasks. The answer is that it depends on the tasks. They are useless for quantum entanglement distribution but can be useful for remote state preparation (RSP). For entanglement distribution, Alice owns qubits A and C initially and sends the qubit C to Bob who owns qubit B, in order to build the entanglement between Alice and Bob. The distributed entanglement is upper bounded by the one-way quantum deficit defined on C [3,5]: $\mathcal{E}^{A|BC} - \mathcal{E}^{AC|B} \leqslant \Delta^{AB|C}$. Now assume that the initial state of the three qubits has zero $\Delta^{AB|C}$, and Alice sends the qubit C through a noisy channel, which generates nonzero one-way deficit $\Delta'^{AB|C}$. However, the output entanglement $\mathcal{E}'^{A|BC}$ cannot be increased by the noisy channel, and consequently $\mathcal{E}'^{A|BC} \leqslant \mathcal{E}^{A|BC} \leqslant \mathcal{E}^{AC|B}$. Therefore, the locally created one-way deficit $\Delta'^{AB|C}$ cannot be used for entanglement distribution.

On the other hand, locally created quantum correlation can be useful in remote state preparation. Consider the general form of a two-qubit state in decomposition of local Pauli matrices $\rho_{AB} = [I\otimes I + \sum_{i=1}^{3} r_i\sigma_i \otimes I + \sum_{j=1}^{3} s_i I \otimes \sigma_i + \sum_{i,j=1}^{3} T_{ij}\sigma_i\otimes\sigma_j]/4$. The correlation tensor $\hat{T}$ can be diagonized by local unitary transformations of qubits A and B, and therefore, $\hat{T} = \mathrm{diag}[T_1,T_2,T_3]$, where $T_2^2, T_3^2 \leqslant T_1^2$. For a classical state, $T_2 = T_3 = 0$. It has been proved that the fidelity of RSP is $\mathcal{F}(\rho) = \frac{1}{2}(T_2^2 + T_3^2)$ [4], which is positive as long as the state is not a classical state. It means that locally created quantum correlation can be used as a resource for remote state preparation. Here we calculate the RSP fidelity for state $\rho = (|00\rangle\langle 00| + |++\rangle\langle ++|)/2$, which can be obtained locally from a classical state, and get $\mathcal{F}(\rho) = 1/8$. It can be proved that it is the maximum RSP fidelity achieved by a separable state. This implies that, even though one cannot prepare most of the separable state locally, the states locally prepared from classical states can achieve as much RSP fidelity as any separable state can do.

V. SUPERACTIVATION OF QCP

We will identify an interesting property of QCP. Consider two classical-quantum states ρ_{AB} and $\rho_{A'B'}$ with qubits A and A' at one site and qubits B and B' at another. A local two-qubit unitary operator acting on qubits A and A' can activate two zero-QCP single-qubit channels into a positive QCP two-qubit channel. We call this phenomenon the superactivation of QCP.

We here give an example of a phase-damping (PD) channel to show exactly how this property works. The Kraus operators of a PD channel are $E_0^{\mathrm{PD}} = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$ and $E_1^{\mathrm{PD}} = \sqrt{p}|1\rangle\langle 1|$. Clearly, PD channel is a mixing channel, which means that quantum correlation cannot be created when a single copy of a classical-quantum state is considered.

Now consider the initial state of qubits A and $B \rho_{AB} = \frac{1}{2}\sum_{i=0}^{1}|i\rangle_A\langle i| \otimes |i\rangle_B\langle i|$. Qubits A' and B' are in the same state; then the total state of the four qubits is

$$\rho = \rho_{AB}\otimes\rho_{A'B'} = \frac{1}{4}\sum_{i,j}|ij\rangle_{AA'}\langle ij| \otimes |ij\rangle_{BB'}\langle ij|. \tag{12}$$

Now apply a two-qubit unitary operation U: $U|ij\rangle = |\psi_{ij}\rangle$ on qubits A and A', where $|\psi_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, $|\psi_{11}\rangle = \frac{1}{\sqrt{2}}(|0+\rangle + |1-\rangle)$, $|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, and $|\psi_{10}\rangle = \frac{1}{\sqrt{2}}(|0-\rangle - |1+\rangle)$. Then qubits A and A' each transmits through a PD channel, and the output state becomes $\rho' = \Lambda_A^{\mathrm{PD}} \otimes \Lambda_{A'}^{\mathrm{PD}} \otimes I_{BB'}(U_{AA'}\rho U_{AA'}^{\dagger})$. Now we check whether quantum correlation defined on AA' is created between the bipartition AA':BB' by using the criterion in Ref. [20]. Notice that $[\Lambda^{\mathrm{PD}}\otimes\Lambda^{\mathrm{PD}}(\psi_{00}), \Lambda^{\mathrm{PD}}\otimes\Lambda^{\mathrm{PD}}(\psi_{11})] = \frac{1}{8}\tilde{i}p\sqrt{1-p}(\sigma^y\otimes\sigma^z + \sigma_z\otimes\sigma^y) \neq 0$, and consequently, quantum correlation is created between the bipartition AA':BB'.

The superactivation of QCP is a collective effect. The reduced two-qubit states $\rho'_{AB} = \mathrm{Tr}_{A'B'}(\rho') = (I_A/2)\otimes\rho_B$ and $\rho'_{A'B'} = \mathrm{Tr}_{AB}(\rho') = (I_{A'}/2)\otimes\rho_{B'}$ are product states, which contain no correlations at all. The local two-qubit unitary operation U does not build correlations between qubits A and A', since a reduced state of qubits A and A' remains completely mixed during the whole process. All in all, no correlation exists between any two qubits of the four-qubit state ρ'. Therefore, we suppose that the effect of superactivation of QCP is due to the genuine quantum correlation.

VI. QCP OF AMPLITUDE DAMPING CHANNEL

Here we calculate the QCP of a single-qubit amplitude damping (AD) channel as a concrete example. The AD channel $\Lambda^{\rm AD} = \sum_{i=0}^{1} E_i^{\rm AD}(\cdot)E_i^{\rm AD\dagger}$ with $E_0^{\rm AD} = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$ and $E_1^{\rm AD} = \sqrt{p}|0\rangle\langle 1|$, describes the evolution of a quantum system interacting with a zero-temperature bath. We will choose quantum discord and one-way quantum deficit as measures of quantum correlation in Eq. (2).

According to Theorem 1, the optimal input state should be of the form $\rho = q_1|\theta\rangle\langle\theta| \otimes |0\rangle\langle 0| + q_2|\theta + \frac{\pi}{2}\rangle\langle\theta + \frac{\pi}{2}| \otimes |1\rangle\langle 1|$, where $|\theta\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. Intuitively, $q_1 = q_2 = 1/2$ should be chosen to maximize the initial classical correlation, while $\theta = \pi/4$ should hold such that the coherence between the two energy levels $|0\rangle$ and $|1\rangle$ of qubit A is maximized. These are verified by numerical results. Depending on the above discussions, the analytical expression of QCP defined on quantum discord and one-way quantum deficit is, respectively,

$$\mathcal{Q}_\delta(\Lambda^{\rm AD}) = h(p) + h(\sqrt{1-p}) - h(\sqrt{1-p+p^2}) - 1,$$
$$\mathcal{Q}_\Delta(\Lambda^{\rm AD}) = \min\left\{h\left[\frac{h(t_1)+h(t_2)}{2}\right], h(\sqrt{1-p}), h(p)\right\} - h(\sqrt{1-p+p^2}), \quad (13)$$

where $t_1 = \sqrt{1-p}\sin 2\chi + p\cos 2\chi$, $t_2 = \sqrt{1-p}\sin 2\chi - p\cos 2\chi$, and χ satisfies

$$\tan 2\chi = \frac{\sqrt{1-p}\log_2 \frac{(1+\sqrt{1-p}\sin 2\chi)^2 - (p\cos 2\chi)^2}{(1-\sqrt{1-p}\sin 2\chi)^2 - (p\cos 2\chi)^2}}{p\log_2 \frac{(1+p\cos 2\chi)^2 - (\sqrt{1-p}\sin 2\chi)^2}{(1-p\cos 2\chi)^2 - (\sqrt{1-p}\sin 2\chi)^2}}. \quad (14)$$

The optimal measurement basis $\{|\chi\rangle, |\chi + \pi/2\rangle\}$ in the definition of one-way quantum deficit changes gradually from $\{|+\rangle, |-\rangle\}$ to $\{|0\rangle, |1\rangle\}$, while for quantum discord, the optimal measurement basis is always $\{|+\rangle, |-\rangle\}$. In Fig. 1 we plot the QCP of an AD channel.

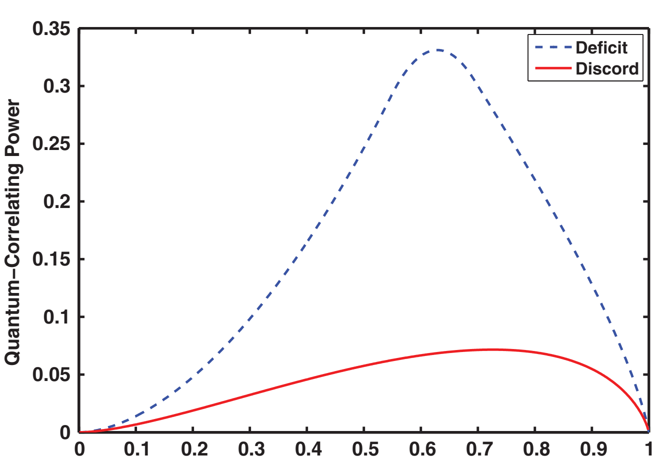

FIG. 1. (Color online) Quantum-correlating power of amplitude damping channel against the parameter p of AD channel. The dashed (blue) and solid (red) lines are, respectively, the QCP based on one-way quantum deficit and quantum discord.

In the following, we will show that a local channel acting on qubit B can increase the geometric discord. Consider AD channel with $0 < p^2 < 1-p$ acting on qubit A of the input state $\rho = (|+0\rangle_{AB}\langle +0| + |-\phi\rangle_{AB}\langle -\phi|)/2$, where $|\phi\rangle = |\tilde{\phi}\rangle$ is a pure state satisfying $\langle\tilde{\phi}|\sigma_3|\tilde{\phi}\rangle = -[3p^2 - (1-p)]/[p^2 + (1-p)]$. The geometric quantum discord that can be created by the channel is $Q_G^{\max} = \frac{p^2(1-p)}{2[p^2+(1-p)]}$. However, when we consider input state with $|\phi\rangle = |1\rangle$, the corresponding geometric quantum discord that can be created is $Q_G = p^2/4 < Q_G^{\max}$. It means that a local operation on B with Kraus operators $E_0 = |0\rangle\langle 0|$ and $E_1 = |\tilde{\phi}\rangle\langle 1|$ can increase geometric discord on A. Consequently, the geometric discord does not satisfy the condition (c) and thus is not a proper quantum correlation measure in the definition of QCP. In fact, there are other works showing that the geometric discord can be increased by simply taking away a mixed ancillary qubit on B side, which is uncorrelated to the system [32,33]. In our example, the geometric discord can be increased even when the dimension of part B does not change.

VII. CONCLUSIONS AND DISCUSSIONS

We have introduced quantum-correlating power in quantifying the ability of a local quantum channel to generate quantum correlation from a classically correlated state. For any channel, the general form of the optimal input state has been proved to be the classical-classical state. Furthermore, the single-qubit channels with maximum QCP can be found in the class of local channels which takes a classical-classical state to a rank-two quantum-classical states. The explicit expression for QCP of a single-qubit AD channel has been obtained. Interestingly, when two zero-QCP channels are used together, a positive-QCP channel can be obtained. We call this effect the superactivation of QCP, which implies that using two channels together is more efficient in creating quantum correlations than using them separately. In the example of PD channel, we find a four-qubit state with genuine four-qubit quantum correlation but zero two-qubit correlation. This result should be helpful in the study of quantum correlating structure in multiqubit states, as well as potentially useful for protocols where two parties have to cooperate to complete the task.

When a channel Λ with positive QCP is applied to a proper classical state, the classical correlation decreases while a quantum correlation is created. In this sense, we may roughly say that the classical correlation can be converted to quantum correlation by a local channel. We know that quantum correlation is the same as the well-accepted entanglement for pure states but closely related to and going beyond it for mixed states. It is responsible for the advantages presented in some quantum algorithms. The QCP proposed in this paper may provide a criteria in classifying quantum channels and further may act as a benchmark in designing quantum channels which can create quantum correlation. It may shed light on the study of the fundamental of quantum correlation and on laboratory state preparation.

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