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Quantum control of spin correlations in ultracold lattice gases

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We describe a technique for the preparation of quantum spin correlations in a lattice gas of ultracold atoms using an atom-light interaction of the kind routinely employed in quantum spin polarization spectroscopy. Our method is based on entropic cooling via quantum nondemolition measurement and feedback, and allows the creation and detection of quantum spin correlations, as well as a certain degree of multipartite entanglement which we verify using a generalization of the entanglement witness described previously M. Cramer *et al.*, Phys. Rev. Lett. **106**, 020401 (2011). We illustrate the procedure with examples drawn from the bilinear-biquadratic Hamiltonian, which can be modeled by a one-dimensional chain of spin-1 atoms.

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Ultracold atomic gases trapped in optical lattices offer an unprecedented playground for studying the quantum phases of many-body systems [1]. In particular, quantum states of ultracold lattice gases with spin degrees of freedom may be used to simulate quantum magnetism and to investigate physics relevant for our understanding of high-T_c superconductivity [2]. While enormous progress has been made towards engineering such systems, achieving the regime of high-T_c superconductivity remains experimentally extremely challenging because of the low temperatures required [1,3].

In this Rapid Communication we propose an alternative approach to preparing quantum spin correlations. We build on studies of quantum polarization spectroscopy (QPS) [4], a promising technique for *detecting* quantum phases in lattice gases via quantum nondemolition (QND) measurement [5]. In particular, spatially resolved QPS that employs standing-wave laser configurations [6] allows direct probing of magnetic structure factors and order parameters of both polarized and unpolarized ensembles [7,8]. On the other hand, recent experimental work has demonstrated the generation of spin squeezing and entanglement in atomic ensembles via QND measurement [9]. Motivated by this work, and by the recent extension of these ideas to unpolarized ensembles [10], we demonstrate that a simple modification of the experimental scheme of Ref. [6] allows for the on-demand *preparation* of spatial spin correlations in a quantum lattice gas.

The proposed technique, illustrated in Fig. 1, works with an unpolarized ensemble of noninteracting spins such as may be obtained by loading ultracold atoms into a deep optical lattice. Spin squeezing via QND measurement using a standing-wave probe, followed by incoherent feedback via optical pumping to maintain the unpolarized state, allows spin entropy to be removed from the ensemble in a mode defined by the wavelength of the probe. By repeating the measurement and feedback procedure with a series of standing-wave probes with varying wavelength and carefully tailoring the atom-light coupling strength of each probe, a nonclassical spin-correlation function with a desired spatial structure is imprinted onto the atoms. The measurement results provide a record of the procedure and can be used to verify multiparticle entanglement among the atoms, which we demonstrate using a generalization of the entanglement witness derived in Ref. [11].

Atom-light interaction. We consider the interaction of atoms trapped in a one-dimensional (1D) optical-lattice potential with a set of standing-wave pulses of near-resonant light with wave numbers k_p. The atoms are described by collective variables $J_{\alpha,i} \equiv \sum_{n=1}^{n_a} j_{\alpha,i}^{(n)}$, where the index n runs over the n_a atoms at lattice site i and $\alpha = x,y,z$ labels the components of the atomic spin operators with length j. With n_s lattice sites and a uniform atomic filling factor, the total number of atoms is $N_A = n_s n_a$. The photons are described by collective Stokes operators S_α with $\alpha = 1,2,3$, defined as $S_\alpha \equiv \frac{1}{2}(a_+^\dagger, a_-^\dagger)\sigma_\alpha(a_+, a_-)^T$, where the σ_α are the Pauli matrices, and $a_\pm$ are annihilation operators for the spatial and temporal mode of the pulse with circular plus or minus polarization.

The atom-light interaction for a single pulse p is described by the effective Hamiltonian

$$H_p = \Omega_p \sum_{i=1}^{n_s} c_i(k_p) J_{z,i} S_3, \tag{1}$$

where $c_i(k_p) = [1 + \cos(2k_p r_i)]/2$ describes the standing-wave intensity profile of the probe pulse [12]. The coupling constants Ω_p depend on the probe detuning and intensity. Equation (1) describes a QND measurement that induces spin squeezing of the J_z component of the collective atomic mode $J_\alpha(k) \equiv \sum_{i=1}^{n_s} J_{\alpha,i} \exp(ikr_i)/\sqrt{n_s}$ with $k = \pm 2k_p$. For multilevel alkali atoms, this effective Hamiltonian can be synthesized using multicolor or dynamical-decoupling probing techniques [9,13].

We model the interaction using methods developed for treating the Gaussian dynamics of collective-variable systems [10,14], assuming that $n_a \gg 1$. The full system is described by the operators $R_m = \{S_1, S_2, S_3;\ J_x(k), J_y(k), J_z(k)\}$ and the covariances $\tilde{\Gamma}(R_m, R_n) \equiv \langle R_m R_n + R_n R_m \rangle/2 - \langle R_m \rangle \langle R_n \rangle$. In the following, we abbreviate $\tilde{\Gamma}_{\alpha\beta}(k_1,k_2) \equiv \tilde{\Gamma}(J_\alpha(k_1), J_\beta(k_1))$, $\tilde{\Gamma}_{\alpha 2}(k) \equiv \tilde{\Gamma}(J_\alpha(k), S_2)$, and $\tilde{\Gamma}_{22} \equiv \tilde{\Gamma}(S_2, S_2)$. The dynamical equations for the covariances can be derived from the Heisenberg equation of motion for the operators: After pulse p, in the small-angle regime, an operator changes as $R_m^{(out)} = R_m^{(in)} - i\tau[R_m^{(in)}, H_p]$, where τ is the pulse duration.

Typically, the atomic and light variables are initially uncorrelated and the atomic covariances show no correlations

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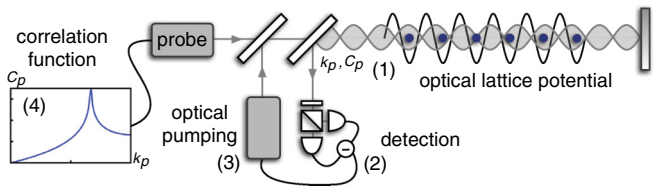

FIG. 1. (Color online) Proposed experimental setup. Atoms trapped in an optical lattice (black) are probed with a far-detuned, linearly polarized standing-wave light pulse with wave vector k_p (1), which is outcoupled to record S_2 (2). The measurement induces spin squeezing, introducing quantum correlations among the atoms in spatial mode $k = 2k_p$. Feedback is applied via optical pumping to set $\langle J_\alpha(k)\rangle = 0$ (3). Successive spin components J_α can then be separately squeezed by coherently rotating the atomic spin between measurements. Steps (1)–(3) are repeated for a set of wave vectors k_p, with interaction strengths C_p weighted according to the cosine Fourier transform of the desired spatial correlation signature (4).

between different spin components, i.e., $\tilde{\Gamma}^{(\mathrm{in})}_{\alpha\beta}(k_1,k_2) = 0\ \forall \alpha \neq \beta$. We assume an initial product state $\Gamma^{(0)}_{zz}(r_1,r_2) = \Gamma^{(0)}\delta_{r_1,r_2}$, which corresponds to a high-temperature state of the spin degree of freedom. Inserting $R^{(\mathrm{out})}_{\mathrm{m}}$ from the Heisenberg equations of motion into the output covariances $\tilde{\Gamma}^{(\mathrm{out})}_{\mathrm{mn}}$, we find that for an input S_1-polarized pulse, the only covariances that change due to the pulse are

$$\tilde{\Gamma}^{(\mathrm{out})}_{z2}(k) = \frac{C_p}{2\sqrt{j}}\tilde{\Gamma}^{(\mathrm{in})}_{zz}(k_p), \tag{2a}$$

$$\tilde{\Gamma}^{(\mathrm{out})}_{22} = \tilde{\Gamma}^{(\mathrm{in})}_{22} + \frac{C_p^2}{8j}\left[2\tilde{\Gamma}^{(\mathrm{in})}_{zz}(0) + \tilde{\Gamma}^{(\mathrm{in})}_{zz}(2k_p) + \tilde{\Gamma}^{(\mathrm{in})}_{zz}(-2k_p)\right]. \tag{2b}$$

Here, we define $\tilde{\Gamma}_{zz}(k) \equiv \tilde{\Gamma}_{zz}(0,k) + \tilde{\Gamma}_{zz}(2k_p,k)/2 + \tilde{\Gamma}_{zz}(-2k_p,k)/2$ and the coupling strength $C_p \equiv \tau\Omega_p S_1\sqrt{N_{\mathrm{A}} j/S_0}$, where S_0 is the probe intensity. The coupling strengths C_p can be adjusted experimentally by choosing detuning, intensity, and duration of the pulse appropriately. With a finite on-resonance optical depth d of the atomic ensemble, the optimal coupling strength is related to the probability of spontaneous emission η_p via $C_p = \sqrt{d\eta_p}$ [12]. For a degenerate quantum gas, $d \gg 1$, allowing a large coupling strength with minimal decoherence due to spontaneous emission [4,6].

Detection of S_2 then transfers the correlations described in Eqs. (1) to the atoms. This is modeled as a projection $\Gamma^{(\mathrm{M})} = \Gamma^{(\mathrm{out})} - \Gamma^{(\mathrm{out})}(\Pi_2\Gamma^{(\mathrm{out})}\Pi_2)^{\mathrm{MP}}\Gamma^{(\mathrm{out})}$, where MP indicates the Moore-Penrose pseudoinverse and $\Pi_2 = \mathrm{diag}(0,0,0,0,1,0)$ [10]. After the measurement, the atomic covariances are

$$\tilde{\Gamma}^{(\mathrm{M})}_{\alpha\beta}(k_1,k_2) = \tilde{\Gamma}^{(\mathrm{out})}_{\alpha\beta}(k_1,k_2) - \frac{\Gamma^{(\mathrm{out})}_{\alpha 2}(k_1)\Gamma^{(\mathrm{out})}_{2\beta}(k_2)}{\Gamma^{(\mathrm{out})}_{22}}. \tag{3}$$

Inserting Eq. (2) in (3) implies that the only atomic covariances changed by the interaction-measurement process are $\tilde{\Gamma}_{zz}(k_1,k_2)$; in other words, the measurement induces spin squeezing of the $J_z(2k_p)$ mode. Furthermore, the process is highly symmetric, preserving $\tilde{\Gamma}_{\alpha\alpha}(k,k') = \tilde{\Gamma}_{\alpha\alpha}(k,-k')$ for $k \neq k'$ and $\tilde{\Gamma}_{\alpha\alpha}(k,k') = \tilde{\Gamma}_{\alpha\alpha}(k',k)\ \forall k,k'$. Decoherence due to spontaneous excitation by the probe pulse is included in the model, following Refs. [10,14,15], by updating the atomic covariances according to $\tilde{\Gamma}^{(\eta_p)}_{\alpha\alpha}(k_1,k_2) = (1-2\eta_p)\tilde{\Gamma}_{\alpha\alpha}(k_1,k_2) + 2\eta_p\Gamma^{(0)}\delta_{k_1,-k_2}$.

The orthogonal spin components $J_\alpha(2k_p)$, $\alpha,\beta = x,y,z$, can be successively squeezed by coherently rotating the atomic spin between measurements. To allow the measurement-induced squeezing to be repeated for each spin component, we require $\langle J_\alpha(2k_p)\rangle = 0$, which allows us to avoid measurement-induced back action due to the Heisenberg uncertainty relation $\Gamma_{\alpha\alpha}(k_1,k_1)\Gamma_{\beta\beta}(k_2,k_2) \equiv [\Delta J_\alpha(k_1)]^2[\Delta J_\beta(k_2)]^2 \geqslant \epsilon_{\alpha\beta\gamma}\frac{1}{4n_{\mathrm{s}}}|\langle J_\gamma(k_1+k_2)\rangle|^2$. Since the QND measurement is projective, this requires using the measurement outcome S_2 in a feedback loop acting on the atomic spins to set the measured component to zero. Moreover, the feedback must be incoherent, since coherent feedback would simply rotate the nonzero component into another axis. A possible choice is optical pumping feedback: A weak pulse of near-resonant light at wave vector k_p with an intensity proportional to $S_2^{(\mathrm{out})}$ will set $\langle J_z(2k_p) + J_z(-2k_p)\rangle = 0$, and a second pulse with a half-period phase shift sets $\langle J_z(2k_p) - J_z(-2k_p)\rangle = 0$, so that $\langle J_z(2k_p)\rangle = \langle J_z(-2k_p)\rangle = 0$. As shown in Ref. [10], this feedback introduces spin noise $\propto N^{1/4}$, which is negligible in the thermodynamic limit.[1]

The outcome of this process is the removal of entropy from the $J_\alpha(k)$ collective spin mode, reducing the variance of all three orthogonal spin components $J_\alpha(2k_p)$ while maintaining an unpolarized ensemble with $\langle J_\alpha(2k_p)\rangle = 0$. We now show that by following this procedure with a series of pulses k_p and controlling the coupling constants C_p, we can manipulate the spatial spin-correlation function of the lattice gas.

The spatial dependence of the atom-light coupling is given by a cosine with well-defined k vector, entering Eq. (1) through $c_i(k_p)$. If we assume that the squeezing induced by a single pulse k_p is not affected by subsequent pulses,[2] this suggests that one can engineer real-space spin-spin correlations $\Gamma_{\alpha\alpha}(r_1,r_2) \equiv \langle J_\alpha(r_1)J_\alpha(r_2)\rangle - \langle J_\alpha(r_1)\rangle\langle J_\alpha(r_2)\rangle$ by applying successive pulses with wave vectors k_p and predetermined coupling constants C_p. A good first approximation is to use the cosine-Fourier transform of the desired spin-spin correlation function: The amplitude of a given Fourier component would then determine the strength of the atom-light coupling C_p at the corresponding wave vector k_p. The coupling strengths C_p that should be used at each wave vector to arrive at a desired output correlation signature $G_{\mathrm{des}}(\delta r) \equiv \Gamma_{zz}(r_1,r_1+\delta r)$ are then

$$C_p = 2\sqrt{j}\sqrt{\frac{\Gamma^{(\mathrm{in})}_{22} f_p}{1 - g_p\Gamma^{(0)} f_p}}, \tag{4}$$

where $f_p = -4\tilde{G}_{\mathrm{des}}(2k_p)/\Gamma^{(0)}$, with $\tilde{G}_{\mathrm{des}}(2k_p)$ the cosine Fourier transform of $G_{\mathrm{des}}(\delta r)$, and $g_p = \frac{9}{2}$ for $k_p = 0$ and $g_p = \frac{3}{2}$ otherwise [16]. In our numerical results below, we demonstrate that this does in fact result in a very good approximation to the desired spatial spin-correlation function.

[1]Alternatively, data with $\langle J_\alpha(2k_p)\rangle \ll \Gamma^{(0)}\ \forall k_p$ could be postselected based on the measurement outcomes.

[2]This assumption holds for weak coupling, and is numerically validated by our results. However, it is not necessary: One could calculate the necessary coupling strengths C_p by taking into account the change in the covariances $\tilde{\Gamma}_{zz}(k_1,k_2)$ after each pulse in the sequence.

Entanglement witness. To show that the proposed technique can create multipartite entanglement, which is particularly important in the context of quantum information processing and many-body systems [17], we derive an entanglement witness for the multimode spatial correlations induced by the procedure described above.

Generalizing the strategy of Refs. [7,11,17], we use the witness $W \equiv \mathcal{S}/n_a - 1$, such that $W < 0$ implies entanglement (see Supplemental Material [16]), where we define

$$\mathcal{S} \equiv \sum_\alpha \mathcal{S}_\alpha = \sum_\alpha \sum_{i,j=1}^{n_s} \langle J_{\alpha,i} J_{\alpha,j} \rangle f^*(r_i) f(r_j). \tag{5}$$

Here, $f(r_i)$ is any normalized function $\sum_{i=1}^{n_s} |f(r_i)|^2 = 1$. This definition encompasses and generalizes the plane waves described in Refs. [11,17], and allows us to calculate the entanglement witness W as a function of spatial separation, which may be of general interest outside this particular example. To probe spatial dependence, we calculate the entanglement between two sets of lattice bins $r_{s=1\ldots m}$ and $r_{w=1\ldots n}$ separated by a distance δr using the witness W with the function

$$f(r_i) = \begin{cases} 1 & \text{if } \ r_i \in r_s, \\ \exp(i\phi) & \text{if } \ r_i \in r_w, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

For a given δr, W can then be minimized with respect to ϕ.

Numerical results. We illustrate this technique using a 1D chain of spin $j = 1$ atoms with $n_s = 200$ sites and $n_a = 10$ atoms per site.[3] An important example for a $j = 1$ chain is the bilinear-biquadratic Hamiltonian, which has a rich phase diagram displaying ferromagnetic, critical, dimerized, and Haldane phases, each with distinctive spatial correlation signatures [18–25]. We note that, since we act on noninteracting spins, the spin-correlation functions that can be imprinted onto the atoms are not limited by this phase diagram, but are in principle arbitrary. Extension to higher dimensional systems should also be possible [4–8].

As a first example, we demonstrate the preparation of predetermined spin correlations $G_{\rm des}(\delta r)$ with (a) an exponential decay $\exp(-r/\xi)$ with a correlation length $\xi = 5$, corresponding to gapped phases, such as spin liquids which are conjectured to appear in the vicinity of high-T_c superconductivity [2,26], and (b) an algebraic decay $r^{-\zeta}$ with $\zeta = 0.7$, similar to those corresponding to critical phases and quantum critical points of the bilinear-biquadratic phase diagram. We compute the C_p corresponding to $G_{\rm des}(\delta r)$ as described in Eq. (4). The only free parameter is the maximum coupling strength $\max_p\{C_p\} = 0.95$, chosen to ensure that the approximations used in deriving Eq. (4) (see Supplemental Material [16]) are valid.[4] We then apply the pulses in sequence to the atoms, compute the resulting covariances using Eq. (3), and calculate the real-space spin correlations:

$$G(\delta r) = \frac{1}{n_s/4} \sum_{i=1}^{n_s/4} \sum_\alpha \Gamma_{\alpha\alpha}(r_i, r_i + \delta r)/\,\Gamma^{(0)}. \tag{7}$$

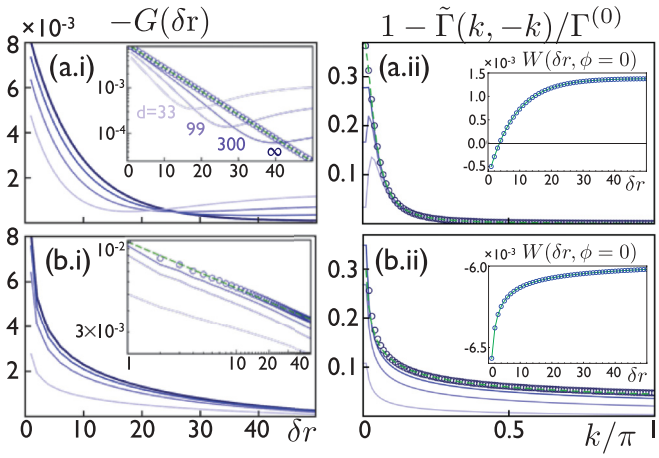

FIG. 2. (Color online) Spin correlation functions for (a) exponential decay and (b) algebraic decay. (i) Real space (in units of lattice sites): The lines from darker to lighter shades are for optical depth $d = \infty$, 300, 99, and 33. The insets are log-linear and log-log plots, respectively, where for clarity we subtract $G(\delta r \to \infty)$, extracted from a fit. In all panels, the $d = \infty$ data is plotted with open circles to allow comparison with fitted curves (green dashed lines). In (a.i), without decoherence, the decay follows an exponential fit. Deviations at large distance become stronger with increasing decoherence (decreasing d), and $G(\delta r \to \infty)$ cannot be reliably determined, yielding deviations from straight lines. In (b.i), the curves are straight lines for all values of d, and algebraic fits are very accurate. (ii) k space (in units of reciprocal lattice sites): The covariances $\Gamma(k, -k)$ closely follow the cosine Fourier transform of the desired correlation signature $G_{\rm des}(\delta r)$, even for small d. Deviations occur only primarily at small k where the pulse strength C_p is high. Insets: Entanglement witness $W(\delta r,\phi)$ for $d = \infty$, minimized at $\phi = 0$. If $W < 0$ the system displays nonclassical correlations.

The numerical results are shown in Fig. 2. At large optical depth the desired correlation signatures $G_{\rm des}(\delta r)$ coincide well with the calculated correlation function (first column). For case (a), the exponential decay is maintained over several orders of magnitude, and fits to the short-range behavior yield a correlation length close to the desired $\xi = 5$. For case (b), a clear algebraic decay is seen with a fitted $\zeta \approx 0.4$. In both cases, deviations from the desired parameters induced by finite optical depth could be further compensated by adjusting the C_p appropriately. The observables measured directly during the procedure are the covariances $\tilde{\Gamma}(k, -k)$, which show good agreement with the Fourier transforms of $G_{\rm des}(\delta r)$. We calculate the entanglement witness W for a single bin entangled with a chain of 106 bins. In both cases, W is minimized for $\phi = 0$. As seen in the insets of Fig. 2(ii) and demonstrated by corresponding fits, $W(\delta r,\phi = 0)$ follows the spatial behavior of the spin correlation function. Notably, the entanglement is both long-range and considerably stronger with algebraic decay.

[3]Note that the same results generalize to a single atom per site, as long as we bin the atoms into n_s bins with n_a atoms per bin, and redefine the coupling constant C_p as an average over the n_a atoms in each bin.

[4]This is a conservative choice: We could increase the strength of the correlations by increasing C_p and taking into account the change in the covariances $\tilde{\Gamma}_{zz}(k_1,k_2)$ after each pulse in the sequence when calculating the f_p.

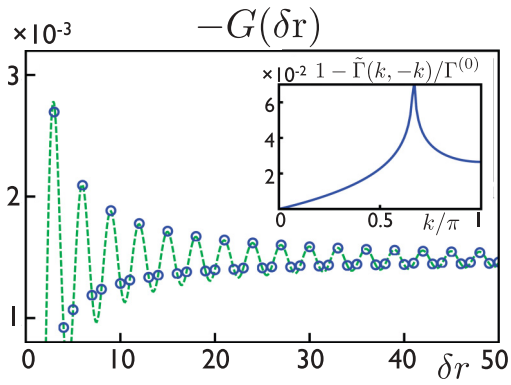

FIG. 3. (Color online) Spin correlations similar to the critical phase of the bilinear-biquadratic Hamiltonian, with an algebraic decay and characteristic period-3 oscillations. The resulting covariances $\tilde{\Gamma}(k,-k)$, shown in the inset, follow the input coupling strengths. These are chosen to heuristically model the structure factor of this phase. Real-space correlations are plotted in units of lattice sites, and k-space correlations in units of reciprocal lattice sites.

The same procedure can be used to prepare more complex correlation signatures. As an example, in Fig. 3 we illustrate the preparation of a correlation signature similar to the critical phase of the bilinear-biquadratic Hamiltonian, which has algebraically decaying correlations with characteristic period-3 oscillations [19]. To obtain such a behavior, we model the characteristic structure factor of this critical phase, which is peaked at $k=\pm 2\pi/3$ [20], by a heuristic function $|k-(2\pi)/3|^{-1+\zeta}\sqrt{2/\pi}\Gamma(1-\zeta)\sin(\pi\zeta/2)$, where Γ is the Euler gamma function, and its mirror image about π at $k=4\pi/3$.[5] This is then used as the input Fourier transform of the spin correlations entering Eq. (4). The resulting covariances $\tilde{\Gamma}(k,-k)$, given in the inset of Fig. 3, closely follow this form. Calculating $W(\delta r,\phi=0)$ also indicates long-range entanglement in this example, with a magnitude slightly smaller than for the algebraic correlations.

[5]This function is chosen to closely resemble the structure factor at the Lai-Sutherland point of the bilinear-biquadratic model. The cosine Fourier transform of this function has two parts, one decaying as $1/\delta r^2$ and one decaying as $\cos(2\pi\delta r/3)/\delta r^{\zeta}$. For $\zeta<2$, this produces the desired algebraic decay with period-3 oscillations at large distances δr.

Outlook. We have demonstrated that, with a simple modification of the experimental scheme discussed in Ref. [6], it is possible to engineer the spatial spin-correlation function of a quantum lattice gas via QND measurement and feedback. Moreover, the procedure is not limited to preparing ground-state correlations, but could in principle be extended to preparing arbitrary spatial spin-correlation functions. We have illustrated the procedure with three examples, motivated by the quantum phases of the bilinear-biquadratic Hamiltonian. In particular, we have demonstrated that it is possible to prepare exponentially and algebraically decaying correlations, as well as spatial correlation signatures of more exotic quantum phases such as quantum criticalities. Furthermore, we have generalized the entanglement witness proposed in Refs. [11] and shown that the engineered spin correlations entail multimode atomic entanglement. We note that it is a strength of our proposal that spin correlations and entanglement can both be prepared and detected with the same procedure. In our calculations, we make conservative assumptions about the experimental parameters, leaving considerable scope for further optimization of the procedure, which is extendible to higher dimensions and larger-spin systems, with both fermionic or bosonic atoms.

The procedure outlined in this Rapid Communication represents a form of measurement-induced entropic cooling that works with an unpolarized ensemble of noninteracting spins, which may be prepared in a deep optical lattice or by tuning the scattering length to zero via a Feshbach resonance. Once the spatial spin-correlation function has been imprinted on the atoms, interactions can then be adiabatically turned on; ground-state spin correlations will then be maintained by the interactions. Alternatively, one could choose to turn on the interactions at a quantum criticality—in which case the spin correlations will in principle be maintained indefinitely due to the critical slowing of relaxation rates—or rapidly switch on the interactions and study spin correlations associated with excited states, or indeed an arbitrary initial state of the interacting system.

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