Building of one-way Hadamard gate for squeezed coherent states

Sergey A. Podoshvedov
Department of General and Theoretical Physics, Southern Ural State University, Lenin Av. 76, Chelyabinsk, Russia
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We present an optical scheme to conditionally generate even or odd squeezed superpositions of coherent states (SSCSs). The optical setup consists of an unbalanced beam splitter whose transmittance tends to unity, and additional balanced beam splitters and photodetectors in auxiliary modes. Squeezed coherent states with different amplitudes are the input states in the optical scheme. The single-qubit operations are probabilistic and employ two- and three-photon subtractions from initial beams as the driving force. Generation of the even or odd SSCSs is observed in a wide diapason of values of used parameters. We consider a possibility to realize a one-way Hadamard gate for the squeezed coherent states when the base states are transformed into superposition states. States approximating the output states of a Hadamard gate with high fidelity can be realized by imposing restrictions on the values of used parameters. Higher-order subtractions from input beams are necessary to generate the SSCSs with larger amplitudes and higher fidelities. The problem is resolved in a Wigner representation to take into account imperfections of the optical devices.

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I. INTRODUCTION

Optical quantum information processing (QIP) in traveling fields is a serious contender among the various physical implementation systems [1]. The optical QIP initially relies on states of single photons [2]. Photons are the base for "flying" qubits which are used for construction of optical quantum networks [1]. Photon qubits have high speed, robustness against decoherence, and can easily be manipulated with optical devices at room temperature. Yet perfect single-photon sources are demanding and photon-photon gates are hard to implement due to extremely weak direct interaction between single photons. An alternative approach is to encode quantum information by macroscopically distinguishable fields or by the states that may give macroscopically distinguishable outcomes, for example, by coherent states with opposite amplitudes rather than using discrete degrees of freedom (e.g., the polarization or spatial modes) of single photons. The original proposal for quantum computing with coherent states [3] suggested that elementary quantum gates for the base coherent states could be implemented by displacement operators and mixing on unbalanced beam splitters followed by projection onto the computational subspace. Interest in quantum computing with coherent states gave rise to the development of the quantum protocols with the states [4–7]. The problem of the realization of quantum protocols with coherent states would be resolved provided that a gigantic Kerr nonlinearity existed. Extremely weak Kerr nonlinearity of modern crystals to rotate the coherent states before they finally decohere within the medium [8] is known to be the main reason holding back quantum information processing with coherent states. Strong interaction with the environment [8] rapidly transforms aborning superposition of coherent states (SCSs) into statistical mixtures.

Conditional quantum operations based on photon detection is an important tool that enables the replacement of weak Kerr nonlinearity in optical QIP. A typical example is the photon subtraction operation, in which an input state is split by a highly transmissive beam splitter (BS) and the reflected state is measured. Selecting some event registered by photodetectors, one may generate desired states. Thus, an unbalanced beam splitter and avalanche photodetectors are the key elements to provide the conditional quantum operations and to generate non-Gaussian states from initially Gaussian ones [9]. This method became the base for many different proposals to approximate either regular or squeezed SCSs to any degree of accuracy by the conditionally generated states [10–18]. All the considerations were based on decomposition of the SCSs in terms of the number states, because the even or odd SCSs always contain an even or odd number of photons. In the following, we are going to present an alternative method of realization of the single-qubit operations. The main feature of the method is the decomposition of an arbitrary one-mode pure state in free-traveling fields in terms of the displaced number states with arbitrary amplitude of displacement [19,20]. We show that the proposed approach is useful to simplify the setup for the conditional generation of arbitrary squeezed superpositions of coherent states (SSCSs). The used approach enables us to show that arbitrary SSCSs can be generated in a wide diapason of values of used parameters. To realize single-qubit gates, in particular the Hadamard gate for the base squeezed coherent states, we need to impose some restrictions on the values of the parameters. The gates are probabilistic, relying on projective measurements (two-, three-, and higher-order photon subtractions) to deliver the nonlinear effect comparable with one of Kerr nonlinearity. It is worth noting we consider a possibility to realize a one-way Hadamard gate for the squeezed coherent states when the base states are transformed into superposition states. The inverse operation transforming the superposition states back into the original basis states is not considered. A new method of implementation of elementary quantum gates for coherent qubits based on single-photon subtractions was developed in [17]. We investigate a possibility to use a higher-order photon subtraction technique to demonstrate that by just in-line linear-optics devices the necessary one-qubit quantum gates for the squeezed coherent states can be realized. Moreover, we show that use of higher-order photon subtraction technique enables us to realize a one-way Hadamard gate

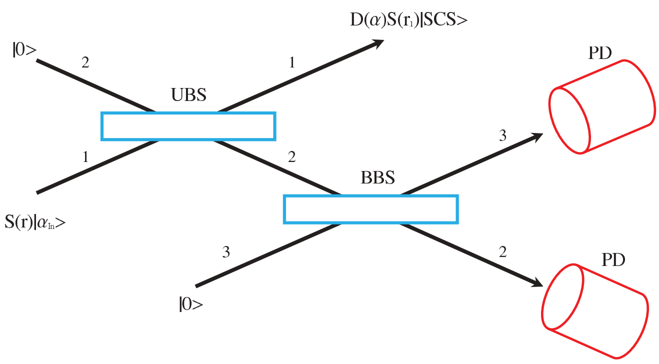

FIG. 1. (Color online) Optical scheme to generate two-photon subtracted squeezed coherent states that can approximate displaced SSCSs under a certain choice of values of used parameters. Only the amplitude of squeezed coherent states is varied to generate the states that approximate the output states of one-way Hadamard gate, provided that two photodetectors fixed two simultaneous clicks. Mode 1 is principal while modes 2 and 3 are auxiliary. (BBS) balanced beam splitter, (UBS) unbalanced beam splitter, (PD) photodetector.

with output superpositions of larger amplitudes and higher fidelities.

II. TWO-PHOTON SUBTRACTION FROM THE BASE SQUEEZED COHERENT STATES

Figure 1 illustrates the optical scheme for two-photon subtraction from initial squeezed coherent states. The modes 1 and 2 are mixed on an unbalanced beam splitter (UBS) that is described by the operator

$$B_{12}(Q) = \exp(QX_{12}), \tag{1}$$

where $X_{12} = a_1^{\dagger}a_2 - a_2^{\dagger}a_1$, a_1 and $a_1^{\dagger}$ (a_2 and $a_2^{\dagger}$) are the bosonic annihilation and creation operators for modes1 and 2, and the BS's parameter Q defines both transmittance $T = \cos^2 Q$ and reflectivity $R = \sin^2 Q$ of the BS. Initially, mode 1 is in a squeezed coherent state while mode 2 is in vacuum state. Consider the following base states,

$$|\varphi_{\pm}\rangle = S(r_{\pm})|\alpha_{In\pm}\rangle = S(r_{\pm})D(\alpha_{In\pm})|0\rangle, \tag{2}$$

with corresponding amplitudes α_{In+} and α_{In+}, where $|\alpha_{In\pm}\rangle = D(\alpha_{In\pm})|0\rangle$ are the coherent states, $D(\alpha) = \exp(\alpha a^{\dagger} - \alpha^{*}a)$ is a displacement operator [21], a ($a^{\dagger}$) is the bosonic annihilation (creation) operator, respectively, and

$$S(r_{\pm}) = \exp[r_{\pm}(a^{\dagger 2} - a^2)/2] \tag{3}$$

is a squeezing operator [21] with $r_{\pm}$ being the squeezing parameters of the squeezing operator (3).

We are interested in building the following transformation U:

$$\begin{aligned} U|\varphi_{\pm}\rangle &= US(r_{\pm})|\alpha_{In\pm}\rangle \rightarrow |\phi_{\pm}\rangle \\ &= N_{\pm}(\alpha_{\mathrm{SCS}})D(\alpha_{\pm})S(r_{1\pm})(|\alpha_{\mathrm{SCS}}\rangle + |-\alpha_{\mathrm{SCS}}\rangle) \\ &= D(\alpha_{\pm})|\mathrm{SSCS}_{\pm}(\alpha_{\mathrm{SCS}})\rangle, \end{aligned} \tag{4}$$

where squeezed superpositions of the coherent states are defined by

$$|\mathrm{SSCS}_{\pm}(\alpha_{\mathrm{SCS}})\rangle = N_{\pm}(\alpha_{\mathrm{SCS}})S(r_{1\pm})(|\alpha_{\mathrm{SCS}}\rangle \pm |-\alpha_{\mathrm{SCS}}\rangle), \tag{5}$$

with $N_{\pm}(\alpha_{\mathrm{SCS}}) = 1/\sqrt{2[1 \pm \exp(-2|\alpha_{\mathrm{SCS}}|^2)]}$ being a normalization factor. Here, α_{SCS} is an amplitude of the SSCSs that is considered to be positive $\alpha_{\mathrm{SCS}} > 0$, magnitudes $r_{1\pm}$ are the squeezing parameters of the SSCSs, and $\alpha_{\pm}$ are the amplitudes of shift of the output superpositions on the phase plane. Here, $\pm$ signs refer to even or odd SSCSs. It follows from the definition of transformation U [Eq. (4)] that the output superpositions are shifted relative to each other by $\alpha_d = \alpha_+ - \alpha_-$. Note, in the general case, the initial amplitudes $\alpha_{In\pm}$ are not equal to α_{SCS} ($\alpha_{In+} \neq \alpha_{\mathrm{SCS}}$ and $\alpha_{In-} \neq \alpha_{\mathrm{SCS}}$) and the squeezing parameters of the initial states $r_{\pm}$ do not coincide with the squeezing parameters $r_{1\pm}$ of the output states of the transformation (4) ($r_+ \neq r_{1+}$ and $r_- \neq r_{1-}$). In the general case, the operation U (4) is nonunitary, but it can resemble a one-way Hadamard transformation in a partial case when $\alpha_d = \alpha_+ - \alpha_- = 0$ ($\alpha_+ = \alpha_- = \alpha$), $r_{1+} = r_{1-} = r_1$, and $r_+ = r_- = r$. Indeed, then, output states of the operation U (4) are the squeezed superpositions of the coherent states (5) shifted by the same value $\alpha_+ = \alpha_- = \alpha$. The output states of the operation U (4) are orthogonal in the case of $\alpha_d = 0$, while the base states (2) may be asymptotically orthogonal with overlap:

$$\begin{aligned} |\langle\varphi_-|\varphi_+\rangle|^2 &= |\langle\alpha_{In-}|S^{+}(r)S(r)|\alpha_{In+}\rangle|^2 = |\langle\alpha_{In-}|\alpha_{In+}\rangle|^2 \\ &= \exp(-2|\alpha_{In+} - \alpha_{In-}|^2). \end{aligned} \tag{6}$$

The overlap (6) approaches zero, when the difference between amplitudes of the base states $\alpha_{dIn} = \alpha_{In+} - \alpha_{In-}$ grows. So, if we introduce an input computational basis $|0\rangle = |\varphi_+\rangle$, $|1\rangle = |\varphi_-\rangle$ and output base states $|0'\rangle = D(\alpha)S(r_1)|\alpha_{\mathrm{SCS}}\rangle$, $|1'\rangle = D(\alpha)S(r_1)|-\alpha_{\mathrm{SCS}}\rangle$, then direct Hadamard transformation,

$$\begin{aligned} H|0\rangle &\rightarrow (|0'\rangle + |1'\rangle)/\sqrt{2}, \\ H|1\rangle &\rightarrow (|0'\rangle - |1'\rangle)/\sqrt{2}, \end{aligned}$$

follows from (4) in the partial case of $\alpha_d = 0$, $\alpha_{\mathrm{SCS}} \rightarrow \infty$, and $\alpha_{dIn} \rightarrow \infty$. It is worth noting that input and output computational base states differ from each other. The transformation (4) in the partial case can be named a one-way Hadamard gate (not full) since only transformation of the base squeezed coherent states into the corresponding superpositions is considered.

To perform transformation (4), we use the method of two-photon subtraction from input beams to approximate the SSCSs under some values of used parameters. The output state of the beam splitter in Fig. 1 is

$$\rho_{12\pm} = B_{12}(\rho_{1\pm} \otimes |0\rangle\langle 0|_2)B_{12}^{+}, \tag{7}$$

where $\rho_{1\pm} = S(r_{\pm})|\alpha_{In\pm}\rangle\langle\alpha_{In\pm}|S^{+}(r_{\pm})$ is a density matrix of the squeezed coherent states and $\otimes$ means the tensor product of the operators. The transmitted beam of the beam splitter is the output signal to be recorded by the homodyne detector with some efficiency. The reflection (mode 2) is sent towards an avalanche photodiode that may mean that two photons have been subtracted from the squeezed coherent mode provided that two photons are fixed under detection. Indeed, the operator of the beam splitter (7) can be decomposed as

$$\begin{aligned} B_{12}(Q) &= \exp(QX_{12}) \\ &= 1 + QX_{12} + Q^2X_{12}^2/2! + Q^3X_{12}^3/3! + \dots. \end{aligned} \tag{8}$$

The term X_{12}^2 proportional to Q^2 contains the term $a_1^2 a_2^{\dagger 2}$, being responsible for the two-photon subtraction a^2 in the first mode provided that two photons are registered in the second auxiliary mode. Thus, if we choose beam splitter parameter $Q \ll 1$ in Fig. 1, a small fraction of the input beam is tapped off via a beam splitter to successfully provide two-photon subtraction from the initial squeezed coherent state when projective measurement onto the state $|2\rangle$ is done in mode 2. If we suppose that it is possible to perform the projective measurement in the auxiliary mode of the BS, then the density operator $\rho_\pm$ of mode 1 conditioned on a two-photon click of the photodetector measuring auxiliary mode 2 becomes

$$\rho_\pm = \frac{\mathrm{tr}_2\left(\rho_{12\pm}\Pi_2\right)}{\mathrm{tr}_{12}\left(\rho_{12\pm}\Pi_2\right)}, \tag{9}$$

where tr_2 is the trace over mode 2, tr_{12} is the trace over modes 1, 2, and $\Pi_2 = |2\rangle\langle 2|$ is an operator of projective measurement onto state $|2\rangle$ in the second mode.

The level of modern technology does not enable one to realize the projective measurement. A more realistic description of the optical scheme must be based on the photodetectors that cannot resolve the number of photons in the mode as is shown in Fig. 1. But before general consideration, it is logical to consider a simplified version of the optical scheme in Fig. 1, in which the operator of the beam splitter (1) is substituted by the operator $a_1^2 a_2^{\dagger 2}$ $[B_{12}(Q) \rightarrow a_1^2 a_2^{\mathrm{tr}2}]$ followed by the projective measurement onto the state $|2\rangle$ in the auxiliary mode. It enables us to deal with pure states instead of treating the density matrix (9). In the case, we have the following:

$$\begin{aligned} a^2 S(r_\pm)\,|\alpha_{In\pm}\rangle &\rightarrow |\Psi_{\pm 2}\rangle \\ &= N_\pm D(\beta_\pm)\, S(r_\pm)\left(|0\rangle + a_{\pm 1}|1\rangle + a_{\pm 2}|2\rangle\right), \end{aligned} \tag{10}$$

where the wave amplitudes are given by

$$a_{\pm 1} = \frac{2\sinh r_\pm(\alpha_{In\pm}\cosh r_\pm + \alpha^*_{In\pm}\sinh r_\pm)}{A_\pm}, \tag{11a}$$

$$a_{\pm 2} = \frac{\sqrt{2}\sinh^2 r_\pm}{A_\pm}, \tag{11b}$$

and

$$\begin{aligned} A_\pm &= \sinh r_\pm \cosh r_\pm (2\,|\alpha_{In\pm}|^2 + 1) \\ &\quad + \alpha^2_{In\pm}\cosh^2 r_\pm + \alpha^{*2}_{In\pm}\sinh^2 r_\pm, \end{aligned} \tag{11c}$$

$$\beta_\pm = \alpha_{In\pm}\cosh r_\pm + \alpha^*_{In\pm}\sinh r_\pm, \tag{11d}$$

and the normalization factor

$$N_\pm = 1/\sqrt{1 + |a_{\pm 1}|^2 + |a_{\pm 2}|^2}. \tag{11e}$$

It is worth noting that we made use of the following relations [21]:

$$S^+(r)\,a^+ S(r) = a^+\cosh r + a\sinh r, \tag{12a}$$

$$S^+(r)\,a\,S(r) = a\cosh r + a^+\sinh r, \tag{12b}$$

to derive formulas (10) and (11).

The states $|\Psi_{\pm 2}\rangle$ (10) are the approximate ones to the target states $|\phi_\pm\rangle$ [Eq. (4)]. The measure which shows how close the approximating states are to the target states is called "fidelity." The fidelity between the pure and mixed states is defined as

$$F_\pm = \mathrm{tr}\left(\rho_\pm \rho_{\pm\mathrm{SSCS}}\right), \tag{13}$$

where $\rho_\pm$ is a density matrix of the generated states (9) and $\rho_{\pm\mathrm{SSCS}} = |\varphi_\pm\rangle\langle\varphi_\pm|$. The fidelity can be calculated as follows for the simplified case:

$$\begin{aligned} F_\pm &= |\langle\phi_\pm \mid \Psi_{\pm 2}\rangle|^2 \\ &= |\langle \mathrm{SCS}_\pm(\alpha_{\mathrm{SCS}})|S^+(r_{1\pm})D^+(\alpha_\pm)N_\pm D(\beta_\pm) \\ &\quad \times S(r_\pm)(|0\rangle + a_{\pm 1}|1\rangle + a_{\pm 2}|2\rangle)|^2 \\ &= |\langle \mathrm{SCS}_\pm(\alpha_{\mathrm{SCS}})|D(\gamma_\pm)N_\pm S(r_\pm - r_{1\pm})(|0\rangle + a_{\pm 1}|1\rangle \\ &\quad + a_{\pm 2}|2\rangle)|^2 \\ &= |\langle \mathrm{SCS}_\pm(\alpha_{\mathrm{SCS}},\gamma_\pm)|N_\pm S(r_\pm - r_{1\pm})(|0\rangle + a_{\pm 1}|1\rangle \\ &\quad + a_{\pm 2}|2\rangle)|^2, \end{aligned} \tag{14}$$

where $\gamma_\pm = (\beta_\pm - \alpha_\pm)\cosh r_{1\pm} - (\beta_\pm - \alpha_\pm)^*\sinh r_{1\pm}$ and $\mathrm{SCS}_\pm(\alpha_{\mathrm{SCS}},\gamma_\pm)$ is a $\gamma_\pm-$ representation of the even or odd SCSs [19,20]. Here, we made use of the $\gamma_\pm-$ representation of the SCSs given in [19,20], where the method of decomposition of coherent states into a series of displaced number states was presented. The fidelity (14) is unity when the states $|\phi_\pm\rangle$ and $|\Psi_{\pm 2}\rangle$ are identical, while it is zero when they are orthogonal to each other.

The next step is to search for the values of the used parameters that provide maximal fidelities $F_{\pm\max}$ (14). The fidelities depend on the parameters of the target states α_{SCS}, $r_{1\pm}$, $\alpha_\pm$ and the ones of the approximating states $r_\pm$, $\alpha_{In\pm}$. Numerical analysis shows that maximal possible fidelities $F_{\pm\max}$ are achieved when parameters $\alpha_{In\pm}$ and $\alpha_\pm$ are pure imaginary, i.e., $\alpha_{In\pm} = i\,|\alpha_{In\pm}|$, $\alpha_\pm = i\,|\alpha_\pm|$. We use only pure imaginary values of the parameters $\alpha_{In\pm}$ and $\alpha_\pm$ in further consideration. Figures 2(a) and 2(b) show dependencies of maximum possible fidelities $F_{\pm\max}$ on absolute values of the seed coherent amplitudes $\alpha_{In\pm} = |\alpha_{In\pm}|$ and amplitudes of the shift $\alpha_\pm = |\alpha_\pm|$ for the case of $\alpha_{\mathrm{SCS}} = 1.5$. The degrees of squeezing $r_\pm$ and $r_{1\pm}$ that optimizes the fidelities are not represented here. Figure 2 shows that there is large diapason of the values of the parameters $\alpha_{In\pm}$ and $\alpha_\pm$ that provide performance of operation (4) with high fidelity. Numerical analysis shows that values of maximal possible fidelities $F_{+\max}$ are larger than maximal possible ones $F_{-\max}$ ($F_{+\max} > F_{-\max}$) in a wider range of the values of the parameters α_{In+} and α_+. But there also are values of the parameters α_{In-} and α_- for which the condition $F_{+\max} < F_{-\max}$ is performed. Similar plots can be constructed for other values of α_{SCS}.

Using the values of the parameters, we can discuss a possibility to build a one-way Hadamard gate for the base squeezed coherent states. To do it, we have to impose the following restrictions, $\alpha_+ = \alpha_- = \alpha$, $r_{1+} = r_{1-} = r_1$, and $r_+ = r_- = r$, on the parameters which also provide maximal possible fidelities of the generated states. Analysis shows that construction of such Hadamard transformation is possible at the expense of reduction of the fidelities of the generated states compared with those presented in Figs. 2(a) and 2(b) due to requirements that are taken to create the single-qubit operation. So, the curves 1 and 2 in Fig. 3(a) show the dependency of the fidelities between approximating and target states on α_{SCS} in the case of $\alpha_d = 0$. The next plots show dependencies of squeezing parameters r (curve 1), r_1 (curve 2) [Fig. 3(b)], the absolute value of $\alpha_+ = \alpha_- = \alpha$ [Fig. 3(c)], and the absolute values α_{In+} (curve 1), α_{In-} (curve 2) [Fig. 3(d)]—that provide the fidelities in Fig. 3(a)—on α_{SCS}. Moreover, the parameters

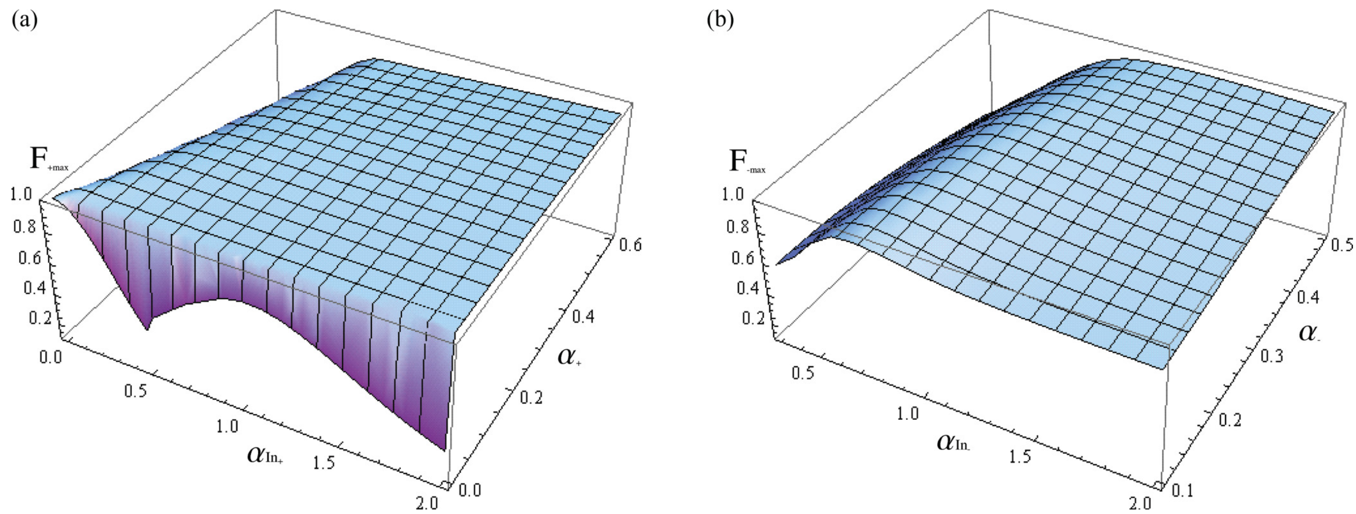

FIG. 2. (Color online) Dependencies of maximal possible fidelities (a) $F_{+\max}$, (b) $F_{-\max}$ on absolute values of the parameters $\alpha_{In\pm} = |\alpha_{In\pm}|$ and $\alpha_{\pm} = |\alpha_{\pm}|$, respectively, in scheme with two-photon subtraction in the case of $\alpha_{SCS} = 1.5$.

are chosen to provide performance of condition $F_+ \cong F_-$ (curves 1 and 2 in Fig. 1). Numerical analysis shows that there is a wide diapason of values of used parameters around those presented in Figs. 3(b)–3(d) for which either $F_- \geqslant F_+$ or $F_+ \geqslant F_-$ are held with $\alpha_d = 0$ (one-way Hadamard gate). We can weaken restrictions on the used parameters $r_{1+} = r_{1-} = r_1$

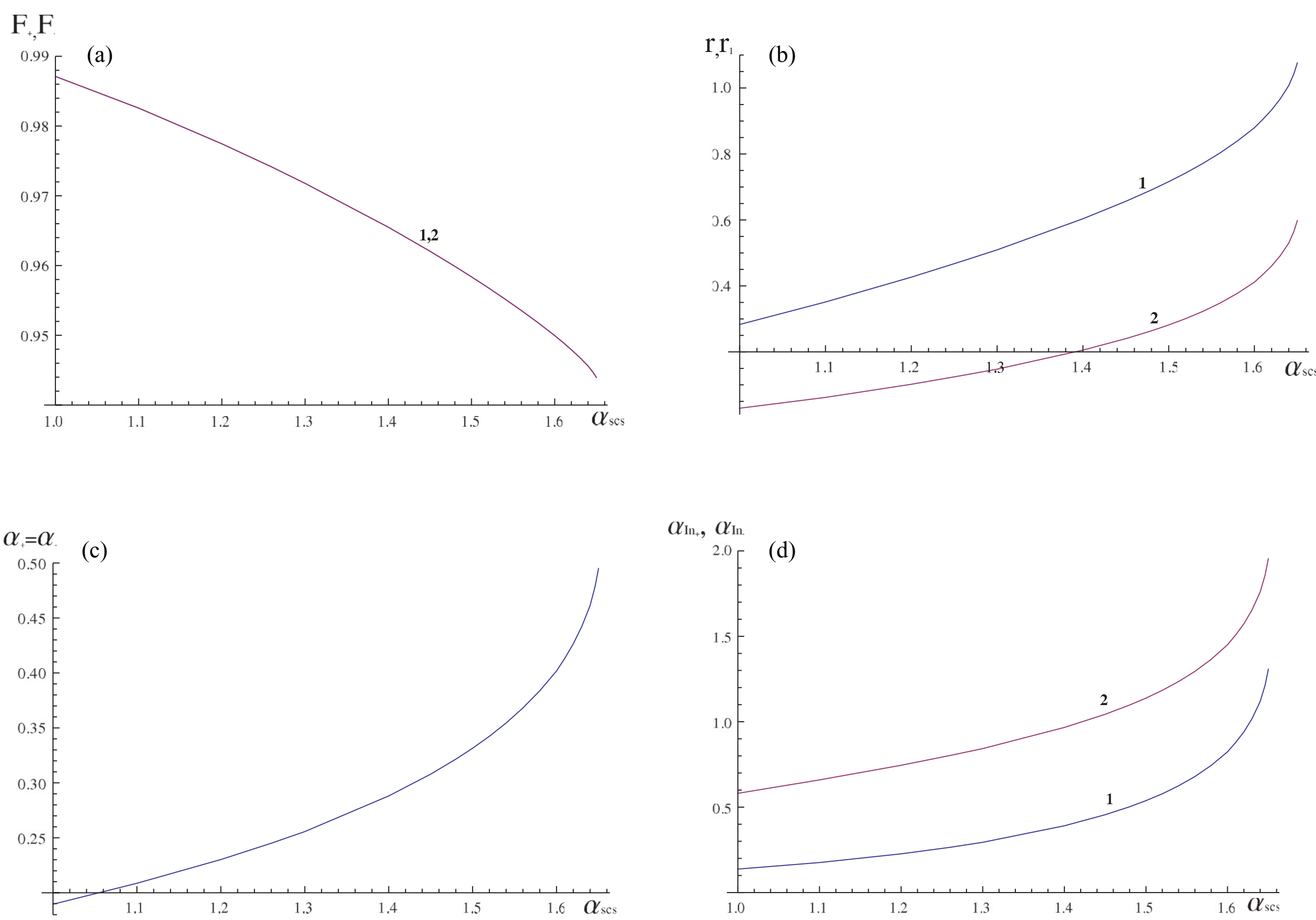

FIG. 3. (Color online) (a) Fidelities $F_{\pm}$ between generated states and output states of the one-way Hadamard gate and parameters: (b) squeezing amplitudes r, r_1, (c) absolute amplitudes of the shift $\alpha_{\pm} = |\alpha_{\pm}|$, and (d) absolute amplitudes of seed coherent states $\alpha_{In\pm} = |\alpha_{In\pm}|$ which provide the fidelities, against an amplitude α_{SCS} of the SSCSs.

and $r_+ = r_- = r$ but $\alpha_+ \neq \alpha_-$ ($\alpha_d \neq 0$). It enables us to increase the fidelities of the generated states [19,20], but such operation may be conditionally named displacing the Hadamard gate for the base states as it displaces its output states by $\alpha_d \neq 0$.

III. PRACTICAL REALIZATION OF ONE-WAY HADAMARD GATE FOR THE BASE SQUEEZED COHERENT STATES

A simplified model of the optical scheme in Fig. 1 considered in the previous section must be modified to involve a more realistic description of the scheme. At first, an unbalanced beam splitter is used. A balanced beam splitter is inserted into the second auxiliary mode of the unbalance BS to split the mode into two auxiliary modes 2 and 3 that are followed by two photodetectors. The squeezed coherent state is mixed with vacuum on the unbalanced BS and, after that, two auxiliary modes 2 and 3 are mixed with vacuum on the balanced BS forming the following state:

$$\rho_{123\pm} = B_{23} B_{12} \left(\rho_{1\pm} \otimes |0\rangle\langle 0|_2 \otimes |0\rangle\langle 0|_3\right) B_{12}^{+} B_{23}^{+}, \tag{15}$$

where B_{12} means unbalanced BS and B_{23} is the balanced one. Now, we start with the covariance matrix $V^{(3)}$ [22] of three-mode state (15) to use it for construction of the Wigner function of the states. Using the covariance matrix, we can construct the Wigner function of three-mode state (15) with the help of the general form of the Gaussian Wigner function [22]:

$$W_{123} = \frac{\exp[-0.5\xi^T (V^{(3)})^{-1}\xi]}{(2\pi)^2 \sqrt{\det V^{(3)}}}, \tag{16}$$

where the matrix $(V^{(3)})^{-1}$ is the inverse of the covariance matrix $V^{(3)}$, $\det V^{(3)}$ is a determinant of the matrix, and ξ is a column vector of the positions and momentums of the corresponding particles. Leaving out details of the calculation, it is possible to write the Wigner functions of the states (15) as

$$\begin{aligned} W_{123\pm}(\alpha_1,\alpha_2,\alpha_3) = {} & W_{1\pm}\left[\alpha_{1\pm}^s \cos Q - \left(\alpha_{2\pm}^s - \alpha_{3\pm}^s\right)\sin Q/\sqrt{2}\right] \\ & \times W_2[\alpha_1 \sin Q + (\alpha_2 - \alpha_3)\cos Q/\sqrt{2}] \\ & \times W_3[(\alpha_2 + \alpha_3)/\sqrt{2}], \end{aligned} \tag{17a}$$

where

$$\begin{aligned} & W_{1\pm}\left[\alpha_{1\pm}^s \cos Q - \left(\alpha_{2\pm}^s - \alpha_{3\pm}^s\right)\sin Q/\sqrt{2}\right] = (2/\pi) \\ & \times \exp\left\{-2\left[\frac{x_1 \cos Q - (x_2 - x_3)\sin Q/\sqrt{2}}{\exp(r_\pm)} - x_{In\pm}\right]^2\right\} \\ & \times \exp\left\{-2\left[\frac{p_1 \cos Q - (p_2 - p_3)\sin Q/\sqrt{2}}{\exp(-r_\pm)} - p_{In\pm}\right]^2\right\}, \end{aligned} \tag{17b}$$

$$\begin{aligned} & W_2[\alpha_1 \cos Q - (\alpha_2 - \alpha_3)\sin Q/\sqrt{2}] \\ & = (2/\pi)\exp\{-2[x_1 \sin Q + (x_2 - x_3)\cos Q/\sqrt{2}]^2\} \\ & \quad \times \exp\{-2[p_1 \sin Q + (p_2 - p_3)\cos Q/\sqrt{2}]^2\}, \end{aligned} \tag{17c}$$

$$\begin{aligned} W_3[(\alpha_2 + \alpha_3)/\sqrt{2}] = {} & (2/\pi)\exp\{-2[(x_2 + x_3)/\sqrt{2}]^2\} \\ & \times \exp\{-2[(p_2 + p_3)/\sqrt{2}]^2\}, \end{aligned} \tag{17d}$$

and $\alpha_{1\pm}^s = \alpha_1 \cosh r_\pm - \alpha_1^* \sinh r_\pm$, $\alpha_{2\pm}^s = \alpha_2 \cosh r_\pm - \alpha_2^* \sinh r_\pm$, $\alpha_{3\pm}^s = \alpha_3 \cosh r_\pm - \alpha_3^* \sinh r_\pm$.

The on (off) photodetector with quantum efficiency η can be described by the positive-operator valued measure (POVM) $\{\Pi_{\text{off}}(\eta), \Pi_{\text{on}}(\eta)\}$ with

$$\Pi_{\text{off}}(\eta) = \sum_{k=0}^{\infty} (1-\eta)^k |k\rangle\langle k|, \tag{18a}$$

$$\Pi_{\text{on}}(\eta) = I - \Pi_{\text{off}}(\eta), \tag{18b}$$

where I is an identity operator. Applying the two-mode on (off) observable,

$$M_{23} = [\Pi_{\text{on}}(\eta)]_2 \otimes [\Pi_{\text{on}}(\eta)]_3, \tag{18c}$$

to the state (15), one obtains the final state

$$\rho_\pm = \frac{\text{tr}_{23}(\rho_{123\pm} M_{23})}{\text{tr}_{123}(\rho_{123\pm} M_{23})}. \tag{19}$$

It is possible to show that the Wigner function of observable M_{23} (18c) is given by

$$\begin{aligned} W_M(\alpha_2,\alpha_3) = {} & (1/\pi^2)\left\{1 - [2/(2-\eta)]\exp\left[-2A\left(x_2^2 + p_2^2\right)\right]\right\} \\ & \times \left\{1 - [2/(2-\eta)]\exp\left[-2A\left(x_3^2 + p_3^2\right)\right]\right\}, \end{aligned} \tag{20}$$

where $A = (2-\eta)/\eta$ and here we suppose that quantum efficiencies of both detectors in the auxiliary modes are equal to each other, $\eta_1 = \eta_2 = \eta$. The Wigner functions of the final states (19) can be calculated as

$$W_\pm(\alpha_1) = N_\pm \pi^2 \int d^2\alpha_2 W_{123\pm}(\alpha_1,\alpha_2,\alpha_3) W_M(\alpha_2,\alpha_3), \tag{21}$$

where $N_\pm$ are the normalization factors corresponding to the term $\text{tr}_{123}(\rho_{123\pm} M_{23})$ in formula (19). We can see from formula (21) the Wigner function (17a) is multiplied by the Wigner function corresponding to the detection operation M_{23} (20) and integrated over modes 2 and 3 to obtain the Wigner function of the final state. It is possible to show that integral (21) is expressed as a sum of four Gaussian integrals with modified covariance matrices. Performing the integration over α_2, α_3 and leaving out details of the calculations, we have the following Wigner functions of two-photon subtracted squeezed coherent states:

$$W_\pm(\alpha_1) = N_\pm [W_{1\pm}(\alpha_1) + W_2(\alpha_1) + W_3(\alpha_1) + W_4(\alpha_1)], \tag{22a}$$

where

$$W_{1\pm}(\alpha_1) = 2/(\pi\sqrt{A_{21x}A_{21p}})\exp\{-2A_{11xtot}[x_1 - x_{In\pm}\exp(r_\pm)\cos Q]^2\}\exp\{-2A_{11ptot}[p_1 - p_{In\pm}\exp(-r_\pm)\cos Q]^2\}, \quad (22b)$$

$$W_2(\alpha_1) = -8A/[2\eta\pi(2A+1)\sqrt{A_{22x}A_{22p}}]\exp[-2A_{12xtot}(x_1 - B_{12xtot}/A_{12xtot})^2]\exp[-2A_{12ptot}(p_1 - B_{12ptot}/A_{12ptot})^2]$$
$$\times\exp\left[-2\left(C_{12xtot} - B_{12xtot}^2/A_{12xtot}\right)\right]\exp\left[-2\left(C_{12ptot} - B_{12ptot}^2/A_{12ptot}\right)\right], \quad (22c)$$

$$W_3(\alpha_1) = -8A/[2\eta\pi(2A+1)\sqrt{A_{23x}A_{23p}}]\exp[-2A_{13xtot}(x_1 - B_{13xtot}/A_{13xtot})^2]\exp[-2A_{13ptot}(p_1 - B_{13ptot}/A_{13ptot})^2]$$
$$\times\exp\left[-2\left(C_{13xtot} - B_{13xtot}^2/A_{13xtot}\right)\right]\exp\left[-2\left(C_{13ptot} - B_{13ptot}^2/A_{13ptot}\right)\right], \quad (22d)$$

$$W_4(\alpha_1) = 8A/[(2\eta)^2\pi(A+1)\sqrt{A_{24x}A_{24p}}]\exp[-2A_{14xtot}(x_1 - B_{14xtot}/A_{14xtot})^2]\exp[-2A_{14ptot}(p_1 - B_{14ptot}/A_{14ptot})^2]$$
$$\times\exp\left[-2\left(C_{14xtot} - B_{14xtot}^2/A_{14xtot}\right)\right]\exp\left[-2\left(C_{14ptot} - B_{14ptot}^2/A_{14ptot}\right)\right], \quad (22e)$$

where the corresponding coefficients and normalization factors $N_\pm$ are not presented here due to their complexity. As operator M_{23} is non-Gaussian, it transforms initially Gaussian states into non-Gaussian ones while BS transformations are Gaussian completely positive maps, and the resulting state $\rho_{123\pm}$ [Eq. (15)] is still a Gaussian state.

To estimate how close the generated states (22a) are to the target ones (5), we present the Wigner functions of the displaced squeezed superpositions of coherent states. The Wigner functions of the SCSs are given by

$$W_{\pm\mathrm{SCS}}(\alpha) = N_\pm(\alpha_{\mathrm{SCS}})[W_0(\alpha) + W_{-0}(\alpha) \pm 2X_{\alpha_{\mathrm{SCS}}}(\alpha)], \quad (23a)$$

where $\alpha_{\mathrm{SCS}} = x_{\mathrm{SCS}} + ip_{\mathrm{SCS}}$ and

$$W_0(\alpha) = \frac{2}{\pi}\exp[-2(x - x_{\mathrm{SCS}})^2 - 2(p - p_{\mathrm{SCS}})^2], \quad (23b)$$

$$W_{-0}(\alpha) = \frac{2}{\pi}\exp[-2(x + x_{\mathrm{SCS}})^2 - 2(p + p_{\mathrm{SCS}})^2], \quad (23c)$$

$$X_{\alpha_{\mathrm{SCS}}}(\alpha) = \frac{2}{\pi}\exp(-2x^2 - 2p^2)\cos[4(xp_{\mathrm{SCS}} - px_{\mathrm{SCS}})], \quad (23d)$$

where $\alpha_{\mathrm{SCS}} = x_{\mathrm{SCS}} + ip_{\mathrm{SCS}}$. Displaced and squeezed versions of the same states can be obtained with the help of the following transformation:

$$W(x,p) \to W\left[\frac{x - x_\xi}{\exp(r)}, \frac{p - p_\xi}{\exp(-r)}\right], \quad (24)$$

where r is a squeezing parameter and ξ is an arbitrary amplitude of the displacement. The fidelities between the generated and target states (13) can be calculated with the help of the Wigner functions as

$$F_\pm = \pi\int d^2\alpha_1 W_\pm(\alpha_1)W_{\pm\mathrm{DSSCS}}(\alpha_1), \quad (25)$$

where $W_\pm(\alpha_1)$, $W_{\pm\mathrm{DSSCS}}(\alpha_1)$ are the Wigner functions of the generated states and displaced squeezed superposition of coherent states, respectively. When inserting the Wigner functions into (25) and carrying out the integrations, we get corresponding fidelities. Corresponding dependencies of the fidelities (25) on α_{SCS} are presented in Fig. 4. We used the following parameters: quantum efficiency $\eta = 0.8$ for both photodetectors, BS's parameter $Q = 0.01$ (curves 1 and 2), and $Q = 0.1$ (curves 3 and 4). Parameters used for the construction of the curves are taken from the plots 3(b)–3(d). Similar dependencies are observed for other values of the parameters η and Q. As can be seen from Fig. 4, increase of the BS's parameter Q leads to decrease of the fidelities of the generated states. Influence of quantum efficiencies of the photodetectors used in the optical scheme in Fig. 1 is negligible. Thus, the model in which the BS operator (1) is substituted by the operator $a_1^2a_2^{\dagger 2}$ can work only in the case of small values

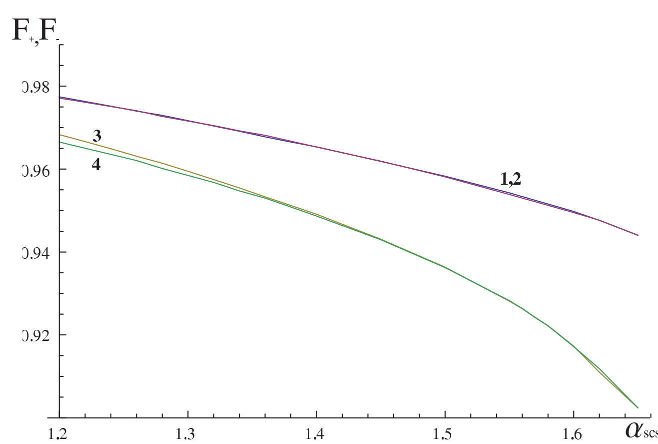

FIG. 4. (Color online) Dependencies of fidelities between generated states and output states of a one-way Hadamard gate on α_{SCS} in a realistic case. Values of the parameters for calculation of the fidelities are taken from Figs. 3(b)–3(d), where a simplified model of the optical scheme in Fig. 1 is used, in which the BS operator is replaced by the operator $a_1^2a_2^{\dagger 2}$ followed by projective measurement onto the state $|2\rangle$ in the second auxiliary mode. The beam splitter parameter is taken to be $Q = 0.01$ (curves 1 and 2 are for F_+ and F_-, respectively) and $Q = 0.1$ (curves 3 and 4 are for F_+ and F_-). The quantum efficiency of detectors is $\eta = 0.8$ for all curves. The fidelities decrease when BS's parameter Q grows, while the influence of the quantum efficiency of detectors η on the quality of the generated states is negligible.

of $Q \ll 1$ when only a negligible part of incident beam is reflected and detected.

IV. POSSIBLE WAYS TO INCREASE AMPLITUDES OF THE GENERATED SSCSs AND FIDELITIES OF OUTPUT STATES OF ONE-WAY HADAMARD GATE

In the previous section we have shown that nonunitary single-qubit operation (4) can be realized in a wide diapason of the values of used parameters [Figs. 2(a) and 2(b)]. The states corresponding to the outcome of the one-way Hadamard gate can be generated by imposing restrictions on used parameters. The restrictions lead to a decrease of the fidelities of the generated states. Moreover, our analysis shows the method with two-photon subtraction from squeezed coherent states is applicable to generate SSCSs with amplitude $\alpha_{\rm SCS} < 1.7$. Let us consider possible ways to increase the fidelities of the output states and their amplitudes $\alpha_{\rm SCS} > 1.7$. One of the possible way to increase fidelities of the generated states is to consider the case $\alpha_+ = \alpha_- = \alpha$, $r_+ \neq r_-$, and $r_{1+} \neq r_{1-}$. Then, the base states (2) may be asymptotically orthogonal under specified values of the parameters with overlap,

$$\begin{aligned} |\langle \varphi_- \mid \varphi_+ \rangle|^2 &= |\langle \alpha_{In-}|S^+(r_-)S(r_+)|\alpha_{In+}\rangle|^2 \\ &= 2/\sqrt{2\left\{1+\cosh\left[2\left(r_+ - r_-\right)\right]\right\}} \\ &\quad \times \exp\left\{-2\frac{\left[x_{In+}\exp\left(r_+ - r_-\right) - x_{In-}\right]^2}{1+\exp\left[2\left(r_+ - r_-\right)\right]}\right\} \\ &\quad \times \exp\left\{-2\frac{\left[p_{In+}\exp\left(r_+ - r_-\right) - p_{In-}\right]^2}{1+\exp\left[2\left(r_+ - r_-\right)\right]}\right\}, \end{aligned} \tag{26a}$$

where $\alpha_{In\pm} = x_{In\pm} + i p_{In\pm}$. The overlap (26a) reduces to

$$\begin{aligned} |\langle \varphi_- \mid \varphi_+ \rangle|^2 &= |\langle \alpha_{In-}|S^+(r_-)S(r_+)|\alpha_{In+}\rangle|^2 \\ &= 2/\sqrt{2\{1+\cosh[2(r_+ - r_-)]\}} \\ &\quad \times \exp\left\{-2\frac{\left[p_{In+}\exp\left(r_+ - r_-\right) - p_{In-}\right]^2}{1+\exp\left[2\left(r_+ - r_-\right)\right]}\right\}, \end{aligned} \tag{26b}$$

in our case since $\alpha_{In\pm} = i p_{In\pm}$. The overlap (26b) may decrease to zero even when $r_{1+} \neq r_{1-}$. Moreover, the states (5) with $r_{1+} \neq r_{1-}$ may also be asymptotically orthogonal under defined values of $\alpha_{\rm SCS}$. Investigation of the problem deserves a separate investigation.

Another possible way to increase amplitudes of the generated states with high fidelity is to consider a possibility to generate three-photon subtracted states. To do it, we have to extend the optical scheme in Fig. 1 by using an additional balanced BS, as is shown in Fig. 5. Then, if three simultaneous clicks are fixed by photodetectors, it means that three-photon subtraction from the initial state is realized in the principal mode of the unbalanced BS. Indeed, it follows from the simplified model of the optical scheme in Fig. 1 when the BS operator (1) is substituted by the term $B_{12}(Q) \rightarrow a_1^3 a_2^{\dagger 3}$ together with projection measurement onto the state $|3\rangle$ in the auxiliary second mode. Then, we have the following states:

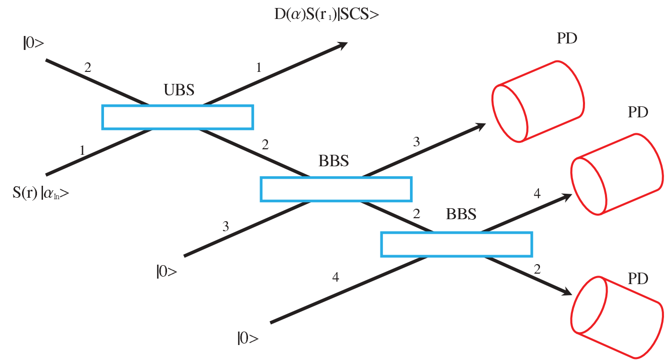

FIG. 5. (Color online) Optical scheme to realize three-photon subtraction from input beam. The optical scheme enables one to realize one-way Hadamard transformation under a certain choice of values of used parameters. Generated states approximate even or odd SSCSs with larger amplitude $\alpha_{\rm SCS}$ and higher fidelity. The same notations are used as in Fig. 1.

$$\begin{aligned} & a^3 S\left(r_\pm\right)\left|\alpha_{In\pm}\right\rangle \rightarrow \left|\Psi_{\pm 3}\right\rangle \\ &\quad = N_\pm D\left(\beta_\pm\right) S\left(r\right)\left(|0\rangle + b_{\pm 1}\,|1\rangle + b_{\pm 2}\,|2\rangle + b_{\pm 3}\,|3\rangle\right), \end{aligned} \tag{27a}$$

where

$$\begin{aligned} b_{\pm 1} &= [a_{\pm 2}\sqrt{2}\cosh r_\pm + a_{\pm 1}(\alpha_{In\pm}\cosh r_\pm + \alpha^*_{In\pm}\sinh r_\pm) \\ &\quad + A_\pm \sinh r_\pm]/B_\pm, \end{aligned} \tag{27b}$$

$$\begin{aligned} b_{\pm 2} &= [a_{\pm 2}(\alpha_{In\pm}\cosh r_\pm + \alpha^*_{In\pm}\sinh r_\pm) \\ &\quad + a_{\pm 1}\sqrt{2}\sinh r_\pm]/B_\pm, \end{aligned} \tag{27c}$$

$$b_{\pm 3} = (a_{\pm 2}\sqrt{2}\sinh r_\pm)/B_\pm, \tag{27d}$$

and

$$B_\pm = a_{\pm 1}\cosh r_\pm + A_\pm(\alpha_{In\pm}\cosh r_\pm + \alpha^*_{In\pm}\sinh r_\pm), \tag{27e}$$

where parameters $a_{\pm 1}$, $a_{\pm 2}$, $A_\pm$, and $\beta_\pm$ are given by Eqs. (11a)–(11d), respectively, and $N_\pm = 1/\sqrt{1 + |b_{\pm 1}|^2 + |b_{\pm 2}|^2 + |b_{\pm 3}|^2}$.

It follows from the expressions for the wave amplitudes (27a)–(27e) that three-photon subtracted states depend on the same parameters that are used in the case of two-photon subtraction. Numerical analysis shows maximal possible fidelities $F_{\pm\max}$ can be obtained when pure imaginary values of input amplitudes of the base states $\alpha_{In\pm} = i\,|\alpha_{In\pm}|$ and amplitudes of shift of the SSCSs $\alpha_\pm = i\,|\alpha_\pm|$ are used. It is important to stress such conditions $\alpha_{In\pm} = i\,|\alpha_{In\pm}|$ and $\alpha_\pm = i\,|\alpha_\pm|$ hold for both two- and three-photon subtraction to achieve maximal possible fidelities $F_{\pm\max}$. High fidelities are also observed in a wide diapason of values of used parameters as in the case of two-photon subtraction. It is worth noting that condition $F_{-\max} > F_{+\max}$ is performed in most cases unlike the two-photon subtraction. The biggest values of $F_{-\max}$ are observed with $|\alpha_{In-}| \ll 1$ and $|\alpha_-| \ll 1$, while achievement of the largest values of $F_{+\max}$ requires the following values of $|\alpha_{In+}| \approx 1$ and $|\alpha_-| \approx 1$. This is diametrically opposite

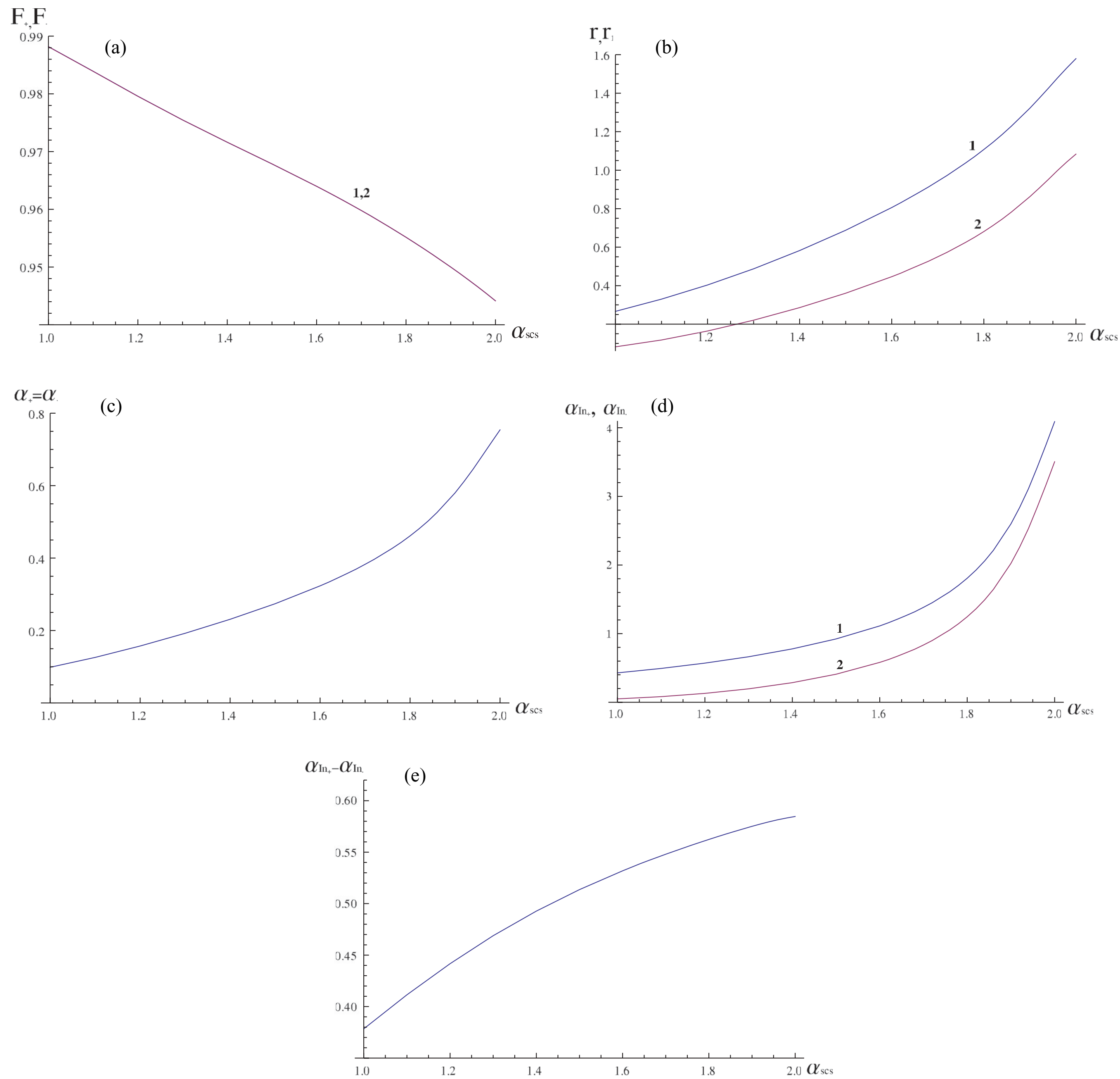

FIG. 6. (Color online) (a) Fidelities $F_{\pm}$ between generated states and output states of the one-way Hadamard gate and parameters: (b) squeezing amplitudes r, r_1, (c) absolute amplitudes of the shift $\alpha_{\pm} = |\alpha_{\pm}|$, and (d) absolute amplitudes of seed coherent states $\alpha_{In\pm} = |\alpha_{In\pm}|$ which provide the fidelities, against an amplitude α_{SCS} of the SSCSs. (e) Dependency of absolute difference of input amplitudes of coherent states $|\alpha_{dIn}| = \alpha_{In+} - \alpha_{In-}$ on α_{SCS}.

to results connected with two-photon subtraction from the initial beam. As examples, we present some maximal possible fidelities $F_{\pm\max}$ that can be attained under certain choice of values of used parameters: $F_{+\max}(\alpha_{SCS} = 1.7) = 0.991\,402$, $F_{-\max}(\alpha_{SCS} = 1.7) = 0.998\,21$, $F_{+\max}(\alpha_{SCS} = 1.8) = 0.987\,595$, $F_{-\max}(\alpha_{SCS} = 1.8) = 0.995\,931$, $F_{+\max}(\alpha_{SCS} = 1.9) = 0.982\,338$, $F_{-\max}(\alpha_{SCS} = 1.9) = 0.991\,608$, $F_{+\max}(\alpha_{SCS} = 2) = 0.974\,791$, $F_{-\max}(\alpha_{SCS} = 2) = 0.984\,989$.

Now we are interested in searching for maximal possible fidelities of the generated states with the following restrictions on the values of the parameters $\alpha_+ = \alpha_- = \alpha$, $r_{1+} = r_{1-} = r_1$, and $r_+ = r_- = r$. Such values can be used to realize one-way Hadamard transformation. So, Fig. 6(a) shows dependency of fidelities $F_+ \cong F_-$ on α_{SCS}. Next, Figs. 6(b)–6(d) show dependencies of r,r_1 [Fig. 6(b)], $\alpha_+ = \alpha_- = \alpha$ [Fig. 6(c)], and α_{In+}, α_{In+} [Fig. 6(d)] on α_{SCS}. The values of the parameters provide performance of condition $F_+ \cong F_-$ in Fig. 6(a). Comparing the fidelities in Figs. 3(a) and 6(a), we can see that, indeed, the method of three-photon subtraction enables one to generate states that can approximate the output states of the Hadamard gate with larger amplitudes and higher fidelities. Especially, the difference between the fidelities in methods of two- and three-photon subtractions is visible starting with amplitudes $\alpha_{SCS} > 1.3$. Figure 6(e) shows the dependency of the difference of initial amplitudes modulo $|\alpha_{dIn}| = \alpha_{In+} - \alpha_{In-}$ that correspond to the fidelities in Fig. 6(a) on α_{SCS}. In the case, the difference is small and can hardly be used to construct inverse transformation transforming the superposition states back into the original basis states. Nevertheless, it is worth noting that value $\alpha_+ = \alpha_- = \alpha$ is crucial in defining the values of the fidelities $F_\pm$. So, if we take values $\alpha_+ = \alpha_- = \alpha$ to be less than those presented in Fig. 6(c), then the condition $F_- > F_+$ is performed and vice versa. The difference between initial amplitudes $\alpha_{In+} - \alpha_{In-}$ can also change, in particular increase, when the values of parameter $\alpha_+ = \alpha_- = \alpha$ become different from those given in Fig. 6(c). The increase of initial amplitudes $|\alpha_{dIn}| = \alpha_{In+} - \alpha_{In-}$ can be used to accomplish transformation inverse to (4), which can be the subject of future investigation. Thus, a simplified model of the optical scheme in Fig. 5 enables us to look for the values of the parameters to realize transformation (4) and, in particular, one-way Hadamard transformation. Analysis of a simplified model of the optical scheme in Fig. 5 shows the utility of the method of three-photon subtraction as it gives a possibility to approximate the superposition states with larger amplitudes and higher fidelities. This may hold even when inefficient photodetectors, such as the standard avalanche photodiodes, are employed in auxiliary modes as is shown in Fig. 5.

The studied methods employ two- and three-photon subtractions as the driving force for realization of a one-way Hadamard gate. The operations are probabilistic and, for example, there are four possible outcomes when no clicks are fixed, when one click by either detectors is fixed, and when clicks by both detectors are observed. Success probabilities to fix the simultaneous clicks can be calculated as $P_{s\pm} = \mathrm{tr}_{123}(\rho_{123\pm}M_{23})$. The success probability is low since only a negligible part of the input beam is reflected and registered by photodetectors. The success probability can be estimated as follows. The probability of reflection of two photons by an unbalanced beam splitter is proportional to Q^2. The probability of registration of three photons is proportional to Q^3. We can take values of Q that were used to calculate fidelities in Fig. 4 to estimate the probability $\sim 10^{-4} - 10^{-2}$ in the case of two-photon subtraction. Success probabilities to generate different conditional superpositions may be also estimated by same magnitudes. Small success probabilities to engineer the conditional single-qubit operations are related with fidelities of generated states. Indeed, it follows from Fig. 4, decrease of the BS parameter Q leads to higher fidelities while increase of the parameter Q decreases the quality of generated states but increases success probability.

Applying the displacement operator $D(i\varepsilon)$, where ε is real and $\ll 1$, to the base states $S(r)|\alpha_{In\pm}\rangle$, we have the following:

$$\begin{aligned} D(i\varepsilon)S(r)|\alpha_{In\pm}\rangle &= S(r)S^+(r)D(i\varepsilon)S(r)|\alpha_{In\pm}\rangle \\ &= S(r)D(i\varepsilon')|\alpha_{In\pm}\rangle, \end{aligned} \tag{28}$$

where $\varepsilon' = \varepsilon\exp(r)$ and we made use of the relations (12a), (12b). According to results [3,6], the displacement operator acts like z rotation $\exp(iQ'/2)U_z(Q/2)$ by an angle $Q = (Q_+ - Q_-)/2$ for the qubit composed of the base states $S(r)|0,\alpha_{In\pm}\rangle$, where $Q' = (Q_+ + Q_-)/2$, $Q_+ = 2\alpha_{In+}\varepsilon'$, and $Q_- = 2\alpha_{In-}\varepsilon'$. Using the Hadamard gate and unitary matrices $U_z(\pm\pi/4)$, it is possible to construct other unitary operations defined by exponentials $U_x(Q_1/2) = \exp(iQ_1\sigma_1/2)$ and $U_y(Q_2/2) = \exp(iQ_2\sigma_2/2)$, where σ_i $(i = 1-3)$ are the Pauli matrices and Q_i $(i = 1-3)$ are the rotation angles around corresponding spatial axes x, y, and z [23], as

$$U_x(Q_1/2) = HU_z(Q_{1'}/2)H, \tag{29a}$$

$$U_y(Q_2/2) = U_z(-\pi/4)HU_z(Q_2/2)HU_z(\pi/4). \tag{29b}$$

Thus, building of a Hadamard gate, rotations around the z axis by $\pi/2$ and $-\pi/2$, and arbitrary rotation $U_z(Q_3/2)$ enable us to realize single-qubit operations for the base squeezed coherent states.

In conclusion, we have shown that the optical schemes (Figs. 1 and 5), consisting of an unbalanced BS in which the transmittance T of the beam splitter is sufficiently close to unity and additional BSs in auxiliary modes, are sufficient to successfully realize output states of a one-way Hadamard gate with high fidelity without use of Kerr nonlinearity. When an unbalanced BS diverts a tiny fraction of the beam passing through the beam splitter towards the photodetectors and they fix simultaneous clicks in auxiliary modes it means either two or three photons have been subtracted from the input beam. We have shown that the optical schemes in Figs. 1 and 5 work with either two or three typical avalanche photodetectors provided that the parameter of the unbalanced BS Q is chosen sufficiently small. The optical schemes (Figs. 1 and 5) enable us to realize operation (4) in a wide diapason of values of used parameters. To build a one-way Hadamard transformation for input squeezed coherent states, we are forced to impose restrictions on values of the parameters that lead to decrease of the fidelities of the generated states (Figs. 3 and 5). The optical schemes allow for one to realize also displacing the Hadamard gate [19,20] where output states are shifted relative to each other by some value $\alpha_d \neq 0$.

The developed approach is promising due to its simplicity since only beam splitters and detectors are used. Calculations show the approach has a potential to be extended to involve higher-order photon $\geqslant 4$ subtractions. Moreover, possible extension can give a possibility to realize a one-way Hadamard gate with larger amplitudes $\alpha_{SCS} \geqslant 2$ and higher fidelity. Possible realization of the optical schemes to produce higher-order photon subtractions from input beam looks simple. It is sufficient to use the basic optical scheme presented in Fig. 1 and introduce additional beam splitters and photodetectors as is demonstrated in Fig. 5. Only the values of parameters that provide realization of a one-way Hadamard gate can differ for two-, three-, and higher-order photon subtractions. Thus, the optical scheme in Fig. 1 may become a base for others. Let us note that preparation of the base squeezed coherent states may be somehow challenging given current technology. To avoid the use of squeezed coherent states as a computational basis, the base optical scheme in Fig. 1 can be slightly modified [16]. Let us use coherent states with corresponding amplitudes in the principal mode of the unbalanced BS as base. Then we have to use squeezed vacuum in the second auxiliary mode that is mixed up with base coherent states on the unbalanced BS. After that, the same optical devices are used as is shown in Fig. 1. Such consideration can be useful from a practical point of view and deserves separate investigation. Building of total Hadamard transformation with the states requires realization of inverse transformation when superposition states are converted back into coherent states. The natural way to perform this is to use photon addition; it directly follows from Eq. (14). Values of $|\alpha_{dIn}| = \alpha_{In+} - \alpha_{In-}$ presented in Fig. 6(e) are hardly useful for construction of inverse transformation. But nevertheless, use of other values of the parameters allows us to increase the value of the parameter $|\alpha_{dIn}| = \alpha_{In+} - \alpha_{In-}$ that may be prospective to realize a full conditional Hadamard gate for squeezed coherent states and, as a consequence, single-qubit transformations.

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