## Rydberg atoms with a reduced sensitivity to dc and low-frequency electric fields

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A nonresonant microwave dressing field at 38.465 GHz was used to eliminate the static electric dipole moment difference between the  $49s_{1/2}$  and  $48s_{1/2}$  Rydberg states of  $^{87}$ Rb in dc fields of  $\approx 1$  V/cm. The reduced susceptibility to electric field fluctuations was measured using two-photon microwave spectroscopy. An anomalous spectral doublet is attributed to polarization ellipticity in the dressing field. The demonstrated ability to inhibit static dipole moment differences, while retaining sensitivity to high frequency fields, is applicable to sensors and/or quantum devices using Rydberg atoms.

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Rydberg atoms have a high sensitivity to resonant oscillating electric fields, typically at GHz frequencies. This property is useful for exploring the quantum nature of radiation-matter interactions [1] and for high-sensitivity rf-microwave detection [2,3]. However, Rydberg atoms also have a potentially problematic sensitivity to lower-frequency nonresonant fields, which can destroy coherence. One source of spurious fields is nearby surfaces, as sensed by Rydberg atoms near metals [4], coplanar resonators [5], and both shielded [6] and unshielded [7] atom chips. The low-frequency fields near surfaces and the high sensitivity of Rydberg atoms to these fields may limit schemes to couple these atoms to surface devices [8–10].

It is often desirable for an atomic transition frequency to have a low sensitivity to electric and/or magnetic fields. For example, the clock transitions between alkali-metal hyperfine levels are insensitive to first-order magnetic field variations [11]. When two-level systems are used to implement qubits, a reduced sensitivity to noisy perturbations helps preserve qubit coherence, as illustrated by Vion *et al.* [12] for superconducting qubits.

To reduce the influence of low-frequency electric fields on Rydberg atoms Hyafil *et al.* [13] proposed using non-resonant dressing microwave fields to modify electric field susceptibilities. A dressing field may be chosen to minimize the dc electric field sensitivity of a transition frequency between two high-angular momentum circular Rydberg states while retaining their sensitivity to higher-frequency fields. Dressing fields are a useful tool for the modification of atomic properties: They can reduce the influence of magnetic fields on optical clock transitions [14], enhance interatomic Rydberg interactions [15–18], and increase Rydberg susceptibility to electric fields [19].

In this work we experimentally demonstrate *dipole nulling*—application of a microwave dressing field to dramatically reduce Rydberg atom susceptibility to varying dc electric fields—using low-angular momentum states. Specifically we study the two-photon  $49s_{1/2} \rightarrow 48s_{1/2}$  transition of <sup>87</sup>Rb in a field of 1 V/cm and find that we can eliminate the first-order dependence on fluctuations about this field. Rydberg atoms may require a dc electric field to break degeneracies [1], to enhance interatomic interactions by making them resonant [20,21], or for trapping [13]. Somewhat surprisingly, we find that the second-order differential shift is also suppressed.

The influence of dressing fields may be considered using exact diagonalization of truncated Floquet matrices [21,22]. However, perturbation theory aids the choice of an appropriate dressing frequency and power for nulling. We use the methods of Zimmerman *et al.* [23] to construct and diagonalize a Hamiltonian in a nonzero dc field  $F_{dc,0}$  with no dressing field. The oscillating field (of amplitude  $F_{ac}$ ) and deviations in the static dc field  $\Delta F_{dc} = F_{dc} - F_{dc,0}$  are then considered as perturbations to the *i*th state, shifting the unperturbed energy  $E_{i,0}$ :

$$E_i \approx E_{i,0} - \mu_i \Delta F_{dc} - \frac{1}{4} \alpha_i(\omega) F_{ac}^2 + \beta_i(\omega) F_{ac}^2 \Delta F_{dc}, \quad (1)$$

where  $\mu_i$  is the undressed  $F_{dc,0}$  induced dipole moment,  $\alpha_i(\omega)$  is the ac polarizability, and  $-\beta_i(\omega)F_{ac}^2$  is an ac field induced contribution to the dipole moment (calculated using third-order perturbation theory). Deviations in the dc field  $\Delta F_{dc}$  are considered to be in the same direction as  $F_{dc,0}$  as these cause the first-order shifts that we would like to suppress, whereas symmetry dictates that fluctuations *transverse* to  $F_{dc,0}$  only result in second-order (and higher) shifts. In our comparison between experiment and theory we establish the limits of this perturbation theory result by diagonalization of Floquet Hamiltonians. Since this technique is likely to be useful near metal surfaces, we consider that the dressing field is applied in the same direction as the static field (normal to the surface), as dictated by boundary conditions.

In the absence of the dressing field, the difference in the dipole moments between two states  $\Delta\mu=\mu_1-\mu_2$  dictates the first-order dependence of the transition frequency on fluctuations in the dc field. The dressing field induced contribution to the dipole moment can be used to counteract this sensitivity: Setting  $-\Delta\mu+\Delta\beta(\omega)F_{ac}^2=0$  eliminates the first-order dependence of the transition frequency to field variations  $\Delta F_{dc}$  about  $F_{dc,0}$ .

To demonstrate dipole nulling we have chosen the  $49s_{1/2} \rightarrow 48s_{1/2}$  transition of <sup>87</sup>Rb in a dc field of 1 V/cm. This choice is based on practical considerations: Given a  $(n+1)s_{1/2} \rightarrow ns_{1/2}$  system in Rb, this is the lowest n for which the involved frequencies (probe and dressing) [24] are below the 40-GHz upper frequency limit of our available synthesizers.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Initial attempts to demonstrate dipole nulling using the  $36s_{1/2}$ - $37s_{1/2}$  levels were unsuccessful due to excessive broadband

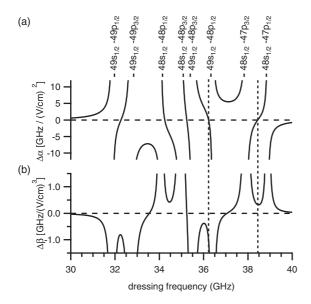


FIG. 1. (a) The ac polarizability and (b) dressing field induced dipole moment differences [see Eq. (1)] between the  $49s_{1/2}$  and  $48s_{1/2}$  states of  $^{87}$ Rb in a 1-V/cm dc field. These diverge near the zero dc-field atomic transition frequencies (labeled).

Figure 1(b) illustrates the results of a perturbative calculation of  $\Delta\beta(\omega)=\beta_{49s_{1/2}}(\omega)-\beta_{48s_{1/2}}(\omega)$  for these two states. As  $\Delta\mu=\mu_{49s_{1/2}}-\mu_{48s_{1/2}}$  is positive [6.0 MHz/(V/cm)] we are confined to dressing frequencies  $\omega$  where  $\Delta\beta(\omega)$  is positive to satisfy  $-\Delta\mu+\Delta\beta(\omega)F_{ac}^2=0$ . Figure 1 shows two frequency ranges where  $\Delta\beta(\omega)>0$ .

The remaining flexibility in the choice of  $\omega$  may be used to satisfy a second constraint (analogous to the magic wavelengths for optical Stark shifts [25]). By choosing the difference in the ac polarizabilities  $\Delta\alpha(\omega)$  to be zero, we can reduce the influence of inhomogeneities in the dressing field over the sample. Satisfying  $\Delta\alpha(\omega)=0$  ensures that inhomogeneities in the dressing field do not contribute to differential energy levels shifts; inhomogeneities will only influence the efficacy of the dipole nulling. The condition  $\Delta\alpha(\omega)=0$  occurs at a frequency of 38.44 GHz (see Fig. 1), where  $\Delta\beta=0.37$  GHz/(V/cm)<sup>3</sup> and thus for nulling  $F_{ac}=\sqrt{\Delta\mu/\Delta\beta}=0.13$  V/cm. This is a relatively weak microwave field amplitude; an enhancement cavity is not necessary. Figure 2 illustrates the relevant energy levels for the particular n we have studied.

Rubidium atoms were gathered in a  $^{87}$ Rb magneto-optical trap (MOT) and optically excited from the  $5\,p_{3/2}$ , F=3 state to the  $49s_{1/2}$ , F=2 Rydberg level using a frequency-doubled Ti:sapphire laser, stabilized using a transfer cavity [26]. dc electric fields were applied using two field plates positioned above and below the MOT center [see Fig. 3(a)]. The dressing microwave field was on continuously during optical excitation,

spectral noise in our dressing field source (an active frequency multiplier) at  $\approx$ 98 GHz, which drove unwanted resonant single-photon transitions (e.g.,  $36s_{1/2}$ - $35p_{1/2,3/2}$ ).

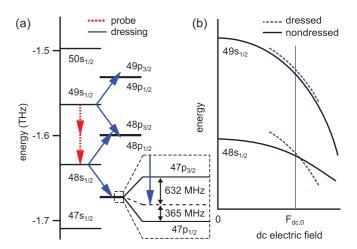


FIG. 2. (Color online) (a) Zero field atomic energy levels and the microwave dressing and probe frequencies. (b) Dipole nulling effect under ideal conditions: With the application of a dressing field there is no first-order dependence of the transition energy on the dc electric field and no differential ac Stark shift at  $F_{dc,0}$ .

while the energy difference between Rydberg states was probed using a second microwave field after optical excitation [see Fig. 3(b)]. Rydberg state populations as a function of probe frequency were measured using selective field ionization [21]. The strength of the probe was chosen to avoid excessive power broadening of the spectral line.

The dressing field was applied through a retroreflecting mirror and vacuum window [see Fig. 3(a)]. This configuration was chosen to reduce reflections from surfaces within the vacuum chamber, as these reflections cause standing waves that decrease the homogeneity of the dressing field. Likewise the Rydberg excitation beam was orthogonal to the microwave propagation direction to reduce the influence of partial standing waves along the direction of microwave propagation.

Figure 4 shows probe spectra of the  $49s_{1/2}$ - $48s_{1/2}$  transition with different dc fields and varying dressing powers (the

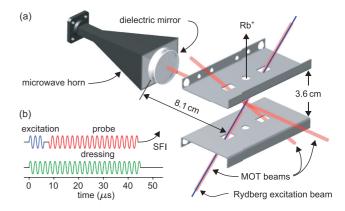


FIG. 3. (Color online) (a) Experimental apparatus. (b) Sequence of applied fields. Prior to Rydberg excitation, the anti-Helmholtz coil current is ramped down and the residual magnetic field nulled using compensation coils; the coil current is switched on again once selective field ionization (SFI) is complete. This sequence is repeated at 10 Hz as the probe frequency is stepped, and the Rydberg populations are measured using SFI.

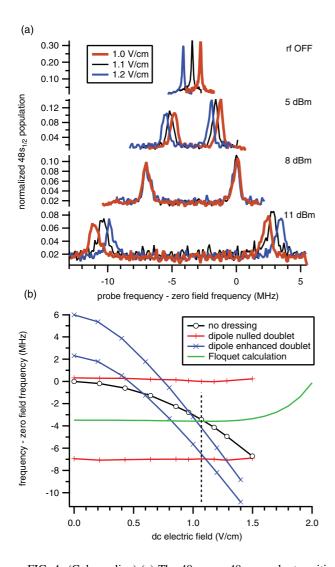


FIG. 4. (Color online) (a) The  $49s_{1/2} \rightarrow 48s_{1/2}$  probe transition for different microwave dressing powers and dc electric fields (labeled). At 8-dBm dressing power the spectral lines for the three different dc fields coincide, illustrating the dipole nulling effect. The dressing frequency is 38.465 GHz, chosen so that the doublet has zero average ac Stark shift at 1.1 V/cm. (b) Observed frequencies of the  $49s_{1/2}$ - $48s_{1/2}$  probe transitions as a function of dc electric field under different dressing field conditions, illustrating the dipole nulling and enhancement effects, compared with the nondressed case. Estimated statistical errors are on the order of the symbol sizes. Also shown are the results of a Floquet diagonalization with a linearly polarized dressing field ( $F_{ac} = 0.13$  V/cm and  $\omega/2\pi = 38.465$  GHz). Since the probe is a two-photon transition, "frequency" in both (a) and (b) refers to twice the applied frequency, for correspondence with energy-level differences.

powers quoted are the synthesizer level settings). As the dressing power is increased the probe line splits into a doublet, which we attribute to a polarization imperfection in the dressing field (discussed further below). Nulling occurs for both lines of the doublet. In particular, we can compare the nondressed to dressed Rydberg state transition frequencies at several dc fields as shown in Fig. 4(b). There is a distinct dipole moment at 1 V/cm in the absence of dressing, whereas the first-order dependence on dc field is eliminated when the

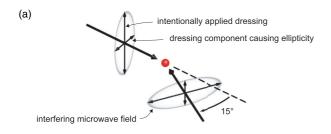
dressing microwave field is present. The nine measured peak centers of the dipole-nulled lower sideband have a standard deviation of 0.06 MHz over the dc field range 0–1.5 V/cm compared with the systematic deviation of 7 MHz over the same range when no dressing field is applied. The strength of the two-photon probe coupling remains unchanged. The observed line broadening with increasing dressing power has a full width at half maximum of 15% of the ac Stark shift and may be attributed to dressing field inhomogeneity.

Both the observations and the full Floquet diagonalization show surprisingly small variations in the dressed energies with dc field, even down to low fields. This implies that  $\Delta\beta$  is very nearly proportional to  $F_{dc,0}$ , so as to match  $\Delta\mu$ . We have effectively nulled the polarizability difference between the  $49s_{1/2}$  and  $48s_{1/2}$  states. This is a difference between our work and that proposed by Ref. [13], where dressing field coupling to a state with a permanent dipole only shows first-order suppression.

Bason *et al.* [19] have demonstrated that nonresonant dressing fields can *enhance* Rydberg susceptibility to dc electric fields. In our case it is possible to increase the dipole moment difference between the  $49s_{1/2}$  and  $48s_{1/2}$  states by using a dressing frequency where  $\Delta\beta(\omega)$  is the opposite sign of  $\Delta\mu$ . Figure 4(b) shows that the  $49s_{1/2}$ - $48s_{1/2}$  transition frequency becomes more sensitive to dc field fluctuations when a dressing frequency of 36.225 GHz is applied, consistent with the calculation of  $\Delta\beta(\omega)$  in Fig. 1; i.e.,  $\Delta\beta(\omega) < 0$  for  $\omega/2\pi = 36.225$  GHz. However, application of a nonzero dc bias electric field, in situations where this is possible, is a more straightforward way to increase Rydberg susceptibility to electric fields.

We now discuss the origin of the doublet in the probe spectra. This is not an Autler-Townes doublet, as the dressing frequency is detuned from the nearest transition by 365 MHz (see the inset to Fig. 2), which is much larger than the observed splitting. This splitting also occurs at zero electric field and does not depend on Rydberg atom density. As we show below, by varying the microwave polarization we have established that this splitting is due to a slight polarization ellipticity in the dressing field. Although the microwaves emerging from the horn should have a high degree of polarization purity, reflections from the vacuum chamber walls, field plates, and windows can introduce ellipticity in the field at the location of atoms. The splitting is similar to the magnetic-field-like vector ac Stark shift observed for ground-state atoms in circularly polarized fields [27].

To understand the splitting, it is useful to consider two extreme cases: (i) a linearly polarized dressing field and (ii) a circularly polarized dressing field. In the linearly polarized field, with an axis of quantization along the oscillating field direction, the  $m_j=\pm 1/2$  states are uncoupled and show the same ac Stark shifts. In a purely circular field, a natural axis of quantization is normal to the plane containing the oscillating electric field. With this choice, the  $m_j=\pm 1/2$  states are also uncoupled, but show different ac Stark shifts, due to their different allowed couplings (i.e., with  $\sigma_+$ :  $s_{1/2}, m_j = 1/2$  is coupled to  $p_{3/2}, m_j = 3/2$ , whereas  $s_{1/2}, m_j = -1/2$  is coupled to  $p_{1/2}, m_j = 1/2$  and  $p_{3/2}, m_j = 1/2$ ). Elliptical polarizations are intermediate between these two extremes and the splitting can be calculated using Floquet theory. Although



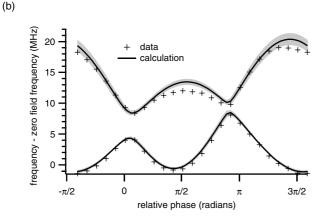


FIG. 5. (Color online) (a) Arrangement of interfering (nearly) orthogonal microwave polarizations for verifying splitting is due to ellipticity. (b) Location of two lines of the  $48s_{1/2}$ - $49s_{1/2}$  doublet as a function of phase between two interfering dressing fields at 38.1 GHz. The experimental data are shifted horizontally by a constant phase for best agreement with the calculation. The shading around the calculation indicates uncertainty related to variations in the transmitted amplitude through the phase shifter. Uncertainties in the measured line positions are on the order of the symbol sizes.

both the  $49s_{1/2}$  and  $48s_{1/2}$  states split, the calculations indicate that splitting of the  $49s_{1/2}$  state is 40 times smaller than that of the  $48s_{1/2}$  state and thus unresolved in our probe spectra.<sup>2</sup>

To confirm that polarization is responsible for the splitting we aimed a second horn towards the atoms [see Fig. 5(a)]. The microwave dressing signal from a synthesizer was split and sent to each of the two horns, with adjustable relative phase and power. The intensity and ellipticity of the fields due to each horn were determined by observing the splitting of the  $49s_{1/2}$ - $48s_{1/2}$  probe transition, using a dressing field frequency where the ac Stark shift is nonzero (38.1 GHz; see Fig. 1) and

selectively blocking one of the horns. The field intensities were determined from the observed average ac Stark shift and the theoretical  $\Delta\alpha$ . The ellipticities were determined by a comparison of the ratio of the splitting to the ac Stark shift (both scale linearly with microwave power), with a Floquet calculation. We characterize the polarization ellipses by the ratio of the minor to major axis: -17 dB for one of the horns and -11 dB for the other.

The relative powers were adjusted so that the intensity at the location of the atoms was the same for both horns and then both beams were allowed to interfere. Figure 5(b) shows that as the relative phase between the two interfering fields is changed (modulating the polarization) the splitting varies in magnitude. We can compute the expected splitting as a function of phase between the two fields using Floquet theory [see Fig. 5(b)]. The relative orientation of the two polarization ellipses is varied for best agreement with the data. The discrepancies (<3 MHz) are possibly due to variation of the relative phase over the sample and/or variations in transmitted power through the phase shifter with varying phase shift.

Since the observed splitting is associated with the  $m_j = \pm 1/2$  degeneracy of the  $ns_{1/2}$  states, this degeneracy could also be lifted in a more controllable way by application of a dc magnetic field. In this case, an elliptical dressing field would cause no further splitting and the dressing frequency could be tuned to avoid any differential ac Stark shift for the component of interest.

Our choice of n for dipole nulling was based on available equipment, but nulling could be achieved for many Rb  $(n+1)s_{1/2}$ - $ns_{1/2}$  pairs. In particular, the dressing frequency [for  $\Delta\alpha(\omega)=0$ ] scales like  $1/n^3$  and the ac field magnitude to obtain zero dc quadratic Stark shifts about  $F_{dc}=0$  scales like  $F_{ac}\propto 1/n^6$  (determined by diagonalization of Floquet Hamiltonians for n=30–55). Nulling with different species and transitions can be examined using the perturbative approach presented here (of particular interest are low-angular momentum states connected by a single-photon transition [5]).

In summary, we have demonstrated that it is possible to selectively inhibit the dipole moment difference between two Rydberg states using nonresonant dressing fields. This technique offers a means to preserve the coherence of Rydberg atom superpositions and is complementary to spin-echo and refocusing techniques [28]. It will be useful in situations where spatially inhomogeneous fields and/or low-frequency fluctuating fields are present, such as near surfaces [4–7,29, 30]. Dressing fields could help maintain resonance between Rydberg atoms and high-Q devices on surfaces, such as superconducting resonators [8], in the presence of uncontrolled low-frequency electric fields.

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 $<sup>^2</sup>$ The second doublet is unresolvable in the spectra of Fig. 4: At the largest dressing field the splitting is predicted to be 350 kHz, whereas the linewidth is 800 kHz. As both the splitting and the broadening due to field inhomogeneities scale linearly with dressing power, the  $49s_{1/2}$  splitting could only be resolved by improving the dressing field homogeneity.

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