Quantum network teleportation for quantum information distribution and concentration

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We investigate the schemes of quantum network teleportation for quantum information distribution and concentration, which are essential in quantum cloud computation and the quantum internet. In those schemes, with the prior shared entanglement in the quantum network, the cloud can send simultaneously identical unknown quantum states to clients located in different places. Additionally, with the same entanglement resource, these clients can concentrate their states to the cloud to reconstruct the original state. The number of clients can be beyond the number of identical quantum states intentionally being sent; this quantum network teleportation can make sure that the quantum states' distribution is optimal in the sense that the fidelity achieves the upper bound. These schemes facilitate the quantum information distribution and concentration in quantum networks in the framework of quantum cloud computation. Potential applications in time synchronization and the photonic implementation of those schemes are discussed.

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I. INTRODUCTION

In the past decades, much progress has been made in the fields of quantum information science and quantum physics. Recently, the quantum network and its extension, the quantum internet, have been attracting a great deal of interest [1–5]. The quantum networks are constituted by quantum nodes where quantum information can be generated, processed, and stored locally. Those nodes are linked by quantum channels and classical channels. With quantum networks, the quantum cloud computation (QCC) seems emergent. In QCC, the constituent quantum nodes may only have moderate capabilities in quantum information processing and some central quantum computers have the full quantum computational power. So the clients at some quantum nodes with limited power can finish all quantum information tasks with the help of the quantum servers, which are assumed to be the cloud. In addition, it is also shown that quantum computers can provide unconditional security in data processing by the quantum blind computation, as proven theoretically and demonstrated experimentally in Refs. [6,7].

An essential feature of a quantum network is that the quantum nodes are linked by both quantum channels and classical channels so that the entanglement can be distributed among them, and thus a fully quantum network has an exponentially large state space. In case there is a largest size attainable for the state space of an individual quantum node, the quantum network provides the infrastructure to link such quantum nodes together as a fully quantum network [1]. Then this full quantum realm with available classical communication can process various quantum information tasks which may not be accomplished if restricted to several local quantum nodes provided they are only classically linked. The quantum network has the capabilities for quantum computation even with a distributed style, such as the blind quantum computation [6], the unconditional secure quantum communication [8], quantum metrology [9,10], and the simulation of quantum many-body systems [11–14]. Those exciting opportunities provide the motivation to examine research related to the quantum network protocols and the physical implementations.

One of the most fundamental functions of a quantum network should be the quantum information transportation from site to site with high fidelity. However, the inevitable decoherence and lossy of flying qubits may induce high errors in the direct transportation of quantum information. Fortunately, quantum information science also provides teleportation for state transportation with a prior shared entanglement [15]. The reduction of entanglement caused by decoherence and lossy of quantum channel, however, can be overcome by various schemes in quantum information science, such as purification and quantum repeaters [16–19]. With a maximally entangled state resource, a quantum state can thus be teleported perfectly from one site to another site only if the "local" quantum operations are perfect. Now we are wondering whether we can have a quantum network teleportation, i.e., many states can be teleported simultaneously across the quantum network with a reduced consumption of the precious entanglement resource. The one-to-many quantum network teleportation is already studied in the framework of telecloning for qubit [20] and qudit [21] and as the programmable processor [22]. In this work, we will study systematically this problem for the general distribution and concentration of quantum information across quantum networks.

II. QUANTUM NETWORK TELEPORTATION FOR QUANTUM INFORMATION DISTRIBUTION

For QCC in a quantum network, we suppose that the cloud tries to teleport N identical but unknown d-level quantum states, i.e., qudits, to N remote clients located in spatially separated quantum nodes. This can be realized by standard teleportation and each qudit is teleported independently. However, we may find an improved method, as presented in the following. With prior shared entanglement, our network

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teleportation protocol is that the cloud performs coherent quantum operation, and each client can recover the qudit locally with the help of the classical information. The case that the cloud tries to distribute quantum information to more than N remote clients is almost the same, and we thus present those results in a unified way.

Suppose N identical qudits $|\varphi\rangle^{\otimes N}$ are in X possessed by the cloud, and M spatially separated clients who will receive the qudit are located in quantum nodes $C_1,C_2,\ldots,C_M$ denoted as C with $M \geqslant N$. Due to the no-cloning theorem [23], when M is strictly larger than N, each retrieved qudit will not be exactly equal to $|\varphi\rangle$, but our protocol is to achieve the optimal fidelity. The cloud at port P first shares entanglement with clients C as a resource; instead of the Bell measurement in standard teleportation, the cloud performs a positive operator-valued measure (POVM) on XP, which is initialed for the qubit case in Ref. [24]. By using the local recovery unitary operators (LRUOs) according to the publicly announced POVM results, each client can obtain their optimal state which is consistent with the optimal quantum cloning [25–31]. The entangled state used in this network teleportation takes the form

$$|\xi\rangle_{PAC} = \frac{1}{\sqrt{d[M]}}\sum_{\vec{m}}^{M}|\vec{m}\rangle_{PA}|\vec{m}\rangle_C, \tag{1}$$

where $d[M] = C^M_{d-1+M}$ is the dimension of the symmetric space $\mathcal{H}_+^{\otimes M}$, which is a subspace of the M-fold Hilbert space $\mathcal{H}^{\otimes M}$, and $|\vec{m}\rangle \equiv |m_0,m_1,\ldots,m_{d-1}\rangle$ is a completely symmetric normalized state with m_j states that are $|j\rangle$; the constraint in summation is $\sum_j m_j = M$, and A represents ancillary states and can be $M-N$ qudits. There is an explicit map between this entangled state and the direct product of M maximally entangled states in [31] presented as $|\xi\rangle_{PAC} = \frac{d^{M/2}}{\sqrt{d[M]}}[\mathbb{I}^{\otimes M}_{PA} \otimes S^M_C]|\Phi^+\rangle^{\otimes M}$, where $|\Phi^+\rangle = \frac{1}{\sqrt{d}}\sum_j |jj\rangle$, $\mathbb{I}$ is the identity operator on Hilbert space $\mathcal{H}$, and $S^M = \sum_{\vec{m}} |\vec{m}\rangle\langle\vec{m}|$ is the symmetric projector that maps states in $\mathcal{H}^{\otimes M}$ onto $\mathcal{H}^{\otimes M}_+$. With the help of a result that the symmetric state $|\vec{m}\rangle$ of M qudits can be divided into two symmetric states of N qudits and $(M-N)$ qudits [31], we rewrite the shared maximally entangled state as

$$\begin{aligned}|\xi\rangle_{PAC} &= \frac{1}{\sqrt{d[N]}}\sum_{\vec{n}}^{N}|\vec{n}\rangle_P\left[\eta\sum_{\vec{m}}^{m_j\geqslant n_j}\sqrt{\prod_j\frac{m_j!}{(m_j-n_j)!n_j!}}\right.\\ &\quad\left.\times|\vec{m}-\vec{n}\rangle_A|\vec{m}\rangle_C\right]\\ &= \frac{1}{\sqrt{d[N]}}\sum_{\vec{n}}^{N}|\vec{n}\rangle_P\left[\eta\sum_{\vec{k}}^{M-N}\sqrt{\prod_j\frac{(n_j+k_j)!}{k_j!n_j!}}\right.\\ &\quad\left.\times|\vec{k}\rangle_A|\vec{n}+\vec{k}\rangle_C\right] = \frac{1}{\sqrt{d[N]}}\sum_{\vec{n}}^{N}|\vec{n}\rangle_P|\phi_{\vec{n}}\rangle_{AC},\end{aligned}$$

where the normalization coefficient is $\eta = \sqrt{\frac{1}{C^N_M}\frac{d[N]}{d[M]}}$. The pure state of qudit is $|\varphi\rangle = \sum_j x_j|j\rangle$, $\sum_j |x_j|^2 = 1$. So, N identical qudits $|\psi\rangle_X = |\varphi\rangle^{\otimes N}$ belong to the symmetric subspace $\mathcal{H}^{\otimes N}_+$

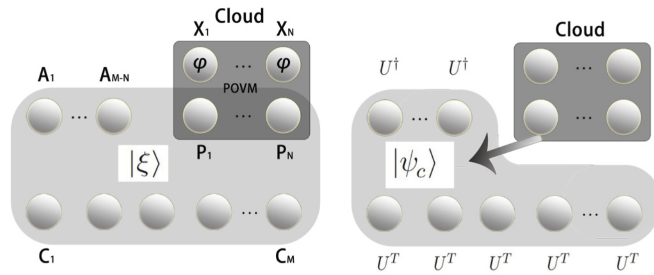

FIG. 1. Procedures of information distribution. The cloud and spatially separated clients share the entanglement resource. POVM is performed and the classical information is sent to clients who can recover their states locally.

as follows:

$$|\psi\rangle_X = \sum_{\vec{n}}^{N}\left(\sqrt{N!}\prod_j\frac{x_j^{n_j}}{\sqrt{n_j!}}\right)|\vec{n}\rangle = \sum_{\vec{n}}^{N} y_{\vec{n}}|\vec{n}\rangle. \tag{2}$$

In our scheme, the cloud performs a POVM as follows:

$$\int d\vec{x}\,F_{\vec{x}} = \int d\vec{x}\,\lambda(\vec{x})|\chi(\vec{x})\rangle\langle\chi(\vec{x})| = S^N_X\otimes S^N_P, \tag{3}$$

$$|\chi(\vec{x})\rangle = \left[\mathbb{I}^{\otimes N}_X\otimes U(\vec{x})^{\otimes N}_P\right]\frac{1}{\sqrt{d[N]}}\sum_{\vec{n}}^{N}|\vec{n}\rangle_X|\vec{n}\rangle_P, \tag{4}$$

where $S^N\otimes S^N$ is the identity operator in the space $\mathcal{H}^{\otimes N}_+\otimes\mathcal{H}^{\otimes N}_+$, $U(\vec{x})$ is an element of the compact Lie group SU(d), and the vector $\vec{x}$ consisting of (d^2-1) parameters determines the unitary matrix.

Interestingly, the order of the symmetric projector S^M and the unitary transformation $U(\vec{x})^{\otimes M}$ can be exchanged, $U(\vec{x})^{\otimes N}S^M = S^M U(\vec{x})^{\otimes M}$. We also have the property $(U\otimes U^*)|\Phi^+\rangle = |\Phi^+\rangle$. Representing the network teleportation, the total system can be expressed as

$$\begin{aligned}|\psi\rangle_X|\xi\rangle_{PAC} &= \sum_{\vec{x}}\lambda(\vec{x})|\chi(\vec{x})\rangle_{XP}\langle\chi(\vec{x})|\psi\rangle_X|\xi\rangle_{PAC}\\ &= \frac{1}{d[N]}\sum_{\vec{x}}\lambda(\vec{x})|\chi(\vec{x})\rangle_{XP}\\ &\quad\times\left[U_A^{\dagger\otimes(M-N)}\otimes U_C^{T\otimes M}\right]^\dagger|\psi_c\rangle_{AC},\end{aligned} \tag{5}$$

where the ultimate output state $|\psi_c\rangle_{AC} = \sum^N_{\vec{n}} y_{\vec{n}}|\phi_{\vec{n}}\rangle_{AC}$ in our scheme is optimal, which is equivalent to the result of optimal universal cloning [29–31]. The procedures of quantum information distribution are described in Fig. 1. Note that $U^T(\vec{x})$ is the LRUO performed by each receiver locally, while $U^\dagger(\vec{x})$ is performed by each ancillary depending on POVM results $\vec{x}$, where superscript T means transposition.

In order to show that Eq. (3) is true, some knowledge of group theory is necessary. According to the Theorem of Weyl Reciprocity [32], the order of an arbitrary permutation P_α and the unitary transformation $U^{\otimes N}$ can be exchanged, and the subspace $\mathcal{Y}^{[\lambda]}_\mu\mathcal{H}^{\otimes N}$ is invariant under transformation $U^{\otimes N}$, where $\mathcal{Y}^{[\lambda]}_\mu$ is a standard Young operator corresponding to the standard Young tableau with N boxes. Also, the symmetric projection $S^N = \frac{1}{N!}\sum_\alpha P_\alpha$ is equal to the standard Young operator $\frac{1}{N!}\mathcal{Y}^{[N]}$; thus we have

$$U(\overrightarrow{x})^{\otimes N}S^N = S^N U(\overrightarrow{x})^{\otimes N}, \tag{6}$$

$$U(\overrightarrow{x})^{\otimes N}|\overrightarrow{n_1}\rangle = \sum_{\overrightarrow{n_2}} D_{\overrightarrow{n_2},\overrightarrow{n_1}}(\overrightarrow{x})|\overrightarrow{n_2}\rangle, \tag{7}$$

where $D(\overrightarrow{x})$ is a unitary representation of Lie group SU(d). In the group theory, there is a theorem [32] that states that if $\mathcal{Y}^{[\lambda]}_{\mu}$ is a standard Young operator, then an irreducible representation of group SU(d) will be induced when $U(\overrightarrow{x})^{\otimes N}$ operates on invariant subspace $\mathcal{Y}^{[\lambda]}_{\mu}\mathcal{H}^{\otimes N}$. So, $D(\overrightarrow{x})$ is an irreducible representation of SU(d). Then, according to Schur's lemmas and the orthogonality relations [32,33], we get

$$\frac{1}{d[N]}\int d\overrightarrow{x}\,\lambda(\overrightarrow{x})D_{\overrightarrow{n_1},\overrightarrow{n_2}}(\overrightarrow{x})D^*_{\overrightarrow{n_3},\overrightarrow{n_4}}(\overrightarrow{x}) = \delta_{\overrightarrow{n_1},\overrightarrow{n_3}}\delta_{\overrightarrow{n_2},\overrightarrow{n_4}}. \tag{8}$$

This equation ensures the integral of the projectors $F_{\overrightarrow{x}}$ is equal to the identity in the space $\mathcal{H}_+^{\otimes N}\otimes\mathcal{H}_+^{\otimes N}$. In the special case when we know the analytical expression of the unitary matrix $U(\overrightarrow{x})$ and its irreducible representation $D(\overrightarrow{x})$, an appropriate finite POVM can be constructed, and then the integral degenerates into summation. It is important to construct the finite POVM because it can be experimentally realizable. Some details for the qubit case will be discussed in the next section.

Thus, by network teleportation, N identical qudits are distributed simultaneously to M spatially separated clients in the quantum networks. If $M = N$, each client can retrieve perfectly this qudit; if $M > N$, each retrieved qudit is optimal. The amount of entanglement used is $\log d[M]$, which is less than $M\log d$ if standard teleportation is performed repeatedly. One may realize that this scheme can distribute arbitrary symmetric state $|\psi\rangle_X = \sum_{\overrightarrow{n}}\alpha_{\overrightarrow{n}}|\overrightarrow{n}\rangle$ in the cloud to remote clients optimally. When $N = 1$, the POVM will reduce to the standard Bell-type measurement [15,20,21].

III. QUANTUM NETWORK TELEPORTATION OF QUBITS

The explicit and finite POVM can be found for the qubit case. In $d = 2$, an arbitrary unitary operator can be expressed by using three Euler angles α, β, and γ, $U(\overrightarrow{x}) = U(\alpha,\beta,\gamma)$, as shown in Refs. [32,33],

$$U(\alpha,\beta,\gamma) = \begin{bmatrix} \cos\frac{\beta}{2}e^{i(\alpha+\gamma)/2} & \sin\frac{\beta}{2}e^{-i(\alpha-\gamma)/2} \\ -\sin\frac{\beta}{2}e^{i(\alpha-\gamma)/2} & \cos\frac{\beta}{2}e^{-i(\alpha+\gamma)/2}\end{bmatrix}. \tag{9}$$

The symmetric state $|\overrightarrow{n}\rangle$ is denoted as $|JM\rangle$, where $|JM\rangle$ denotes that $J-M$ states are $|0\rangle$ and $J+M$ states are $|1\rangle$,($J = N/2$, $M = \{-J,-J+1,\dots,J-1,J\}$). And the irreducible representation is given by the following analytical form [32,33]:

$$\begin{aligned}
&U(\alpha,\beta,\gamma)^{\otimes N}|JM\rangle \\
&\quad= \sum_{M'} e^{-i(M\alpha+M'\gamma)}d^J_{M',M}(\beta)|JM'\rangle, \\
&\quad\times\sum_{\nu}(-1)^{\nu}\frac{[(J+M')!(J-M')!(J+M)!(J-M)!]^{1/2}}{(J+M'-\nu)!(J-M-\nu)!\nu!(\nu+M-M')!} \\
&\quad\times\left(\cos\frac{\beta}{2}\right)^{2J+M'-M-2\nu}\left(\sin\frac{\beta}{2}\right)^{2\nu-M'+M} = d^J_{M',M}(\beta).
\end{aligned}$$

In order to ensure that Eq. (8) is satisfied, we choose the following parameters to construct a finite POVM:

$$\begin{aligned}
\alpha &= j\frac{2\pi}{N+1}, \quad j = 0,1,\dots,N, \\
\gamma &= j'\frac{2\pi}{N+1}, \quad j' = 0,1,\dots,N, \\
\lambda(\alpha,\beta,\gamma) &= \lambda(\beta), \\
\sum_{\beta}\lambda(\beta)\big[d^J_{M',M}(\beta)\big]^2 &= \frac{1}{N+1}.
\end{aligned} \tag{10}$$

The next task is to simplify the equations which determine β and $\lambda(\beta)$, and we find that the result is surprisingly concise. It is important to obtain the simplified expression

$$\begin{aligned}
\big[d^J_{M',M}(\beta)\big]^2 &= \sum_{\mu,\nu'}(-1)^{\mu+\nu'+M'-M}C^{\mu}_{J+M'}C^{\mu+M-M'}_{J-M'} \\
&\quad\times C^{\nu'}_{J+M}C^{\nu'+M'-M}_{J-M}\left(\cos\frac{\beta}{2}\right)^{2N-2(\mu+\nu')} \\
&\quad\times\left(\sin\frac{\beta}{2}\right)^{2(\mu+\nu')}.
\end{aligned} \tag{11}$$

(i) We can choose $M' = J$, $M = \{-J,-J+1,\dots,J-1,J\}$, then $[d^J_{J,M}(\beta)]^2 = C^{J-M}_N(\cos\frac{\beta}{2})^{2N-2(J-M)}(\sin\frac{\beta}{2})^{2(J-M)}$, and therefore

$$\begin{aligned}
\sum_{\beta}\lambda(\beta)\big[d^J_{M',M}(\beta)\big]^2 &= \frac{1}{N+1} \\
&\Rightarrow \sum_{\beta}\lambda(\beta)C^i_N\left[\cos\frac{\beta}{2}\right]^{2(N-i)}\left[\sin\frac{\beta}{2}\right]^{2i} \\
&= \frac{1}{N+1}.
\end{aligned} \tag{12}$$

(ii) Using Schur's Lemma and the orthogonality relations [32,33], we have

$$\int_0^{\pi} d\beta\sin\beta\big[d^J_{M',M}(\beta)\big]^2 = \frac{2}{2J+1}. \tag{13}$$

And according to the well-known Euler integral, B function and Γ function, we get

$$\begin{aligned}
&\int_0^{\pi} d\beta\sin\beta\left(\cos\frac{\beta}{2}\right)^{2N-2(\mu+\nu')}\left(\sin\frac{\beta}{2}\right)^{2(\mu+\nu')} \\
&\quad= \frac{2}{N+1}\times\frac{1}{C^{\mu+\nu'}_N}.
\end{aligned} \tag{14}$$

Thus,

$$\begin{aligned}
&\sum_{\mu,\nu'}\frac{(-1)^{\mu+\nu'+M'-M}}{C^{\mu+\nu'}_N}C^{\mu}_{J+M'}C^{\mu+M-M'}_{J-M'}C^{\nu'}_{J+M}C^{\nu'+M'-M}_{J-M} \\
&\quad= 1, \\
&\quad\sum_{\beta}\lambda(\beta)C^i_N\left[\cos\frac{\beta}{2}\right]^{2(N-i)}\left[\sin\frac{\beta}{2}\right]^{2i} \\
&\quad= \frac{1}{N+1} \Rightarrow \sum_{\beta}\lambda(\beta)[d^J_{M',M}(\beta)]^2 = \frac{1}{N+1}.
\end{aligned} \tag{15}$$

TABLE I. Values for β and weight factors $\lambda(\beta)$ to construct a finite POVM.

	$N=1$		$N=2$			$N=3$			
$\beta/2$	0	$\pi/2$	0	$\pi/4$	$\pi/2$	0	$\pi/6$	$\pi/3$	$\pi/2$
$\lambda(\beta)$	1/2	1/2	1/6	2/3	1/6	1/18	4/9	4/9	1/18

So, we have proven that the concise independent equations which determine β and $\lambda(\beta)$ are

$$\sum_{\beta}\lambda(\beta)C_N^i\left[\cos\frac{\beta}{2}\right]^{2(N-i)}\left[\sin\frac{\beta}{2}\right]^{2i}=\frac{1}{N+1}, \tag{16}$$

where $i=0,1,\ldots,N$. We can choose the parameters $\beta=j\frac{\pi}{N}$ $(j=0,1,\ldots,N)$, and it is convenient to find the weight factors $\lambda(\beta)$ from the linear equations (16). Given these parameters, we have $\dim\mathcal{H}_+^{\otimes N}\otimes\mathcal{H}_+^{\otimes N}=(N+1)^2<\sharp(F_{\vec{x}})=(N+1)^3$, which is different from the results in Ref. [24]. For $N=1,2,3$, we explicitly present these factors in Table I. If we use standard teleportation N times, then $2N$ bits of classical information and N ebits of entanglement are required. Our scheme requires $3N\log_2(N+1)$ bits of classical information and $\log_2(N+1)$ ebits of entanglement; the precious entanglement resource is saved.

IV. QUANTUM INFORMATION CONCENTRATION IN THE NETWORKS

Remote quantum information concentration is a reverse process of the quantum information distribution. It begins with a situation where the spatially separated clients hold the clones that the cloud distributed. The aim is to concentrate the distributed quantum information by a network teleportation scheme. The process can be done, trivially for example, by repeatedly using standard teleportation so that all states are teleported to the cloud, then a reverse unitary transformation on all states is performed to recover the original state. The drawbacks of this method are that lots of entanglement resources are needed and a coherent operation on a large Hilbert space including all states is necessary. It is already shown that bound entanglement can be used as a resource in quantum information concentration [34], and some different schemes are also proposed in some contexts [35–38].

Next is our general network teleportation scheme for quantum information concentration, which can be accomplished by two different methods. By distribution state $|\psi\rangle_X=|\varphi\rangle=\sum_j x_j|j\rangle$ to M clients $C_1,\ldots,C_M$ with $M-1$ ancillary states $A_1,\ldots,A_{M-1}$, we have [30]

$$|\psi_c\rangle_{AC}=\sqrt{\frac{d}{d[M]}}\left(\sum_j\alpha_{jP}\langle j|\right)\left(\sum_{\vec{m}}^{M}|\vec{m}\rangle_{PA}|\vec{m}\rangle_C\right).$$

First, we suppose M maximally entangled states $|\Phi^+\rangle$ are shared between pairs of ancillary states and clients (C'_j,A'_j), $j=1,\ldots,M$, where $A'_M=$ cloud; then the total system is expressed as

$$\begin{aligned}&|\psi_c\rangle_{AC}\prod_j|\Phi^+\rangle_{C'_jA'_j}\\&\quad=\frac{1}{d^M}\sum_{m_i,n_i}\prod_j|\Phi_{m_j,n_j}\rangle_{C_jC'_j}\left(U_{m_j,n_j}\right)^{\dagger}_{A'_j}|\psi_c\rangle_{AA'}.\end{aligned} \tag{17}$$

The universal cloning state $|\psi_c\rangle_{AC}$ can be transferred to $|\psi_c\rangle_{AA'}$ by using standard teleportation [15]. We also have the equation $\sum_{\vec{m}}|\vec{m}\rangle_{A'}|\vec{m}\rangle_A=\sum_{m',n'}f(m',n')\delta_{\sum m'_i,0}\delta_{\sum n'_i,0}\prod_j|\Phi_{m'_j,n'_j}\rangle_{A'_j,A_j}$, where $m'=(m'_1,\ldots,m'_M)$, $n'=(n'_1,\ldots,n'_M)$, and module d is always assumed. Thus, Bell measurements on qubits A'_iA_i $(i=1,\ldots,M-1)$ with outcomes $\{m'_i,n'_i\}$ can let the cloud recover the original state by local operation $U_{m,n}$, where $m=(\sum m'_i)$, $n=(\sum n'_i)$. The entanglement and procedures of concentration scheme can be represented by Fig. 2.

Additionally, we can show that entanglement in Eq. (1) can be a universal resource which can also accomplish network concentration. In the second scheme, the clients and A perform Bell measurements, $\prod_i|\Phi_{m_i,n_i}\rangle_{C_i,C'_i}\prod_j|\Phi_{x_j,y_j}\rangle_{A'_j,A_j}$, then the cloud performs the local operation $U_{m,n}$ on its qudit according to the measurement results, where $m=\sum m_i+\sum x_j, n=\sum n_i+\sum y_j$. The resource $|\Phi^+\rangle^{\otimes M}$ can be used in this scheme because $U^{\dagger}_{m_j,n_j}|\Phi_{m'_j,n'_j}\rangle\propto|\Phi_{m'_j-m_j,n'_j-n_j}\rangle$. On the other hand, the entanglement resource used in the distribution can also accomplish this concentration task, $|\xi\rangle=\sqrt{\frac{1}{d[M]}}(\sum_{\vec{m}}|\vec{m}\rangle|\vec{m}\rangle)=\frac{1}{\sqrt{d[M]M!}}\sum_{\sigma}\prod_j|\Phi^+\rangle_{A'_jA_{\sigma_j}}$, where σ is a permutation. It is obvious that the amount of entanglement in this scheme is reduced.

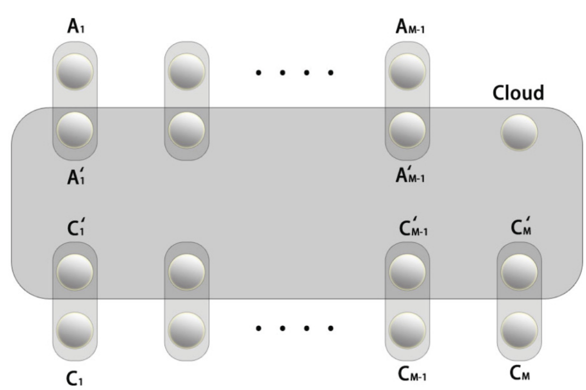

FIG. 2. Procedures of remote information concentration. The cloud and spatially separated clients share the entanglement resource, Bell-type measurement is performed, and the classical information is sent to the cloud.

V. POTENTIAL APPLICATIONS AND DISCUSSIONS

For qubits, the maximally entangled bipartite state used in our scheme has been generated experimentally by using stimulated parametric down-conversion and has been used in the 1-to-3 + 2 information distribution [39]. Some other experiments also produce such entanglement and realize the quantum information distribution [40]. With the schemes presented in this paper, it is possible that the many-to-many information distribution and the many-to-one information concentration can also be realized experimentally.

The quantum information distribution is no doubt very useful as a fundamental function of a quantum network. The application of quantum information concentration seems obscure. Here we try to propose one application for both distribution and concentration functions. It is known that the timekeeping of International Atomic Time [41] is operated jointly by several atomic clocks located in different places around the world with different environments and accuracies. We suppose that its next generation might be operated by using a quantum network. The quantum information distribution can be used as the time synchronization method. In case all duty atomic clocks run independently, the quantum information concentration thus can collect all different times and form one average (with different weights) standard time. It then can be distributed again for time synchronization. From this viewpoint, the concentration will be very useful for cases where only one standard or only the average value is important. This function is expected to be broadly useful in QCC and quantum networks.

In conclusion, we present a general network quantum teleportation for quantum information distribution and concentration with a universal entanglement resource. Our scheme can play a key role in quantum networks and in QCC, and it might be useful in time synchronization. Since teleportation plays a fundamental role in many protocols in quantum information science, the network teleportation can be modified and extended to other cases.

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