PHYSICAL REVIEW A **87**, 012132 (2013)

# Effects of detuning on tunneling and traversal of ultracold atoms through vacuum-induced potentials

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(Received 10 August 2012; published 31 January 2013)

In this paper, we study the tunneling and traversal of ultracold two-level atoms through the potential induced by the vacuum cavity mode. In particular, we discuss the effects of off-resonant interaction between the cavity mode and atomic transition on tunneling time of the ultracold atoms through a high-$Q$ mazer cavity. The phase time which may be considered as an appropriate measure of the time required for the atom to cross the cavity, exhibits some interesting features in the presence of off-resonant interaction. For example, switching between the sub and superclassical behaviors in phase time occurs for proper choice of detuning. Similarly, negative phase time appears for the transmission of atoms in both excited and ground states in the presence of off-resonant interaction.

DOI: 10.1103/PhysRevA.87.012132 PACS number(s): 03.65.Xp, 42.50.−p, 03.75.Lm

## I. INTRODUCTION

Tunneling is one of the most fundamental and important phenomenon in quantum mechanics, which provides the physical basis for many useful semiconductor devices and scanning tunneling microscope. Soon after the stimulating work of MacColl and Hartman [1,2] on the dynamics of wave packets through potential barriers, many tunneling time definitions were introduced [3–5]. Among all, phase time [2,6,7] is widely studied and well established, which measures how long it takes for the peak of the transmitted wave packet to emerge from the exit of the barrier. It is related to the energy derivative of the phase shift, and has been studied using numerical, experimental, and analytical methods [5,8–15], in quite detail.

More recently, interesting effects related to the tunneling problem were studied. These include bounds and enhancement for the Hartman effect derived from the causality principle [16], superluminal tunneling as a weak measurement effect [17], the speedup effect due to the entanglement between the spin and the spatial degree of freedom in a magnetic field [18], and the reshaping mechanism of quantum tunneling [19].

The interaction of ultracold atoms with a high-$Q$ microwave cavity has been a problem of considerable interest in recent years. The quantum theory of induced emission due to the quantized motion of the ultracold atoms passing through a micromaser cavity was established in a seminal paper by Scully *et al.* [20]. The drastic change in the atom-field coupling which results when the cold atom enters the cavity leads to an entirely different kind of emission known as microwave amplification via $z$-motion-induced emission of radiation (mazer). The dressed state analysis of the problem shows that the interaction of cold atoms with a single-mode cavity is equivalent to a combination of potential barrier and a well [21–23].

Recently, Arun and Agarwal [24] discussed the tunneling of ultracold two-level atoms through the vacuum-induced potential. It was shown that phase tunneling time for ultracold atoms exhibit both superclassical and subclassical behaviors which can be understood in terms of the momentum dependence of the transmission amplitudes. The passage of the atoms through the cavity involves a coherent addition of the transition amplitudes corresponding to both barrier and well; as a result it is unique. In our earlier study [25] we discussed the passage of ultracold three-level atoms through a high-$Q$ bimodal cavity. It was shown that the presence of dark states and interference effects in cascade atomic configuration affect the phase tunneling time.

In some recent studies, the effects of detuning have also been investigated in the context of the emission probability for two-level ultracold atoms passing through a high-$Q$ microwave cavity [26]. It was shown that detuning adds a potential step effect that is not present for the resonant case. It results in a well-defined acceleration or deceleration (depending upon the sign of the detuning) of the excited atom that contributes a photon inside the cavity. The use of positive detuning provides a well-controlled cooling mechanism. It was also shown that the photon emission can be completely blocked by appropriate choice of the detuning [26]. The problem of mazer action is closely related to the velocity selection of ultracold atoms [27]. It was shown that the velocity selection for ultracold atoms can be very easily tuned and enhanced using off-resonant interaction [28–30].

In the earlier studies, the tunneling times of ultracold atoms were discussed in the resonant cases where the mode frequency is equal to the atomic transition frequency [24,25]. In this paper we discuss the effects of off-resonant interaction on the tunneling or traversal time for ultracold two-level atoms passing through a high-$Q$ cavity. The atoms, which are assumed to be initially in their excited state, after interaction with the cavity field (initially in a vacuum state) may be transmitted or reflected while remaining in the same state or making a transition to the ground state. We calculate the phase tunneling time for both situations using stationary phase approximation. Our results show some interesting features of phase time in the presence of off-resonant interaction. For example, negative phase time is obtained for transmission of the atoms in both excited as well as in the ground state. In particular, for a proper set of parameters, we find that change in the sign of detuning switches the tunneling time

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from subclassical to superclassical. Although the results are obtained using stationary phase approximation, however, the actual envelope of the wave function corresponding to the ground state was also solved using numerical integration techniques. It is shown that the peak of the transmitted wave packet appears at the same time, which is obtained using stationary phase approximation.

The paper is organized in the following manner. In Sec. II we discuss our system and study the interaction of ultra cold atoms with the vacuum field in a high-$Q$ mazer cavity. In Sec. III we discuss the phase tunneling time. In Sec. IV we present our numerical results for the phase tunneling time against mean atomic momentum. Finally, we conclude our results in Sec. V.

## II. MODEL AND DYNAMICS

We consider a beam of ultracold two-level atoms having $|e\rangle$ and $|g\rangle$, as excited and ground states, respectively, as shown in Fig. 1. The atoms are supposed to be moving along $z$ direction and are injected into a mazer cavity of length $L$. We assume that the cavity mode frequency $\omega$ is detuned from the atomic transition frequency $\omega_0$ by an amount $\Delta$, i.e., $\Delta = \omega - \omega_0$, where $\omega_0 = (\Omega_e - \Omega_g)$. The atoms are prepared initially in their excited states, which interact with the single mode of the cavity field.

The atomic center-of-mass (c.m.) motion is treated quantum mechanically and the corresponding Hamiltonian of the atom-field system under dipole and rotating wave approximations is given by

$$H = \frac{p_z^2}{2m} + \hbar\Omega_e|e\rangle\langle e| + \hbar\Omega_g|g\rangle\langle g| + \hbar\omega a^\dagger a + \hbar g u(z)(a^\dagger\sigma + \sigma^\dagger a). \tag{1}$$

Here $p_z$ is the atomic center-of-mass momentum along the $z$ axis, $m$ is mass of the atom, $\sigma = |g\rangle\langle e|(\sigma^\dagger = |e\rangle\langle g|)$ are the lowering (raising) operators of the atom, $a(a^\dagger)$ corresponds to annihilation (creation) operators of the field, and $g$ is the atom-field coupling strength. The parameter $u(z)$ is the cavity mode function, which is assumed to be a mesa mode function and is given by

$$\begin{aligned} u(z) &= 1 \quad \text{for} \quad 0 < z < L \\ &= 0 \quad \text{elsewhere}. \end{aligned} \tag{2}$$

It is clear that coupling is uniform within the cavity and is zero everywhere outside.

While passing through the cavity, the atoms interact with the field and are transmitted (reflected) being in the final state

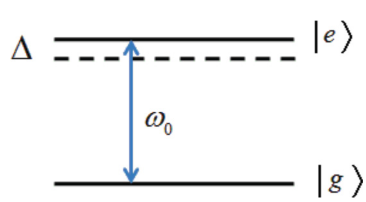


FIG. 1. (Color online) Schematic energy-level diagram of a two-level atom having angular frequency $\omega_0$ with atom-field detuning $\Delta$.

$|i\rangle(i = e,g)$. The corresponding transmission and reflection amplitudes for excited (ground) states are $\tau_n^e(\tau_{n+1}^g)$ and $\rho_n^e(\rho_{n+1}^g)$, respectively. Here the subscript $(n)$ represents the occupation number of the cavity field while the superscripts $(e)$ and $(g)$ denote excited and ground states, respectively. In the case of detuning $\Delta$, the atom found in the ground state $|g\rangle$ propagates with a momentum $\hbar k_g$ instead of the initial value $\hbar k$. It should be noted that the change in momentum does not take place for the resonant case [24]. Here, in the presence of detuning, the atom-field-induced transition $|e,n\rangle \to |g,n+1\rangle$ is responsible for the change in momentum. Thus the exchange of energy between the atom and the field causes the atom to be speed up for $\Delta < 0$ or to slow it down for $\Delta > 0$. These are the consequences of the energy conservation.

The photon number in the cavity field is increased by 1 if the atom leaves the cavity in the ground state $|g\rangle$ after making a transition from the upper level $|e\rangle$. During this process, the atom-field internal energy changes by an amount $\hbar\Delta$. The change in the internal energy is counterbalanced by the kinetic energy of the atom. In case the initial kinetic energy of the atom $\hbar^2k^2/2m$ is less than the change in the internal energy $\hbar\Delta$, the transition from the $|e,0\rangle \to |g,1\rangle$ is no longer possible. Thus the emission process is blocked completely as the required energy is not available in the system. Such type of behavior was discussed earlier in the context of emission and transmission probabilities and the velocity selection for nonresonant two and three-level atoms [26,28,29].

The atom initially in the excited state $|e\rangle$, after interaction with the cavity which is in a vacuum state (i.e., $n = 0$), can be transmitted in the same excited state or in the ground state with transmission probabilities given by [28]

$$T_{e,n} = \left|\tau_n^e(k)\right|^2 \tag{3}$$

and

$$\begin{aligned} T_{g,n+1} &= \left\{ \frac{k_g}{k}\left|\tau_{n+1}^g(k)\right|^2 \quad \text{if} \quad \left(\frac{k}{k_0}\right)^2 > \tilde{\Delta}, \right. \\ &= 0 \text{ otherwise;} \end{aligned} \tag{4}$$

where

$$k_g^2 = k^2 - k_0^2\tilde{\Delta}, \tag{5}$$

$$k_0^2 = \frac{2mg}{\hbar}, \tag{6}$$

and

$$\tilde{\Delta} = \frac{\Delta}{g}. \tag{7}$$

The expressions for $\tau_n^e(k)$ and $\tau_{n+1}^g(k)$ are the same as given in Appendix.

In order to calculate the transmission probabilities, the Schrödinger equation needs to be solved over the entire $z$ axis, which is rather cumbersome inside the cavity. However, for the special case of mesa mode function defined through Eq. (2), the problem is considerably simplified. The normalized eigenstates of the operator $H - p_z^2/2m$ are given by [26]

$$|\gamma_n^+(\theta)\rangle = \cos\theta|e,n\rangle + \sin\theta|g,n+1\rangle, \tag{8}$$

$$|\gamma_n^-(\theta)\rangle = -\sin\theta|e,n\rangle + \cos\theta|g,n+1\rangle. \tag{9}$$

Here the parameter $\theta$ is arbitrary. For $\theta = 0$, the state $|\gamma_n^{\pm}(\theta)\rangle$ coincides with the uncoupled states $|e,n\rangle$ and $|g,n+1\rangle$ and for $\theta = \theta_n$ with dressed states and is given by

$$\cot 2\theta_n = -\frac{\Delta}{\Omega_n} \tag{10}$$

with

$$\Omega_n = 2g\sqrt{n+1}. \tag{11}$$

Therefore, in the basis $|\gamma_n^{\beta}\rangle(\beta = \pm)$ and in the $z$ representation, the Schrödinger equation takes the form

$$i\hbar\frac{\partial}{\partial t}\psi_n^{\beta}(z,t) = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V_n^{\beta}\right)\psi_n^{\beta}(z,t), \tag{12}$$

with

$$\psi_n^{\beta}(z,t) = e^{iE_e t/\hbar}\langle z,\gamma_n^{\beta}|\psi(t)\rangle, \tag{13}$$

$$E_e = \hbar\omega_0 + n\hbar\omega, \tag{14}$$

$$V_n^{+} = \sin^2\theta_n\hbar\Delta + \hbar g\sqrt{n+1}\sin 2\theta_n, \tag{15}$$

$$V_n^{-} = \hbar\Delta - V_n^{+}. \tag{16}$$

The general solution of Eq. (12) related to the energy eigenstate $|\phi_k\rangle$ is given by

$$\psi_n^{\beta}(z,t) = e^{-i(\hbar k^2/2m)t}\varphi_n^{\beta}(z), \tag{17}$$

with

$$\varphi_n^{\beta}(z) = A_n^{\beta}e^{ik_n^{\beta}z} + B_n^{\beta}e^{-ik_n^{\beta}z}. \tag{18}$$

Here, $A_n^{\beta}$ and $B_n^{\beta}$ are complex variables and

$$\left(k_n^{\beta}\right)^2 = k^2 - \frac{2m}{\hbar^2}V_n^{\beta}. \tag{19}$$

Inside the cavity, components of the wave function can be written over the noncoupled state basis. The transmission coefficients $\tau_n^e(\tau_{n+1}^g)$ and $A_n^{\beta}(B_n^{\beta})$ are then derived by applying the continuity conditions of the wave function itself and of its first derivative at the cavity interfaces (i.e., $z = 0$ and $z = L$).

## III. THE PHASE TUNNELING TIME

In order to derive an expression for phase tunneling time, we take the transmission amplitude say in excited state $\tau_0^e(k) \equiv |\tau_0^e(k)|e^{i\phi(k)}$. It is given by Eq. (A1), which incorporates the effects of detuning and depends upon the vacuum coupling energy as $\hbar g \equiv \hbar^2 k_0^2/2m$. Here $k_0$ is the momentum for which the kinetic energy of the incoming atom becomes equal to the height of the potential barrier. We consider a Gaussian wave packet associated with the atom having amplitude $A(k) = \exp[-(k-\overline{k})^2/\sigma^2]$, with $\overline{k}$ and $\sigma$ as mean momentum and width of the wave packet, respectively. Therefore, the transmitted wave function for $z \geqslant L$ and for two-level atom (initially prepared in the excited state) interacting off-resonantly with the cavity field is given by

$$|\Psi_T(z,t)\rangle = \frac{1}{(2\pi)^{3/4}}\sqrt{\frac{2}{\sigma}}\int_{-\infty}^{\infty}dk\,\exp[-(k-\overline{k})^2/\sigma^2] \times e^{-i(\hbar k^2/2m)t}\left|\tau_0^e(k)\right|e^{i\phi(k)}e^{ikz}|e,0\rangle. \tag{20}$$

If the width of the wave packet $\sigma$ is small, then the integrand in Eq. (20) survives only in a small range of wave numbers $k$ centered about the mean momentum $\overline{k}$. Therefore the envelope of the transmitted wave packet $|\langle e,0|\Psi_T(z,t)\rangle|^2$ reaches a maximum when the total phase $\Theta(k)$ of the integrand exhibits extremum at the wave number $k = \overline{k}$. Now, under the assumption that the peak of the incident wave packet enters the cavity at time $t = 0$ and by using the stationary phase condition, the time at which the wave packet peaks at the exit of the cavity $z = L$ can be obtained as [24]

$$\left.\frac{\partial\Theta(k)}{\partial k}\right|_{k=\overline{k}} = \left.\frac{\partial}{\partial k}[kL + \phi(k) - (\hbar k^2/2m)t]\right|_{k=\overline{k}} = 0. \tag{21}$$

The phase tunneling time comes out to be

$$t_{ph} = \left.\frac{m}{\hbar k}\left(\frac{\partial\phi}{\partial k} + L\right)\right|_{k=\overline{k}}. \tag{22}$$

The integral in Eq. (20) can be calculated approximately by using the Taylor expansion of the phase associated with the transmission amplitude about the mean wave number $k = \overline{k}$. An approximate expression for the transmitted wave function is obtained by Arun and Agarwal [24] by keeping the terms up to second order in the Taylor expansion and assuming $\sigma \ll \overline{k}$. The phase time has no significance when the Taylor expansion of the phase does not converge or additional terms more than the second order are important in the expansion.

In free space, the time taken by the peak of the wave packet to travel a distance $L$ is given by $t_{cl} \equiv mL/\hbar\overline{k}$, which is the classical traversal time. In the absence of the cavity there will be no reflection; therefore transmission probability gets to its peak value, i.e., $|\tau_0^e(k)|^2 = 1$, with invariant phase $\frac{\partial\phi}{\partial k} = 0$. As a result, in free space, as clear from Eq. (22), the phase time becomes equal to the classical time.

## IV. DISCUSSION

Here we consider the transmission of ultracold two-level atoms initially prepared in their excited state, passing through a high-$Q$ mazer cavity which is prepared initially in a vacuum state. The atoms which are assumed to be detuned from the cavity mode frequency undergo reflection or transmission after interaction with the cavity. In case of transmission in the excited $|e\rangle$ or ground $|g\rangle$ state, the atoms may contribute zero or one photon inside the cavity mode. In Fig. 2 we show the plots of the phase time for atoms transmitted in their excited state $|e\rangle$ as a function of mean momentum $\overline{k}/k_0$ for two different choices of detuning. Here we consider the cavity length $k_0L = 10\pi$. The solid line represents phase time while the dashed line is for the transmission probability. For zero detuning, we obtain the same results [see Fig. 2(a)] as discussed in Ref. [24]. Here it is important to note that the phase time follows resonances of the transmission probability and remains positive for all values of momentum. The resonances in the transmission probability appear whenever the de Broglie wavelength associated with the ultracold atoms satisfies the condition for the standing wave in a high-$Q$ mazer cavity, i.e., $L = m(\lambda_{dB})/2$, where $m = 1,2,3,\ldots$ as mentioned in Ref. [21]. It is interesting to mention here that a similar resonant behavior was observed in the evolution of phase time

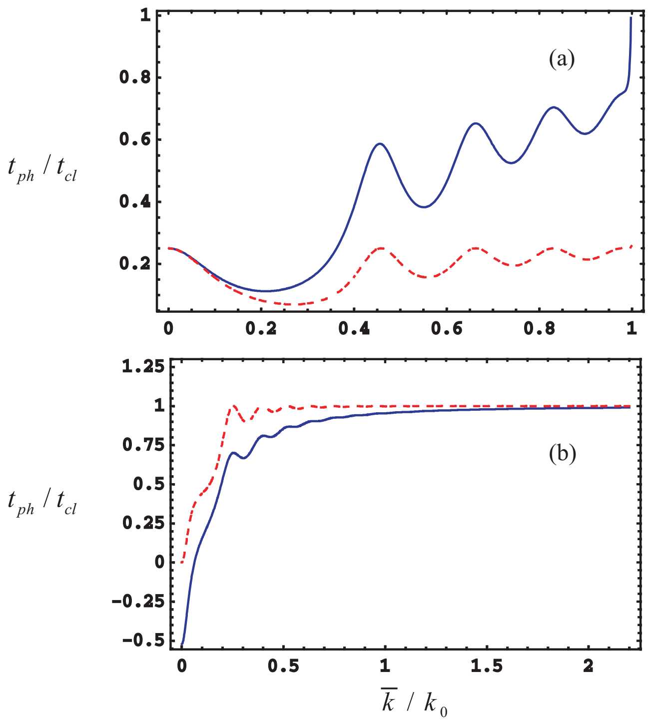


FIG. 2. (Color online) Dimensionless phase time (solid curve) for transmission of a two-level atom in excited state vs the mean momentum for the parameters $k_0L = 10\pi$ (a) $\tilde{\Delta} = 0$ and (b) $\tilde{\Delta} = +10$. The transmission probability $T_{e,0}$ of the atom in excited state is shown in dashed curves.

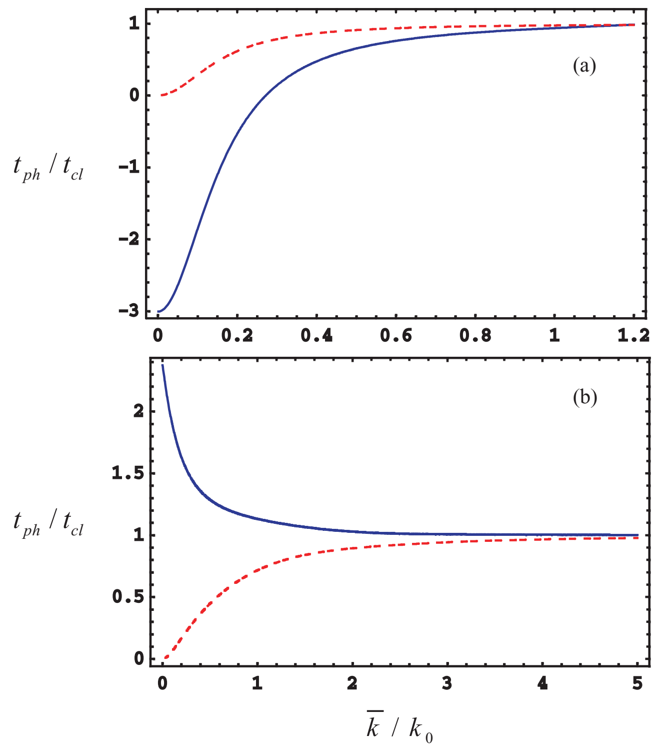


FIG. 3. (Color online) Dimensionless phase time (solid curve) for transmission of a two-level atom in excited state vs the mean momentum for the parameters $k_0L = \pi/2$ (a) $\tilde{\Delta} = +3$ and (b) $\tilde{\Delta} = -3$. The transmission probability $T_{e,0}$ of the atom in excited state is shown in dashed curves.

associated with the passage of electrons or photons through a finite superlattice by Pereyra [31].

The results for the off-resonant case with $\tilde{\Delta} = 10$, using the same parameters as in Fig. 2(a), is shown in Fig. 2(b). A comparison of this result with Fig. 2(a) indicates a clear change in the behavior of phase tunneling time. For example, here we obtain a negative phase time corresponding to smaller values of the mean momentum. Furthermore, phase time increases and approaches to the classical time due to the increase in mean momentum. The transmission probability asymptotically reaches toward the maximum value of 1, which is a consequence of the fact that due to the increase in the mean momentum, the atomic center of mass energy increases and becomes larger than the atom field interaction energy. The appearance of negative phase time is an important feature of our results. This indicates that the transmitted wave packet emerges even before the entrance of the incident wave packet. It may be a consequence of the interference between the incoming wave and the wave which is reflected from the inner wall of the cavity, as discussed in [24]. In an earlier study, such a behavior was reported for the propagation of electromagnetic pulses through dielectric media [32].

In Fig. 3 we consider a different choice of the cavity length, i.e., $k_0L = \pi/2$, and detuning $\tilde{\Delta} = \pm 3$. For positive detuning ($\tilde{\Delta} = +3$), the phase time is negative or superclassical and approaches the classical time with the increase in the mean momentum as shown in Fig. 3(a). It is interesting to note that as the sign of detuning is reversed, i.e., $\tilde{\Delta} = -3$, phase time becomes positive or subclassical as shown by solid curve in Fig. 3(b). This shows that detuning plays an important role in tunneling and traversal of ultracold atoms through the vacuum-induced potential. A change in the sign of the detuning switches the phase time from superclassical to subclassical.

We have discussed so far the results for phase time for atoms transmitted in their excited state. Next we consider the behavior of the phase tunneling time for transmission of the atoms in their ground state. In Fig. 4 we show the results of the transmission probability and phase time for the same set of parameters as in Fig. 3, except that the atoms are now transmitting in their ground state after undergoing a transition from the initial excited state and contributing a single photon inside the cavity. The transition from excited to the ground state does not take place if the initial kinetic energy of the atoms is smaller than $\hbar\Delta$, as clear from Eq. (4). Thus the emission process is completely blocked for $(k/k_0)^2 < \tilde{\Delta}$, because the required energy is no more available. Therefore the transmission probability of the atoms in their ground state remains zero; as a result, phase time becomes meaningless until $(k/k_0)^2 > \tilde{\Delta}$ is satisfied, as shown by the inset in Fig. 4(a). The corresponding phase time for $(k/k_0)^2 > \tilde{\Delta}$ is shown in Fig. 4(a), which exhibits subclassical behavior and approaches the classical value with the increase in mean momentum. For negative detuning $\tilde{\Delta} = -3$, phase time remains subclassical, as shown in Fig. 4(b).

Next we show the plots of phase time for transmission of two-level atoms in their ground state for cavity length $k_0L = 3\pi/2$ and $\tilde{\Delta} = +2$ (see Fig. 5). Here the dimensionless phase time is shown in Fig. 5(a), while transmission probability is

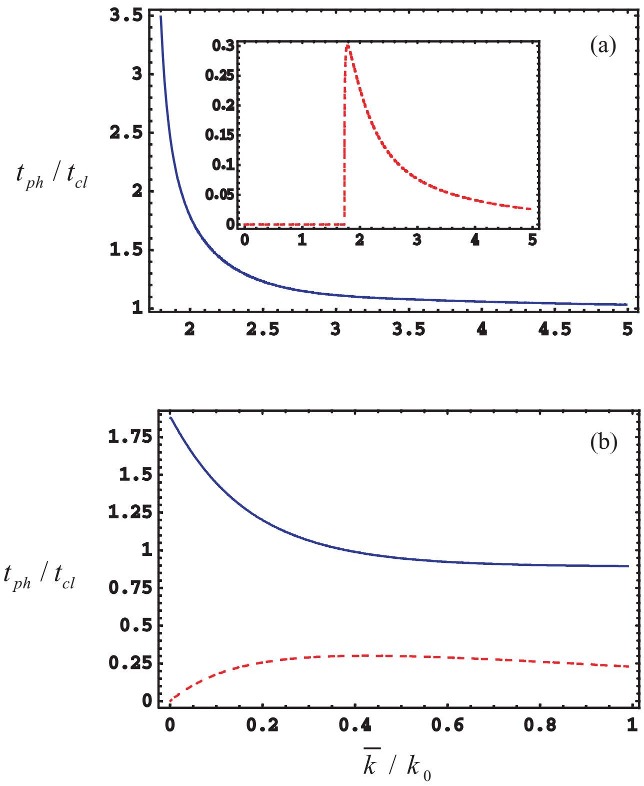


FIG. 4. (Color online) Dimensionless phase time (solid curve) for transmission of a two-level atom in ground state vs the mean momentum for the parameters $k_0L = \pi/2$, (a) $\tilde{\Delta} = +3$ and (b) $\tilde{\Delta} = -3$. The transmission probability in the ground state $T_{g,1}$ is shown in the inset of (a) and with a dashed curve in (b).

shown in Fig. 5(b). In order to show the results more clearly, the two graphs are plotted in separate figures. In Fig. 6 we present the results for phase time and the corresponding transmission probability when $\tilde{\Delta} = -2$, while the rest of the parameters are the same as in Fig. 5. We obtain negative phase time for both positive and negative values of detuning for transmission of the atoms in their ground state. In an earlier study it was shown that the phase time remains positive for atoms leaving the cavity in their ground state after resonantly interacting with the vacuum cavity field [24].

The results discussed so far are obtained using stationary phase approximation. However, for further affirmation, the behavior of the actual envelope of the transmitted wave function in the ground state is solved using numerical integration techniques. The result in the form of normalized probability density $|\langle g,1|\psi_T(z,t)\rangle|^2/\sigma$ against the dimensionless time is shown in Fig. 7. The parameters used for the calculations are $k_0L = 3\pi/2$, $\tilde{\Delta} = 2$, $\sigma/k_0 = 0.01$, and $\bar{k}/k_0 = 1.6$. It is clear from the plot that the peak of the transmitted wave packet appears at $t/t_{cl} \approx -1.9$, which matches with the phase time in Fig. 5(a) at $\bar{k}/k_0 = 1.6$.

It may be mentioned here that the definition of phase time is valid only under the condition that the transmission amplitude varies slowly with wave number $k$. For rapidly varying transmission probability it is no longer valid. Throughout our numerical calculations, we have assumed that the modulus of the transmission amplitude is a slowly varying function of the

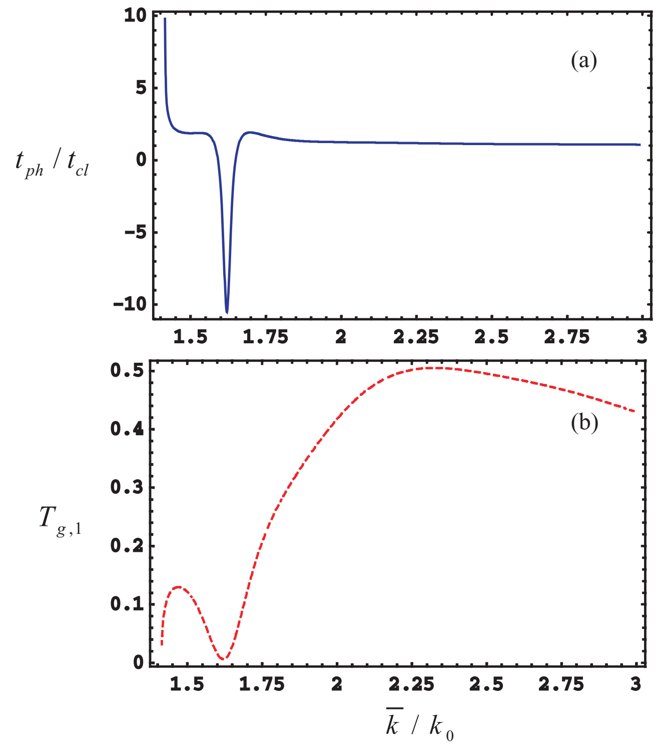


FIG. 5. (Color online) Dimensionless phase time (solid curve) and transmission probability (dashed curve) for transmission of a two-level atom in ground state vs the mean momentum for the parameters $k_0L = 3\pi/2$ and $\tilde{\Delta} = +2$.

wave vector. We also assume that the width of the Gaussian packet associated with the ultracold atoms is very narrow, i.e., $\frac{\sigma}{\bar{k}} \ll 1$, otherwise the transmitted wave packet deformed from the Gaussian shape and the idea of following the peak of the wave packet to calculate the phase time becomes meaningless. This fact is clearly shown in Fig. 8, where the splitting of the transmitted wave packet occurs after passing through the high-$Q$ mazer cavity.

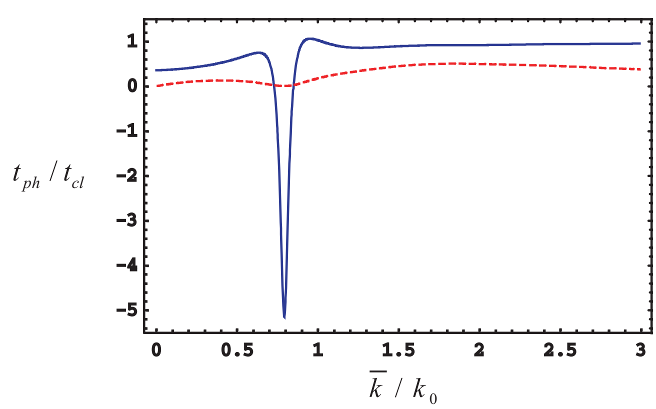


FIG. 6. (Color online) Dimensionless phase time (solid curve) for transmission of a two-level atom in ground state vs the mean momentum for the parameters $k_0L = 3\pi/2$ and $\tilde{\Delta} = -2$. The dashed curve represents the transmission probability $T_{g,1}$ of the atom in ground state.

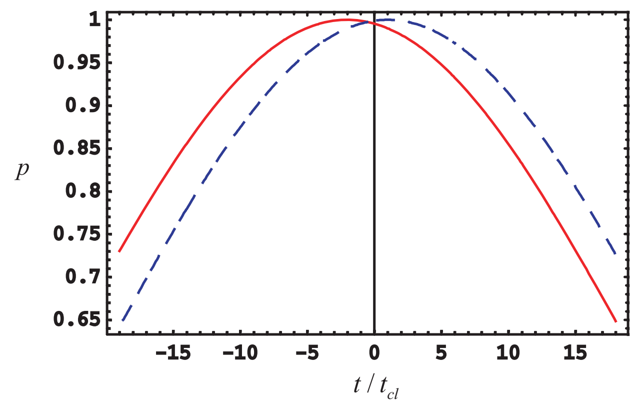


FIG. 7. (Color online) Normalized probability density $P \equiv |\langle g,1|\psi_T(z,t)\rangle|^2/\sigma$ at $Z = L$ as a function of dimensionless time $t/t_{cl}$. The solid (dashed) curve represents $P$ after transmission through the cavity (in free space). The parameters used for the calculations are $k_0L = 3\pi/2$, $\tilde{\Delta} = 2$, $\sigma/k_0 = 0.01$, $\bar{k}/k_0 = 1.6$. Both the solid and dashed curves are normalized to unity.

## V. CONCLUSIONS

In conclusion, we have studied the tunneling and traversal of ultracold two-level atoms interacting off-resonantly with a single-mode field in a high-$Q$ cavity. The atoms are assumed to be initially in their excited state while the cavity field is in a vacuum state. After interaction with the cavity field, the atom may either be reflected or transmitted in excited or ground states. We have studied the effects of detuning on phase tunneling time for transmission of ultracold two-level atoms through the vacuum-induced potential. Our results show that the behavior of phase tunneling time exhibits some interesting features in the presence of off-resonant interaction. For example, detuning can be used to switch the phase time from subclassical to superclassical for appropriate choice of the parameters. The appearance of negative phase time is similar to the negative group velocities studied for the case of electromagnetic pulse propagation through a dielectric media [32]. Our results show that in the presence of off-resonant interaction, negative phase time can be obtained even for atoms leaving the cavity in their ground state for an appropriate choice of parameters. Throughout our numerical results, we have assumed that the modulus of the transmission amplitude is a slowly varying function of the wave vector $k$. Our results are presented under stationary phase approximation; however, we have also studied the propagation of the actual envelope of the Gaussian wave packet associated with the two-level atoms using numerical integration techniques. The results are in good agreement with the one obtained using stationary phase approximation.

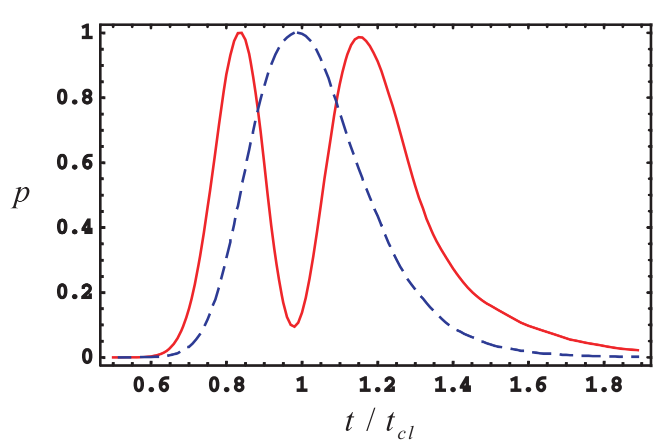


FIG. 8. (Color online) Normalized probability density $P \equiv |\langle g,1|\psi_T(z,t)\rangle|^2/\sigma$ at $Z = L$ as a function of dimensionless time $t/t_{cl}$. The solid (dashed) curve represents $P$ after transmission through the cavity (in free space). The parameters used for the calculations are $k_0L = 10\pi$, $\tilde{\Delta} = -0.55$, $\sigma/k_0 = 0.55$, $\bar{k}/k_0 = 2.055$. Both the solid and dashed curves are normalized to unity.

## ACKNOWLEDGMENT

F.B. would like to express his gratitude to the Higher Education Commission of Pakistan for support through a fellowship under the Indigenous Ph.D. program batch-IV. Sh.Q. would like to thank Prof. B. G. Englert for fruitful discussion.

## APPENDIX

After interaction with the high-$Q$ cavity, the ultracold atoms can be transmitted in excited or ground states with transmission probabilities as given in Eqs. (3) and (4) [28], with

$$\tau_n^e(k) = \frac{\cos^2\theta_n \frac{\tau_n^-(k)}{\tilde{\tau}_n^-(k_g)}\tau_n^+(k_g) + \sin^2\theta_n \tau_n^-(k)}{\left(\cos^2\theta_n \frac{k-k_g}{k_n^c} - 1\right)\left(\cos^2\theta_n \frac{k-k_g}{k_n^t} - 1\right)} \tag{A1}$$

and

$$\tau_{n+1}^g(k) = \frac{\sin 2\theta_n}{4}\left(1 + \frac{k}{k_g}\right) \times \frac{\frac{\tau_n^-(k)}{\tilde{\tau}_n^-(k,k_g)}\tau_n^+(k_g) - \frac{\tau_n^+(k_g)}{\tilde{\tau}_n^+(k,k_g)}\tau_n^-(k)}{\left(\cos^2\theta_n \frac{k-k_g}{k_n^c} - 1\right)\left(\cos^2\theta_n \frac{k-k_g}{k_n^t} - 1\right)}. \tag{A2}$$

Here

$$\tau_n^\pm(k) = e^{-ikL}[\cos(k_n^\pm L) - i\,\Sigma_n^\pm(k)\sin(k_n^\pm L)]^{-1}, \tag{A3}$$

$$\tilde{\tau}_n^\pm(k,k_g) = e^{-ikL}[\cos(k_n^\pm L) - i\,\tilde{\Sigma}_n^\pm(k,k_g)\sin(k_n^\pm L)]^{-1}, \tag{A4}$$

with

$$k_n^+ = \sqrt{k^2 - k_0^2 \tan\theta_n}, \tag{A5}$$

$$k_n^- = \sqrt{k^2 + k_0^2 \cot\theta_n}, \tag{A6}$$

$$\Sigma_n^\pm(k) = \frac{1}{2}\left(\frac{k_n^\pm}{k} + \frac{k}{k_n^\pm}\right), \tag{A7}$$

$$\tilde{\Sigma}_n^\pm(k,k_g) = \left(\frac{k_n^\pm}{k+k_g} + \frac{k_g}{k+k_g}\frac{k}{k_n^\pm}\right), \tag{A8}$$

$$k_n^c = i\,\frac{\left[k + i\cot\left(\frac{k_n^- L}{2}\right)k_n^-\right]\left[k_g + i\cot\left(\frac{k_n^+ L}{2}\right)k_n^+\right]}{\cot\left(\frac{k_n^- L}{2}\right)k_n^- - \cot\left(\frac{k_n^+ L}{2}\right)k_n^+}, \tag{A9}$$

$$k_n^t = i\,\frac{\left[k - i\tan\left(\frac{k_n^- L}{2}\right)k_n^-\right]\left[k_g - i\tan\left(\frac{k_n^+ L}{2}\right)k_n^+\right]}{\tan\left(\frac{k_n^+ L}{2}\right)k_n^+ - \tan\left(\frac{k_n^- L}{2}\right)k_n^-}. \tag{A10}$$

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