PHYSICAL REVIEW A **87**, 021602(R) (2013)

Emergence of atomic semifluxons in optical Josephson junctions

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(Received 3 November 2012; published 13 February 2013)

We propose to create pairs of semifluxons starting from a flat-phase state in long, optical 0-π-0 Josephson junctions formed with internal electronic states of atomic Bose-Einstein condensates. In this optical system, we can dynamically tune the length a of the π junction, the detuning δ of the optical transition, or the strength Ω_0 of the laser coupling to induce transitions from the flat-phase state to such a semifluxon-pair state. Similarly as in superconducting 0-π-0 junctions, there are two, energetically degenerate semifluxon-pair states. A linear mean-field model with two internal electronic states explains this degeneracy and shows the distinct static field configuration in a phase diagram of the junction parameters. This optical system offers the possibility to dynamically create a coherent superposition of the distinct semifluxon-pair states and observe macroscopic quantum oscillation.

DOI: 10.1103/PhysRevA.87.021602 PACS number(s): 03.75.Lm, 37.10.Vz, 74.50.+r

The phenomenon of coupling coherent oscillators happens ubiquitously in mechanical and electrical systems [1], in condensed-matter physics [2,3], in nonlinear optics [4,5], and in high-energy physics [6]. For many spatially extended fields, such as laser pulses [4,5], superconducting Josephson junctions [7,8], or atomic Bose-Einstein condensates [9–16], one can approximate the dynamics by the nonlinear sine-Gordon equation [17] for the real 2π-periodic relative phase field $\phi(t,x)$.

A particularly interesting realization of such coupled oscillatory fields are superconducting 0-π Josephson junctions. Such junctions consist of segments where the ground state of the phase field $\phi(x)$ is either 0 or π. A 0-π junction can have semifluxons as local topological excitation which, in contrast to the well-known fluxons, carry only half of the magnetic-flux quantum Φ_0 [18–21]. The slightly more complicated 0-π-0 junction can exhibit energy-degenerate pairs of semifluxons as solutions. While the classical dynamics of semifluxons can be described well by sine-Gordon–type equations [22,23], to address tunneling or macroscopic oscillations between the field configurations a requantization of the sine-Gordon phase field is required [3,6,24–26].

In the present Rapid Communication, we will demonstrate the appearance of the semifluxon-pair states starting from the flat-phase state in optical 0-π-0 Josephson junctions implemented with two-level Bose-condensed atoms on a line x. Instead of the approximative sine-Gordon phase field, we will consider a linear Schrödinger equation as our mean-field model. As usual, we decompose the two complex field amplitudes $\psi_\sigma(t,x) = \sqrt{n_\sigma(t,x)}e^{i\phi_\sigma(t,x)}$ with densities n_σ and phases ϕ_σ. Then, we identify the phase difference $\phi \equiv \phi_e - \phi_g$ with the relative phase field of the sine-Gordon equation. For magnetically trapped ^{87}Rb Bose-Einstein condensates, it is well known that the relative phase is insensitive to density dependent energy shifts [27–29]. Thus even a linear mean-field model exhibits the same energy-degenerate semifluxon states known from continuous [24–26] or discrete [30] quantum models.

We consider an atomic Bose-Einstein condensate with two internal states modeled by the Schrödinger equation $i\partial_t\psi(t,x) = H\psi(t,x)$, where the atomic Hamiltonian operator

$$H = -\partial_x^2 + U, \quad U(x) = \begin{pmatrix} \delta & \Omega_0(x) \\ \Omega_0^*(x) & -\delta \end{pmatrix} \tag{1}$$

consists of kinetic energy and the dipole interaction $U(x)$ in the presence of an external laser field [31,32]. Here, δ is the detuning of the laser frequency from the atomic transition and the Rabi frequency $\Omega_0(x) = \Omega_0 e^{i\theta(x)}$ is a spatially dependent dipole coupling derived from a laser field with constant modulus $|\Omega_0|$, but an abruptly jumping optical phase $\theta(x)$ as depicted in Fig. 1. In general, this phase $\theta(x)$ could take on any value κ [25,26], but in order to model the 0-π-0 Josephson junctions, we assume $\theta(x) = 0,\pi$ or 0, depending on zone 1, 2, or 3. We emphasize that such a phase change can be realized by optical phase-imprinting techniques [33–37].

The physics of the semifluxons in a single junction is given by an interplay between the mechanical motion as well as the coherent oscillation between the internal levels. Therefore, we introduce the dressed basis states $V = (V_+, V_-)$ of the local potential $U(x)V[\xi(x)] = V[\xi(x)]\Lambda$, with

$$V(\xi) \equiv \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix}, \quad \tan[\xi(x)] \equiv \frac{\Omega_0(x)}{\delta + \Omega}, \tag{2}$$

and the generalized Rabi frequency $\Omega = \sqrt{|\Omega_0|^2 + \delta^2}$ defines the diagonal eigenvalue matrix $\Lambda = \mathrm{diag}(\Omega, -\Omega)$.

In a general Josephson-junction array with j zones, there are j different V_j eigenmatrices. However, it is a feature of this system that the eigenvalues $\Lambda = \Lambda_j$ are all identical. Clearly, this fact is related to the light pressure forces considered in atom trapping and cooling [38].

Before discussing the exact solution of the stationary Schrödinger equation $H\psi = E\psi$, it is important to find length scales. By equating the energy of the internal motion (Ω) with the mechanical energy ($1/a^2$), one can identify a characteristic

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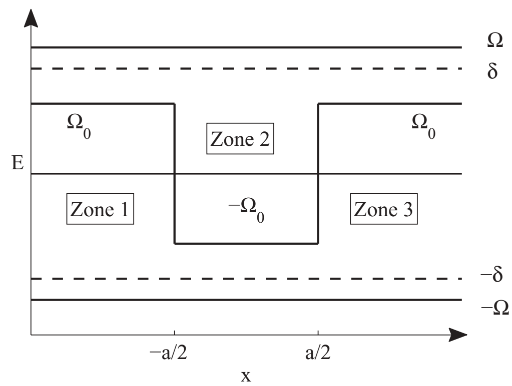

FIG. 1. Schematic drawing of spatial variation of frequencies in an optical 0-π-0 Josephson junction for cold atoms: the Rabi frequency $\Omega_0(x)$, the detuning δ, and the generalized Rabi frequencies $\pm\Omega$, which are constant throughout the system. The junction's domains are localized at $\pm a/2$ (all variables dimensionless).

length as $a_c = 1/\sqrt{\Omega}$. In each zone j, the physical solution

$$\psi(x) = V(e^{iKx}\psi^r + e^{-iKx}\psi^l) \tag{3}$$

is a superposition of left- or right-propagating or attenuated waves with upper (+) and lower (−) dressed state amplitudes $\psi^{h=r/l} = (\psi_+^h,\psi_-^h)$ according to Eq. (2). The compact matrix notation also extends to the wave number $K \equiv \sqrt{E - \Lambda + i0^+}$, which is a diagonal matrix with entries $k_\pm \equiv \sqrt{E \mp \Omega + i0^+}$. For definiteness, we have shifted the square root into the upper complex plane and cut it along the negative real axis. This fact is relevant as there are two distinct energy ranges: $-\Omega < E < \Omega$ and $\Omega < E$. In the former case, k_+ has a positive imaginary part and $k_- > 0$, while in the latter case both $k_\pm > 0$.

First, let us consider a single 0-π junction at $x_1 = -a/2$. There, we have to match the solutions

$$\begin{aligned} \psi_1(x) &= V_1\left(e^{iKx}\psi_1^{\text{in}} + e^{-iKx}\psi_1^{\text{out}}\right), \\ \psi_2(x) &= V_2\left(e^{iKx}\psi_2^{\text{out}} + e^{-iKx}\psi_2^{\text{in}}\right) \end{aligned} \tag{4}$$

in zone 1 and 2 requiring continuity $\psi_1(x_1) = \psi_2(x_1)$ and differentiability $\partial_x\psi_1(x_1) = \partial_x\psi_2(x_1)$. Quantitatively, we use the current density $j(x) \equiv \text{Im}\{\psi^\dagger\partial_x\psi\}$ (imaginary part) to decide whether left- or right-propagating waves in zone j are counted as incoming ψ_j^{in} or outgoing ψ_j^{out} relative to the location of the junction at x_j. These definitions lead to a four-dimensional scattering matrix S of the 0-π junction given by

$$\varphi_\beta^{\text{out}} = \sum_\alpha S_{\beta\alpha}\varphi_\alpha^{\text{in}}. \tag{5}$$

This relation quantifies the energy-dependent response of the system $\varphi^{\text{out}} = (\psi_2^{\text{out}},\psi_1^{\text{out}})$ to input signals $\varphi^{\text{in}} = (\psi_1^{\text{in}},\psi_2^{\text{in}})$ in the four different collision channels labeled by $\alpha \equiv (j = 1,2;\sigma = \pm)$. In our Hamiltonian system, currents are conserved at all junctions. This implies the unitarity of the S matrix, that is $S^\dagger g S = g$, in all open channels with respect to the diagonal metric $g = \text{Re}\{\text{diag}(k_+,k_-,k_+,k_-)\}$.

This simple model can be solved analytically. In the energy range $-\Omega < E < \Omega$, the excited dressed state channels are energetically closed, i.e., $\psi_i^{\text{in/out}} = (0,\psi_{i-}^{\text{in/out}})$ and the S matrix between the open collision channels reads

$$\begin{pmatrix} \psi_{2-}^{\text{out}} \\ \psi_{1-}^{\text{out}} \end{pmatrix} = \begin{pmatrix} t & r \\ r & t \end{pmatrix}\begin{pmatrix} \psi_{1-}^{\text{in}} \\ \psi_{2-}^{\text{in}} \end{pmatrix}, \tag{6}$$

where $r = (k_-^2 - k_+^2)\sin^2(2\xi)/f$, $t = -4(k_-k_+)\cos(2\xi)/f$, and $f = [k_-^2 + 6k_-k_+ + k_+^2 - (k_- - k_+)^2\cos(4\xi)]/2$. The energy-dependent transmission $|t(E)|^2$ vanishes at $E = \pm\Omega$ and the simple maximum in between depends on the laser parameter $\xi(\Omega_0,\delta)$. In Fig. 2(a), we observe the expected π-phase flip between left- and right-asymptotic state amplitudes. Mathematically, this property is reflected in the negative sign in the transmission amplitude $t(E)$ defined in Eq. (6). In the limit of very weak coupling this becomes $\lim_{\xi\to 0} t(E) = -1$.

The classical part of the kinetic energy of the excited-state population is proportional to $n_e(x)(\partial_x\phi_e)^2$. In order to minimize the energy change along the junction, a steep phase gradient has to be accompanied by a node in the excited-state population as seen in Fig. 2(b). This is the same physical mechanism as the vanishing core density of two- or higher-dimensional superfluid vortices [39].

In the atomic 0-π-0 Josephson junction of Fig. 1, we can now find a qualitatively new feature: as before semifluxons occur on each location of the junctions, but only if the length a of the π junction exceeds a characteristic value $a > a_c(\Omega)$, in analogy to superconductivity [24]. Thus a different motional topological state emerges. By generalizing the previous calculation, we add a wave function for the middle

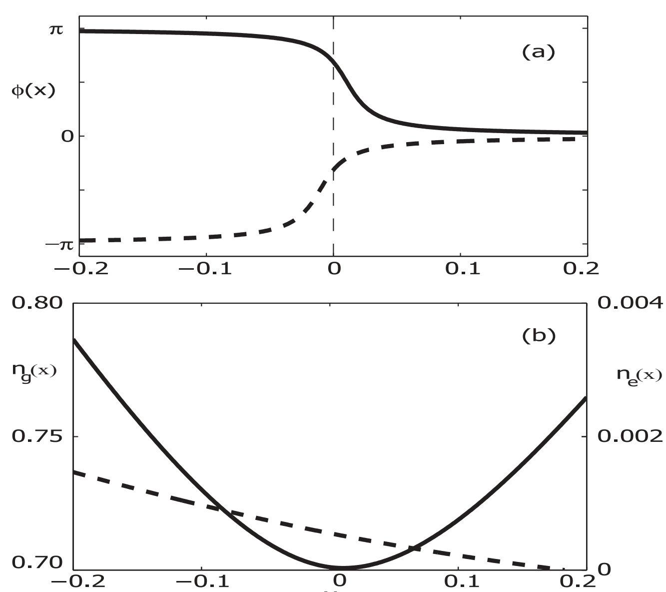

FIG. 2. (a) Relative phase $\phi(x) \equiv \phi_e(x) - \phi_g(x)$, (b) ground-state population $n_g(x)$ (left axis, dashed), and excited-state populations $n_e(x)$ (right axis, solid) vs position x along the 0-π junction. In (a), we depict two energy-degenerate solutions, which have left (solid) and right (dashed) incoming plane-wave asymptotics. The corresponding populations (b) are identical for both energy-degenerate solutions. As parameters, we use $\Omega_0 = 1$, $\delta = 3$, and a very low kinetic energy near the scattering threshold $E = -\Omega + k_-^2$ with $k_-^2 = 0.01$. The node of the excited-state population n_e at the location of the phase change resembles the physics of vortices in higher dimensions (all variables dimensionless).

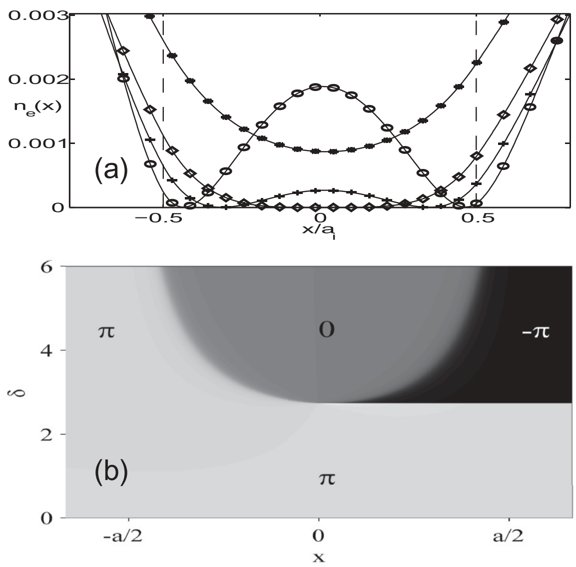

FIG. 3. (a) Excited-state density $n_e(x;a_i)$ vs a scaled length coordinate x/a_i. With the boundary condition of left-incoming plane waves impinging on the 0-π-0 junction, we picked four different values a_i for the length of the π zone: $a_1 = 0.4$ (*), $a_2 = a_c \approx 0.571$ ($\Diamond$), $a_3 = 0.7$ (+), and $a_4 = 1$ (o). The laser parameters are $\delta = 3$, $\Omega_0 = 1$, and we have an energy near the scattering threshold $E = -\Omega + k_-^2$ with $k_-^2 = 0.01$. (b) Gray-scale density plot of the relative phase $\phi(x;\delta)$ versus position and detuning δ with $a = 0.6$. By varying the detuning δ, we can switch from the semifluxon regime ($\delta_c \approx 2.74$) to the flat-phase state domain (all variables dimensionless).

zone and obtain the scattering solutions of the Schrödinger equation by matching the pieces

$$\begin{aligned} \psi_1(x) &= V_1\left(e^{iKx}\psi_1^{\text{in}} + e^{-iKx}\psi_1^{\text{out}}\right), \\ \psi_2(x) &= V_2\left(e^{iKx}\psi_2^{r} + e^{-iKx}\psi_2^{l}\right), \\ \psi_3(x) &= V_3\left(e^{iKx}\psi_3^{\text{out}} + e^{-iKx}\psi_3^{\text{in}}\right), \end{aligned} \tag{7}$$

at $x = \pm a/2$ as in the case of the π junction.

In Fig. 3(a), we display the excited-state density $n_e(x;a_i)$ as a function of position in scaled coordinates x/a_i for different values a_i of the length of π junction. We clearly see a qualitative change in the shape of the density when we increase the length to values above $a > a_c$. In the former case, the density is nonzero everywhere. By increasing the length of the junction to $a = a_c$ the density touches zero. A further increase of the length to $a > a_c$ leads to the formation of two nodes located at $x = \pm a/2$ and a nonvanishing density in between.

Already this static picture for the density suggests the formation of semifluxon pairs, when the length exceeds a critical length. However, this effect is also seen by directly studying the relative phase as a function of position and any system parameter a, Ω_0, or δ. If any one of the parameters varies while keeping the others fixed, we can observe the emergence of different quantum states in a two-dimensional phase diagram. In particular, we have varied the laser detuning δ at fixed values for a and Ω_0 in Fig. 3(b). This gray-scale density plot depicts the relative phase $\phi(x;\delta)$. In essence, we find a semiannular phase boundary limited by $|x| < a/2$ and $\delta > \delta_c$, which separates flat-phase domains (π) from semifluxon pair regions [π-0-($-\pi$)].

So far, we have confined the discussion to low-energy scattering $k_-^2 = 0.01$, as seen in Figs. 2 and 3. But this is no limitation for the experimental realizations of this system. Therefore, we also analyze the high-energy scattering behavior with the S matrix for the 0-π-0 junction. It is defined as in Eq. (5) and can be found explicitly from the solution of Eqs. (7). We only have to specify which outgoing amplitudes $\varphi^{\text{out}} = (\psi_3^{\text{out}}, \psi_1^{\text{out}})$ are connected by the S matrix to the incoming amplitudes $\varphi^{\text{in}} = (\psi_1^{\text{in}}, \psi_3^{\text{in}})$ in the four different collision channels of zones 1 and 3, now labeled by $\alpha = (j = 1,3, \sigma = \pm)$.

If we consider the scattering solutions for energies $-\Omega < E < \Omega$, then the lower dressed state is an oscillatory and the excited component is an exponentially decaying state. Thus we define the transmission amplitude $t(E) \equiv S_{3-,1-}$ as the forward-scattering amplitude for a left-incoming wave $\varphi^{\text{in}} = (0,1,0,0)$. This energy-dependent transmission is shown in Fig. 4(a) for two different lengths of the π junction. One observes the typical transmission behavior with vanishing or low transmission at both sides of the energy range and resonances in between. It is intuitively clear that there are more resonances with increasing junction length. This feature can be explained from an in-depth mathematical analysis of the poles of the S matrix, or a qualitative physical reasoning.

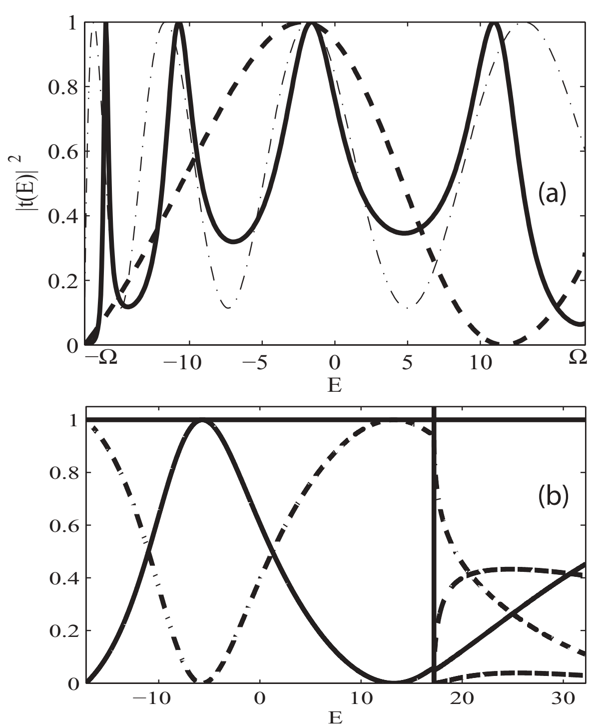

FIG. 4. (a) Transmission $|t(E)|^2$ in a 0-π-0 junction versus energy $-\Omega < E < \Omega$ for a left-incoming plane wave with $\Omega_0 = 14$, detuning $\delta = 10$, π-zone length $a_1 = 0.4$ (dashed line), $a_2 = 2$ (solid line), and the square-well approximation $|t_{sqw}(E;a_2)|^2$ (thin dashed line). (b) Transmission $|t(E)|^2$ (solid line) and reflection $|r(E)|^2$ (dashed dotted line) versus energy for $a = 0.5$. Above the energy border $E = \Omega$ (solid vertical line), we depict also excited channels transmission $|S_{1+,1-}|^2$ (dashed line) and $|S_{3+,1-}|^2$ (long dashed line). The constant line at 1 (solid line) proves unitarity for all energies (all variables dimensionless).

Indeed assuming an oscillatory solution in the lower dressed state manifold between the π-junction walls suggests the condition $\cos[k_-(E)a] = 0$, like in a square well. This analogy leads to several discrete resonances at the energies $E_n = (n + 1/2)^2\pi^2/a^2 - \Omega$. With this approximation, we find the elementary but analytical expression

$$|t_{sqw}(E)|^2 = \cos^2(k_-a)\cos^4(2\xi) + \sin^2(k_-a) \tag{8}$$

for the transmission coefficient. This square-well approximation $|t_{sqw}(E; a_2)|^2$ is depicted for the length a_2 in Fig. 4(a) with a thin dashed line. It does match the exact solution quite well and reproduces the resonances up to some minor energy shifts.

If we lift the limitations on scattering energies $E > \Omega$, then all four collision channels are energetically accessible and will be occupied. This situation is depicted in Fig. 4(b), where we scan the whole energy range for a short π-junction length. With the incoming state $\varphi^{\text{in}} \equiv (0,1,0,0)$, we get for the outgoing amplitudes $\varphi^{\text{out}} = (S_{3+,1-}, t = S_{3-,1-}, S_{1+,1-}, r = S_{1-,1-})$. They satisfy the current conservation rule known as the unitarity condition,

$$|t|^2 + |r|^2 + \text{Re}\left(\frac{k_+}{k_-}\right)(|S_{3+,1-}|^2 + |S_{1+,1-}|^2) = 1. \tag{9}$$

In this Rapid Communication, we have provided an atomic model of a 0-π-0 Josephson junction, today realized with superconductors. We have studied a linear two-component Schrödinger equation for a bosonic atomic gas, which is coupled by a phase-flipping laser field. On the mean-field level, this model demonstrates the emergence of macroscopic energy-degenerate quantum states, which are topologically distinct from flat-phase states. Their domain of existence is studied with phase diagrams and as a function of the external system parameters, such as the π-junction length a, the Rabi frequency Ω_0, or the detuning δ.

To observe these states on the mean-field level experimentally, one can use trapped prolate Bose-Einstein condensates. A release and time-of-flight measurement should reveal typical diffraction patterns of a finite-size phase mask in the case of semifluxon-pair states and none for flat-phase states.

By further quantizing the Schrödinger field, one can study the quantum evolution of these macroscopic energy-degenerate states, like quantum- and thermally induced tunneling, or coherent oscillations, eventually.

We gratefully acknowledge financial support by the SFB/TRR 21 "Control of quantum correlations in tailored matter" funded by the Deutsche Forschungsgemeinschaft (DFG). R.W. thanks the Deutsche Luft- und Raumfahrtagentur (DLR) for support from Grant No. 50WM 1137.

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