Generation of a two-mode squeezed vacuum field in forward four-wave-mixing process in an ensemble of Λ atoms

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We develop the theory to describe the generation of entangled-photon pairs in the parametrically induced transparency regime in an ensemble of Λ atoms. A highly effective conversion of the monochromatic drive field into entangled-photon pairs is predicted. In the suggested regime, the generation of entangled photons has some advantages as compared to the well-known methods, such as a narrow frequency band and a low threshold for the drive power. The two-mode squeezed vacuum generation is thorouhly investigated.

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I. INTRODUCTION

Nowadays, entangled photons are considered to be the most suitable method for quantum information transfer [1–5]. These quantum states are of great importance for some fundamental problems (e.g., Bell's inequalities tests [6]), quantum cryptography [7], quantum teleportation [8], and high-precision measurements [9–11]. Of special interest are the entangled photons which form the so-called squeezed vacuum and, particularly, the two-mode squeezed vacuum [12–16]. The squeezed vacuum can be used for quantum information processing, e.g., for quantum cryptography [17], and for quantum noise suppression in interferometers [18,19].

Most schemes of entangled light-state generation are based on nonlinear processes. In particular, the nonlinear decay of the drive photon into two signal (probe) photons can be used for squeezed vacuum generation [20–25]. One of the intensively developing methods of entangled-photon generation is based on the electromagnetically induced transparency (EIT) regime in simple and doubled Λ schemes [26–35]. This method uses resonantly enhanced nonlinearities [36]. We believe that forward four-wave mixing of copropagating waves in an ensemble of Λ atoms controlled by a monochromatic drive field is promising for generation of intense entangled photons. This case was discovered experimentally in [37], and the complete theory was developed in [38,39] for classical correlated fields. We consider a standard three-level Λ scheme with the transition frequencies $\omega_{31,32,21}$ (see Fig. 1), which are driven by an intense wave with the frequency $\omega_d = \omega_{32}$ (D wave). In such a system the signal (probe) wave with the frequency of an anti-Stokes drive satellite $\omega_p \approx \omega_d + \omega_{21} \approx \omega_{31}$ (P wave) propagates in the EIT regime. Drive scattering by the low-frequency (LF) transition with the frequency ω_{21} also generates a Stokes satellite with the frequency $\omega_s \approx \omega_d - \omega_{21}$ (S wave) (see Fig. 1). Earlier studies of the EIT effect showed that propagation of a P wave in a medium, which is opaque for it, is possible due to the nonlinear coupling of the P and D waves with any fairly long-lived mode of collective oscillations of the medium. As in our previous papers, we will call this regime "parametrically induced transparency"(PIT). Unlike the standard variant of EIT, in this regime the condition of synchronism of wave vectors, which is required for the nonlinear wave processes, is not fulfilled automatically, but is significantly dependent on the dispersion characteristics of the waves under consideration. PIT is different from the standard Raman scattering in that the medium is opaque for the P wave in the absence of the nonlinear coupling with other waves. The generation of bichromatic waves (combination of P and S waves) is most effective in the four-wave-mixing regime, which is determined by the energy-conservation law

$$2\hbar\omega_d = \hbar\omega_p + \hbar\omega_s \tag{1}$$

and the momentum-conservation law

$$2\hbar\boldsymbol{k}_d = \hbar\boldsymbol{k}_p + \hbar\boldsymbol{k}_s. \tag{2}$$

The linear theory from [38] explains previous experiments on the Stokes satellite generation in the EIT propagation regime for the probe field [40–42]. The nonlinear theory was developed in [39], which predicted special regimes with an almost complete transformation of drive radiation into correlated polychromatic radiation.

In this paper, we present theoretical analysis of generation of entangled photons, and, especially, the squeezed vacuum, in the process of forward four-wave mixing in an optical dense three-level Λ medium. Some benefits of this method are revealed.

The paper is structured as follows. In Sec. II, we discuss approximations and obtain the field operator equations. In Sec. III, we construct a solution in the form of biharmonic normal modes. In Sec. IV, we estimate the dissipation effects. In Secs. V and VI, we consider the regimes of the biphoton generation and the squeezed vacuum generation. In Sec. VII, we make some numerical estimations and compare our results with the parametric down-conversion method. In Sec. VIII, we formulate our results and discuss them.

II. FORMALISM

At first, we formulate general relations, which determine one-dimensional propagation of quantized fields with a relatively narrow spectrum. The simplest way to obtain such equations is to apply the "slowly varying amplitude" method to the well-known wave equation for the Heisenberg quantum field operator [46,47]:[1]

$$\frac{\partial^2}{\partial t^2}(\hat{\boldsymbol{E}} + 4\pi\hat{\boldsymbol{P}}) - c^2\frac{\partial^2\hat{\boldsymbol{E}}}{\partial z^2} = 0. \tag{3a}$$

[1]The derivation of Eq. (3a) from the quantum Hamiltonian of the field and medium system is presented, e.g., in [56].

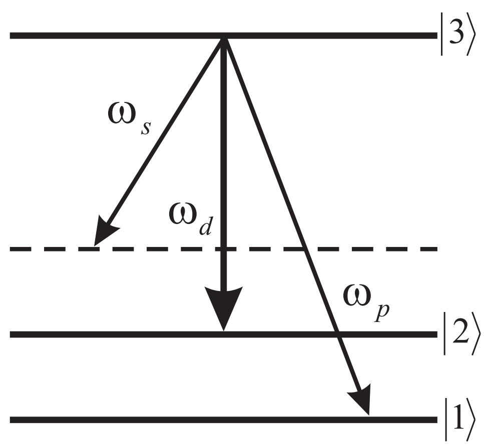

FIG. 1. Λ scheme with excitation of a Stokes wave.

This equation contains the electric-field operator $\hat{\boldsymbol{E}}$ and the polarization operator $\hat{\boldsymbol{P}}$. We assume that all fields are x polarized. If the spectrum is narrow and concentrated near the carrier frequency ω_0, we can represent the field operator $\hat{\boldsymbol{E}}$ in the following form:

$$\hat{\boldsymbol{E}} = \boldsymbol{x}_0[\hat{E}^{(+)}(z,t)\,e^{ik_0z-i\omega_0t} + \hat{E}^{(-)}(z,t)\,e^{-ik_0z+i\omega_0t}]; \tag{3b}$$

here, $\boldsymbol{x}_0$ is a unit polarization vector, $\hat{E}^{(+,-)}(z,t)$ are the Hermitian conjugate Heisenberg operators for the slowly varying amplitudes:

$$\hat{E}^{(+)}(z,t) = \sum_\delta \hat{E}_\delta^{(+)}(z)\,e^{-i\delta t}, \quad \hat{E}^{(-)}(z,t) = \sum_\delta \hat{E}_\delta^{(-)}(z)\,e^{i\delta t},$$
$$\hat{E}_\delta^{(+)}(z) = E_\delta \hat{c}_\delta(z), \quad \hat{E}_\delta^{(-)}(z) = E_\delta \hat{c}_\delta^\dagger(z), \tag{3c}$$

the set of the frequency detuning δ defines the quantization modes set, E_δ are the normalization constants, and $\hat{c}_\delta(z)$ and $\hat{c}_\delta^\dagger(z)$ are the Fock state annihilation and creation operators, which in our case depend on the coordinate. Thus, in our description, the "slow" coordinate dependence of the field corresponds to the similar dependence of operators. As in [47], we represent the medium polarization operator in the following form:

$$\hat{\boldsymbol{P}} = \boldsymbol{x}_0[\hat{P}^{(+)}(z,t)\,e^{ik_0z-i\omega_0t} + \hat{P}^{(-)}(z,t)\,e^{-ik_0z+i\omega_0t}], \tag{4a}$$

$$\hat{P}^{(+)}(z,t) = \sum_\delta \hat{P}_\delta^{(+)}(z)\,e^{-i\delta t},$$
$$\hat{P}^{(-)}(z,t) = \sum_\delta \hat{P}_\delta^{(-)}(z)\,e^{i\delta t}, \tag{4b}$$

where

$$\hat{P}_\delta^{(+)}(z) = \chi(\omega_0+\delta)\hat{E}_\delta^{(+)}(z) + \hat{P}_{N,\delta}^{(+)}(z) + \hat{P}_{L,\delta}^{(+)}(z),$$
$$\hat{P}_\delta^{(-)}(z) = \chi^*(\omega_0+\delta)\hat{E}_\delta^{(-)}(z) + \hat{P}_{N,\delta}^{(-)}(z) + \hat{P}_{L,\delta}^{(-)}(z), \tag{4c}$$

$\chi(\omega_0+\delta)$ is the linear susceptibility at the frequency $\omega = \omega_0 + \delta$, $\hat{P}_{N,\delta}^{(+,-)}$ and $\hat{P}_{L,\delta}^{(+,-)}$ are the spectral polarization terms, which correspond to the nonlinear (or parametrical) processes and Langevin noise respectively. The Langevin noise depends on the dissipative effects through the imaginary component of susceptibility $\mathrm{Im}\chi$—see, e.g. [12,43–45,47]. We define the wave number k_0 by the relation $c^2k_0^2/\omega_0^2 = n^2(\omega_0)$, where $n^2(\omega_0) = 1 + 4\pi\mathrm{Re}\chi(\omega_0)$ is the linear refractive index at the "carrier" frequency ω_0. We use the known results of the procedure of quantization in a transparent medium [48] to determine the normalization constants:[2]

$$E_\delta = \sqrt{4\pi\hbar\omega^2\left(\frac{\partial(\omega^2\mathrm{Re}n^2(\omega))}{\partial\omega}\right)^{-1}}\Bigg|_{\omega=\omega_0+\delta}. \tag{5a}$$

With this normalization operator $\hat{n}_\delta = \hat{c}_\delta^\dagger\hat{c}_\delta$ corresponds to the spatial density of the photon number in the medium at the frequency $\omega_0+\delta$ [48]. As in [48], we consider the quantization volume V, which is much less than the scale of spatial variation for the field intensity.[3] According to [48] and field representations (3b) and (3c), Eq. (5a) corresponds to the following expression for the energy field operator in the volume V (without consideration of the dissipation and/or nonlinear processes),

$$\hat{W} = \hbar\sum_\delta(\omega_0+\delta)\left(V\hat{n}_\delta + \frac{1}{2}\right), \tag{5b}$$

and the following commutation relation:

$$[\hat{c}_\delta,\hat{c}_{\delta'}^\dagger] = \delta_{\delta\delta'}/V. \tag{5c}$$

It is significant that field energy operator (5b) includes both the field energy and the medium excitation energy (see also [48]).

We assume that the frequency band is narrow, so we simplify relation (3c) by using one and the same normalization constant $E_0 = E_{\delta=0}$ for all values of δ. In this case, we obtain

$$\hat{E}^{(+)} \approx E_0\hat{c}_0(z,t), \quad \hat{E}^{(-)} \approx E_0\hat{c}_0^\dagger(z,t), \tag{6a}$$

where

$$\hat{c}_0(z,t) = \sum_\delta \hat{c}_\delta(z)\,e^{-i\delta t}, \quad \hat{c}_0^\dagger(z,t) = \sum_\delta \hat{c}_\delta^\dagger(z)\,e^{i\delta t}. \tag{6b}$$

Here, $\hat{c}_0(z,t)$, $\hat{c}_0^\dagger(z,t)$ are the "slowly varying" amplitudes for the Heisenberg annihilation and creation operators. For them, we obtain the commutation relations, which do not depend on the arbitrary quantization volume V. Let us see the volume equal to a ray tube with square ΔS and length $l = V/\Delta S$ along the ray axis. The number of z-propagating modes dZ in the wave vector interval dk is determined by the following expression [45]:[4]

$$dZ = \frac{l}{2\pi}dk = \frac{l}{2\pi}k'_\omega d\omega \tag{7a}$$

(here, $d\omega$ is the frequency interval corresponding to the wave number interval). Using relations (5c), (6b), and (7a), we obtain the following commutation relation:

$$[\hat{c}_0(z,t),\hat{c}_0^\dagger(z,t')] = \frac{\delta(t-t')}{\Delta S v_{gr}}, \tag{7b}$$

[2] At the "vacuum" limit $n^2 = 1$, this normalization corresponds to the standard one for the unit quantization volume case: $E_\delta = \sqrt{2\pi\hbar(\omega_0+\delta)}$.

[3] The quantization volume is large according to the wavelength.

[4] One can obtain the same expression by multiplying the state density $(Vk^2/8\pi^3)k'_\omega$ by the beam spectral angular width $4\pi^2/k^2\Delta S$.

where

$$v_{gr} = \left(\frac{\partial \omega}{\partial k}\right)_{\omega=\omega_0} = 2k_0c^2\left(\frac{\partial(\omega^2 \mathrm{Re}n^2(\omega))}{\partial\omega}\right)^{-1}_{\omega=\omega_0} \tag{8}$$

is the group velocity. Relation (7b) is of great generality. For example, an analogous relation was obtained in [45] for a particular case of a medium without frequency dispersion and with a different field normalization.

The simplest way to obtain the evolution equation for annihilation and creation operators is to substitute relations (3b), (4a), (4b), (6a), and (6b) into operator wave equation (3a). For further evaluations, we need equations for the z dependence of the harmonic field components $E_0\hat{c}_\delta(z)\,e^{ik_0z-i(\delta+\omega_0)t}$ and $E_0\hat{c}_\delta^\dagger(z)\,e^{-ik_0z+i(\delta+\omega_0)t}$. As in [48], we take the "additional" polarization components $\hat{P}^{(+,-)}_{N,\delta}$ and $\hat{P}^{(+,-)}_{L,\delta}$ into account by the perturbation method. Here, we make the following approximations: (i) narrow spectrum condition, $|\delta| \ll \omega_0$; (ii) transparent medium approximation, $\mathrm{Re}n^2 \gg \mathrm{Im}n^2$; (iii) operators $\hat{c}_\delta(z)$ and $\hat{c}_\delta^\dagger(z)$ depend slowly on the coordinate on the wavelength scale $2\pi/k_0$.

We use the relations

$$\frac{\partial^2}{\partial t^2}[n^2(\omega_0+\delta)\hat{c}_\delta e^{-i(\delta+\omega_0)t}] \approx -e^{-i(\delta+\omega_0)t}\left[\omega_0^2 n^2(\omega_0) + \delta\left(\frac{\partial(\omega^2 n^2(\omega))}{\partial\omega}\right)_{\omega=\omega_0}\right]\hat{c}_\delta,$$

$$-\frac{\partial^2}{\partial z^2}[\hat{c}_\delta(z)\,e^{ik_0z}] \approx e^{ik_0z}\left(k_0^2 - 2ik_0\frac{\partial}{\partial z}\right)\hat{c}_\delta(z)$$

and use expression (8) as well, for group velocity, and obtain the following equation for the annihilation operator $\hat{c}_\delta(z)$:

$$\left(-i\delta + v_{gr}\frac{\partial}{\partial z} + \gamma_\delta\right)\hat{c}_\delta = \frac{i}{\hbar}\left(\hat{P}^{(+)}_{N,\delta} + \hat{P}^{(+)}_{L,\delta}\right)E_0. \tag{9a}$$

Here

$$\gamma_\delta = \frac{\omega_0^2 \mathrm{Im}n^2(\omega_0+\delta)}{[\partial(\omega^2 \mathrm{Re}n^2(\omega))/\partial\omega]_{\omega=\omega_0}}. \tag{9b}$$

We also obtain a Hermitian conjugated equation for the creation operator $\hat{c}_\delta^\dagger$. In the case of low optical density, relation (9a) is simplified by substitution of $v_{gr} = c$ and $\mathrm{Re}n^2 = 1$ (see, e.g. [43,44]). The case of a dense medium is specified, and a relation similar to Eq. (9a) is obtained in [48] for ideal resonator modes, i.e., for the case with no convective and dissipative loss. We use the above-derived general expressions (9a) and (9b) for the Heisenberg field operators in medium and results of papers [38] and [39] to analyze the four-wave interaction in the Λ system.

The consistency of Eqs. (1) and (2) depends on wave dispersion in the medium. In this case, rapid dependence of wave vector $\boldsymbol{k}_p(\omega_p)$ on the frequency in the EIT "transparency window" is of great importance. The corresponding one-dimensional (1D) theory was developed in [38] and [39], where the formula for the resonant frequencies[5] ω_p^0 and ω_s^0 was obtained:

$$\omega_p^0 = \omega_{31} + \Delta\omega_{\mathrm{PIT}}, \quad \omega_s^0 = 2\omega_{32} - \omega_{31} - \Delta\omega_{\mathrm{PIT}}. \tag{10}$$

Here, $\Delta\omega_{\mathrm{PIT}} = 5\Omega_R^2/2\omega_{21}$, $\Omega_R = \frac{|d_{32}\xi_d|}{2\hbar}$ is the Rabi frequency for the drive field with the amplitude ξ_d, and d_{32} is the electric dipole moment of corresponding transition. It is of great importance that condition (10) does not depend on either the Λ-atom density or the background refractive index.[6] In the so-called parametrically induced transparency (PIT) band [38], there is convective instability for wave pairs with frequencies $\omega_{\delta;p} = \omega_p^0 + \delta$ and $\omega_{\delta;s} = \omega_s^0 - \delta$. The instability takes place in the case of the frequency band which is much narrower than the EIT transparency window:

$$-\tfrac{4}{5}\Delta\omega_{\mathrm{PIT}} \leqslant \delta \leqslant \tfrac{4}{5}\Delta\omega_{\mathrm{PIT}}. \tag{11}$$

If we allow for the relaxation processes, the instability band broadens (dissipative instability regime), but the maximum of the increment corresponds to the weak relaxation case [38,39].

We assume that all waves propagate in the z direction and the x polarized, and the Λ-atom layer is plane parallel. We describe the drive field classically:

$$\boldsymbol{E}_d = \boldsymbol{x}_0(\xi_d e^{ik_dz-i\omega_dt} + \xi_d^* e^{-ik_dz+i\omega_dt}). \tag{12a}$$

It is a standard approximation [43–45]. Here, the signal field corresponds to two relatively narrow spectral intervals near the frequencies ω_p^0 and ω_s^0. We call this field the biband wave (BBW). We use an operator form similar to Eq. (3b) for signal wave fields:

$$\hat{\boldsymbol{E}} = \boldsymbol{x}_0\big[\hat{E}_p^{(+)}(z,t)\,e^{ik_p^0z-i\omega_p^0t} + \hat{E}_s^{(+)}(z,t)\,e^{ik_s^0z-i\omega_s^0t} + \hat{E}_p^{(-)}(z,t)\,e^{-ik_p^0z+i\omega_p^0t} + \hat{E}_s^{(-)}(z,t)\,e^{-ik_s^0z+i\omega_s^0t}\big]. \tag{12b}$$

Here, $k_p^0 = k_p(\omega_p^0)$, $k_s^0 = k_s(\omega_s^0)$ and the Heisenberg operators $\hat{E}_p^{(+,-)}$ and $\hat{E}_s^{(+,-)}$ correspond to the P and S waves. For these operators, we use a representation similar to Eqs. (6a) and (6b):

$$\hat{E}^{(+)}_{p,s} = \sum_\delta E_{p,s}\hat{c}_{\delta;p,s}(z)\,e^{\mp i\delta t}, \qquad \hat{E}^{(-)}_{p,s} = \sum_\delta E_{p,s}\hat{c}^\dagger_{\delta;p,s}(z)\,e^{\pm i\delta t}. \tag{12c}$$

Here, the index "p" corresponds to the upper sign under exponents and the index "s" corresponds to the lower sign, and $E_{p,s}$ are the normalization constants similar to E_0 in formula (6a):

$$E_{p,s} = \sqrt{4\pi\hbar\omega^2\left(\frac{\partial\omega^2 \mathrm{Re}n^2_{p,s}(\omega)}{\partial\omega}\right)^{-1}_{\omega=\omega^0_{p,s}}}. \tag{13}$$

Here, $n^2_{p,s}(\omega)$ are the frequency-dependent refractive indexes for the corresponding waves, which will be specified below. Annihilation and creation operators $\hat{c}_{\delta;p,s}(z)$ and $\hat{c}^\dagger_{\delta;p,s}$ correspond to the resonant frequency pairs $\omega_{\delta;p,s} = \omega^0_{p,s} \pm \delta$,

[5] We consider copropagating waves, because conditions (1) and (2) cannot be fulfilled, if the signal and the drive waves counterpropagate.

[6] The background refractive index can be determined by other levels neglected in the resonant scheme, the buffer gas, the material of the matrix doped with atoms in condensed media, etc.

which satisfy synchronism condition (1). Operators $\hat{c}_{\delta;p,s}(z)$ and $\hat{c}^{\dagger}_{\delta;p,s}$ depend on the z coordinate slowly on the wavelength scale $2\pi/k^0_{p,s}$.

It should be emphasized that the classic drive field with constant power implies that the refractive indexes $n^2_{p,s}$ in Eq. (13) may depend on the drive field amplitude as a parameter. In the case similar to (5b), the expressions for the energy field operators do not simply correspond to a sum of the "vacuum" signal field energy and the corresponding medium excitation energy, but also contain a reversible exchange of energy between the medium and the drive field. However, this detail does not change any formulas in the linear in quantum field theory (see also [49,50]).

We use papers [38] and [39] to determine the polarization $\hat{\boldsymbol{P}}$ of the medium under the influence of P and S waves in the presence of drive field. These papers present a solution of the equation system for the Λ -system density operator in the classical electromagnetic field case. In the case of a quantum description of the P and S waves, there are similar formulas for the density operator (and, consequently, the polarization operator), if (i) the drive field is nonquantum and (ii) the theory is linear on quantum field.

The only difference from the classic field case is the substitution of the complex amplitudes of the P and S waves by the corresponding Heisenberg field operators (12c). Moreover, because of the dissipation we should, in principle, take into consideration the polarization thermal noise. As in [12], we treat the thermal fluctuation effect by simply introducing Langevin noise sources directly in the final equations for the Heisenberg field operators.

At first, we obtain the "dynamic" components of the medium polarization operator at the frequencies of the P and S waves. We represent them in the form similar to formulas (4a) and (4b):

$$\hat{\boldsymbol{P}} = \boldsymbol{x}_0\big[\hat{P}^{(+)}_p(z,t)\,e^{ik^0_p z - i\omega^0_p t} + \hat{P}^{(-)}_p(z,t)\,e^{-ik^0_p z + i\omega^0_p t} + \hat{P}^{(+)}_s(z,t)\,e^{ik^0_s z - i\omega^0_s t} + \hat{P}^{(-)}_s(z,t)\,e^{-ik^0_s z + i\omega^0_s t}\big], \quad (14a)$$

where

$$\hat{P}^{(+)}_{p,s}(z,t) = \sum_\delta \hat{P}^{(+)}_{\delta;p,s}(z)\,e^{\mp i\delta t}, \qquad \hat{P}^{(-)}_{p,s}(z,t) = \sum_\delta \hat{P}^{(-)}_{\delta;p,s}(z)\,e^{\pm i\delta t} \quad (14b)$$

(the index "p" corresponds to the upper sign under exponent and the index "s" corresponds to the lower one). Papers [38] and [39] state that, if synchronism relations (1) and (2) are fulfilled, the spectral components of the medium polarization operator at the frequencies of the P and S waves are determined by the following general relations:

$$\begin{aligned}
\hat{P}^{(+)}_{\delta;p} &= \chi_{pp}E_p\hat{c}_{\delta;p} + \chi_{ps}e^{2i\theta}E_s\hat{c}^{\dagger}_{\delta;s},\\
\hat{P}^{(+)}_{\delta;s} &= \chi_{ss}E_s\hat{c}_{\delta;s} + \chi_{sp}e^{2i\theta}E_p\hat{c}^{\dagger}_{p;s},\\
\hat{P}^{(-)}_{\delta;p} &= \chi^*_{pp}E_p\hat{c}^{\dagger}_{\delta;p} + \chi^*_{ps}e^{-2i\theta}E_s\hat{c}_{\delta;s},\\
\hat{P}^{(-)}_{\delta;s} &= \chi^*_{ss}E_s\hat{c}^{\dagger}_{\delta;s} + \chi^*_{sp}e^{-2i\theta}E_p\hat{c}_{\delta;p},
\end{aligned} \quad (15)$$

where θ is the drive field income phase. Similar to papers [38] and [39], we define χ_{pp}, χ_{ss}, χ_{ps}, and χ_{sp} in the following model approximations.

(i) The "rotating wave" approximation:

$$|\omega_{\delta;p} - \omega_{31}| = |\omega_{\delta;s} - \omega_{32} + \omega_{21}| \ll \omega_{21} \ll \omega_{31,32}; \quad (16)$$

(ii) a hierarchy of relaxation constants:

$$\gamma_{21} \ll \gamma_{31,32} \ll \omega_{21}; \quad (17)$$

(iii) some conditions for the drive field Rabi frequency

$$\Omega^2_R \gg \gamma_{31}\cdot\gamma_{21}, \quad (18)$$

$$\Omega_R \ll \gamma_{31,32}. \quad (19)$$

Condition (18) provides the EIT-regime realization. Due to condition (19), we assume that the drive field does not change populations.

(iv) Only state $|1\rangle$ is populated.

We refer to [38] and [39] for detailed calculations and write down the result: [7]

$$\begin{aligned}
\chi_{pp} &= \frac{\eta}{4\pi\Omega_R}\bigg[\frac{\omega_{\delta;p}-\omega_{31}}{\Omega_R} - \frac{|d_{31}|^2}{|d_{32}|^2}\frac{\Omega_R}{\omega_{21}} + i\bigg(\frac{\gamma_{21}}{\Omega_R} + \frac{\gamma_{31}(\omega_{\delta;p}-\omega_{31})^2}{\Omega^3_R}\bigg)\bigg],\\
\chi_{ss} &= \frac{\eta}{8\pi\omega_{21}}\bigg(1 - \frac{3}{2}\frac{\omega_{\delta;s}-\omega_{32}+\omega_{21}}{\omega_{21}} - \frac{3i}{2}\frac{\gamma_{31}}{\omega_{21}}\bigg),\\
\chi_{ps} &= \chi_{sp} = -\frac{\eta}{4\pi\omega_{21}}.
\end{aligned} \quad (20)$$

Here, $d_{31,32}$ are the dipole moments of corresponding transitions, $\eta = 4\pi|d_{31}|^2N/\hbar$, and N is the Λ-atoms density.[8] Note that the factors χ_{ps} and χ_{ss} do not depend on the drive field due to inequality (18), because in the EIT-PIT regime these dependences are determined by the factor $\frac{\Omega^2_R}{\Omega^2_R+\gamma_{31}\gamma_{21}} \approx 1$ (see [38] for more detail).

We substitute formulas (12b), (12c), (13), (14a), (14b), and (15) into operator wave equation (3a). So we obtain equations like (9a) and (9b) for the P and S waves. The last terms in the right parts of Eqs. (15) correspond to the polarization components $\hat{P}^{(+)}_{N;\delta}$ in the right-hand parts of field operator equations (9a); the terms guarantee the parametrical coupling of P and S waves. First terms in the right-hand parts of Eqs. (15) determine the corresponding refractive indexes $n^2_{p,s} = 1 + 4\pi\chi_{pp,ss}$. These indexes determine normalization (13), group velocities, and wave decrements (or increments).[9] We also take into consideration the condition for the Λ-atoms density and level splitting in the three-level

[7]Relations (20), according to the same in paper [38], have some nonessential corrections in formulas for $\mathrm{Re}\chi_{pp}$ and $\mathrm{Re}\chi_{ss}$.

[8]Unlike in [38], we neglect effects of the nonresonant medium (background refractive index, etc.) for simplicity.

[9]Previously used formulas (10) and (11) also follow from the corresponding expressions for the refractive indexes $n^2_{p,s} = 1 + 4\pi\chi_{pp,ss}$.

system:

$$\eta \ll \omega_{21}. \tag{21}$$

Condition (21) means that the four-wave shift influence on the partial wave dispersion laws is weak to a certain extent.

Similar to [38], it is appropriate to write the resulting equations for operators, which describe photon fluxes. For that purpose, we introduce operators $\hat{p}_\delta$,$\hat{s}_\delta$ and $\hat{p}_\delta^\dagger$,$\hat{s}_\delta^\dagger$; thereby the dyads $\hat{p}_\delta^\dagger\hat{p}_\delta$ and $\hat{s}_\delta^\dagger\hat{s}_\delta$ are the P,S-photon flux density operators. Finally, we obtain quantum operator equations by adding the corresponding Langevin noise operators (Langevin forces) to the right-hand parts of the equations:

$$\begin{aligned}\left(\frac{\partial}{\partial z}-\frac{i\delta}{V_{\rm EIT}^{gr}}+\kappa_{\rm EIT}\right)\hat{p}_\delta &= -i\chi_{\rm PIT}e^{2i\theta}\hat{s}_\delta^\dagger+\hat{F}_{\delta;p},\\ \left(\frac{\partial}{\partial z}-\frac{i\delta}{V_s^{gr}}-\mu_s\right)\hat{s}_\delta^\dagger &= i\chi_{\rm PIT}e^{-2i\theta}\hat{p}_\delta+\hat{F}_{\delta;s}^\dagger,\end{aligned} \tag{22}$$

where $\hat{F}_{\delta;p}$ and $\hat{F}_{\delta;s}^\dagger$ are the spectra for the Langevin operators, which are described in detail in [12],

$$V_{\rm EIT}^{gr}\approx\frac{c}{1+\eta\omega_{\rm HF}/2\Omega_R^2},\quad V_s^{gr}\approx\frac{c}{1-3\eta\omega_{\rm HF}/8\omega_{21}^2}, \tag{23}$$

where $V_{\rm EIT}^{gr}$ is the group velocity for signal (probe) wave at the EIT conditions, V_s^{gr} is the Stokes satellite group velocity, and $\omega_{\rm HF}\approx\omega_d\approx\omega_{\delta;p}\approx\omega_{\delta;s}$ is the characteristic frequency of radiation. One can see that according to Eq. (23), V_s^{gr} is a group velocity that exceeds the speed of light and can be negative at certain parameters. This effect is well known for active and dissipative media and does not lead to any paradox (see [51–54]). However, within the scope of our paper, we restrict ourselves to the "positive" group velocities case for simplicity.

"Flux" operators $\hat{p}_\delta$, $\hat{p}_\delta^\dagger$ and $\hat{s}_\delta$, $\hat{s}_\delta^\dagger$ relates to previously introduced field operators $\hat{c}_{\delta;p}$, $\hat{c}_{\delta;p}^\dagger$ and $\hat{c}_{\delta;s}$, $\hat{c}_{\delta;s}^\dagger$ the following way:

$$\begin{aligned}\hat{p}_\delta &= \sqrt{V_{\rm EIT}^{gr}}\hat{c}_{\delta;p},\quad \hat{p}_\delta^\dagger=\sqrt{V_{\rm EIT}^{gr}}\hat{c}_{\delta;p}^\dagger,\\ \hat{s}_\delta &= \sqrt{V_s^{gr}}\hat{c}_{\delta;s},\quad \hat{s}_\delta^\dagger=\sqrt{V_s^{gr}}\hat{c}_{\delta;s}^\dagger.\end{aligned} \tag{24}$$

Other notations in Eqs. (22) are as follows:

$$\kappa_{\rm EIT}\approx\frac{\eta\omega_{31}}{2c\Omega_R^2}\left(\gamma_{21}+\gamma_{31}\frac{(\omega_{\delta;p}-\omega_{31})^2}{\Omega_R^2}\right)$$

is the absorption coefficient for the P wave with the frequency $\omega_{\delta;p}=\omega_p^0+\delta$ in the EIT regime; $\mu_s\approx\frac{3\omega_{\rm HF}\eta\gamma_{31}}{8c\omega_{21}^2}$ is the spatial increment for dissipative instability transformation $\hbar\omega_d\to h\omega_s+\hbar\omega_{21}$ which was reported in [38]; $\chi_{\rm PIT}\approx\frac{\omega_{\rm HF}\eta}{2c\omega_{21}}$ is the parametric wave interaction coefficient under resonance conditions (1, 2); and θ is the drive field income phase. In a transparent medium (i.e., $\kappa_{\rm EIT}=\mu_s=\hat{F}_{\delta;p}=\hat{F}_{\delta;s}=0$), Eqs. (22) have an integral, which expresses conservation of the difference of P,S-photon fluxes in process (1, 2): $\frac{\partial}{\partial z}(\hat{p}_\delta^\dagger\hat{p}_\delta-\hat{s}_\delta^\dagger\hat{s}_\delta)=0$ (Manley-Rowe relation [48]).

Equations (22) need the corresponding boundary conditions. Let us denote the border between the Λ medium and the vacuum by $z=z_b$. Similar to [38], we neglect the weak energy flux reflection from the boundary. In this case, we suppose that the operators $\hat{p}_\delta$ and $\hat{s}_\delta$ are continuous and get the following relations for the field operators $\hat{c}_{\delta;p}$ and $\hat{c}_{\delta;s}$ in the vacuum and the "flux" operators $\hat{p}_\delta$ and $\hat{s}_\delta$ in the medium:

$$\begin{aligned}\hat{c}_{\delta;p}(z_b)\Big|_{\rm vacuum} &= \frac{1}{\sqrt{c}}\hat{p}_\delta(z_b)\Big|_{\rm medium},\\ \hat{c}_{\delta;s}(z_b)\Big|_{\rm vacuum} &= \frac{1}{\sqrt{c}}\hat{s}_\delta(z_b)\Big|_{\rm medium}.\end{aligned} \tag{25}$$

Relations (22) and (25) make a complete description of the transportation of quantum states of light through the Λ-atoms layer.

We present some useful expressions for photon flux operators in vacuum $\hat{N}_{p,s}$ in a light beam with a constant square of the cross section:

$$\hat{N}_p\approx c\Delta S\sum_\delta\hat{c}_{\delta;p}^\dagger\hat{c}_{\delta;p},\quad \hat{N}_s\approx c\Delta S\sum_\delta\hat{c}_{\delta;s}^\dagger\hat{c}_{\delta;s}$$

(ΔS is the square of the beam cross section). Moving to the continuous interval of the frequency detuning $\Delta\omega$, we use relation (7a). For the unit quantization volume $l\Delta S=1$, we obtain formulas

$$\hat{N}_p=\frac{1}{2\pi}\int_{\Delta\omega}\hat{c}_{\delta;p}^\dagger\hat{c}_{\delta;p}d\delta,\quad \hat{N}_s=\frac{1}{2\pi}\int_{\Delta\omega}\hat{c}_{\delta;s}^\dagger\hat{c}_{\delta;s}d\delta. \tag{26a}$$

Here, the operators $\hat{c}_{\delta;p}^\dagger$, $\hat{c}_{\delta;p}$ and $\hat{c}_{\delta;s}^\dagger$, $\hat{c}_{\delta;s}$ correspond to the creation and annihilation operators in the standard unit volume normalization.

Using formulas (12b) and (12c), we similarly obtain a convenient formula for the beam aperture average of the electric-field correlator in vacuum:

$$\langle\hat{E}(z,t)\hat{E}(z,t')\rangle=\frac{\hbar}{c\Delta S}\int_{\Delta\omega}\langle\hat{G}_\delta(z,t,t')\rangle d\delta, \tag{26b}$$

where

$$\begin{aligned}\langle\hat{G}_\delta(z,t,t')\rangle &= \omega_p^0\{2\langle\hat{c}_{\delta;p}^\dagger(z)\hat{c}_{\delta;p}(z)\rangle\cos[\omega_{\delta;p}(t-t')]+e^{i\omega_{\delta;p}(t'-t)}\}+\omega_s^0\{2\langle\hat{c}_{\delta;s}^\dagger(z)\hat{c}_{\delta;s}(z)\rangle\cos[\omega_{\delta;s}(t-t')]+e^{i\omega_{\delta;s}(t-t')}\}\\ &\quad+\sqrt{\omega_p^0\omega_s^0}\langle\hat{c}_{\delta;p}^\dagger(z)\hat{c}_{\delta;s}(z)\rangle e^{i(k_s^0-k_p^0)z}(e^{i(\omega_{\delta;p}t-\omega_{\delta;s}t')}+e^{i(\omega_{\delta;p}t'-\omega_{\delta;s}t)})\\ &\quad+\sqrt{\omega_p^0\omega_s^0}\langle\hat{c}_{\delta;s}^\dagger(z)\hat{c}_{\delta;p}(z)\rangle e^{i(k_p^0-k_s^0)z}(e^{i(\omega_{\delta;s}t-\omega_{\delta;p}t')}+e^{i(\omega_{\delta;s}t'-\omega_{\delta;p}t)})\\ &\quad+\big[\omega_p^0\langle\hat{c}_{\delta;p}(z)\hat{c}_{\delta;p}(z)\rangle e^{2ik_p^0z-i\omega_{\delta;p}(t+t')}+\omega_s^0\langle\hat{c}_{\delta;s}(z)\hat{c}_{\delta;s}(z)\rangle e^{2ik_s^0z-i\omega_{\delta;s}(t+t')}\\ &\quad+\sqrt{\omega_p^0\omega_s^0}\langle\hat{c}_{\delta;p}(z)\hat{c}_{\delta;s}(z)\rangle e^{i(k_s^0+k_p^0)z}(e^{-i(\omega_{\delta;p}t+\omega_{\delta;s}t')}+e^{-i(\omega_{\delta;p}t'+\omega_{\delta;s}t)})+\text{H.c.}\big].\end{aligned} \tag{26c}$$

When finding the correlator of the electric field, we are not interested in the terms with a different frequency detuning (e.g., $\hat{c}^{\dagger}_{\delta;p}\hat{c}_{\delta';p}$). We can see from Eqs. (22) that such operators are completely independent and the average of their product for nonentangled initial values is identically equal to zero. Equation (22) means that the operators of P and S waves, which correspond to one and the same index δ (e.g., $\hat{c}_{\delta;p}$ and $\hat{c}^{\dagger}_{\delta;s}$), become dependent on each other after transmission through the atom layer.

III. ANALYTIC SOLUTION FOR FIELD OPERATOR EQUATIONS

Equations (22) have the following general solution:

$$\begin{pmatrix}\hat{p}_\delta\\ \hat{s}^{\dagger}_\delta\end{pmatrix} = \begin{pmatrix}1\\ Q_X(\delta)\end{pmatrix} e^{iq_X(\delta)z}\left(\hat{u}_X + \int_0^z e^{-iq_X(\delta)\xi}\hat{f}_{\delta;X}d\xi\right) + \begin{pmatrix}1\\ Q_O(\delta)\end{pmatrix} e^{iq_O(\delta)z}\left(\hat{u}_O + \int_0^z e^{-iq_O(\delta)\xi}\hat{f}_{\delta;O}d\xi\right), \tag{27}$$

where

$$q_{O,X}(\delta) = \frac{\delta}{2}\left(\frac{1}{V^{gr}_{\mathrm{EIT}}} + \frac{1}{V^{gr}_{s}}\right) + i\frac{\kappa_{\mathrm{EIT}} - \mu_s}{2} \pm i\chi_{\mathrm{PIT}}\sqrt{1-\sigma^2}, \tag{28}$$

$$\sigma = \frac{\delta}{2\chi_{\mathrm{PIT}}}\left(\frac{1}{V^{gr}_{\mathrm{EIT}}} - \frac{1}{V^{gr}_{s}}\right) + \frac{i}{2\chi_{\mathrm{PIT}}}(\kappa_{\mathrm{EIT}} + \mu_s), \tag{29}$$

and

$$Q_{O,X}(\delta) = (\sigma \mp \sqrt{\sigma^2 - 1})e^{-2i\theta}. \tag{30}$$

Solution of Eq. (22) has the form of a superposition of two bichromatic normal modes in the medium. As in [38], we denote them the O and X modes. The coefficients $Q_{O,X}$ determine the amplitude ratio of the harmonic components in the corresponding normal mode (they are similar to the polarization factor in an anisotropic medium; see also [38]). Formula (28) means that the X mode is always unstable. Arbitrary operators $\hat{u}_X$ and $\hat{u}_O$ are determined by boundary conditions (25). We specify the input field operators $\hat{c}_{\delta;p}(0)$ and $\hat{c}^{\dagger}_{\delta;s}(0)$ at the layer boundary $z = 0$ in vacuum and obtain

$$\begin{aligned}\hat{u}_X &= \sqrt{c}\frac{Q_O(\delta)\hat{c}_{\delta;p}(0) - \hat{c}^{\dagger}_{\delta;s}(0)}{Q_O(\delta) - Q_X(\delta)},\\ \hat{u}_O &= \sqrt{c}\frac{\hat{c}^{\dagger}_{\delta;s}(0) - Q_X(\delta)\hat{c}_{\delta;p}(0)}{Q_O(\delta) - Q_X(\delta)}.\end{aligned} \tag{31}$$

In (27), $\hat{f}_{\delta;X}$ and $\hat{f}_{\delta;O}$ are the Langevin sources for the O and X modes:

$$\hat{f}_{\delta;X} = \frac{Q_O(\delta)\hat{F}_{\delta;p} - \hat{F}^{\dagger}_{\delta;s}}{Q_O(\delta) - Q_X(\delta)}, \quad \hat{f}_{\delta;O} = \frac{\hat{F}^{\dagger}_{\delta;s} - Q_X(\delta)\hat{F}_{\delta;p}}{Q_O(\delta) - Q_X(\delta)}. \tag{32}$$

From Eqs. (27) and (31), it is clear that uncorrelated photons, which passed throw the driven Λ-atoms layer, became entangled in pairs, being coupled in compliance with relation (1). Formally, it follows from functional connections $\hat{p}_\delta(z) = \hat{p}_\delta(\hat{c}_{\delta;p}(0),\hat{c}^{\dagger}_{\delta;s}(0),z)$ and $\hat{s}^{\dagger}_\delta(z) = \hat{s}^{\dagger}_\delta(\hat{c}_{\delta;p}(0),\hat{c}^{\dagger}_{\delta;s}(0),z)$ which are determined by formulas (27) and (31).

IV. ESTIMATIONS OF DISSIPATIVE EFFECTS

Using the well-known estimations for the power of Langevin sources (see [12,47]) and exact solution (27), one can easily get the following estimation:

$$\frac{n_L}{n_E} \approx \frac{\max(\kappa_{\mathrm{EIT}},\mu_s)}{|\mathrm{Im}q_X|}\frac{n_T}{n_0}. \tag{33}$$

Here, n_0 is the number of "seed" photons at the layer input, n_E is the number of the entangled photons at the layer output, n_L is the number of the Langevin "noise" photons, and n_T is the number of the thermal photons corresponding to the layer temperature T. Quantities $n_{T,0}$ also contain the so-called zero-point motion, e.g., $n_{T=0} = 1/2$ (in a case of a specific normalization). Estimation (33) corresponds to the paper [55], which describes the influence of the finite temperature reservoir on the generation of entangled quantum states in an unstable system of two oscillators with harmonically modulated coupling. Formula (33) means that the contribution of Langevin sources is weak if $n_T \ll |\mathrm{Im}q_x| / \max(\kappa_{\mathrm{EIT}},\mu_s)$. The regime of strong instability leads to the optimal generation of entangled photon:

$$|\mathrm{Im}q_X| \approx \chi_{\mathrm{PIT}} \gg \max(\kappa_{\mathrm{EIT}},\mu_s). \tag{34}$$

For the sake of simplicity we do not consider fluctuations of the drive field amplitude and phase. Under typical experimental conditions, they are of less importance than the Langevin noise sources. It is derived from the independence of the amplifying coefficient χ_{PIT} of the drive power.

V. GENERATION OF ENTANGLED PHOTONS

The most intense entangled-state generation takes place in the narrow band of the PIT-resonance frequencies (11). In this band, the condition of comparatively weak dissipative effects Eq. (34) is automatically fulfilled if

$$\Omega_R^2 \gg \omega_{21}\gamma_{21} \tag{35}$$

(see [38,39]). Thus, near the PIT resonance, $|\delta| \leqslant \frac{4}{5}\Delta\omega_{\mathrm{PIT}}$, we can neglect the dissipative effects and assume that κ_{EIT}, μ_s, $\hat{f}_{\delta;X}$, $\hat{f}_{\delta;O} \approx 0$. We take into account the strong inequalities $V^{gr}_{\mathrm{EIT}} \ll V^{gr}_s$ and $V^{gr}_{\mathrm{EIT}} \ll c$ (it is the well-known EIT slow light effect[10]), which leads to the following relations: $q_{O,X} \approx \frac{\delta}{2V^{gr}_{\mathrm{EIT}}} \pm i\chi_{\mathrm{PIT}}\sqrt{1-\sigma^2}$ and $\sigma \approx \frac{5}{4}\frac{\delta}{\Delta\omega_{\mathrm{PIT}}}$. Then, solution (27) at the layer output takes on the

[10]The condition of extremely low group velocity and the inequality (21) defines the following feasible interval for Λ-atoms density: $\omega_{21} \gg \eta \gg \frac{\Omega_R^2}{\omega_{\mathrm{HF}}}$.

following form:

$$\begin{aligned}\frac{\hat{p}_\delta(L)}{\sqrt{c}} &= \frac{e^{i\phi\delta}}{\sqrt{1-\sigma^2}}(c_{\delta;p}(0)[\sqrt{1-\sigma^2}\cosh(\tau\sqrt{1-\sigma^2}) + i\sigma\sinh(\tau\sqrt{1-\sigma^2})] - ic^\dagger_{\delta;p}(0)e^{2i\theta}\sinh(\tau\sqrt{1-\sigma^2})),\\ \frac{\hat{s}^\dagger_\delta(L)}{\sqrt{c}} &= \frac{e^{i\phi\delta}}{\sqrt{1-\sigma^2}}(c^\dagger_{\delta;s}(0)[\sqrt{1-\sigma^2}\cosh(\tau\sqrt{1-\sigma^2}) - i\sigma\sinh(\tau\sqrt{1-\sigma^2})] + ic_{\delta;p}(0)e^{-2i\theta}\sinh(\tau\sqrt{1-\sigma^2})).\end{aligned} \tag{36}$$

Here, L is the depth of the layer, $\tau = \chi_{\rm PIT}L$, and $\phi = \frac{L}{2V^{gr}_{\rm EIT}}$; the variation interval δ of the frequency detuning is determined by the condition $\sigma^2 \ll 1$. Note that formulas (36) are finite at the limit $\sigma^2 \to 1$ (at the boundary of the PIT region).

At the center of the PIT-resonance line $|\delta| \ll \Delta\omega_{PIT}$ ($\sigma^2 \ll 1$), we use boundary conditions (25), definition (12c), and solution (36) and obtain the following expression for transformation of the slowly varying amplitudes of field operators during transmission through the layer:

$$\begin{aligned}\hat{E}^{(+)}_p(L,t) &= \hat{E}^{(+)}_p(0,\tilde{t})\cosh(\tau) - ie^{2i\theta}\hat{E}^{(-)}_s(0,\tilde{t})\sinh(\tau),\\ \hat{E}^{(+)}_s(L,t) &= \hat{E}^{(+)}_s(0,\tilde{t})\cosh(\tau) - ie^{2i\theta}\hat{E}^{(-)}_p(0,\tilde{t})\sinh(\tau),\end{aligned} \tag{37}$$

where $\tilde{t} = t - \frac{L}{2V^{gr}_{\rm EIT}}$. Formula (37) means that such a system may amplify a field with nonclassical statistics. The P-harmonic and the S-harmonic "exchange" their statistics during the transmission. Under certain conditions, "cloning" of nonclassical statistics from one frequency to another occurs (there are two fields with different frequencies and similar statistics at the output). If, e.g., the field with one frequency is more intense than the field with the other frequency at the input, at the output the both fields will be similar to the more intense input field.

VI. GENERATION OF THE BIBAND SQUEEZED VACUUM

We describe the case of the vacuum input field in more detail. It is convenient to use constant Schrödinger operators for the boundary conditions and "empty" Fock states $|\Psi\rangle = \Pi_\delta|0_{\delta;p}\rangle|0_{\delta;s}\rangle$ to perform the quantum-mechanical averaging. We define the frequency detuning range $\Delta\omega$ as follows:

$$-\Delta\omega/2 < \delta < \Delta\omega/2, \quad \Delta\omega \leqslant \tfrac{8}{5}\Delta\omega_{\rm PIT}.$$

Actually, the frequency detuning range $\Delta\omega$ is determined by the frequency filter or spectral susceptibility of the detector.

Substitution of solution (36) into Eq. (26a) gives the following expression for the output photon flux operators:

$$\langle \hat{N}_{p,s}\rangle = \frac{\Delta\omega}{2\pi}\frac{1}{2\sigma_{\max}}\int_{-\sigma_{\max}}^{\sigma_{\max}}\frac{\sinh^2(\tau\sqrt{1-\sigma^2})}{1-\sigma^2}d\sigma, \tag{38a}$$

where $\sigma = \frac{5\delta}{4\Delta\omega_{\rm PIT}}$ and $\sigma_{\max} = \frac{5\Delta\omega}{8\Delta\omega_{\rm PIT}} \leqslant 1$. The photon flux fluctuations are relatively large:[11]

$$\begin{aligned}&\sqrt{\langle(\hat{N}_{p,s} - \langle\hat{N}_{p,s}\rangle)^2\rangle}\\ &\quad= \frac{\Delta\omega}{2\pi}\frac{1}{2\sigma_{\max}}\int_{-\sigma_{\max}}^{\sigma_{\max}}\frac{\cosh(\tau\sqrt{1-\sigma^2})\cdot\sinh(\tau\sqrt{1-\sigma^2})}{\sqrt{1-\sigma^2}}d\sigma\\ &\quad\propto \langle\hat{N}_{p,s}\rangle.\end{aligned} \tag{38b}$$

This means that the field state is not the classical coherent field. At the high amplification limit ($e^{2\tau} \gg 1$) and a relatively narrow frequency band ($\Delta\omega \ll \Delta\omega_{\rm PIT}$), we can use Eqs. (37) and simplify Eq. (38b):

$$\sqrt{\langle(\hat{N}_{p,s} - \langle\hat{N}_{p,s}\rangle)^2\rangle} \approx \langle\hat{N}_{p,s}\rangle \approx \frac{\Delta\omega}{8\pi}e^{2\tau}.$$

The photon flux difference $\langle\Delta\hat{N}\rangle = \langle\hat{N}_p - \hat{N}_s\rangle$ is an unfluctuated (exactly measurable) quantity: $\langle\Delta\hat{N}\rangle = \langle\Delta\hat{N}^2\rangle = 0$. This expression corresponds to the well-known Manley-Rowe relation for resonance processes with synchronism conditions (1, 2).

The average electric field is $\langle\hat{E}(L)\rangle = 0$. We obtain the field fluctuations $\langle\hat{E}^2(L)\rangle$ using Eqs. (26b) and (26c) and the "vacuum" boundary condition which simplifies the calculations substantially (quantum averaging makes some terms equal to zero). We substitute solution (36) into Eqs. (26b) and (26c), take into consideration the frequency hierarchy $\omega_{\rm HF} \gg \omega_{21} \gg \Delta\omega_{\rm PIT}$, and finally get

$$\langle\hat{E}^2(L)\rangle = \frac{E_0^2}{2\sigma_{\max}}\int_{-\sigma_{\max}}^{\sigma_{\max}}G_\delta d\delta, \tag{39a}$$

where

$$\begin{aligned}G_\delta \approx\ & \frac{\cosh^2(\tau\sqrt{1-\sigma^2}) + \sinh^2(\tau\sqrt{1-\sigma^2}) - \sigma^2}{1-\sigma^2} + \left(2 - \frac{1}{4}\frac{\omega_{21}^2}{\omega_{HF}^2}\right)\frac{\sinh(\tau\sqrt{1-\sigma^2})\sqrt{\cosh^2(\tau\sqrt{1-\sigma^2}) - \sigma^2}}{1-\sigma^2}\\ &\times \sin\left[2k_dL - 2\omega_dt + 2\theta + \arcsin\left(\frac{\sigma\sinh(\tau\sqrt{1-\sigma^2})}{\sqrt{\cosh^2(\tau\sqrt{1-\sigma^2}) - \sigma^2}}\right)\right],\end{aligned} \tag{39b}$$

and $E_0^2 \approx \frac{2\hbar\omega_{\rm HF}\Delta\omega}{c\Delta S}$ is the vacuum fluctuations level for the uncorrelated bichromatic field in the corresponding range of frequencies and spatial angles.

[11] We consider the averaging time is less than the characteristic signal correlation time $T_{\rm corr} \propto \Delta\omega^{-1}$. In the opposite situation, the relative fluctuations of photon counting will decrease.

Phase characteristics of field (39b) can be described by the squeezing and antisqueezing coefficients K_{SQV} and $K_{\text{anti-SQV}}$. These values correspond to the ratio of the minimal or maximum fluctuation level over the vacuum level. The minimal and maximum fluctuations correspond to the phase values $\omega_d t - k_d L + \theta - \pi N = \frac{\pi}{4}, \frac{3\pi}{4}$, respectively. The definitions and values of the squeezing and antisqueezing coefficients are

$$\frac{\langle \hat{E}^2\rangle_{\min}}{E_0^2} = \frac{1}{K_{\mathrm{SQV}}}, \quad \frac{\langle \hat{E}^2\rangle_{\max}}{E_0^2} = K_{\text{anti-SQV}}$$

$$\frac{1}{K_{\mathrm{SQV}}} \approx \frac{1}{2\sigma_{\max}}\int_{-\sigma_{\max}}^{\sigma_{\max}} \left\{ \frac{e^{-2\tau\sqrt{1-\sigma^2}} - \sigma^2}{1-\sigma^2} + \left[\frac{1}{4}\frac{\omega_{21}^2}{\omega_{\mathrm{HF}}^2}\sqrt{1-\sigma^2} + 2(1-\sqrt{1-\sigma^2})\right] \frac{\cosh(\tau\sqrt{1-\sigma^2})\cdot\sinh(\tau\sqrt{1-\sigma^2})}{1-\sigma^2} \right\}, \tag{40a}$$

$$\frac{1}{K_{\text{anti-SQV}}} \approx \frac{1}{K_{\mathrm{SQV}}} + \frac{1}{2\sigma_{\max}}\int_{-\sigma_{\max}}^{\sigma_{\max}} \left(4 - \frac{1}{2}\frac{\omega_{21}^2}{\omega_{\mathrm{HF}}^2}\sqrt{1-\sigma^2}\right) \frac{\cosh(\tau\sqrt{1-\sigma^2})\sinh(\tau\sqrt{1-\sigma^2})}{1-\sigma^2} d\sigma. \tag{40b}$$

Formula (40a) means that for sufficiently large amplification factor ($e^{2\tau} \gg 1$) the squeezing coefficient is big $K_{\mathrm{SQV}} \gg 1$ and the state is "squeezed."Assuming $\sigma_{\max} \ll 1$ in Eq. (40a), we simplify it:

$$K_{\mathrm{SQV}} = \frac{K_{\mathrm{SQV}}^{\max}}{1 + \frac{1}{12}\left(K_{\mathrm{SQV}}^{\max}\sigma_{\max}\right)^2 + \left(\frac{K_{\mathrm{SQV}}^{\max}\omega_{21}}{4\omega_{\mathrm{HF}}}\right)^2}, \quad K_{\mathrm{SQV}}^{\max} \approx e^{2\tau}. \tag{41}$$

Comparing Eq. (40b) and Eqs. (38a) and (38b) we notice that in the case of high squeezing level ($K_{\mathrm{SQV}} \gg 1$) and low Λ-system splitting ($\omega_{\mathrm{HF}} \gg \omega_{21}$) the antisqueezing coefficient $K_{\text{anti-SQV}}$ is proportional to spectral intensity per frequency $\Delta f = \Delta\omega/2\pi$: $K_{\text{anti-SQV}} \propto \frac{\langle \hat{N}_{p,s}\rangle}{\Delta f}$.

For a narrow frequency band $\Delta\omega \ll \Delta\omega_{\mathrm{PIT}}$ and a sufficiently high amplification ($e^{2\tau} \gg 1$), we get the following estimation:

$$K_{\text{anti-SQV}} \approx K_{\mathrm{SQV}} \approx \frac{1}{4}\frac{\langle \hat{N}_{p,s}\rangle}{\Delta f} \approx e^{2\tau}.$$

Figures 2 and 3 illustrate the dependence of $K_{\text{anti-SQV}}$ and K_{SQV} on the relative spectral bandwidth $\sigma_{\max} = \frac{5\Delta\omega}{8\Delta\omega_{\mathrm{PIT}}}$ for different amplification coefficients $e^{2\tau}$ and characteristic parameter $\frac{\omega_{\mathrm{HF}}}{\omega_{21}} = 3\times 10^5$ according to expressions (40a) and (40b).

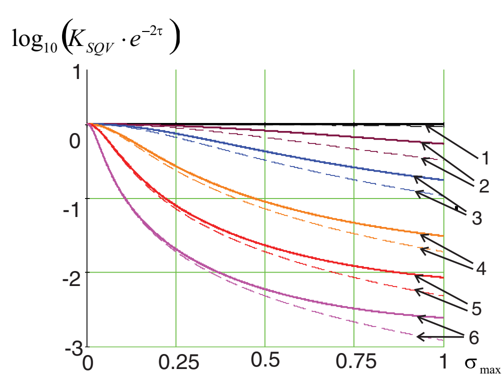

FIG. 2. (Color online) Form of the normalized squeezing coefficient $\log_{10}(K_{\mathrm{SQV}}e^{-2\tau})$ as a function of the relative bandwidth $\sigma_{\max} = \frac{5\Delta\omega}{8\Delta\omega_{\mathrm{PIT}}}$. For lines 1–6, the amplifying coefficients $e^{2\tau}$ are 1, 5, 10, 25, 50, and 100, respectively. The solid lines represent expression (40a); the dashed lines represent approximation (41).

The antisqueezing coefficient depends weakly on the spectral bandwidth. The squeezing coefficient is close to its maximum value in the frequency band which is narrower for greater values of the amplifying coefficient.

VII. NUMERICAL ESTIMATIONS

We make some numerical estimations for the characteristic parameter range of the Rb ($D1$-line) experiment: $\omega_{\mathrm{HF}} \approx 2\times\ 10^{15}\ \mathrm{s}^{-1}$ ($\lambda = 794$ nm), $\omega_{21} \approx 6.83\times\ 10^9\ \mathrm{s}^{-1}$, $\gamma_{31} \approx 10^8\ \mathrm{s}^{-1}$, $\gamma_{21} \approx 1{,}5\times\ 10^4\ \mathrm{s}^{-1}$. Key condition (35) is satisfied automatically for, e.g., the drive power $P \approx 10$ mW and the

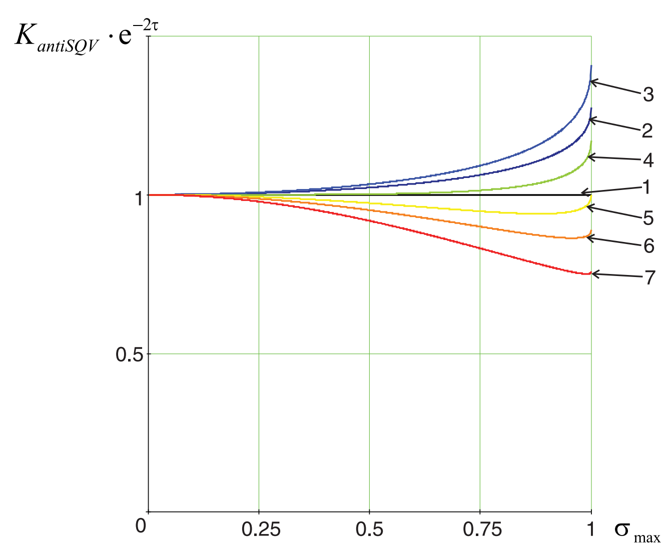

FIG. 3. (Color online) Form of the normalized antisqueezing coefficient $K_{\text{anti-SQV}}e^{-2\tau}$ as a function of the relative bandwidth $\sigma_{\max} = \frac{5\Delta\omega}{8\Delta\omega_{\mathrm{PIT}}}$. For lines 1–7, the amplifying coefficients $e^{2\tau}$ are 1, 1.4, 2.5, 10, 22, 40, and 100, respectively.

TABLE I. Latest numerical results in squeezing.

Paper	K_{SQV} (dB)	$K_{\text{anti-SQV}}$ (dB)	Drive power (mW)	Squeezing bandwidth (MHz)	Productivity (photon pairs per second)
Here	20	20	10	0.016	10^5
[19]	11.5	16	600	170	
[21]	11		0.08 (mean power, pulsed mode)		
[15]	9	15	100	1	
[25]	7	11.6	120		
[24]	3.8		1100	0.1	
[16]	2.75	7	290	1	
[14]	2.5		600	0.08	
[4]			160	0.3	10^3
[5]			100	0.75	10^5

beam focusing diameter is $d \approx 2$ mm ($\Omega_R \approx 2.4 \times 10^7\ \mathrm{s}^{-1} \approx 6\gamma_{21}\omega_{21}$). In this case, we obtain $\Delta\omega_{\mathrm{PIT}} \approx \frac{\Omega_R^2}{\omega_{21}} \approx 10^5\ \mathrm{s}^{-1}$. The energy amplifying coefficient $e^{2\tau}$ corresponds to the product of the layer length and the atom density $LN \approx \tau \times 3\ \mathrm{cm} \times 6.6 \times 10^{11}\ \mathrm{cm}^{-3}$; we can see that, e.g., amplification values of $e^{2\tau} \approx 10^2$–10^3 correspond to some experimentally appropriate parameters. As a pessimistic estimation, we consider that in any experiment the amplification coefficient $e^{2\tau}$ is much less than $\frac{\omega_{\mathrm{HF}}}{\omega_{21}} = 3 \times 10^5$. Then we can neglect the last term in the denominator in Eqs. (41). We set the maximum squeezing coefficient $K_{\mathrm{SQV}}^{\max} \sim e^{2\tau} \sim 10^2$ (maximum squeezing in the modern experiments with nonlinear crystals is about 12 dB [19,21,25]), and for the squeezing coefficient interval $K_{\mathrm{SQV}}^{\max} \geqslant K_{\mathrm{SQV}} \geqslant 0.5K_{\mathrm{SQV}}^{\max}$ we get the frequency bandwidth $\Delta\omega_{\mathrm{SQV}} \approx 0.1\Delta\omega_{\mathrm{PIT}}$, that corresponds to $\Delta f = \frac{\Delta\omega_{\mathrm{SQV}}}{2\pi} \approx 1.6$ KHz. In this case, the antisqueezing coefficient is $\propto$20 dB and, consequently, the generation rate is $\propto 10^5$ photon pairs per second at the squeezing frequency band Δf.

As compared with the known biphoton generation methods, this scheme allows one, in theory, to get a narrower frequency band with a lower drive power, a larger squeezing level, and high or same productivity. For instance, we present the following data in Table I.

VIII. SUMMARY

The theoretical analysis of the discussed PIT scheme in the case of a quantum signal field demonstrates great potential of the scheme for generation of entangled photons. Along with the squeezed vacuum generation (we discuss it in this paper), such a system can be used for amplification of a field with nonclassical statistics and/or for the "statistics exchange" between P,S harmonics. The efficiency and productivity of this scheme may be limited by the inhomogeneous broadening effects, the noise level, and the nonlinear effects in a specific setup. We believe that noise analysis results in this paper and the classical field nonlinear theory in [39] give grounds for optimism. The principal question about the maximum power of the fields with nonclassical statistics is more complicated and needs some additional analysis (some aspects of it are discussed, e.g., in [12]). Another important question concerns the boundaries of experimentally achievable squeezing degrees. The main limiting factors are the squeezed state phase fluctuation and dissipation [15]. The drastic decrease in the dissipation in the EIT regime and strict binding of the generated squeezed vacuum phase to the drive phase make this regime rather promising, though it surely needs more detailed investigation for specific equipment and experiments.

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