Frequency-modulated few-cycle optical-pulse-train-induced controllable ultrafast coherent population oscillations in two-level atomic systems

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We report a study on the ultrafast coherent population oscillations (UCPOs) in two-level atoms induced by a frequency-modulated few-cycle optical pulse train. The phenomenon of UCPOs is investigated by numerically solving the optical Bloch equations beyond the rotating wave approximation. We demonstrate that the quantum state of the atoms and the frequency of the UCPOs may be controlled by controlling the number of pulses in the pulse trains and the pulse repetition time, respectively. Moreover, the robustness of the population inversion against the variation of the laser pulse parameters is also investigated. The proposed scheme may be useful for the creation of atoms in selected quantum states for desired time duration and may have potential applications in ultrafast optical switching. The scheme may also be used to measure pulse repetition rate.

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Recent progress in the generation of well-controlled shaped few-cycle optical pulses with durations of only a few periods of the optical radiation gives a new boost to the study of the so-called light-matter interaction [\[1–4\]](#page-2-0). With the advent of these few-cycle pulses, the research in quantum coherent control is getting tremendous attention owing to the high peak powers, the enormous spectral bandwidth, and the selectivity offered by the few-cycle pulses [\[5–7\]](#page-2-0). In particular, controlling the population transfer among the quantum states of atoms has remained one of the foremost areas of research in optical physics. This is primarily due to many well-known potential applications including collision dynamics, atomic interferometry, spectroscopy and optical control of chemical reactions, etc. In the context of coherent population transfer, several schemes, such as stimulated Raman adiabatic passage (STIRAP) [\[8–10\]](#page-2-0), adiabatic rapid passage (ARP) [\[11\]](#page-2-0), Raman chirped adiabatic passage (RCAP) [\[12\]](#page-2-0), temporal coherent control (TCC) [\[13\]](#page-2-0), etc., have been proposed and exploited by many authors. Recently, it has been demonstrated both experimentally and theoretically that femtosecond pulses and trains of femtosecond pulses may be used for efficient and robust controlling of the population transfer among the quantum states of atoms $[14–16]$. Recently, coherent population oscillations (CPOs) have been studied by many authors in *-*-like three-level atoms, in the context of electromagnetically induced transparency (EIT) [\[17\]](#page-2-0), spatial optical memory [\[18\]](#page-2-0), superluminal light [\[19\]](#page-2-0), and ultraslow light [\[20\]](#page-2-0). The present brief report is largely motivated by the work of Scalora and Bowden [\[21\]](#page-2-0) and Crenshaw *et al.* [\[22\]](#page-2-0), in which CPOs in two-level atomic systems as induced by varying the Rabi frequency of the interacting pulse are investigated. In this work, we report a scheme of near complete population transfer with the trains of few-cycle pulses in which the atoms may be put on hold in selected states for the desired time duration with the judicious choice of pulse parameters. The phenomenon of CPOs induced by a train of few-cycle pulses is investigated. This kind of CPO is termed ultrafast coherent population oscillations (UCPOs) owing to the ultrafast nature of the

oscillations. The novel feature of the reported UCPOs is that the frequency of oscillations may be controlled by controlling the pulse repetition time. The model atomic system for this work is chosen to be a two-level Na atom. However, the proposed scheme may be applicable to all other atoms which could be modeled as two-level atoms. It may be noted that, recently, it has been pointed out by many authors that the so-called rotating wave approximation (RWA) does not hold when one deals with few-cycle pulse-related phenomena and should work in the non-RWA regime [\[16,23,24\]](#page-2-0). Hence, in this work we are working in the non-RWA regime and assume that all the atomic relaxation times are considerably longer than the interaction times with the laser pulses.

Our analysis is based on the scheme depicted in Fig. [1.](#page-1-0) In Fig. $1(a)$, the states $|1\rangle$ and $|2\rangle$ refer to 3*s* and 3*p* quantum states of neutral sodium atoms, respectively. The total electric field for the train of pulse can be written as $\vec{E}(t - nt_r) =$ $\sum_{n=0}^{N-1} \hat{\epsilon} E_0 f(t - nt_r) \cos[\omega(t - nt_r) + \alpha(t - nt_r)^2].$ Here, $\overline{f(t - nt_r)} = \exp\{-[(t - nt_r)/\tau]^2\}$ is the Gaussian-shaped pulse envelope; $\tau = 1.177\tau_p$, where τ_p is the temporal pulse width at full width at half maximum (FWHM); *N* is the number of pulses; and t_r , α , and $\hat{\varepsilon}$ are the pulse repetition time, the linear chirp rate, and the electric field polarization direction, respectively. In this work, the states $|1\rangle$ and $|2\rangle$ refer to the down state and the up state, respectively. The optical Bloch equations, without invoking the so-called RWA, describing the temporal evolution of the density matrix elements, are

$$
\begin{aligned}\n\frac{du}{dt} &= \Omega v - \frac{u}{T_2},\\
\frac{dv}{dt} &= -\Omega u - 2\Omega_R(t)w - \frac{v}{T_2},\\
\frac{dw}{dt} &= 2\Omega_R(t)v - \frac{(w+1)}{T_1}.\n\end{aligned}
$$
(1)

Here $u, v,$ and w (population inversion) are the three components of the Bloch vector. T_2 and T_1 are, respectively, the dipole dephasing and spontaneous decay times, Ω is the transition frequency of the two-level atoms, and $\Omega_R(t)$ is the Rabi frequency, defined as $\Omega_R(t) = \vec{\mu} \cdot \vec{E}(t)/\hbar$. In the present

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FIG. 1. (Color online) (a) Two-level atomic system. (b) Timedependent frequency of up-chirped $(+\alpha)$ and down-chirped $(-\alpha)$ pulses.

study, we have neglected the terms associated with T_2 and T_1 . This may be attributed to the fact that the atom-field interaction time, owing to the extremely short duration of the few-cycle laser field, is negligibly small compared to T_2 and T_1 .

We solve Eq. [\(1\)](#page-0-0) numerically using a standard fourthorder Runge-Kutta method. We assume that initially all the atoms are in the ground state $|1\rangle$. We use the following typical parameters: $\Omega = \omega = 3.19$ rad/fs, $\tau = 25$ fs, $\mu_{12} =$ 1.85×10^{-29} Cm [\[25\]](#page-2-0), peak Rabi frequency, $\Omega_{21} = \mu E_0 / \hbar =$ 0.50 rad/fs, and $\alpha = 0.015$ fs ⁻². The laser pulse parameters such as linear chirp rate, pulse duration, and peak Rabi frequency have been investigated numerically in order to achieve the maximum population inversion. In fact, it may be noted that a laser pulse with parameters similar to the ones chosen here could be generated experimentally [\[26,27\]](#page-2-0). Figure 2 depicts the temporal evolution of the population inversion with respect to the pulse repetition time.

It can be observed from Fig. $2(a)$ that the all the atoms are transferred to the up state during the interaction with the initial pulse $(n = 0)$ in the pulse train with peak Rabi frequency at $t = 0$. On the other hand, all the atoms are transferred to the down state, when the next pulse $(n = 1)$ in the pulse train interacts with the up-state atoms. The holding time of atoms in the up state is nearly equal to the pulse repetition time (t_r) , chosen to be 500 fs in Fig. $2(a)$. Moreover, it can be seen from Fig. 2 that every odd number of pulses acts as a pump pulse, leading to the atoms in the up state while every even number of pulses acts as a dump pulse, leading to the atoms in the down state. Therefore, the final state of atoms may be controlled by manipulating the number of pulses in the pulse train. A careful inspection of Fig. 2 reveals that the hold on time of atoms (τ_a) in up and down states follows the relation $\tau_a \approx t_r$ and the frequency of UCPOs (*f*ucpo) between the up state and the down state is $f_{\text{ucpo}} \approx 1/2t_r$. Hence the hold on time of atoms in down states and up states and the frequency of population oscillations may be controlled by just controlling the pulse repetition time. Physically speaking, the pumping and dumping of atoms to the up states and the down states occurs via the so-called stimulated absorption and stimulated emission, respectively, as the spontaneous processes may not take place at such a short interaction time scale. It may be noted from Fig. 2 that the system exhibits a steplike transition, from absorbing $(w < 0)$ to amplifying $(w > 0)$ and from amplifying $(w > 0)$ to absorbing $(w < 0)$ as a function of the number of the pulses in the pulse train. The atomic medium, for an odd number of pulses in the pulse train may be interpreted as an "on" state and for an even number of pulses in the pulse train may be interpreted as an "off" state. Therefore the present scheme may serve as a unique ultrafast optical switch, in which switching time may be controlled as follows. When a switching signal enters into the absorbing medium $(w < 0)$, the interaction may take place between the switching signal and the absorbing medium; thereby the switching signal may get absorbed in the medium due to the absorptive character of the medium. This corresponds to the off state of the optical switch for a switching signal. On the other hand, when a switching signal enters into the amplifying medium $(w > 0)$, the interaction may take place between the switching signal and the amplifying medium; thereby the switching signal may not get absorbed in the medium due to the amplifying character of the medium. This corresponds to the on state of the optical switch for a switching signal. It is worthwhile to mention that the reported

FIG. 2. (Color online) Temporal evolution of population inversion for $N = 5$, with pulse repetition time: (a) $t_r = 500$ fs, (b) $t_r = 1000$ fs, (c) $t_r = 1500$ fs, and (d) $t_r = 2000$ fs.

FIG. 3. (Color online) Evolution of $w(\infty)$ against the variation of chirp rate (α) .

UCPO is almost similar to the one reported by Scalora and Bowden [21] and Crenshaw *et al.* [22] for ultrafast optical switching. Therefore the present scheme may also find similar applications. However, in the present work we are considering a dilute atomic medium and hence the effect of near-dipoledipole (NDD) interaction is not taken into account. In order to verify the robustness of the scheme, in Fig. 3, we depict the evolution of $w(\infty)$ against the variation of the chirp rate for $t_r = 1000$ fs and $N = 5$.

It is clear from Fig. 3 that the final population inversion is sufficiently robust against the variation of the chirp rate

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for the chosen t_r and N . From numerical study, the final population inversion is found to be robust against the variation of the pulse duration and the Rabi frequency as well. The final population transfer is found to be robust against the variation in the chirp rate, the pulse duration, and the Rabi frequency to a sufficiently large range, e.g., $\alpha \approx \pm 0.01 - \pm 0.03$ fs⁻², $\tau_p \approx 19 - 110$ fs, and $\Omega_{12} \approx 0.35 - 1.50$ rad/fs, respectively. The remarkable features, namely, $\tau_a \approx t_r$ and $f_{\text{ucpo}} \approx 1/2t_r$, of the present study remain invariant to the pulse envelopes of various shapes, such as sech-, sinc-, and Lorentz-shaped pulse envelopes. Since the final population inversion is sufficiently robust against the variation of the laser pulse parameters, the proposed scheme may enable efficient generation of complete population inversion in atoms of an ensemble located in different spatial points covered by the laser pulse. Hence the proposed scheme may be explored experimentally as well. The scheme may also be exploited to measure pulse repetition rate.

To conclude, we have demonstrated the phenomenon of controllable UCPOs in the two-level atoms by utilizing the train of frequency-modulated few-cycle pulses. It is shown that the hold on time for atoms in the up state and the down state may be controlled by controlling the pulse repetition time. The possible applications of the proposed scheme in ultrafast optical switching are also explored. The final population transfer to the target quantum state is found to be sufficiently robust against the variation of the Rabi frequencies and the chirp rates. The proposed scheme may find new applications in the area of ultrafast optical switching.

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