Beable-guided quantum theories: Generalizing quantum probability laws

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Beable-guided quantum theories (BGQT) are generalizations of quantum theory, inspired by Bell's concept of beables. They modify the quantum probabilities for some specified set of fundamental events, histories, or other elements of quasiclassical reality by probability laws that depend on the realized configuration of beables. For example, they may define an additional probability weight factor for a beable configuration, independent of the quantum dynamics. Beable-guided quantum theories can be fitted to observational data to provide foils against which to compare explanations based on standard quantum theory. For example, a BGQT could, in principle, characterize the effects attributed to dark energy or dark matter, or any other deviation from the predictions of standard quantum dynamics, without introducing extra fields or a cosmological constant. The complexity of the beable-guided theory would then parametrize how far we are from a standard quantum explanation. Less conservatively, we give reasons for taking suitably simple beable-guided quantum theories as serious phenomenological theories in their own right. Among these are the possibility that cosmological models defined by BGQT might in fact fit the empirical data better than any standard quantum explanation, and the fact that BGQT suggest potentially interesting nonstandard ways of coupling quantum matter to gravity.

I. INTRODUCTION

Weinberg's paper [1] exploring ideas about nonlinear generalizations of quantum theory led to some fundamental insights about the relationship between quantum theory and special relativity [2,3]. In introducing his work, Weinberg commented [1] that "Considering the pervasive importance of quantum mechanics in modern physics, it is odd how rarely one hears of efforts to test quantum mechanics experimentally with high precision...The trouble is that it is very difficult to find any logically consistent generalization of quantum mechanics."

In fact, applied to nonrelativistic quantum mechanics, this was already contradicted by dynamical collapse models [4,5]. Nonetheless, the belief that generalizations of quantum theory must be inconsistent or at least suffer from some fundamental problem remains widespread. In particular, the fact that Weinberg's nonlinear generalizations of quantum theory allow superluminal signaling [2,3] (or, it was argued, interuniverse signaling [2]) seems to have persuaded many that relativistic quantum theory, at least, is probably in some strong sense an isolated point in the space of theories. Recent no-go theorems [6–8] may have helped reinforce this impression.

None of these no-go results actually imply this conclusion, however, nor do their authors argue that they do. In fact, if one is flexible about the mathematical constructions that can be used, the rules one can postulate, as well as the ways in which quantum laws can be generalized, it is not hard to define logically consistent and potentially interesting generalizations of quantum mechanics that do not conflict with special relativity. For example, there exists an infinite class of consistent nonlinear theories that neither violate Lorentz invariance nor allow superluminal signaling [9].

This paper describes another infinite class of generalizations of quantum theory. This class includes arbitrarily baroque theories, but also includes subclasses of relatively simple theories that seem both potentially phenomenologically useful and interesting in their own right. We discuss their potential scientific implications, expanding an earlier discussion [10] and setting it in a more general and fundamental context.

No specific beable-guided quantum theories (BGQT) are advocated or compared to empirical data here. The aim is rather to make some simple conceptual points in order to expand the boundaries of future research in several directions.

II. BEABLE MODELS

Arguably,[1] the key problem in quantum foundations—and one of the key problems in modern physics—is the apparent impossibility of deriving, from unitary quantum theory alone, an explanation of the appearance of a quasiclassical world.[2]

The most interesting attempts to make progress on this problem do so by adding extra mathematical structure to unitary quantum theory and defining what Bell called beables [14,15]. Beables are mathematical representations of particle trajectories, or processes, or events, or histories, or whatever the right concept is for the elementary quantities from which we can build a description of possible quasiclassical worlds

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[1]Some claim that quantum decoherence explains, or points toward an explanation of, the appearance of a single quasiclassical world following quantum probabilistic laws within a many-worlds picture defined by the universal wave function. For arguments and counterarguments on this point see, e.g., Refs. [11,12]; some reasons why I disagree can be found in Ref. [13].

[2]That is, a world with macroscopic variables following mostly deterministic classical equations, and also influenced by definite but unpredictable outcomes of quantum events.

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(if the idea works). Beable models are indeterministic—they assign a probability measure to configurations of beables. This (if it works) implies a probability measure on possible quasi-classical worlds and hence implies probabilistic predictions about the quasiclassical world we experience. In particular, if the beable model is intended to replicate the predictions of quantum theory precisely, it should be possible in principle to derive the Born rule for quantum experiments from the beable configuration probability measure. So, according to this view, our quasiclassical world is the one that nature randomly chose to be realized from among all the possible worlds, and is fundamentally defined by some randomly chosen configuration of beables from among all the possible configurations.

Examples of beables in beable models or protomodels include the particle trajectories in de Broglie–Bohm theories, the collapse centers in discrete dynamical collapse models, and the real history defined by a time-dependent density matrix in a real world interpretation [16]. The consistent or decoherent histories approach [17,18] can and arguably should [19–21] also be viewed as a so far unsuccessful attempt at an honest beable model, in which the beables would be elementary histories from a preferred consistent set defined by an (alas as yet undiscovered) appropriate quasiclassical set selection rule.

A. Beables for skeptics

Let us assume that the final theory we are heading for is as compellingly beautiful as most physicists hope. Still, it is not yet in sight. Why should we be so confident that *every* step on the path to it involves expressing physical insights in mathematically beautiful ideas? Maybe a theory unifying quantum theory and gravity, or some other successor to quantum theory, will emerge in something like the way quantum theory did, from incomplete *ad hoc* ideas, toy models and partial insights.

Especially at the moment, when progress on fundamental problems in physics is so stalled, it seems a mistake to dismiss ideas that achieve *something*, however unaesthetically. Beable models do have some problems.[3] However, we can make scientific use of them as they give a logically straightforward way of resolving the tension between classical and quantum physics—a tension that has to be resolved *somehow*. Also in their favor is the fact that—unlike attempts to make sense of many-worlds quantum theory—they work within the only tried and tested scientific paradigm we have, in which the aim of a scientific theory is to define a single objective reality and make standard probabilistic predictions about our observations of that reality. Those seem good enough reasons to explore whether beable models lead to interesting and testable new

scientific ideas. This paper adds more reasons for believing that they do, and also for believing that these ideas may be valuable even if beable models eventually turn out to be only rough approximations to a deeper theory framed using different concepts.

III. BEABLE PROBABILITIES IN STANDARD BEABLE MODELS

A. Nonrelativistic beable models

We start by characterizing abstractly a class of beable models of nonrelativistic quantum mechanics, namely, *time-local* beable theories. In these, we suppose we are given quantum theory with some fixed initial state $|\psi(0)\rangle$ at some initial time $t = 0$, and some fixed Hamiltonian H, which for simplicity and definiteness we take here to be time independent. Our aim is to describe physics, at the most fundamental level, for all times $t > 0$. The possible beable configurations take the form of collections B of pairs of beables and (corresponding) times:

$$B = \{(B_t, t) : B_t \in \Lambda(t), t \geqslant 0\},$$

where each $\Lambda(t)$ is a set of the beables (some stipulated mathematical quantities) defined for each $t \geqslant 0$. The set $\Lambda(t)$ may be empty at some or even generic times t, and need not depend continuously on t.

If the model respects standard quantum dynamics for the state vector, it also includes the standard time-evolved quantum state $|\psi(t)\rangle = \exp(-iHt/\hbar)|\psi(0)\rangle$ as a mathematical object. The quantum state may itself also be defined to be one of the beables, but it need not necessarily be so defined.

Perhaps the most familiar example of a time-local beable model respecting standard quantum dynamics is standard nonrelativistic de Broglie–Bohm theory [22,23] applied to a system of N distinguishable particles. Here the quantum state at time t is $|\psi(t)\rangle$, where

$$\langle x_1, \ldots, x_N \mid \psi(t) \rangle = \psi(x_1, \ldots, x_N; t).$$

We will take the beables at time t to be the position coordinates of the N particle trajectories. (We will not take the quantum state here to be a beable, although many authors choose to. Both options are possible, and both are questionable: See footnote 3 on double ontologies.) So we have

$$\Lambda(t) = \{x_1(t), \ldots, x_N(t)\}.$$

These trajectories follow continuous equations, so in this case $\Lambda(t)$ depends continuously on t.

Standard de Broglie–Bohm theory makes experimental predictions indistinguishable from those of Copenhagen quantum theory, where both apply, in the following sense. We can always recover the Bohmian predictions for quasiclassical physics in the Copenhagen formalism by finding some suitably macroscopic system A that undergoes an effectively irreversible measurement interaction with the measured quantum system S, and treating A as though it follows quasiclassical dynamical laws rather than quantum laws. However, de Broglie–Bohm theory gives precise dynamical equations from which the quasiclassical behavior of such objects can be derived, rather than separately postulated. De Broglie–Bohm theory can also be straightforwardly applied to closed quantum

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[3]The apparent difficulty in constructing beable models that respect Lorentz invariance is one. Another is the problem of a double ontology. Do the beables and the quantum state give apparently equally valid alternative pictures of reality? If they do, to claim that the beable picture resolves any problem you have to be willing to postulate that *they* represent reality, or at least the reality we experience, and the quantum state does not. One way of saying this is to say the initial quantum state and Hamiltonian play lawlike roles in the theory, while the beables are the real physical variables. Whether this resolves or just restates the problem is debatable.

systems and can describe the emergence of quasiclassical physics within closed systems. Copenhagen quantum theory, on the other hand, relies on the imprecise concept of a separate classical realm through which the quantum realm is probed and measured, and does not apply to closed quantum systems.

An example of a time-local beable model that respects standard quantum dynamics for the state vector but makes distinct predictions from standard quantum theory is Valentini's modified de Broglie–Bohm theory [24,25], which follows the de Broglie–Bohm guidance equation but has a nonstandard initial condition on the particle trajectories at $t = 0$:

$$P(x_1(0), \ldots, x_N(0)) \neq |\psi(x_1, \ldots, x_N; 0)|^2 .$$

An example of a time-local beable model that does *not* respect standard quantum dynamics, and that also makes distinct predictions from standard quantum theory, is the original Ghirardi-Rimini-Weber (GRW) discrete dynamical collapse model [4]. We consider it here in the form first suggested by Bell [26] (see also, e.g., Refs. [27,28]), where the beables are the (isolated) space-time points about which the collapses are centered. Here, for a system of N distinguishable particles, at generic times t there are no collapses and hence no beables: $\Lambda(t) = \{\}$. If a collapse centered about x occurs for particle i at time t, then $\Lambda(t) = \{(x,i)\}$. In cases where $M \geqslant 2$ collapses occur at exactly the same time t collapses at time t, we have $\Lambda(t) = \{(x_1, i_1), \ldots, (x_M, i_M)\}$. (We include these cases for completeness, although the total probability measure for such multiple collapse events, integrated over all time, is zero.) As the GRW model illustrates, $\Lambda(t)$ need not necessarily depend continuously on t in a physically sensible time-local beable model.

In summary, a time-local beable model defines the possible sets of time-labeled beables,

$$B = \{(B_t, t) : B_t \in \Lambda(t), t \geqslant 0\} ,$$

takes as input the initial quantum state $|\psi(0)\rangle$ and the Hamiltonian H, and from these data computes as output a probability measure $\mu(B)$ on the sample space of allowed sets B. The measure depends on $|\psi(0)\rangle$ and H. If we think of these as fixed by some particular theory T, we may write $\mu \equiv \mu_T$ to emphasize that the beable configuration probabilities depend on the (quantum) theory.

While these familiar examples of beable models are time-local, we can also imagine more general types of beable models. For example, each beable might be associated with an extended region of space-time.[4] We can extend the above characterization to more general beable models, since nothing in our abstract definitions relies on time-locality. Thus, given a theory T defining $|\psi(0)\rangle$ and H, a *beable model* defines the possible sets of beables B and a probability measure $\mu_T(B)$.

B. Relativistic beable models

Relativistic beable models have proved harder to construct, perhaps unsurprisingly, given that we have no mathematically

rigorous construction even of nontrivial relativistic quantum field theories in $(3 + 1)$-dimensional Minkowski space-time.[5]

We can straightforwardly extend our abstract characterization of beable models to quantum field theory in Minkowski space and quantum cosmology. This turns out to be useful, despite the paucity of familiar concrete examples. (See the discussion of coarse-grainings in cosmological models below.)

A Lorentz-invariant beable model defines the possible sets of beables B and computes a probability measure $\mu_T(B)$ from a theory T defining the Hamiltonian H and some asymptotic past boundary condition $\psi_{-\infty} = \lim_{S \to -\infty} |\psi_S\rangle$ on the quantum state associated with spacelike hypersurfaces S tending to past infinity, *by Lorentz-covariant rules*.

Similarly, a generally covariant beable model in quantum cosmology uses generally covariant rules to define the possible sets of beables B and compute a probability measure $\mu_T(B)$, given a generally covariant theory T defining the quantum evolution law and some initial condition postulate (for example, the no-boundary condition).

IV. BEABLE-GUIDED QUANTUM THEORY

There is something unsettlingly epiphenomenal about the status of the beables in standard quantum beable theories. The quantum state evolution does all the mathematical work in defining the beable probability distribution; the beables, so to speak, hitch a free ride. As James put it [29] in another context: "Inert, uninfluential, a simple passenger in the voyage of life, it is allowed to remain on board, but not to touch the helm or handle the rigging."

While some nonstandard beable theories give the beables at least a little more of a role, it is still a secondary one.

For example, in Valentini's modified Bohmian theory [24,25], the evolution of the Bohmian particle trajectories throughout time is determined by the quantum state, just as in ordinary de Broglie–Bohm theory. The only difference is that the probability distribution of the initial Bohmian particle positions is chosen independently of the initial quantum state. Granted, this is a significant difference, with intriguing consequences, but the dynamics remain determined by the quantum state throughout.

In Ghirardi-Rimini-Weber-Pearle dynamical collapse models [4,5], there is a genuine interplay between the quantum state and collapse events. The quantum state at time t depends on all previous collapse events as well as on the initial state and Hamiltonian. Again, this is a significant generalization of standard quantum theory. Still, in at least one sense the quantum state still plays a dominant role. The probability of a collapse taking place at any given point in Galilean space-time is entirely determined by the quantum state at that time. Indeed, all of physics, including the beable probability distributions after time t, is determined by the quantum state at time t.

Each of these pioneering examples of generalizations of quantum theory has its own internal logic that provides

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[4]A model involving histories of generalized quantum events would be an example.

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[5]There are various proposals for relativistic beable models (e.g., [16,28]). Assessing these works in progress is beyond our scope here. Our aim is to describe a general class of theories rather than focusing on specific examples.

motivation for the beables playing precisely the role they do. Maybe one of these theories, or a theory in which the beables play a similar and similarly secondary role, will indeed turn out to be a better description of nature than standard nonrelativistic quantum theory.

However, we see a compelling motivation to explore ways of setting the beables on a still more equal footing with the quantum state. Hence the idea of beable-guided quantum theories, to which we now turn.

A. Beable configuration probability weights and probabilities

The theory T that defines the initial quantum state and Hamiltonian still defines a probability measure $\mu_T(B)$, as above. However, $\mu_T(B)$ no longer defines the probability of the beable configuration B.

Instead, we take the actual probability measure of the beable configuration B in a BGQT to be some function

$$\mu_T'(B) = f(\mu_T(B), B)$$

that depends on the quantum probability measure and on the beable configuration.

This gives a very large class of possibilities indeed. To be a little more concrete, while still allowing a large class of possibilities that includes many interesting generalizations of quantum theory, in what follows we will illustrate the idea by considering product functions of the form

$$f(\mu_T(B), B) = C\mu_T(B)w(B),$$

where $w(B)$ is a real non-negative weight function. The normalized probability measure is then

$$\mu_T'(B) = \mu_T(B)w(B)\left(\int_{B'}\mu_T(B')w(B')\right)^{-1}.$$

What defines the weight function $w(B)$? We have defined $w(B)$ to be a function only of the beable configuration B, independent of the initial quantum state and Hamiltonian, and to be non-negative and real. Modulo these constraints, in principle, *any rule at all* is allowed. We have (even in this restricted class) an uncountably infinite class of theories. However, in the most interesting cases, the rules defining $w(B)$ should be relatively simple.

It is worth stressing again that the wave function $|\psi(t)\rangle$ may be included in the beable set B. If it is, a beable-guided quantum theory may be governed by probability rules that explicitly depend on the set $\{|\psi(t)\rangle\}_{-\infty<t<\infty}$ or on selected subsets. Similarly, relativistic BGQT that include hypersurface-dependent wave functions $|\psi(S)\rangle$ as beables may be governed by probability rules that explicitly depend on $\{|\psi(S)\rangle\}_S$.

B. Examples of rules for weight functions
1. Simple Bohmian examples

To give a simple example, we could define a beable-guided quantum theory from a nonrelativistic Bohmian model of two particles with Bohmian trajectories $B = \{x_1(t), x_2(t)\}_t$, taking

$$w(B) = \exp\left(-\{\lim\sup_t[x_1(t) - x_2(t)]^2/a^2\}\right).$$

This is fairly easy to understand intuitively, although rather *ad hoc*. Compared to the standard Bohmian model, pairs of trajectories are more or less likely to be selected depending on the closest separation they ever attain.

A variation that perhaps might appear a little more natural, for a model universe of finite duration, from time 0 to T, is

$$w(B) = T^{-1}\int_0^T dt\exp\left(-\{[x_1(t) - x_2(t)]^2/a^2\}\right),$$

which prefers pairs of trajectories that stay close over time, with respect to a simple measure. Note however that the limit

$$w(B) = \lim_{T\to\infty} T^{-1}\int_0^T dt\exp\left(-\{[x_1(t) - x_2(t)]^2/a^2\}\right)$$

may not lead to well-defined beable configuration probabilities in general, since in many examples almost all configurations (with respect to the standard measure) have $w(B) = 0$.[6]

To reiterate, any rule at all is allowed in principle. A more baroque, less intuitively interpretable, and presumably correspondingly less physically interesting example is given by

$$\begin{aligned}
w(B) = {}& \alpha\exp\left(-\int_{0\leqslant t\leqslant 1}[x_1(t) - x_2(t)]^2/a^2\right) \\
& + \beta\exp\left(-\max_{2\leqslant t\leqslant 6}[x_1(t) - x_2(t)]^2/b^2\right) \\
& + \gamma\theta(\max_t[x_1(t)]^2 - c^2) \\
& + \delta\cos^2\left(\max_t\left[x_1^2(t) - x_2^2(t + T)\right]\right),
\end{aligned}$$

where $\alpha, \beta, \gamma, \delta, a, b, c, T$ are positive constants and θ is the Heaviside step function.

2. A simple collapse model example

A simple beable-guided version of a nonrelativistic Ghirardi-Rimini-Weber model for two distinguishable particles for times $t \geqslant 0$, with collapse events taking place at $\{x_i^1, t_i^1\}_{i=1}^\infty$ and $\{x_i^2, t_i^2\}_{i=1}^\infty$, is defined by

$$\begin{aligned}
w\left(\{x_i^1, t_i^1\}, \{x_i^2, t_i^2\}\right) = \inf_{i,i'}\big\{&\exp\left[-\left(t_i^1 - t_{i'}^2\right)^2/T^2\right] \\
& \times\exp\left[-\left(x_i^1 - x_{i'}^2\right)^2/X^2\right]\big\},
\end{aligned}$$

for constants X, T. Roughly speaking, this tends to favor collapse event histories that include a pair of collapse events for the two particles that are nearby in space and time with respect to the scales X, T.

V. COARSE-GRAININGS OF BEABLES IN COSMOLOGICAL MODELS

Simple nonrelativistic beable-guided quantum theories based on de Broglie–Bohm theory, the GRW model, or other familiar beable theories make testably different predictions from standard quantum theory. They suggest new ways of parametrizing experimental and observational tests of quantum mechanics.

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[6]This model could be varied by defining $\mu'(B)$ directly via a limit, rather than $w(B)$.

However, the class of theories, and hence parametrizations, is infinite. No one beable-guided quantum theory leaps out as a clearly more compelling candidate than the rest. Also, while intrinsically nonrelativistic theories might possibly still suggest interesting cosmological tests, they are obviously fundamentally flawed, and so at best of limited use, as cosmological models. Can we get any further?

A. Beable-guided quantum cosmological theories: Problems

Cosmology poses the sternest test of quantum theory as a universal theory, and so seems the likeliest arena where observation might help select potentially physically relevant beable-guided quantum theories. Among the problems are that we do not have a quantum theory of gravity; we do not have anything approaching a standard quantum cosmological model that starts from a simple theory of initial conditions and fits all the data; ideas about Lorentz covariant beable models are works in progress; and ideas about generally covariant beable models are less substantial still.

B. Phenomenological BGQT

Building a BGQT cosmological theory from fundamental first principles may not necessarily be the most fruitful approach. To test a cosmological beable model—or any quantum cosmological theory—we do not necessarily need a fine-grained description of the beables. The first key test is whether the model explains (insofar as a probabilistic theory can) the features of the observed universe. For this we need to characterize the possible (mostly) quasiclassical worlds that might be realized in any given theory, which we can describe in terms of higher-level physical quantities that the elementary beables must characterize. In any successful beable quantum theory, quasiclassical parameters—the approximate density of matter in a small region, the average distance between galaxies, the size of the universe (if finite) at any given cosmological time—must be characterized by the beables, at least to a very good approximation. In other words, quasiclassical parameters must, to good approximation, be functions of the elementary beables, and hence must effectively be higher-order beables. *We can define higher-level phenomenological beable-guided quantum theories directly in terms of these parameters.*

So, insofar as we can talk about quantum cosmological models at all (which we do, despite all the theoretical and conceptual problems), we can also talk about beable-guided quantum cosmological theories. If we have covariantly defined quasiclassical parameters, we can use them to construct covariantly defined beable-guided quantum cosmological theories. Theories of this sort were proposed in Ref. [10]; the present discussion sets them in a more general and maybe more fundamentally appealing context.

For example, any of the covariant definitions of quasiclassical events explored in the consistent histories literature might be used to define a beable-guided cosmological theory. In particular, we can use covariant notions of events defined via path integrals [30]. We could, for instance [10], define quantum cosmologies for an expanding universe with a cosmological time coordinate in which we stipulate in advance that when

the compact three-metric has volume V_i, the matter inhomogeneities are of scale somewhere in the range $(\delta_i^{\min}, \delta_i^{\max})$, for some sequence $V_1 < V_2 < \cdots < V_n < \cdots$ of increasing volumes. More generally, we could define a probabilistic theory of this type, where the probability distribution for the sequence $\{\delta_i\}$ is $p(\{\delta_i\})$. We can also consider continuous versions of such theories, defined by appropriate limits.

Similarly, we can define models for a finite universe that deterministically or probabilistically constrain the scale of the universe, or the average separation between galaxies, or any other quasiclassical quantity, as a function of cosmological time.

Again, of course, this gives us an infinite class of theories, including arbitrarily baroque ones as well as some quite simple ones. Collectively, these theories seem ideally designed as foils against which to test the postulate that initial causes and standard evolution laws together explain everything that can be explained in physics [10]. They can also be used as foils for earlier nonstandard theories, for example, in testing the alternative postulate explored in the two-time cosmology literature, that initial and final causes, together with the Hamiltonian, suffice. They are also potentially interesting nonstandard theories in their own right.

VI. BEABLE-GUIDED QUANTUM THEORY AND GRAVITY

The problems in unifying quantum theory and gravity are notoriously deep. We do not have a consistent quantum theory of gravity, nor a clear conceptual understanding of how a picture of macroscopic events taking place in a space-time with an apparently relatively well-defined large-scale structure could emerge from one if we did. We do not know that a quantum theory of gravity (in any conventional sense) is even what we should be looking for.

Beable-guided quantum theories suggest a different way of thinking about quantum theory and gravity. Quantum theory appears to be a good description of the behavior of matter, at least at small scales. The gravitational field appears to define the structure of a definite associated space-time, at least at large scales. Whether the gravitational field is *fundamentally* quantum or classical or something else, it seems to behave, at large scales, like a higher-level beable, giving a unique and definite picture of reality associated with the quantum evolution of the universe. Supposing this is correct, i.e., that large-scale features of space-time are defined as higher-level functions of fundamental beables, we can use BGQT as a framework for defining consistent theories in which the gravitational field satisfies interesting constraints. These constraints need not necessarily arise from standard expectations or intuitions about quantum theory and gravity. We can also consider constraints that only make sense if one is looking for a new physical principle embodied in a new type of theory, such as a BGQT.

One example of such a constraint, which is a useful foil against which to test standard expectations, is to impose by fiat that the gravitational field must be *locally causal*, in a sense that naturally generalizes Bell's definition of local causality to metric theories [31].

Define a *past region* in a metric space-time to be a region which contains its own causal past, and the *domain of dependence* of a region R in a space-time S to be the set of points p such that every endless past causal curve through p intersects R.

Suppose that we have identified a specified past region of space-time Λ, with specified metric and matter fields, and let κ be any fixed region with specified metric and matter fields.

Let Λ' be another past region, again with specified metric and matter fields. (In the cases we are most interested in, $\Lambda \cap \Lambda'$ will be nonempty, and thus necessarily also a past region.) Define

$$\mathrm{Prob}(\kappa | \Lambda \perp \Lambda')$$

to be the probability that the domain of dependence of Λ will be isometric to κ, given that $\Lambda \cup \Lambda'$ form part of space-time, and given that the domains of dependence of Λ and Λ' are spacelike separated regions.

Let κ' be another fixed region of space-time with specified metric and matter fields.

Define

$$\mathrm{Prob}(\kappa | \Lambda \perp \Lambda'; \kappa')$$

to be the probability that the domain of dependence of Λ will be isometric to κ, given that $\Lambda \cup \Lambda'$ form part of space-time, that the domain of dependence of Λ' is isometric to κ', and that the domains of dependence of Λ and Λ' are spacelike separated.

We say a metric theory of space-time is *locally causal* if for all such $\Lambda, \Lambda', \kappa$, and κ' the relevant conditional probabilities are defined by the theory and satisfy

$$\mathrm{Prob}(\kappa | \Lambda \perp \Lambda') = \mathrm{Prob}(\kappa | \Lambda \perp \Lambda'; \kappa') .$$

The standard expectation is that our space-time is not locally causal. A Bell experiment in which the measurement outcomes are amplified macroscopically so that the gravitational fields in spacelike separated regions depend on the outcomes ought to produce nonlocally causal correlations in the gravitational fields as well as the measurement outcomes. A beautiful experiment in this direction by Salart *et al.* [32] addressed the related question [33] of whether we can ensure that measurements in the two wings of a Bell experiment produce definite outcomes in spacelike separated regions, if we follow the intuitions proposed by Penrose [34] and Diosi [35] and assume that a definite outcome of a quantum measurement requires a gravitationally macroscopic superposition to be created.

Salart *et al.* show the answer is affirmative, if we also accept Penrose and Diosi's estimates for what constitutes a gravitationally macroscopic superposition. However, the nonlocality of the gravitational field has not yet been directly tested. It would be good to do so, since there is *some* motivation [31] for exploring the idea that the gravitational field might be locally causal, strange though such a theory would be, and small though our sliver of doubt on the point may be. It would be easier to see how to write dynamical equations for a quasiclassical metric theory—easier to see how space-time puts itself together from locally determined pieces—if it were

locally causal. BGQT models incorporating gravity gives a useful way of defining models that serve as the requisite foils.

VII. DISCUSSION

Any generalization of quantum theory can be seen as a foil for testing standard theories, a way of parametrizing how well any given experiment tests the theory, or how well standard quantum explanations currently fit observational cosmological data. This is certainly sufficient motivation for thinking about beable-guided quantum theories. Perhaps these and other generalizations of quantum theory will indeed turn out to be mere foils. Perhaps quantum dynamics will indeed survive all experimental tests. Perhaps some elegant Lorentz and generally covariant beable extension of quantum theory and quantum gravity will also explain the appearance of quasiclassicality and the probabilistic and deterministic laws governing our quasiclassical world.

There is, though, another less conservative motivation for considering beable-guided quantum theories. Beables give the best way we have of explaining the appearance of quasiclassical physics within a quantum world. But in existing beable models—even nonstandard models such as those of Valentini and Ghirardi-Rimini-Weber-Pearle—the beables seem unsettlingly epiphenomenal. The quantum dynamics does most or all of the work in defining the beable probability distribution, yet it is the beables that are supposed to represent physical reality, not the quantum state. While this is not logically inconsistent, it seems odd that the beables should be simultaneously so physically important and so passive.

Beable-guided quantum theories may not completely eliminate this sense of unease. The quantum state still plays the major role, at least in simple beable-guided theories. And since quantum theory works so well, this is a fairly inescapable feature, at least in theories describing laboratory experiments. But they do at least reduce the imbalance: The beables behave more like independent physical quantities.

In short, then, the case for taking beable-guided quantum theories seriously as fundamental theories in their own right is that we need beables, and then, once we have beables, they should play an active role in physics.

One obvious reason to be skeptical is that, with the crucial exception of explaining the appearance of quasiclassical physics, standard quantum theory appears to work very well. It seems that any corrections due to a beable-guided quantum theory must be very small, and yet nothing in known physics suggests any obvious reason to expect a small correction parameter. But then, similar arguments apply to, for example, the cosmological constant, the ratio between gravitational and electromagnetic force strengths, the degree of parity violation—and yet the parameters are small in each case.

There is also a danger of overstating the successes of quantum theory. Mainstream cosmological theories tend to assume quantum theory applies to the universe, and for understandable reasons. But we do not actually have a good quantum theory of gravity, let alone a tested quantum theory of cosmology. On cosmological scales, it is not so clear that quantum theory *does* actually explain all the data well.

Another obvious criticism is that we have infinitely many beable-guided quantum theories and no compelling principle

for picking out a small number of them as contenders for fundamental theories. If we think of beable-guided quantum theories as only stepping stones towards a more compelling successor to quantum theory, though, this might not be so much of a concern either. On this view, perhaps some reasonably simple beable-guided quantum theory will turn out to be a better theory of nature than quantum theory, but if so, we can only find out which one empirically, and we will only understand why that particular beable-guided quantum theory is a good approximation once we have the deeper successor theory.

Another interesting speculative possibility is that a guiding condition of the sort we have explored might be necessary to *define* a quantum theory of gravity, or to rigorously define physically relevant relativistic quantum field theories, in the first place. The idea here is that by rescaling the probabilities of quantum events (expressed in terms of beables), and perhaps excluding some classes of events altogether, guiding conditions could allow rigorously defined theories to be constructed, although the underlying unguided quantum theory is not rigorously defined or even renormalizable. For example, in principle one could try to define a beable-guided quantum field theory that modifies the contributions to a scattering amplitude so as to remove divergences.

Note that, even if a BGQT is constructed in a Lorentz-invariant (or generally covariant) way from a similarly invariant beable theory, it might allow superluminal signaling. Indeed, we already know that cosmological theories with independent initial and final boundary conditions can (not surprisingly, given that they break all standard notions of causality) allow superluminal signaling [36]. But as such theories remain consistent, and evade causal paradoxes [36], it is not so clear that this should be seen as a disastrous problem.

It would, in any case, be very interesting to clarify which types of nontrivial guidance conditions prohibit superluminal signaling, which allow it in theory but impose strong practical constraints, and which would allow it in practice with current technology.

We have presented beable-guided quantum theories in a form that perhaps fits most naturally within a block universe picture, in which reality is defined by one configuration of beables, chosen randomly from among all the possibilities. To put it picturesquely, on this view, nature's random choice is made once, before or outside any physical reality is created, and this choice brings into existence (in some approximate beable representation) space-time and all events therein.

That said, nothing in the definition of beable-guided quantum theories logically implies any stronger commitment to a block universe picture than already implied by standard quantum theory. Although the appearance of a present time in physics is arguably puzzling in both, it is not inconsistent with either. In each case, we can calculate the probabilities of present or near future events, conditioned on past events, for successive present times, and so recover a dynamical picture of beable events randomly happening over time.

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