Tuning non-Markovianity by spin-dynamics control

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We study the interplay between forgetful and memory-keeping evolution enforced on a two-level system by a multi-spin environment whose elements are coupled to local bosonic baths. Contrarily to the expectation that any non-Markovian effect would be *buried* by the forgetful mechanism induced by the spin-bath coupling, one can actually induce a full Markovian–to–non-Markovian transition of the two-level system's dynamics, controllable by parameters such as the mismatch between the energy of the two-level system and of the spin environment. For a symmetric coupling, the amount of non-Markovianity surprisingly grows with the number of decoherence channels.

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I. INTRODUCTION

The understanding of the implications of non-Markovianity and the reasons for its occurrence are still largely elusive. Yet, they are stimulating a growing interest in light of their potential impact on many disciplines, from quantum information and nanotechnology up to quantum biology [1,2]. An important contribution to this quest came from the formulation of quantitative measures of the degree of non-Markovianity of a process [3–6]. In general, these tools address different *features* of non-Markovianity, from the lack of divisibility of a map [4] to the ability of the environment to reciprocate the information transfer from the system. This process occurs unidirectionally in a Markovian dynamics [5], while the refocusing of information on the system is the signature of memory effects, as verified in all-optical set-ups [7]. The handiness of such instruments has recently triggered the analysis of non-Markovianity in quantum many-body systems such as quantum spin chains [8] or impurity-embedded ultracold atomic systems [9] and in excitation-transfer processes in photosynthetic complexes [2]. While these studies relate non-Markovian features to the critical behavior of a quantum many-body system [8,10], they also provide a promising arena where the roots for non-Markovianity can be researched in physically motivated contexts.

In this paper we explore the competition between two profoundly different mechanisms in a simple open quantum model that is relevant for the physics of nitrogen-vacancy centers in diamonds [11] and molecular nanomagnets [12]. Specifically, we address the interplay between the dynamics induced on a two-level system by its coherent interaction with other (environmental) spins, and the Markovian process describing the relaxation of the latter. One would expect that, when such memoryless dissipative coupling determines the shortest dynamical timescale of the system, Markovianity should emerge preponderantly, especially as the number of environmental spins increases. Indeed, one could imagine that a sort of "Markovianity-mixing" property would hold as a result of the increasing difficulty to rebuild the coherence of the system when many decoherence channels are open. Quite strikingly, we show that this is not generally true. In order to do this using a physically relevant model, general enough to encompass the unexpected features that we would like to highlight, we consider a spin-star configuration whose peripheral sites are coupled to *rigid* boson environments, assumed to induce a memoryless dissipative dynamics. While certainly not exhausting the possible scenarios that can be tackled, our choice is illustrative since the degree of non-Markovianity (as defined in Ref. [5]) *can actually increase* with the number of peripheral spins, while stronger interactions with the boson baths only affect its rate of growth. The features of the system at hand are quite complex and a rich non-Markovianity phase diagram emerges, spanning degrees of memory-keeping effects all the way down to zero values. This can be exploited to qualitatively modify the character of the dynamics by engineering its features via accessible control parameters such as the detuning between the central and the outer spins. In turn, this opens up the possibility to implement qubit-state preparation protocols in an open-system scenario that exploits non-Markovinity, along the lines of Ref. [13] and beyond the well-established Markovian dissipative framework [14,15].

In the following, we first present the model and its solution in the simplest terms in Sec. II, while the microscopic description and more sophisticated solution method are presented in the Appendices. We then proceed to the analysis of the non-Markovianity of the dynamics in Secs. III and IV. Some concluding remarks are given in Sec. V.

II. THE MODEL AND ITS SOLUTION

The physical set-up that we describe is sketched in Fig. 1(a), which shows a central spin (labelled 0) coupled to N outer spins, with bonds along the branch of a star. Each environmental spin is further coupled to a local boson reservoir. The evolution of the central spin is ruled by the master equation

$$\partial_t \rho_0(t) = \mathrm{Tr}_S \left\{ -i[\hat{H}, \rho(t)] + \sum_{j=1}^{N} \hat{\mathcal{L}}_j[\rho(t)] \right\} \qquad (1)$$

with $\rho(t)$ the density matrix of the whole system. Each Lindblad superoperator $\hat{\mathcal{L}}_j$ describes local dissipation at

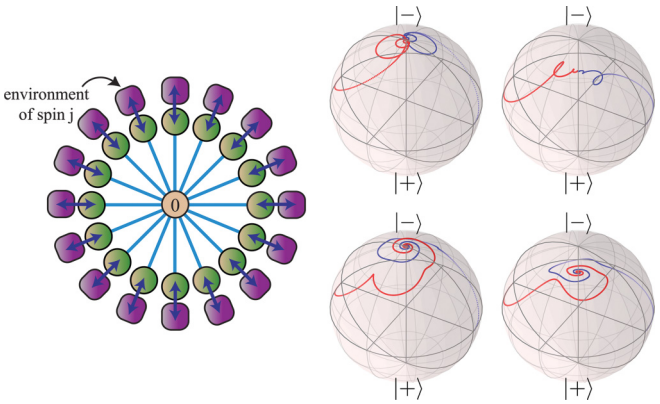

FIG. 1. (Color online) (a) Scheme of the system: The central spin 0 interacts with N peripheral spins, each affected by its own local environment. (b) Evolution of states $|+\rangle_x$ (red trajectory) and $|-\rangle_x$ (blue one) for a star with $N = 4$ peripheral sites. The Bloch spheres in the left (right) column correspond to the isotropic (anisotropic) spin-spin coupling. The top (bottom) row is for the resonant (off-resonant at $\Delta = 1/2$) case with $\gamma/J = 1$ and $T = 0$. In the isotropic cases, the final state of spin 0 is pure, while for $\lambda = \pm 1$ it is mixed. $\Delta \neq 0$ prevents the intersections of the trajectories, which are the dynamical points at which the trace distance is strictly null.

temperature T (the same for all the baths) as [16]

$$\hat{\mathcal{L}}_j(\rho) = \gamma(\bar{n}+1)(\hat{\sigma}_j^- \rho \hat{\sigma}_j^+ - \{\hat{\sigma}_j^+\hat{\sigma}_j^-, \rho\}/2) + \gamma\bar{n}(\hat{\sigma}_j^+ \rho \hat{\sigma}_j^- - \{\hat{\sigma}_j^-\hat{\sigma}_j^+, \rho\}/2), \quad (2)$$

where γ describes the effective coupling of each external spin to its thermal reservoir, populated by $\bar{n} = (e^{\beta\omega_j} - 1)^{-1}$ excitations ($\beta = 1/k_b T$, where k_b is the Boltzmann constant). In what follows, we will consider the peripheral spins to be initially prepared in $\otimes_{j=1}^N |-\rangle_j$.

To solve the master equation, we use the damping basis [17] made out of tensor products of eigenoperators of $\hat{\mathcal{L}}_j$. In this basis, the density matrix of the system reads

$$\rho(t) = \sum_{n=1}^{4} \sum_{m=1}^{4^N} c_{nm}(t) \hat{\mu}_0^n \otimes \hat{O}_m, \quad (3)$$

where $\hat{\mu}_j^1 = (\hat{\mathbb{1}} - \frac{1}{2\bar{n}+1}\hat{\sigma}_j^z)/2$, $\hat{\mu}_j^2 = \hat{\sigma}_j^z/2$, $\hat{\mu}_j^3 = \hat{\sigma}_j^+$ and $\hat{\mu}_j^4 = \hat{\sigma}_j^-$ are right eigenoperators of $\hat{\mathcal{L}}_j$ with eigenvalues $\lambda_j^1 = 0, \lambda_j^2 = 2, \lambda_j^{3,4} = -(2\bar{n}+1)$. The set of operators $\{\hat{O}_m\}$ is composed of the tensor product of N damping-basis elements, one for each peripheral spin. Due to the symmetry of the Hamiltonian, if $\hat{O}_r$ and $\hat{O}_s$ consist of the same elements of the damping bases (although differing for their order), the respective coefficients must satisfy $c_{nr} = c_{ns}$. This simple observation allows us to reduce the number of relevant operators from 4^N to $\tilde{N} \approx N^3$. With the help of the single-spin dual damping basis $\{\check{\mu}_j^n\}(n = 1, \ldots, 4)$, made of left eigenoperators of $\hat{\mathcal{L}}_j$'s, and using the orthogonality condition $\mathrm{Tr}[\hat{\mu}_j^k, \check{\mu}_l^{k'}] = \delta_{kk'}\delta_{jl}$, we find

$$\dot{c}_{rs}(t) = \sum_{n=1}^{4} \sum_{m=1}^{\tilde{N}} c_{nm}(t) \mathcal{M}_{nmrs} \quad (4)$$

with $\mathcal{M}_{nmrs} = -i\mathrm{Tr}\{(\check{\mu}_0^r \otimes \check{O}_s)[\hat{H}, \hat{\sigma}_0^n \otimes \hat{O}_m]\} + \Lambda_m \delta_{rn}\delta_{ms}$ and $\Lambda_m = \sum_{j=1}^N \lambda_j^m$. By calling $\mathcal{K}(t) = e^{\mathcal{M}t}$, the state of the spin star at time t is

$$\rho(t) = \sum_{r,s,n,m} \mathcal{K}_{nmrs}(t) c_{rs}(0) \hat{\sigma}_r^0 \otimes \hat{O}_s. \quad (5)$$

Tracing over the degrees of freedom of the peripheral spins, we find

$$\rho_0(t) = \sum_r \left(\sum_{nm} \mathcal{K}_{nmr1}(t) \right) c_{r1}(0) \hat{\sigma}_r^0 = \begin{pmatrix} \frac{\bar{n}}{1+2\bar{n}} + \frac{c_{21}(t)}{2} & c_{31}(t) \\ c_{31}(t) & \frac{(1+\bar{n})}{1+2\bar{n}} - \frac{c_{21}(t)}{2} \end{pmatrix}. \quad (6)$$

This gives the exact solution for the dynamics of the central spin, valid for any N once the expressions for $c_{rs}(t)$ are taken. With this at hand, in the next section we evaluate the amount non-Markovianity of the time evolution.

III. NON-MARKOVIANITY

To quantify the degree of non-Markovianity of the dynamical evolution of the central spin described in Eq. (6), we employ the measure put forward in Ref. [5], which is based on the idea that memory effects can be characterized by the information flowing out of the open system 0 and quantified in terms of the trace distance $D[\rho_{0,1}(t), \rho_{0,2}(t)] = \mathrm{Tr}|\rho_{0,1}(t) - \rho_{0,2}(t)|/2$ between any two of its states $\rho_{0,j}(t)(j = 1,2)$. The trace distance quantifies the distinguishability of two states and leads to measure non-Markovianity as

$$\mathcal{N} = \max_{\rho_{0,j}(0)} \int_{\Omega_+} \partial_t D[\rho_{0,1}(t), \rho_{0,2}(t)], \quad (7)$$

where Ω_+ is the union of the intervals where $\partial_t D > 0$. To provide a general assessment of the dynamics of spin 0, we consider the coupling with the external spins to be described by the anisotropic XY model

$$\hat{H}_S = J \sum_{j=1}^{N} \left[(1+\lambda)\hat{\sigma}_0^x \hat{\sigma}_j^x + (1-\lambda)\hat{\sigma}_0^y \hat{\sigma}_j^y \right], \quad (8)$$

where λ is an anisotropy parameter and J is the spin-spin coupling strength. For isotropic coupling ($\lambda = 0$) and zero temperature, we obtain a simple scaling law [18]: for any $N > 1$ $\rho_0(t)$ is obtained from the expression valid for $N = 1$ with the redefinition $J \to J\sqrt{N}$. This enables the analytic optimization over the input states entering $\mathcal{N}$. By calling $\rho_{0,i}^{kl} = \langle k|\rho_{0,i}|l\rangle$, we have

$$D[\rho_{0,1}(t), \rho_{0,2}(t)] = \sqrt{\delta\rho^{00}(t)|g_0(t)|^2 + \delta\rho^{01}(t)|g_0(t)|}, \quad (9)$$

where $\delta\rho^{kl}(t) = \rho_{0,1}^{kl}(t) - \rho_{0,2}^{kl}(t)$ and we have introduced $g_{\bar{n}}(t) = \exp[-\frac{1}{2}(G + i\Delta)t][(G + i\Delta)\sinh(zt) + z\cosh(zt)]/2z$, $z = \sqrt{(G + i\Delta)^2 - J^2 N}/2, G = \gamma(\bar{n} + 1/2)$ and the energy mismatch $\Delta = \epsilon - \epsilon_0$ between the central and outer spins. The maximum in Eq. (7) is achieved for the pure states $\rho_{0,i} = |\psi_i\rangle_0\langle\psi_i|$ with $|\psi_i\rangle_0 = \cos(\theta_i/2)|-\rangle_0 + e^{i\phi_i}\sin(\theta_i/2)|+\rangle_0$. Here, (θ_i, ϕ_i) are the angles that identify the respective Bloch vector. $\mathcal{N}$ is optimized by equatorial antipodal states (*i.e.* states with

$\theta_{1,2} = \pi/2$ and $\phi_2 = \pi - \phi_1$). In Appendix B, we provide an alternative analytic approach to the evolution of spin 0 and the dependence of the trace distance on such angles.

The trajectories described on the Bloch sphere by the evolved states are shown in Fig. 1(b) (top row, left-most sphere) where we see that the states tend to intersect, giving $D = 0$. For $\Delta \neq 0$, the states that optimize the measure of non-Markovianity are those with $(\theta_1,\theta_2) = (\pi,0)$ (the phases being immaterial) as shown in Fig. 2(b). Interestingly, non-zero values of Δ hinder the intersections of the state trajectories [cf. Fig. 1(b)]. However, this does not prevent the dynamics to become Markovian at proper working points, as we show later on.

The evolution of spin 0 can be characterized using $\mathcal{N}$. When the peripheral spins are detached from their respective baths, any information seeded in the central site undergoes coherent oscillations from the center to the periphery of the star and back. For $\lambda = 1$ and peripheral spins prepared in $\mathbb{1}/N$, the dynamics induced by $\hat{R}\hat{H}_S\hat{R}$ with $\hat{R} = \hat{\sigma}_0^y \otimes_{j=1}^N \hat{\sigma}_j^y$ is (strongly) non-Markovian at all times [5]. In our case, the interaction of the outer spins with their environments radically modifies this picture. As an example, in Fig. 2(a) we plot the trace distance for the optimal states at $\Delta = 0$. We ramp up the spin-bath interaction strength γ, at set values of the intra-star coupling J, looking for the influences that an explicitly Markovian mechanism has on the degree of non-Markovianity that arises from the dynamical environment to which particle 0 is exposed. We find a non-monotonic behavior of the trace distance that results in non-Markovianity. The quantitative features of D depend on the actual strength of the Markovian process: as γ increases, the revivals of the trace distance become less pronounced. As $\mathcal{N}$ depends on the number of temporal regions where $\partial_t D > 0$, Fig. 2(a) tells us that $\mathcal{N}$ decreases as γ increases, thus showing that, at resonance, a strong influence from the rigid environmental baths over the peripheral spins is sufficient to make the whole process Markovian.

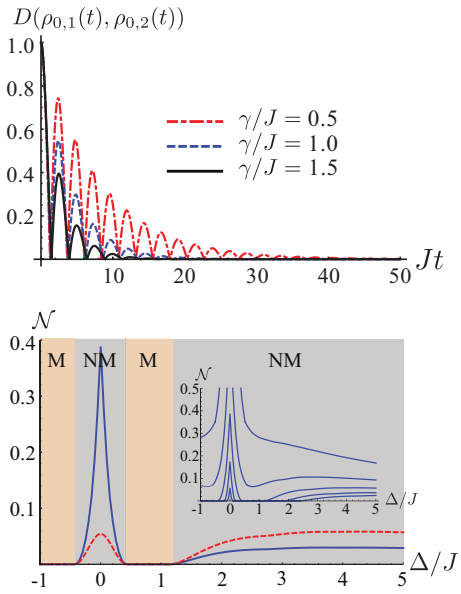

FIG. 2. (Color online) (a) Trace distance between the optimal states $\rho_{0,j}(t)(j = 1,2)$ for $N = 6$ peripheral spins with $\lambda = 0$ and $\gamma/J = 0.5$ (dot-dashed line), $\gamma/J = 1$ (dashed line), and $\gamma/J = 1.5$ (solid line). As the relaxation time becomes shorter, the revivals of $D(\rho_{0,1}(t),\rho_{0,2}(t))$ are suppressed as a result of a reduction of information back-flow from the baths. (b) $\mathcal{N}$ against Δ for $\gamma/J = \sqrt{N}$. The two lines correspond to $\theta_{1,2} = \pi/2$,$(\phi_1,\phi_2) = (0,\pi)$ (solid blue curve) and $(\theta_1,\theta_2) = (0,\pi)$ (dashed red curve), which are the optimal states in different detuning regions: $\mathcal{N}$ is the topmost curve in each region. There is a finite window of detunings (light-shadowed region marked as M) where $\mathcal{N} = 0$ (NM marks regions where $\mathcal{N} \neq 0$). Inset: $\mathcal{N}$ against Δ for $\gamma/J = 0.5,0.75,1,1.25$, and 1.5 (from top to bottom curve).

This is expected as the excitations distributed to the peripheral spins by spin 0 find the *sink* embodied by the baths. The reduced ability to feed back information sets $\mathcal{N} = 0$. However, the general picture is more involved: it is sufficient to move to the off-resonant case to face a rather rich *phase diagram* of non-Markovianity. Figure 2(b) considers the case of coupling mechanisms such that $\gamma/J = \sqrt{N}$ and explores the effect that an energy mismatch between spin 0 and the peripheral sites has on $\mathcal{N}$. We find two ranges of values of Δ for which $\mathcal{N} = 0$, symmetrically with respect to $\Delta = 0$. In between and beyond such regions, $\mathcal{N}$ behaves quite distinctively: at resonance, the measure of non-Markovianity achieves a global maximum (equatorial states realize the maximum upon which $\mathcal{N}$ depends). For larger detunings, $\mathcal{N}$ changes slowly with Δ ($|\pm\rangle$ being the optimal states). Clearly, the trend followed by $\mathcal{N}$ also depends on γ/J: small values of γ/J push the dynamics towards strong non-Markovianity, regardless of Δ, as many coherent oscillations occur between site 0 and the periphery before the initial excitation is lost into the environments. At the same time, the range of detunings for which $\mathcal{N} = 0$ increases with γ [cf. inset of Fig. 2(b)]. However non-Markovianity persists, both on and off resonance, even when γ becomes the largest parameter. This demonstrates an effective control of the degree of non-Markovianity of the dynamics undergone by spin 0, which can be tuned by both the energy mismatch between the outer spins and the central one, Δ, and the intra-star coupling strength, J.

Our discussions so far were restricted to the isotropic coupling at zero temperature, $T = \lambda = 0$. When the peripheral spins interact with baths populated by $\overline{n}$ thermal excitations, the Markovianity regions disappear. This is seen in Fig. 3(a) where we show a typical case of the behavior of $\mathcal{N}$ against Δ and N. The anisotropy of the intra-star coupling is crucial for the determination of the dynamics: for $\lambda \neq 0$ the pair of states that maximize $\mathcal{N}$ changes with the number of peripheral spins. A numerical search for the optimal states can be performed, leading to quite surprising results concerning the scaling of $\mathcal{N}$ with the size of the spin environment. Intuitively, one would conclude that, as N grows, the dynamics of spin 0 will be pushed towards Markovianity. This is not the case: as shown in Fig. 3(a), $\mathcal{N}$ *increases* with N if $\lambda = 0$, regardless of Δ. This shows that the non-Markovian character resists such Markovianity-enforcing mechanisms and, counterintuitively, overcomes them. We have checked this behavior for the exact analytical expression obtained at $\Delta = \lambda = 0$ [cf. Fig. 3(b)]. The picture somehow changes for $\lambda \neq 0$: $\mathcal{N}$ decreases with the growing dimension of the star. However, even for $N \gg 1$ the non-Markovian character is preserved and $\mathcal{N}$ achieves a non-null quasi-asymptotic value.

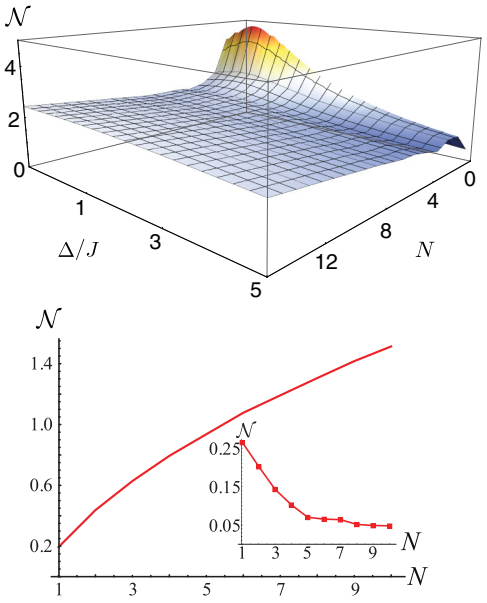

FIG. 3. (Color online) (a) $\mathcal{N}$ against N and Δ for $\gamma/J = 0.1$ and $\overline{n} = 3$. Differently from $T = 0$, except for a small range of values, the detuning has no effect on the *character* of the dynamics of spin 0. Strikingly, $\mathcal{N}$ grows with N (almost linearly for $N \gg 1$). (b) Analytic behavior of $\mathcal{N}$ versus N for $\lambda = 0$, $\Delta = 0$, $\gamma = J$ at $T = 0$ ($\overline{n} = 0$). Inset: we present the case corresponding to $\lambda = -1$ [other parameters as in (b)].

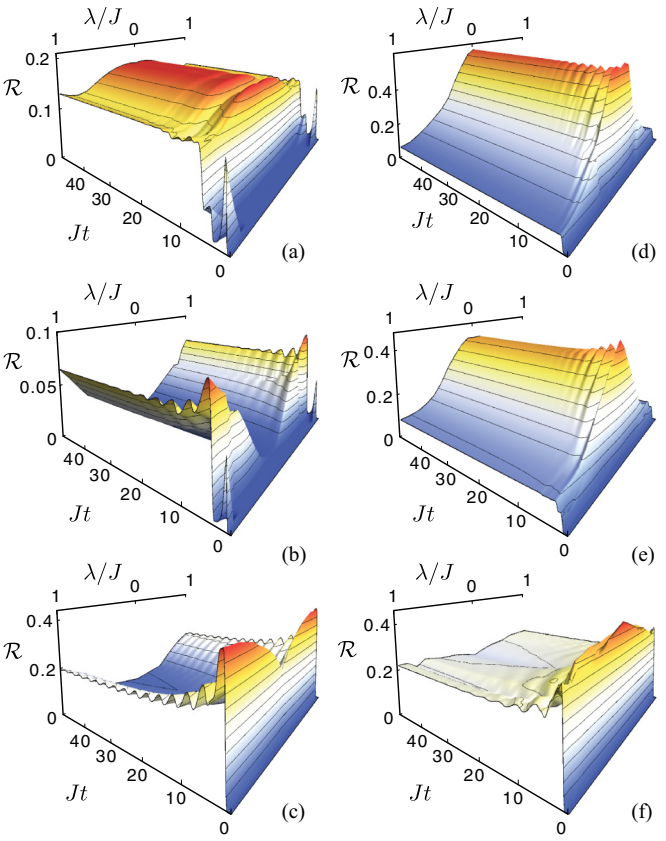

FIG. 4. (Color online) Ratio $\mathcal{R}(t) = \mathcal{N}^{+}(t)/\mathcal{N}^{-}(t)$ versus λ for a star of $N = 8$ sites at $T = 0$, with $\gamma/J = \sqrt{8}$ (left plots) and $\gamma/J = 1$ (right plots) for three different values of the detuning: $\Delta/J = 0$ for (a) and (d), $\Delta/J = 0.7$ for (b) and (e), while $\Delta/J = 3$ for (c) and (f).

IV. TIME DEVELOPMENT OF NON-MARKOVIANITY

The non-Markovianity measure gives an integral characterization of the dynamics. More details on the time dependence of the system-environment information-exchange process is obtained by considering the ratio of in-flowing to out-flowing information, up to a given value τ of the evolution time. To this end, we define $\mathcal{R}(\tau) = \frac{\mathcal{N}^{+}(\tau)}{\mathcal{N}^{-}(\tau)}$, where the in-flow [out-flow] $\mathcal{N}^{+}(\tau)$ [$\mathcal{N}^{-}(\tau)$] is defined as (minus) the integral of $\partial_t D$, over the time intervals in which it is positive (negative), but only up to τ. To evaluate these quantities explicitly, we chose as input states the same $\rho_{0,i}$ that optimize the non-Markovianity measure $\mathcal{N} \equiv \lim_{\tau\to\infty} \mathcal{N}^{+}(\tau)$. The ratio $\mathcal{R}(\tau)$ gives the fraction of the lost information that returns to the system within τ, and its behavior is quite different in the various dynamical regimes that we have identified so far. In Fig. 4, $\mathcal{R}(\tau)$ is shown for three values of Δ corresponding to the three regions of Fig. 2(b). The diverse evolutions of $\mathcal{R}(\tau)$ signal qualitatively different dynamical behaviors of the system, depending on both the detuning and the anisotropy parameter. At short times, $\mathcal{R}(\tau)$ is always zero (information has to flow out of the system before it can come back), while its first peak is determined by the first revival of the trace distance [see Fig. 2(a)]. Then, its features become strongly dependent on Δ. At long times and at resonance, where a maximum of $\mathcal{N}$ is found for $\lambda = 0$, information oscillates between the star and spin 0 and $\mathcal{R}(\tau) \neq 0$ [cf. Fig. 4(a)]. The overall dynamics is non-Markovian also for the case of Fig. 4(c), where the time behavior of $\mathcal{R}(\tau)$ is shown for a large detuning. In this case, however, $\mathcal{R}(\tau)$ decays to zero at long times. Thus, the regions of non-Markovianity in Fig. 2(b) correspond to different behaviors: near resonance, a fraction of information comes back to the system, different input states remain distinguishable even at long times and thus no equilibrium state is found. For large detunings, non-Markovianity is built up at short times, while different input states converge towards a long-time equilibrium. On the other hand, for intermediate values of the detuning [i.e., for Δ in the Markovianity region of Fig. 2(b)] and $\lambda = 0$, there is no back-flow. Even for $\lambda \neq 0$, the fraction of information that comes back is quite small. The picture changes when J increases, the evolution becoming increasingly non-Markovian and the role of the anisotropy being fully reversed: $\lambda = 0$ implies a larger $\mathcal{R}(\tau)$, persisting for longer times at resonance.

V. CONCLUDING REMARKS

We have used a measure of non-Markovianity to show the possibility to control the dynamics of an open quantum system coupled to many independent decohering channels. We have highlighted the key role played by the detuning and the degree of anisotropy of the system-environment coupling: both can be used to explore a rich non-Markovianity phase diagram, where qualitatively different scaling laws with the number of decoherence channels are found. The ability to switch from a Markovian to a non-Markovian regime by

means of a local parameter could be used to prepare a quantum system in a desired state: indeed the Markovian character of processes can be employed for state engineering and information manipulation [14,15]. On other hand, while the formation of a steady entangled state is supported by non-Markovianity, a purely Markovian dynamics produces separable steady states [13].

ACKNOWLEDGMENTS

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APPENDIX A

The total Hamiltonian of the spin-star system $\hat{H} = \hat{H}_0 + \hat{H}_S + \hat{H}_B$ consists of a few contributions. The first one is the system's free energy (we take units such that $\hbar = 1$ throughout the paper) $\hat{H}_0 = \sum_{j=0}^{N} \epsilon_j \hat{\sigma}_j^z + \sum_{j=1}^{N} \sum_k \omega_k \hat{b}_{k,j}^\dagger \hat{b}_{k,j}$, describing the free evolution of $N+1$ spin-1/2 particles [here, $\hat{\sigma}_j^s$ is the s-Pauli matrix of spin j $(s = x,y,z)$], each with transition frequency ϵ_j between spin states $|-\rangle_j$ and $|+\rangle_j$. The second term in $\hat{H}_0$ describes the energy of N sets of M_j harmonic modes (one set per peripheral spin of the star) with creation (annihilation) operators $\hat{b}_{k,j}^\dagger$ $(\hat{b}_{k,j})$ which satisfy the commutation relations $[\hat{b}_{k,j}, \hat{b}_{k',l}^\dagger] = \delta_{kk'}\delta_{jl}$. The central and peripheral spins are coupled by $\hat{H}_S$, whose explicit form will be specified later on. Each peripheral spin interacts with its own bath as $\hat{H}_B = \sum_{j=1}^{N} \hat{H}_{B,j}$, where

$$\hat{H}_{B,j} = \sum_k (g_{k,j} \hat{\sigma}_j^+ \hat{b}_{k,j} + g_{k,j}^* \hat{\sigma}_j^- \hat{b}_{k,j}^\dagger) \tag{A1}$$

with $\hat{\sigma}_j^\pm = (\hat{\sigma}_j^x \pm i\hat{\sigma}_j^y)/2$. We assume that the their local bath induce a Markovian dynamics of the peripheral spins and take uniform couplings, so that the evolution of the central spin is ruled by

$$\partial_t \rho_0(t) = \mathrm{Tr}_S \left\{ -i[\hat{H}, \rho(t)] + \sum_{j=1}^{N} \hat{\mathcal{L}}_j[\rho(t)] \right\}. \tag{A2}$$

APPENDIX B

Here we provide an alternative solution to the dynamics of the system. In the interaction picture with respect to $\hat{H}_0$ the Schrödinger equation reads

$$\partial_t |\Psi(t)\rangle = -i \hat{H}_I(t) |\Psi(t)\rangle, \tag{B1}$$

where the interaction Hamiltonian is given by

$$\begin{aligned} H_I(t) = {} & J \sum_{j=1}^{N} [\sigma_0^+(t)\sigma_j^-(t) + \sigma_0^-(t)\sigma_j^+(t)] \\ & + \sum_{j=1}^{N} \sum_k [g_k \sigma_+^j(t) b_k(t) + g_k^* \sigma_-^j(t) b_k^\dagger(t)] \end{aligned} \tag{B2}$$

with

$$\begin{aligned} \sigma_j^\pm(t) &= \sigma_j^\pm e^{\pm i\epsilon_j t} (j = 0, \ldots, N) \\ b_k(t) &= g_k b_k e^{-i\omega_k t} \\ b_k^\dagger(t) &= g_k b_k^\dagger e^{+i\omega_k t} \end{aligned} \tag{B3}$$

The operator $N = \sum_j [\sigma_j^z + (\sum_k b_k^\dagger b_k)_j]$ counts the number of excitations in the system and commutes with the total Hamiltonian H, so that any initial state of the form

$$\begin{aligned} |\Psi(0)\rangle = {} & (c_0|-\rangle^0 + c_1(0)|+\rangle^0)|\mathbf{0}\rangle^S|\mathbf{0}\rangle^B \\ & + \sum_{j=1}^{N} c_j(0)|-\rangle^0|\mathbf{j}\rangle^S|\mathbf{0}\rangle^B \\ & + \sum_{j=1}^{N} \sum_k c_{kj}(0)|-\rangle^0|\mathbf{0}\rangle^S|\mathbf{k}\rangle_j^B \end{aligned}$$

evolves after time t into the state

$$\begin{aligned} |\Psi(t)\rangle = {} & (c_0|-\rangle^0 + c_1(0)|+\rangle^0)|\mathbf{0}\rangle^S|\mathbf{0}\rangle^B + \sum_{j=1}^{N} c_j(t)|-\rangle^0|\mathbf{j}\rangle^S|\mathbf{0}\rangle^B \\ & + \sum_{j=1}^{N} \sum_k c_{kj}(t)|-\rangle^0|\mathbf{0}\rangle^S|\mathbf{k}\rangle_j^B, \end{aligned} \tag{B4}$$

where the state $|\mathbf{0}\rangle^S$ denotes the product state $\otimes_{j=1}^{N}|-\rangle^j$ and $|\mathbf{j}\rangle^S = \sigma_j^+|\mathbf{0}\rangle^S$ for the sites on the star; $|\mathbf{0}\rangle^B$ is the vacuum state of all the reservoirs, and $|\mathbf{k}\rangle_j^B = b_k^\dagger|0\rangle_j$ the state with one particle in mode k in the jth reservoir. The amplitude c_0 is constant in time because of $H_I(t)|-\rangle^0|\mathbf{0}\rangle^S|\mathbf{0}\rangle^B = 0$.

Substituting Eq. (B4) into the Schrödinger equation (B1) one finds

$$\begin{aligned} \frac{d}{dt}c_1(t) &= -iJ \sum_{j=1}^{N} c_j(t) e^{i(\epsilon_0 - \epsilon_j)t}, \\ \frac{d}{dt}c_j(t) &= -iJc_1(t)e^{-i(\epsilon_0-\epsilon_j)t} - i\sum_k c_{kj}(t) g_{kj} e^{i(\epsilon_j - \omega_{kj})t}, \\ \frac{d}{dt}c_{kj}(t) &= -ig_{kj}^* c_j(t) e^{-i(\epsilon_i - \omega_{kj})t}. \end{aligned} \tag{B5}$$

We assume in the following that $c_j(0) = c_{kj}(0) = 0$. This means that the two level systems on the star are in the $|-\rangle$ state and that each environment is in the vacuum state initially.

The total initial state is given by the product state

$$|\Psi(0)\rangle = (c_0|-\rangle^0 + c_1(0)|+\rangle^0)|\mathbf{0}\rangle^B|\mathbf{0}\rangle^S. \tag{B6}$$

Formally integrating Eq. (B5) and substituting into Eq. (B5) one obtains the system for the amplitude $c_1(t), c_j(t)$,

$$\begin{aligned} \frac{d}{dt}c_1(t) &= -iJ \sum_{j=1}^{N} c_j(t) e^{i(\epsilon_0 - \epsilon_j)t} \\ \frac{d}{dt}c_j(t) &= -iJc_1(t)e^{-i(\epsilon_0-\epsilon_j)t} \\ & \quad - \int_0^t c_j(t_1) \sum_k |g_{kj}|^2 e^{i(\epsilon_j - \omega_{kj})(t-t_1)} dt_1. \end{aligned} \tag{B7}$$

We can define the kernels $f_j(t-t_1)$ describing the two-point correlation function of each reservoir, which are the Fourier transform of the respective environmental spectral density

$$f_j(t-t_1)=\sum_k |g_{kj}|^2 e^{i(\epsilon_j-\omega_{kj})(t-t_1)}. \tag{B8}$$

For the moment, we do not make any restrictive hypothesis on the form of f_j, so that our results will be valid for an environment with a generic spectral density. In order to solve the system above it is convenient to pass in the Laplace domain:

$$\begin{aligned} s\tilde{c_1}[s] &= c_1(0) - iJ\sum_{j=1}^{N}\tilde{c_j}[s+i(\epsilon_0-\epsilon_j)] \\ s\tilde{c_j}[s] &= -iJ\tilde{c_1}[s-i(\epsilon_0-\epsilon_j)] - \tilde{c_j}[s]\tilde{f_j}[s]. \end{aligned} \tag{B9}$$

Solving the second of Eq. (B9) respect to $\tilde{c_j}[s]$,assuming that all the reservoirs are the same ($f_j(t)=f(t)\forall j$), and substituting in the first we get

$$\tilde{c_1}[s]=c_1(0)\frac{s-i\Delta-f[s-i(\epsilon_0-\epsilon)]}{s^2-is(\epsilon_0-\epsilon)-isf[s-i(\epsilon_0-\epsilon)]+J^2N},$$

where $\Delta=(\epsilon_0-\epsilon)$ ($\epsilon_j=\epsilon\ \ \forall\ j$).

To specify the model, but still retaining a general enough description, we consider a Lorentzian spectral density for each bath (which gives rise to an exponentially decaying correlation function):

$$J(\omega)=\frac{1}{2\pi}\frac{\gamma\lambda^2}{(\epsilon_j-\delta-\omega)^2+\lambda^2}. \tag{B10}$$

Here $\delta=\epsilon_j-\omega_c$ is the detuning of the center frequency of the bath ω_c and the frequency of the two-level system ϵ_j, the parameter λ defines the spectral width of the environment, which is associated with the reservoir correlation time by the relation $\tau_B=\lambda^{-1}$ and the parameter γ is related to the relaxation time scale τ_R by the relation $\tau_R=\gamma^{-1}$.

We will consider $\delta=0$, and in this case we may distinguish between the Markovian and the non-Markovian regimes (for the dynamics of the environmental spins themselves) using the ratio of γ and λ: $\gamma<\frac{\lambda}{2}$ gives a Markovian regime and $\gamma>\frac{\lambda}{2}$ corresponds to non-Markovian regime.

Substituting in Eq. (B10) and anti-transforming we have $c_1(t)=c_1(0)G(t)$ with

$$G(t)=c_1(0)\frac{\sum_{i=1}^{3}(-1)^{i-1}e^{t\alpha_i}(\alpha_j-\alpha_k)\left[\delta_i^2+(\delta_i+\gamma/2)\lambda\right]}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)(\alpha_1-\alpha_3)}, \tag{B11}$$

where $\delta_i=\alpha_i-i\Delta$, $j,\ k=1,2,3$ and for $j<k$. Here, α_i's are the roots of the equation

$$\begin{aligned} p(s) = s^3 &+ (2i\Delta+\lambda)s^2 + (J^2N+\Delta^2+i\Delta\lambda+\lambda\gamma/2)s \\ &+ J^2N(i\Delta+\lambda). \end{aligned} \tag{B12}$$

Already at this point, it is evident how the only effects of increasing N is to redefine the coupling constant J.

The solution of the Schrödinger equation of the total system with initial states of the form (B6) lies in the sector of the Hilbert space corresponding to zero or one excitations.

We can construct the exact dynamical map describing the time-evolution of the reduced density matrix of the central spin which is given by

$$\rho(t)=\mathrm{Tr}_{S+B}\{|\Psi(t)\rangle\langle\Psi(t)|\}=\begin{pmatrix}\rho_{11}(t) & \rho_{10}(t)\\ \rho_{01}(t) & \rho_{00}(t)\end{pmatrix}, \tag{B13}$$

where $\rho_{ij}(t)=\langle i|\rho(t)|j\rangle$ for $i,j=0,1$. Using Eqs. (B4) and (B11) we find

$$\rho_{11}(t)=1-\rho_{00}(t)=|c_1(0)G(t)|^2, \tag{B14}$$

$$\rho_{10}(t)=\rho_{01}^*(t)=c_0^*c_1(0)G(t). \tag{B15}$$

The optimization of the initial states in Eq. (7) obtains the maximally possible non-Markovianity of a particular quantum evolution.

In our case, the maximization is achieved by pure states, thus we choose as initial states for Eq. (B6)

$$|\Psi_1(0)\rangle=(\cos(\theta_1)|-\rangle+e^{i\phi_1}\sin(\theta_1)|+\rangle)|\mathbf{0}\rangle_B|\mathbf{0}\rangle_S, \tag{B16}$$

$$|\Psi_2(0)\rangle=(\cos(\theta_2)|-\rangle+e^{i\phi_2}\sin(\theta_2)|+\rangle)|\mathbf{0}\rangle_B|\mathbf{0}\rangle_S. \tag{B17}$$

With these states, the trace distance takes the form

$$\tfrac{1}{2}|G(t)|\sqrt{|G(t)|^2[\cos(\theta_1)-\cos(\theta_2)]^2+[\sin(\theta_1)+\sin(\theta_2)]^2}, \tag{B18}$$

where we used the fact that, since H is invariant under rotations along the z axis, the maximum is obtained for $\phi_1-\phi_2=\pi$.

[1] F. Caruso, A. W. Chin, A. Datta, S. F. Huelga, and M. B. Plenio, Phys. Rev. A **81**, 062346 (2010); M. B. Plenio and S. F. Huelga, New J. Phys. **10**, 113019 (2008); M. Mohseni *et al.*, J. Chem. Phys. **129**, 174106 (2008); A. Olaya-Castro, C. F. Lee, F. F. Olsen, and N. F. Johnson, Phys. Rev. B **78**, 085115 (2008); P. Rebentrost *et al.*, New J. Phys. **11**, 033003 (2009); F. Caruso *et al.*, J. Chem. Phys. **131**, 105106 (2009); M. Thorwart *et al.*, Chem. Phys. Lett. **478**, 234 (2009); P. Rebentrost, R. Chakraborty, and A. Aspuru-Guzik, J. Chem. Phys. **131**, 184102 (2009); S. Lorenzo, F. Plastina, and M. Paternostro, Phys. Rev. A **84**, 032124 (2011).

[2] P. Rebentrost and A. Aspuru-Guzik, J. Chem. Phys. **134**, 101103 (2011).

[3] M. M. Wolf, J. Eisert, T. S. Cubitt, and J. I. Cirac, Phys. Rev. Lett. **101**, 150402 (2008).

[4] Á. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010).

[5] H.-P. Breuer, E. M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).

[6] X.-M. Lu, X. Wang, and C. P. Sun, Phys. Rev. A **82**, 042103 (2010).

[7] B.-H. Liu *et al.*, Nat. Phys. **7**, 931 (2011); A. Chiuri *et al.*, Sci. Rep. **2**, 968 (2012).

[8] T. J. G. Apollaro, C. Di Franco, F. Plastina, and M. Paternostro, Phys. Rev. A **83**, 032103 (2011).
[9] P. Haikka, S. McEndoo, G. De Chiara, G. M. Palma, and S. Maniscalco, Phys. Rev. A **84**, 031602(R) (2011).
[10] P. Haikka, J. Goold, S. McEndoo, F. Plastina, and S. Maniscalco, Phys. Rev. A **85**, 060101(R) (2012).
[11] F. Reinhard *et al.*, Phys. Rev. Lett. **108**, 200402 (2012).
[12] A. Ardavan, O. Rival, J. J. L. Morton, S. J. Blundell, A. M. Tyryshkin, G. A. Timco, and R. E. P. Winpenny, Phys. Rev. Lett. **98**, 057201 (2007); A. Candini *et al.*, *ibid.* **104**, 037203 (2010).
[13] S. F. Huelga, Á. Rivas, and M. B. Plenio, Phys. Rev. Lett. **108**, 160402 (2012).
[14] B. Kraus, H. P. Buchler, S. Diehl, A. Kantian, A. Micheli, and P. Zoller, Phys. Rev. A **78**, 042307 (2008).
[15] F. Verstraete, M. M. Wolf, and J. I. Cirac, Nat. Phys. **5**, 633 (2009).
[16] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2007).
[17] H. J. Briegel and B. G. Englert, Phys. Rev. A **47**, 3311 (1993).
[18] H. P. Breuer, D. Burgarth, and F. Petruccione, Phys. Rev. B **70**, 045323 (2004).