Atom lens without chromatic aberrations

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We propose a lens for atoms with reduced chromatic aberrations and calculate its focal length and spot size. In our scheme a two-level atom interacts with a near-resonant standing light wave formed by two running waves of slightly different wave vectors, and a far-detuned running wave propagating perpendicularly to the standing wave. We show that within the Raman-Nath approximation and for an adiabatically slow atom-light interaction, the phase acquired by the atom is independent of the incident atomic velocity.

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A crucial element of the toolbox for atom optics [1] is a lens to focus atom waves. Such an atom lens plays a crucial role in the realm of atom lithography [2], which is important nowadays for a multitude of technological applications. For this reason, many theoretical suggestions [3] and their realizations in experiments [4] have been made using laser fields. However, most of these realizations suffer from chromatic aberrations. In the present paper we propose a lens that is free of this type of aberration by using a special combination of light waves. Our lens is the atom-optics analog of a conventional achromatic lens [5].

We start our analysis by recalling the key features of a conventional thin lens [2,6] where a two-level atom interacts with a standing light field detuned by Δ, giving rise to the Rabi frequency Ω_0. This interaction creates the optical potential $U_{\rm opt}(x) \equiv (\hbar\Omega_0^2/\Delta)\sin^2(k_x x)$ for the motion along the x axis, which we treat quantum-mechanically. In contrast, the velocity v_y of the atom in the direction of the y axis is large and remains almost constant during the scattering process. For this reason we consider this motion classically, which allows us to set $y \equiv v_y t$.

Moreover, due to the small interaction time w_0/v_y determined by the waist w_0 of the standing wave and the longitudinal velocity v_y, and the large detuning Δ, we neglect the spontaneous emission, provided that

$$\Gamma w_e\left(\frac{w_0}{v_y}\right) \lesssim 1, \tag{1}$$

where Γ and w_e are the spontaneous emission rate and the occupation probability of the excited state, respectively. In the case of $|\Delta| > \Omega_0$, the maximum value of the population probability is $w_e \sim (\Omega_0/\Delta)^2$.

In the Raman-Nath approximation [7] for the transverse center-of-mass motion of the atom the displacement of the atom along the x axis caused by atom-field interaction is small compared to $1/k_x$, corresponding to

$$\omega_r\,\Omega_0\,\frac{w_0^2}{v_y^2} \ll 1, \tag{2}$$

where $\omega_r \equiv \hbar k_x^2/(2M)$ denotes the recoil frequency.

Within these approximations we imprint the phase

$$\phi(x) \sim -\frac{U_{\rm opt}}{\hbar}\frac{w_0}{v_y} = -\frac{\Omega_0^2}{\Delta}\frac{w_0}{v_y}\sin^2(k_x x) \tag{3}$$

onto the wave function of the center-of-mass motion of the atom in the ground state, which for $k_x|x| \ll 1$ is a quadratic function of x, that is,

$$\phi(x) \approx -\frac{\Omega_0^2}{\Delta}\frac{w_0}{v_y}k_x^2 x^2. \tag{4}$$

This quadratic variation is the origin of a thin lens [6,7] with the focal length

$$\mathcal{F}_0 = \frac{M v_y^2}{2\hbar\Omega_0}\frac{\Delta}{k_x^2 w_0\Omega_0} \equiv \kappa v_y^2\Delta. \tag{5}$$

The spot size $\mathcal{S} \equiv \alpha_0\mathcal{F}$ is determined by the focal length $\mathcal{F}$ and the angular divergence $\alpha_0 \equiv \delta v/v_y$ of the atomic beam, with δv being the uncertainty of the transverse atomic velocity. For a Gaussian wave packet of width δx the uncertainty $\delta v = \hbar/(M\delta x)$ gives rise to the angular divergence $\alpha_0 = \hbar/(M v_y \delta x)$ and the spot size

$$\mathcal{S}_0 = \frac{v_y\Delta}{2k_x^2\delta x\, w_0\Omega_0^2}. \tag{6}$$

According to Eqs. (5) and (6) in a thin conventional lens both the focal length $\mathcal{F}_0$ and the spot size $\mathcal{S}_0$ depend on the atomic velocity v_y, namely, $\mathcal{F}_0 \propto v_y^2$ and $\mathcal{S}_0 \propto v_y$, resulting in large chromatic aberrations. These scaling laws serve as our motivation to engineer a phase element for atoms and, in particular, a lens with reduced chromatic aberrations. Our suggestion relies on the interaction of a two-level atom with a near-resonant standing wave, providing us with the optical potential inducing the focusing, and a far-detuned traveling light wave removing the achromatic aberrations.

We create the lens field by the superposition

$$\mathbf{E}_s(t,\mathbf{r}) = \mathbf{E}_l(\mathbf{r})e^{-i\omega t}(e^{i\mathbf{k}_2\mathbf{r}} - e^{i\mathbf{k}_1\mathbf{r}}) + \text{c.c.} \tag{7}$$

of two traveling waves of wave vectors $\mathbf{k}_1 \equiv (k_x,k_y) \equiv (k\cos\alpha, k\sin\alpha)$ and $\mathbf{k}_2 \equiv (-k_x,k_y)$, which form an angle α relative to the x axis shown in Fig. 1. Here $\mathbf{E}_l(\mathbf{r})$ describes the position-dependent real-valued amplitude of the waves, which, throughout the article, is assumed to be of the form of the TEM_{01} Hermite-Gauss mode with a node along the y

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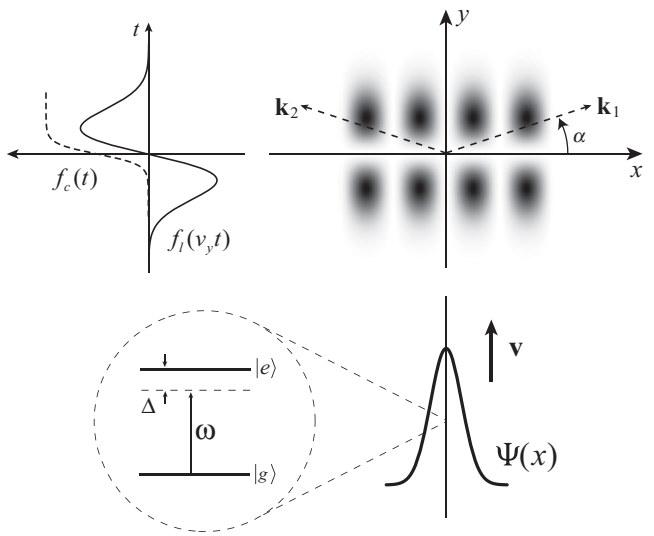

FIG. 1. Scattering of the wave packet $\Psi = \Psi(x)$ of a two-level atom by a combination of a standing electromagnetic field formed by two propagating waves of wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and a traveling wave propagating orthogonal to the x-y plane serving as a control field. As the atom propagates along the y axis with the velocity $\mathbf{v} = v_y\,\mathbf{e}_y$, the envelope of the two running waves translates according to the relation $y = v_y t$ into the time-dependent function $f_l = f_l(v_y t)$. During the atom-field interaction the effective detuning of the field frequencies from the atomic transition changes its sign due to the control field with an envelope $f_c(t)$ in the shape of a "top hat."

axis. The frequency ω is detuned from the frequency of the atomic transition between the ground $|g\rangle$ and excited $|e\rangle$ states of the corresponding energies $E_g \equiv \hbar\omega_g$ and $E_e \equiv \hbar\omega_e$ by an amount $\Delta \equiv \omega - \omega_e + \omega_g$ as shown in Fig. 1.

A running control wave

$$\mathbf{E}_r(t,\mathbf{r}) = \mathbf{E}_c(x,y)e^{i(k_c z-\omega_c t)} + \text{c.c.} \tag{8}$$

with the position-dependent amplitude $\mathbf{E}_c(x,y)$ and the shape of a top-hat propagates along the z axis perpendicular to the x-y plane. The frequency ω_c is far detuned by $\Delta_c \equiv \omega_c - \omega_{e'} + \omega_e$ from the atomic transition between the exited state $|e\rangle$ and some other state $|e'\rangle$. We suppose that the control field is weak enough to be considered perturbatively, resulting in the Stark shift $\Delta E_e = |\tilde{\boldsymbol{\wp}}\mathbf{E}_c|^2/(\hbar\Delta_c)$ of the atomic exited state $|e\rangle$, where $\tilde{\boldsymbol{\wp}} \equiv \langle e|\mathbf{d}|e'\rangle$ is the dipole matrix element.

The time evolution of the state vector

$$|\Psi(t)\rangle = a_e(t;\mathbf{r})e^{-i\omega_e t}|e\rangle + a_g(t;\mathbf{r})e^{-i\omega_g t}|g\rangle \tag{9}$$

follows from the Schrödinger equation. Indeed, within the rotating-wave approximation the time-dependent amplitudes a_g and a_e, which depend on the position $\mathbf{r}$ of the atom as a parameter, obey the system of equations

$$i\hbar\frac{d}{dt}\begin{pmatrix} a_e \\ a_g \end{pmatrix} = \hat{H}\begin{pmatrix} a_e \\ a_g \end{pmatrix}. \tag{10}$$

The Hamiltonian

$$\hat{H} \equiv \begin{pmatrix} \Delta E_e & V_l^* e^{-i\Delta t} \\ V_l e^{i\Delta t} & 0 \end{pmatrix} \tag{11}$$

contains the complex-valued coupling matrix elements

$$V_l(\mathbf{r}) = 2\boldsymbol{\wp}\mathbf{E}_l(x,y)e^{-ik_y y}\sin(k_x x), \tag{12}$$

with $\boldsymbol{\wp} \equiv \langle g|\mathbf{d}|e\rangle$ being the dipole matrix element.

We assume that the x and y dependence of $\mathbf{E}_l$ and $\mathbf{E}_c$ can be separated and, since the atomic motion along the y axis is treated classically, $y = v_y t$, we find the forms $\mathbf{E}_l(x,y) \equiv \boldsymbol{\mathcal{E}}_l(x)f_l(y) = \boldsymbol{\mathcal{E}}_l(x)f_l(v_y t)$ and $\mathbf{E}_c(x,v_y t) = \boldsymbol{\mathcal{E}}_c(x)f_c(t)$ for the electric field amplitudes. Here the envelope function

$$f_l(y) \equiv \frac{\sqrt{2}}{\pi^{1/4}}\frac{y}{w_0}\exp\left(-\frac{y^2}{2w_0^2}\right) \tag{13}$$

of the standing wave results from the TEM_{01} Hermite-Gauss mode along the y axis and satisfies the normalization conditions

$$\int_{-\infty}^{\infty} dy f_l^2(y) = w_0. \tag{14}$$

In contrast, the envelope

$$f_c(t) \equiv \theta(t) = \begin{cases} 1, & t \geqslant 0, \\ 0, & t < 0, \end{cases} \tag{15}$$

of the control field has a top-hat profile, that is, a stepwise dependence as expressed by the Heaviside function $\theta(t)$ [8].

Moreover, the detunings Δ and Δ_c are assumed to have the same sign and we can then set the amplitude of the Rabi frequency $\Omega_c \equiv |\tilde{\boldsymbol{\wp}}\mathcal{E}_c|/\hbar$ of the control field to $\Omega_c = \sqrt{2\Delta\Delta_c}$. As a result, the Stark shift ΔE_e induced by the control light field is given by $\Delta E_e = 2\hbar\Delta f_c^2(t)$.

The Hamiltonian Eq. (11) finally takes the form

$$\hat{H} \cong \begin{pmatrix} 2\hbar\Delta f_c^2(t) & \hbar\Omega(x)f_l(v_y t)e^{-i(\Delta-\omega_\alpha)t} \\ \hbar\Omega(x)f_l(v_y t)e^{i(\Delta-\omega_\alpha)t} & 0 \end{pmatrix}, \tag{16}$$

where

$$\omega_\alpha \equiv k_y v_y \tag{17}$$

and

$$\Omega(x) \equiv (2|\boldsymbol{\wp}\boldsymbol{\mathcal{E}}_l|/\hbar)\sin(k_x x) \equiv \Omega_0\sin(k_x x) \tag{18}$$

are the velocity-dependent Doppler and position-dependent Rabi frequencies, respectively.

We now solve the Schrödinger equation (10) with the Hamiltonian Eq. (16) in the case of an adiabatically slow atom-field interaction. For this purpose we substitute the second equation of the system (10) for the amplitude a_g into the first one for a_e and get the second-order differential equation

$$\frac{d^2}{dt^2}a_g - \left(i\tilde{\Delta} + \frac{1}{f_l}\frac{df_l}{dt}\right)\frac{d}{dt}a_g + \Omega^2 f_l^2 a_g = 0 \tag{19}$$

with the initial conditions

$$a_g(t_0) = 1, \quad \left.\frac{da_g}{dt}\right|_{t_0} = 0 \tag{20}$$

at time t_0. Here we have introduced the time-dependent effective detuning

$$\tilde{\Delta}(t) \equiv \Delta - \omega_\alpha - 2\Delta f_c^2(t) = \Delta - \omega_\alpha - 2\Delta\theta(t). \tag{21}$$

In the case of a slowly varying envelope $f_l(t)$ with

$$\left|\frac{1}{f_l}\frac{df_l}{dt}\right| \ll |\tilde{\Delta}| \quad \text{or} \quad |\tilde{\Delta}|\frac{w_0}{v_y} \gg 1, \tag{22}$$

we can neglect its time derivative in the second term of Eq. (19) and arrive at the approximate equation

$$\frac{d^2}{dt^2}a_g - i\tilde{\Delta}\frac{d}{dt}a_g + \Omega^2 f_l^2 a_g \simeq 0. \tag{23}$$

For each time interval, that is, for $-\infty < t \leqslant 0$ and $0 \leqslant t < \infty$, when the detuning $\tilde{\Delta}$ is constant, the solution of Eq. (23) with the initial conditions Eq. (20) reads

$$a_g(t) = \frac{1}{2}\left[1 - \frac{\tilde{\Delta}(t_0)}{\lambda(t_0)}\right]e^{i\phi_+(t)} + \frac{1}{2}\left[1 + \frac{\tilde{\Delta}(t_0)}{\lambda(t_0)}\right]e^{i\phi_-(t)} \tag{24}$$

with

$$\phi_\pm(t) = \frac{1}{2}\int_{t_0}^{t} dt'[\pm\lambda(t') + \tilde{\Delta}(t')] \tag{25}$$

and

$$\lambda(t) \equiv \sqrt{\tilde{\Delta}^2(t) + 4\Omega^2 f_l^2(v_y t)}. \tag{26}$$

For the two time intervals $-\infty < t \leqslant 0$ and $0 \leqslant t < \infty$ the initial time t_0 corresponds to $t_0 = -\infty$ and $t_0 = 0$, respectively, where the envelope $f_l(v_y t)$ vanishes. From Eq. (26) we find $\lambda(t_0) = |\tilde{\Delta}(t_0)|$ and

$$\frac{1}{2}\left[1 \pm \frac{\tilde{\Delta}(t_0)}{\lambda(t_0)}\right] = \frac{1}{2}\left[1 \pm \frac{\tilde{\Delta}(t_0)}{|\tilde{\Delta}(t_0)|}\right] = \theta[\pm\tilde{\Delta}(t_0)]. \tag{27}$$

With the definitions Eqs. (21), (25), and (26) of $\tilde{\Delta}$, $\phi_\pm$ and λ together with the explicit form Eq. (27) for coefficients, we can cast Eq. (24) into the compact form $a_g \equiv \exp(i\varphi_g)$, where the phase

$$\varphi_g(t;x) = \frac{1}{2}\int_{-\infty}^{t} dt'[\tilde{\Delta} - \mathrm{sgn}(\tilde{\Delta})\sqrt{\tilde{\Delta}^2 + 4\Omega^2(x) f_l^2(v_y t')}\,] \tag{28}$$

depends on the transverse coordinate x of the atom. Since we are interesting in engineering a lens for matter waves, we can ignore the phase $-\omega_g t$ in the Schrödinger picture, which is independent of x, and the total phase Φ_g acquired by the atom during its interaction with the two light fields is the sum

$$\Phi_g \equiv \varphi_g(t \to \infty; x) = \phi_g(x; \Delta - \omega_\alpha) + \phi_g(x; -\Delta - \omega_\alpha) \tag{29}$$

of the two contributions determined by the time intervals $-\infty < t \leqslant 0$ and $0 \leqslant t < \infty$, where

$$\phi_g(x;\delta) \equiv \frac{\mathrm{sgn}(\delta)}{2}\int_0^\infty dt\Big[|\delta| - \sqrt{\delta^2 + 4\Omega^2(x) f_l^2(v_y t)}\Big]. \tag{30}$$

In the case of $\omega_\alpha \ll |\Delta|$, the total phase Φ_g given by Eq. (29) reduces to

$$\Phi_g(x) = \omega_\alpha \int_0^\infty dt\left[\frac{|\Delta|}{\sqrt{\Delta^2 + 4\Omega^2(x) f_l^2(v_y t)}} - 1\right]. \tag{31}$$

When we introduce the integration variable $y \equiv v_y t$, and recall the definitions Eqs. (17) and (18) of ω_α and Ω, we arrive at

$$\Phi_g(x) = k_y \int_0^\infty dy\left\{\left[1 + 4\frac{\Omega_0^2}{\Delta^2}\sin^2(k_x x) f_l^2(y)\right]^{-1/2} - 1\right\}. \tag{32}$$

We emphasize that Φ_g is proportional to k_y, which is a consequence of the noncollinearity of the two wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. Moreover, Φ_g is independent of v_y. Hence, the combination of the lens field and the control wave acting on the atom creates an achromatic phase element.

We now use this phase element to construct a lens with reduced chromatic aberrations. For this purpose we consider a position of the atom close to a node of the standing wave, which allows us to expand the square root in Eq. (32), and we arrive at

$$\Phi_g(x) \approx \frac{k_y v_y}{\Delta}\left(-\frac{\Omega_0^2}{\Delta}\frac{w_0}{v_y}k_x^2 x^2\right) = \frac{k_y v_y}{\Delta}\phi(x). \tag{33}$$

Here we have recalled the normalization condition Eq. (14) for the profile function f_l and the form Eq. (4) of the phase ϕ induced by a regular optical potential.

Due to the control field Φ_g is the product of the phase ϕ corresponding to a conventional optical potential and the ratio $(k_y v_y)/\Delta$. Hence, the focal length and the spot size of our lens read

$$\mathcal{F} = \frac{\Delta}{k_y v_y}\mathcal{F}_0 \quad \text{and} \quad \mathcal{S} = \frac{\Delta}{k_y v_y}\mathcal{S}_0. \tag{34}$$

Since according to Eqs. (5) and (6) $\mathcal{F}_0$ and $\mathcal{S}_0$ depend quadratically and linearly on v_y, in our lens the focal length $\mathcal{F}$ is proportional to v_y and the spot size $\mathcal{S}$ is independent of it. This scaling implies a reduction of the chromatic aberrations in comparison with the conventional technique of focusing atoms. Moreover, in our lens both the focal length $\mathcal{F}$ and the spot size $\mathcal{S}$ are larger by a factor $|\Delta|/(k_y v_y)$ than those of the conventional lens.

The reduced chromatic aberrations are due to the symmetry of the TEM_{01} Hermite-Gauss mode with respect to a node at $y = 0$. Indeed, our combination of light waves acts as two thin optical lenses covering the domains $-\infty < y \leqslant 0$ and $0 \leqslant y < \infty$ and contributing to the total phase Φ_g given by Eq. (29). Depending on the sign of Δ, the first lens is converging, whereas the second one is diverging, or vice versa. According to Eq. (5), the corresponding focal lengths

$$\mathcal{F}_\pm \equiv 2\kappa v_y^2(\pm\Delta - \omega_\alpha) = 2\kappa v_y^2(\pm\Delta - k_y v_y) \tag{35}$$

give rise with the familiar identity

$$\frac{1}{\mathcal{F}} = \frac{1}{\mathcal{F}_+} + \frac{1}{\mathcal{F}_-} \tag{36}$$

to the total focal length

$$\mathcal{F} = 2\kappa v_y^2\frac{(\Delta - k_y v_y)(\Delta + k_y v_y)}{2k_y v_y} \approx \frac{\Delta}{k_y v_y}\mathcal{F}_0, \tag{37}$$

which coincides with Eq. (34). Here we have recalled the definition Eq. (5) and used the fact that $k_y v_y \ll |\Delta|$. Thus,

the suggested atom lens is designed in a similar manner as a conventional achromatic lens in optics [5].

The conditions Eqs. (1) and (2) are satisfied in experiments [2]. Indeed, for metastable helium with the velocity $v_y = 2000$ m/s, the angle $\alpha = 10^{-3}$, the waist $w_0 = 50$ μm, the wavelength $\lambda = 1083$ nm, the wave packet width $\delta x = \lambda$, the detuning $\Delta = 2\pi \times 30$ MHz, the rate $\Gamma = 10^7$ s^{-1}, and the Rabi frequency $\Omega_0 = |\Delta|$, we obtain the interaction time $2w_0/v_y = 50$ ns and the Doppler frequency $k_y v_y = 2\pi \times 1.85$ MHz. For these values, we obtain the focal length $\mathcal{F}_0 = 390$ μm, the spot size $\mathcal{S}_0 = 3$ nm, and $\mathcal{F}_0/\mathcal{F} = \mathcal{S}_0/\mathcal{S} = (k_y v_y)/\Delta = 0.06$.

In summary we have proposed a lens with reduced chromatic aberrations. Our scheme differs from a conventional lens by the use of *two* rather than a single light field. The improvement factor is given by the ratio of the detuning and a Doppler shift.

It is interesting to note that we can interpret [9] Φ_g as a sum of two Berry phases [10–12] acquired by the atom during the two interaction regions $-\infty < y \leqslant 0$ and $0 \leqslant y < \infty$. Since the Berry phase is purely of geometrical nature, it is insensitive to small perturbations in the control parameters [13]. For this reason, we expect that our lens designed in this way is more robust against small fluctuations of the system parameters, such as the intensity of the light fields.

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