Abelian geometric phase due to the presence of an edge dislocation

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We study the appearance of an Abelian geometric phase in relativistic and nonrelativistic quantum dynamics of a neutral particle due to the presence of an edge dislocation. We demonstrate that the nonminimal coupling of fermions with torsion introduces a geometric phase in the wave function of the neutral particle in relativistic and in nonrelativistic quantum dynamics.

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I. INTRODUCTION

The role of the quantum mechanical potential has a fundamental importance in studies of quantum interference effects. The most famous quantum effect of interference is the Aharonov-Bohm (AB) effect [1], where the presence of a quantum mechanical potential gives rise to the appearance of an Abelian quantum phase [2]. Other well-known topological effects are the dual of effect of the Aharonov-Bohm effect [3], the Aharonov-Casher (AC) effect [4] and the He-McKellar-Wilkens (HMW) effect [5], where these last two topological effects are characterized by the appearance of non-Abelian quantum phases [2]. Other interesting studies of the topological nature of quantum phases are the appearance of an Abelian quantum phase for an induced electric dipole [6], the equivalence between AB and AC effects [7], and nonlocality and nondispersivity [8–10]. It worth mentioning other studies of the AC and HMW effects such as those in Refs. [11–13].

Geometric quantum phases are related to unitary transformations called holonomies [14] which have a purely geometrical origin and play an important and fundamental role in various areas of physics. In the early eighties, Berry [15] discovered that a slowly evolving (adiabatic) quantum system retains information of its evolution when it returns to its original physical state. This information is termed the Berry phase. In Ref. [16], the study of the Berry phase has been generalized to the case of a nonadiabatic evolution of a quantum system. In any case (adiabatic or nonadiabatic), the phase depends only on the geometrical nature of the pathway along which the system evolves. Several experiments have been reported in recent years concerning the appearance of adiabatic and nonadiabatic geometric phases, including observations in systems with photons [17], neutrons [18], and nuclear spins [19].

In recent decades there have been many discussions about the influence of torsion on several physical systems [20–26]. The Dirac equation coupled with torsion is discussed in Ref. [21]. The interaction between fermions and a torsion field, conservations laws, and possible physical effects due to the presence of torsion are treated in Refs. [22–24]. The study of torsion has also been made to describe the continuum picture of defects in solid crystals [27–31], where the formulation of the differential geometry is applied to describe the strain and the stress induced by the defect in an elastic medium. In the context of nonrelativistic particles in an elastic medium, the influence of torsion is investigated in Refs. [32,33]. In the relativistic context, the influence of torsion is discussed in Ref. [34]. Recently, the interest of torsion in condensed matter systems has attracted a great deal of attention due to the studies of the influence of a topological defect in graphene and related systems [35,36]. The influence of disclinations, dislocations, and the Stone-Wales defects on the carbon structure has been investigated, and it has been experimentally observed in graphitic structures [37]. A series of interesting papers [38,39] have used the general relativity formalism to investigate the influence of a curved portion of graphene on the electronic properties of these systems. In these works, the electronic properties are investigated by using the coupling of Dirac fermions to the corresponding curved space. In Refs. [40,41], the authors investigate the electronic properties of graphene sheets with a density of dislocations.

In this work, we study the appearance of an Abelian topological quantum phase in the relativistic and nonrelativistic dynamics of a neutral particle due to the presence of a linear topological defect. We claim that this study can be applied to graphene and related systems where the presence of topological defects introduces the concept of torsion, and in the study of fermions in curved space including torsion is necessary.

The structure of this paper is as follows. In Sec. II, we present the topological defect background. In Sec. III, we discuss the appearance of a relativistic Abelian topological phase in the wave function of a spin-1/2 particle. In Sec. IV, we discuss possible applications of the relativistic system in $(2+1)$ dimensions in condensed matter systems. In Sec. V, we discuss the appearance of a nonrelativistic Abelian topological defect in the wave function of a spin-1/2 particle. In Sec. VI, we present our conclusions.

II. TOPOLOGICAL DEFECT BACKGROUND

In this section, we present the topological defect which is the background of this work. Inspired by the description of an edge dislocation in crystalline solids, we construct a generalization of this topological defect in gravitation. We can see that an edge dislocation is a distortion of a circle into a spiral, which can be called a spiral dislocation [31]. The line element that describes the space-time background containing

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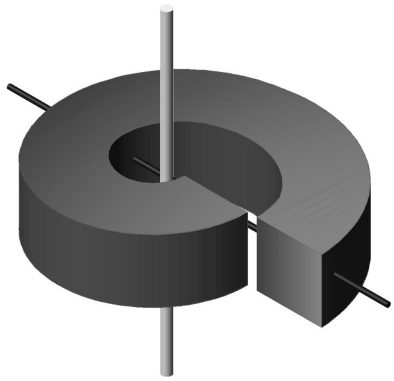

FIG. 1. Distortion of a circle into a spiral or edge dislocation [31].

this topological defect is (we consider the units $\hbar = c = 1$)

$$ds^2 = -dt^2 + d\rho^2 + 2\chi\, d\rho\, d\varphi + (\chi^2 + \rho^2) d\varphi^2 + dz^2, \quad (1)$$

where χ is a constant parameter related to the distortion of the defect. The parameter χ is also related to the Burgers vector $\vec{b}$ by $\chi = |\vec{b}|/2\pi$.

In the study of the relativistic dynamics of a quantum particle in the presence of this topological defect, it is convenient to build a local reference frame where we can define the spinors in a way analogous to the definition of the spinors in a curved space-time background [42,43]. We can build the local reference frame through the noncoordinate basis $\hat{\theta}^a = e^a_{\ \mu}(x)\, dx^\mu$, whose components $e^a_{\ \mu}(x)$ satisfy the relation $g_{\mu\nu}(x) = e^a_{\ \mu}(x)\, e^b_{\ \nu}(x)\, \eta_{ab}$ [42,44], where $\eta_{ab} = \mathrm{diag}(- + + +)$ is the Minkowski tensor. The components of the noncoordinate basis $e^a_{\ \mu}(x)$ form the local reference frame of the observers and are called *tetrads*. The inverse of the tetrads are defined as $dx^\mu = e^\mu_{\ a}(x)\, \hat{\theta}^a$, where $e^a_{\ \mu}(x)\, e^\mu_{\ b}(x) = \delta^a_{\ b}$ and $e^\mu_{\ a}(x)\, e^a_{\ \nu}(x) = \delta^\mu_{\ \nu}$. In this way, we can write the local reference frame $e^a_{\ \mu}(x)$ and its inverse as

$$e^a_{\ \mu}(x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \chi & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad e^\mu_{\ a}(x) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{\chi}{\rho} & 0 \\ 0 & 0 & \frac{1}{\rho} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

With the choice of the local reference frame given in Eq. (2), we have to solve the Maurer-Cartan structure equations [44] to obtain the non-null components of the connection one-form $\omega^a_{\ b} = \omega_\mu{}^a{}_b(x) dx^\mu$, and the torsion two-form $T^a = T^a_{\ \mu\nu}\, dx^\mu \wedge dx^\nu$. The Maurer-Cartan structure equation is $T^a = d\hat{\theta}^a + \omega^a_{\ b} \wedge \hat{\theta}^b$, where the operator d corresponds to the exterior derivative and the symbol $\wedge$ means the wedge product [44]. Thus, the non-null components of the torsion two-form and the connection one-form are

$$T^1 = 2\pi\, \chi\, \delta(\rho)\, \delta(\varphi)\, d\rho \wedge d\varphi, \quad \omega_\varphi{}^2{}_1(x) = -\omega_\varphi{}^1{}_2(x) = 1. \quad (3)$$

In this form, we study in the next section the arising of a relativistic Abelian geometric phase in the quantum dynamics of a spin-1/2 particle from the presence of the linear topological defect called spiral dislocation described by the line element (1).

III. RELATIVISTIC ABELIAN QUANTUM PHASE

In this section, we work the Dirac equation in the presence of the topological defect described by the line element (1). By using the Dirac phase factor method, we obtain the quantum phase obtained for the quantum particle when it encircles the topological defect. Moreover, we demonstrate that this quantum phase acquired by wave function is an Abelian geometric phase. In order to discuss the nature of the geometric phase, we use the same approach adopted by Berry [2,15] extended to the relativistic case. In the presence of a topological defect, it is convenient to write the Dirac equation with the same approach used in the spinor theory in curved space-time [42,43] as introduced in the previous section. In curved space-time and in the presence of a torsion field, the covariant derivative is defined in the form [23,45]

$$\tilde{\nabla}_\mu = \partial_\mu + \frac{i}{4}\, \omega_{\mu ab}(x) \Sigma^{ab} + \frac{i}{4} K_{\mu ab}(x) \Sigma^{ab}, \quad (4)$$

where the term $\Gamma_\mu = \frac{i}{4}\, \omega_{\mu ab}(x)\, \Sigma^{ab}$ is called the spinorial connection [42,44] and $\Sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$. The indices $(a,b,c = 0,1,2,3)$ indicate the local reference frame. The γ^a matrices are defined in the local reference frame and correspond to the standard Dirac matrices in the Minkowski space-time [46]:

$$\gamma^0 = \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \hat{\beta}\, \hat{\alpha}^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$
$$\Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}, \quad (5)$$

with $\vec{\Sigma}$ being the spin vector. The matrices σ^i are the Pauli matrices and satisfy the relation $(\sigma^i\, \sigma^j + \sigma^j\, \sigma^i) = 2\, \eta^{ij}$. The indices $(i,j,k = 1,2,3)$, indicating the spatial indices of the local reference frame. The object $K_{\mu ab}(x)$ in Eq. (4) is related to the contortion tensor by [23]

$$K_{\mu ab}(x) = K_{\beta\nu\mu}(x)\big[e^\nu_{\ a}(x)\, e^\beta_{\ b}(x) - e^\nu_{\ b}(x)\, e^\beta_{\ a}(x)\big], \quad (6)$$

and the contortion tensor is related to the torsion tensor via

$$K^\beta_{\ \nu\mu} = \tfrac{1}{2}\big(T^\beta_{\ \nu\mu} - T_\nu{}^\beta{}_\mu - T_\mu{}^\beta{}_\nu\big). \quad (7)$$

In this work, we use the definitions given in Ref. [23], where the torsion tensor is antisymmetric in the last two indices and the contortion tensor is antisymmetric in the first two indices. Following Ref. [23], we can rewrite the torsion tensor in terms of three irreducible components:

$$T_\mu = T^\beta_{\ \mu\beta}; \quad S^\alpha = \epsilon^{\alpha\beta\nu\mu}\, T_{\beta\nu\mu}; \quad (8)$$

and the tensor $q_{\beta\nu\mu}$, which satisfy the conditions $q^\beta_{\ \mu\beta} = 0$ and $\epsilon^{\alpha\beta\nu\mu}\, q_{\beta\nu\mu} = 0$. In this way, the torsion tensor is written in the form: $T_{\alpha\beta\mu} = \frac{1}{3}[T_\beta\, g_{\alpha\mu}(x) - T_\mu\, g_{\alpha\beta}(x)] - \frac{1}{6}\epsilon_{\alpha\beta\mu\nu}\, S^\nu + q_{\alpha\beta\mu}$. As pointed out in Refs. [22,23], the trace four-vector T_μ and the tensor $q_{\beta\nu\mu}$ decouple with fermions while the axial four-vector couples with fermions. However, taking the local reference frames (2) and the torsion two-form (3), we obtain that the only non-null component of the irreducible components of the torsion tensor is

$$T_\varphi = T^{\rho}_{\ \varphi\rho} = -T^{\rho}_{\ \rho\varphi} = -2\pi\,\chi\,\delta(\rho)\delta(\varphi), \tag{9}$$

which is a component of the trace four-vector T_μ. From the result given in Eq. (9), we have that all components of the axial four-vector S^μ are null. Therefore, there is no explicit contribution from the interaction between torsion and spinors that stems from the covariant derivative (4). This kind of behavior is expected as discussed in Refs. [36,40] because the axial four-vector S^μ is a four-vector associated with a distribution of screw dislocations (see examples in Refs. [45,47]). As we can see in Eq. (9), the interaction between spinors and torsion cannot be described by the axial four-vector, but it can be given via the trace four-vector T_μ agreeing with Refs. [36,40]. In Refs. [36,40] it is shown that the trace four-vector T_μ is a four-vector associated with a distribution of edge dislocations. Thereby, in order to investigate the interaction between spinors and torsion that stems from the presence of an edge dislocation, we need to build a coupling from the product of T_μ and γ^μ. As shown in Ref. [23], this kind of coupling is introduced into the Dirac equation as a nonminimal coupling since it cannot be included in the Dirac equation via the spinorial connection (minimal coupling) as for S^μ in Eq. (4). Hence, to study the influence of the topology of the defect (1) on the relativistic quantum dynamics of the neutral particle, we must introduce a nonminimal coupling [23],

$$i\gamma^\mu\,\nabla_\mu \to i\gamma^\mu\,\nabla_\mu + \nu\,\gamma^\mu\,T_\mu, \tag{10}$$

with $\nabla_\mu = \partial_\mu + \frac{i}{4}\,\omega_{\mu ab}(x)\Sigma^{ab}$, and ν being an arbitrary nonminimal coupling parameter (dimensionless). By using the tetrads field (2), the Dirac equation becomes

$$m\psi = i\gamma^0\frac{\partial\psi}{\partial t} + i\gamma^1\left(\frac{\partial}{\partial\rho}+\frac{1}{2\rho}\right)\psi + i\frac{\gamma^2}{\rho}\left(\frac{\partial}{\partial\varphi}-\chi\,\frac{\partial}{\partial\rho}-i\,\nu\,T_\varphi\right)\psi + i\gamma^3\frac{\partial\psi}{\partial z}. \tag{11}$$

We should note, as pointed out in Ref. [23], that the interaction between the trace four-vector $T_\mu(x)$ and fermions is identical to the interaction between the electromagnetic field $A_\mu(x)$ and fermions. This means that the interaction of torsion with fermions described by the introduction of the nonminimal coupling (10) does not break the gauge invariance of the Dirac equation (11). Furthermore, since the background described by Eq. (1) is stationary in time and symmetric under translations along the z axis, we can write the solution of Eq. (11) in the following form,

$$\psi(t,\rho,\varphi,z) = \exp(-iEt + ikz)\psi(\rho,\varphi), \tag{12}$$

where E is the energy eigenvalue and k corresponds to the wave vector in the z direction. Thereby, from the Dirac phase factor method [48], we can write the solution of Eq. (11) in the following form,

$$\psi(\rho,\varphi) = \exp\left[-i\int_{\Phi_0}^{\Phi} T_\varphi\,d\varphi\right]\psi_0(\rho,\varphi), \tag{13}$$

where the term in the exponential corresponds to the relativistic phase acquired by the wave function of the quantum particle,

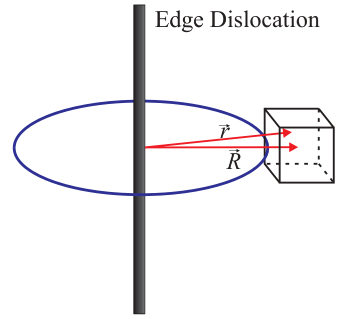

FIG. 2. (Color online) Pictorial representation of physical setup in the space-time of edge dislocation.

and $\psi_0(\rho,\varphi)$ is the solution of the Dirac equation,

$$m\,\psi_0(\rho,\varphi) = i\gamma^1\left(\frac{\partial}{\partial\rho}+\frac{1}{2\rho}\right)\psi_0(\rho,\varphi) + i\frac{\gamma^2}{\rho}\left(\frac{\partial}{\partial\varphi}-\chi\,\frac{\partial}{\partial\rho}\right)\psi_0(\rho,\varphi) - \gamma^3 k\,\psi_0(\rho,\varphi) + \gamma^0 E\,\psi_0(\rho,\varphi). \tag{14}$$

Now, we wish to investigate the nature of the phase acquired by a spin-1/2 particle in the presence of an edge dislocation. We use the same formalism adopted in Refs. [49–51] in the space-time of a cosmic string for a scalar particle. The procedure is as follows. We first confine the quantum system to a perfect reflecting box. Let the box be transported around a circuit C threaded by the defect. The vector that localizes the box in relation to the defect is labeled $\vec{R}$, where this vector is orientated from the origin of the coordinate system (localized on the defect) to the center of the box, and the components R_i of the vector $\vec{R}$ are given by $R_i = (\rho_0,\phi_0,z_0)$. The spinor $\psi(\rho,\phi)$ is nonzero only in the interior of the box and is given by a superposition of different eigenfunctions. When the coupling T_φ is absent, the spinor has the form $\psi_0(\vec{r}-\vec{R})$, and the eigenvalues E do not depend on R_i. In the presence of the coupling T_φ, the spinor is obtained by using the Dirac phase method [48] inside the box through Eq. (13). Note that in the case where $T_\varphi \neq 0$, the spinorial wave function is sensitive to the coupling with the torsion, and the geometric phase can be obtained inside the box by using Eq. (13).

A way of obtaining the geometric phase for a Dirac neutral particle is to use the Dirac phase method [48] to calculate the Berry connection [2]. The relativistic version of the Berry phase was obtained in Refs. [52,53], and it can be written as

$$\Phi = i\oint_C \langle\psi(R)|\vec{\nabla}_R|\psi(R)\rangle\,dR, \tag{15}$$

where R is an adiabatic parameter that can be expressed as $R = \{R_1\cdots R_i\}$, and $\vec{\nabla}_R$ is the gradient with respect to the parameter R. Note that we proceed in the same way as Berry [15] in the nonrelativistic Aharonov-Bohm effect, where the external parameter $\vec{R}$ localizes the box, and we consider the spin-1/2 particle inside a perfectly reflecting box. This means that the particle is described by a wave packet corresponding to a linear combination of different eigenfunctions of the free Hamiltonian, $\psi(\rho,\varphi) = \sum_n e^{\Phi}\,\psi_n(\rho,\varphi) = \sum_n e^{i\int_{\vec{R}}^{\vec{r}}\mathcal{A}_n^{IJ}\,dR}\,\psi_n(\vec{r}-\vec{R})$, where we can write the integrand term of Eq. (15) (which is known as the non-Abelian analog of the the Mead-Berry connection one-form [2]) in the form

$$\mathcal{A}_n^{IJ} = \langle \psi_n^I(r_i - R_i)|\nabla_R|\psi_n^J(r_i - R_i)\rangle, \tag{16}$$

where I and J stand for possible degeneracy labels. Note that the indices I and J determine the matrix elements of an $\mathcal{N} \times \mathcal{N}$ Hermitian matrix ($1 \leqslant I, J \leqslant \mathcal{N}$). Thus, the connection one-form (16) gives rise to the unitary transformation $\mathrm{U}(\mathcal{N}) = \mathcal{P}\exp(\oint A_n^{IJ}\, dR)$ of a symmetry gauge group. For instance, $\mathcal{N} = 1$ corresponds to the U(1) symmetry gauge group and then the geometric phase yielded by the connection one-form (16) is an element of the Abelian gauge group U(1). For $\mathcal{N} = 2$, we have that the geometric phase is an element of the non-Abelian gauge group U(2), that is, a 2×2 matrix. Returning to the physical system described from Eq. (11) to Eq. (14), the inner product given in Eq. (16) may be evaluated as follows:

$$\begin{aligned}&\langle \psi_n^I(x_i - R_i)|\vec{\nabla}_R|\psi_n^J(r_i - R_i)\rangle\\ &\quad = -i\oint_\Sigma \psi_n^{*I}(r_i - R_i)\{T_\varphi \psi_n^J(r_i - R_i) + \vec{\nabla}_R\, \psi_n^J(r_i - R_i)\}dS\\ &\quad = -i T_\varphi\, \delta_{IJ},\end{aligned} \tag{17}$$

where $dS = \rho\, d\rho\, d\phi\, dz$, and ψ_n is assumed to be normalized. Therefore, putting the above results into Eq. (15), we obtain

$$\Phi_n(C) = \nu \oint T_\varphi\, d\varphi = \nu \iint \frac{\partial T_\varphi}{\partial x^\mu}\sqrt{-g}\, dx^\mu \wedge d\varphi = 2\pi\nu\chi, \tag{18}$$

where C is a closed curve which involves the defect. We observe that this result depends on the Burgers vector of the edge dislocation χ and the coupling parameter ν (which is dimensionless). As pointed out in Ref. [54], for the screw dislocation case, this effect can be interpreted as interference between the wave functions associated with the spinorial particle in the transported box and with a spinorial particle in a box that follows the orbit along the circuit C. Note that the geometric phase obtained in Eq. (18) is an element of the symmetry gauge group U(1). Hence, the topology of the defect gives rise to the appearance of an *Abelian geometric phase* in the wave function of a relativistic quantum particle. Therefore, we can observe that the nature of the topological defect determines the kind of topological quantum phase that will be acquired by the wave function in the relativistic dynamics of a neutral particle. In this case of an edge dislocation, the line of the dislocation is parallel to the plane of motion of the quantum particle and the relativistic geometric phase is Abelian. In Ref. [45], the line of the screw dislocation is perpendicular to the plane of motion of the quantum particle and the relativistic geometric phase is non-Abelian. Moreover, we can note that the relativistic geometric quantum phase (18) does not depend on the velocity of the quantum particle, which means that the relativistic Abelian geometric phase (18) is a nondispersive quantum phase [9,10]. The nondispersivity nature of the quantum phase shows us that the quantum phase is independent of the energy of the neutral particle, which we can verify directly in Eq. (18). In the following, we show that the same phase shift can be obtained in the nonrelativistic regime. Therefore one can expect that the nondispersivity nature of the Abelian quantum phase yielded by the presence of an edge dislocation can be obtained in an interferometry experiment over a wide range of energy as discussed in Refs. [9,10]. Note that, the phase (18) is not sensitive to smooth deformations of the path in the parameter space, which shows us that it is really a topological phase.

IV. APPLICATION IN (2 + 1) DIMENSIONS

Let us discuss the possibility of using the results obtained in the previous section in condensed matter systems, for example, a graphene layer. Graphene is a layer structure of carbon atoms arranged in a hexagonal lattice. By using a tight-binding description for a graphene layer, the system reduces to a coupling of fermions with the honeycomb lattice. The behavior of electrons near the Fermi points obeys the Dirac equation for massless fermions in $(2 + 1)$ dimensions. In this approximation, the spectrum of energy is a linear dispersion relation with respect to momenta. This system has been widely studied as a good laboratory technique for field theory since it is described by relativistic equations. A series of interesting papers has used the general relativity formalism to investigate the influence of a curved portion of graphene on the electronic properties of these systems [38,39]. In these works, the electronic properties have been investigated by using the coupling of Dirac fermions with the corresponding curved space. Moreover, the influence of a topological defect (a disclination) has been investigated, and it has been shown that the presence of a disclination modifies the electronic properties of the system. The presence of a disclination in a graphene layer introduces five or seven membered rings in the honeycomb lattice, which corresponds to introducing curvature in a graphene layer. In this way, a graphene layer with this type of defect is a curved surface described by the coupling of Dirac fermions with the corresponding curved space. Furthermore, dislocations have also been observed in the graphene structure [37]. For instance, a graphene edge dislocation is represented by the presence of a pentagon-heptagon pair in the graphene structure. In Refs. [40,41], the electronic properties of graphene sheets with a density of dislocations have been investigated through the coupling of fermions with a curved space and torsion.

In the present work, by introducing the nonminimal coupling with torsion (10) in the $(2 + 1)$-dimensional effective graphene theory, we obtain the same Abelian geometric phase obtained in Eq. (18) in contrast to the phase shift obtained by Mesaros *et al.* [41]. In Ref. [41], a non-Abelian geometric phase was obtained due to the change of the local frame in the quantum dynamics of a massless Dirac particle in the geometry of an edge dislocation. Comparing the results of Ref. [41] with ours, we have that both the Abelian geometric phase (18) and the non-Abelian geometric phase obtained in Ref. [41] are present in Eq. (11). In the following, we show that we can obtain both contributions to the geometric phase in the $(2 + 1)$-dimensional massless Dirac theory for graphene. First of all, the $(2 + 1)$-dimensional metric that describes a radial edge dislocation in a graphene layer is given by

$$ds^2 = -dt^2 + d\rho^2 + 2\chi\, d\rho d\varphi + (\chi^2 + \rho^2)d\varphi^2, \tag{19}$$

where χ is a constant parameter related to the distortion of the defect, and it is related to the Burgers vectors $\vec{b}$ by $\chi = |\vec{b}|/2\pi$ as in Eq. (1).

In the geometric theory of quasiparticles in a graphene layer with defects, the massless Dirac equation in the presence of an edge dislocation (19) is given by

$$i\gamma^0\frac{\partial\psi}{\partial t}+i\gamma^1\left(\frac{\partial}{\partial\rho}+\frac{1}{2\rho}\right)\psi + i\frac{\gamma^2}{\rho}\left(\frac{\partial}{\partial\varphi}-\chi\frac{\partial}{\partial\rho}-i\nu\,T_\varphi\right)\psi=0. \quad (20)$$

Note that, for the $(2+1)$-dimensional massless Dirac equation, the Dirac spinors can be four-component spinors and the γ^a matrices can be written as 4×4 matrices as in Eq. (5). However, the covariant form of the Dirac equation (20) is independent of the representation of the γ^a matrices. Indeed, we can build the chiral representation [55,56] of the 4×4 Dirac matrices as in Ref. [41] from the 2×2 matrices τ^i and σ^i that act on the Fermi-point indices and $\mathcal{A}/\mathcal{B}$ labels, respectively:

$$\gamma^0=\tau^1\otimes I=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\quad \gamma^i=-i\tau^2\otimes\sigma^i=\begin{pmatrix}0 & -\sigma^i\\ \sigma^i & 0\end{pmatrix},$$
$$\Sigma^i=I\otimes\sigma^i=\begin{pmatrix}\sigma^i & 0\\ 0 & \sigma^i\end{pmatrix},\quad \gamma^5=\tau^3\otimes I=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}, \quad (21)$$

where I is the 2×2 identity matrix and $\vec{\Sigma}$ is the spin vector. The matrices σ^i (τ^i) are the standard Pauli matrices that satisfy the relation $(\sigma^i\sigma^j+\sigma^j\sigma^i)=2\,\eta^{ij}$ $(i,j,k=1,2,3)$. The representation of the Dirac matrices (21) is known in the literature as the Weyl or chiral representation [55,56].

Next, we employ the same procedure adopted in the previous section, from Eq. (13) to Eq. (18), to obtain the geometric quantum phase. Our first contribution to the geometric phase is the same geometric phase given in Eq. (18). Now, we investigate another contribution to geometric phase by writing the solution of Eq. (20) as in Eq. (13), where $\psi_0(\rho,\varphi)$ is the solution of the following equation:

$$i\gamma^1\left(\frac{\partial}{\partial\rho}-i\frac{\chi\Sigma^3}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{2\rho}\right)\psi_0(\rho,\varphi) + i\frac{\gamma^2}{\rho}\frac{\partial}{\partial\varphi}\psi_0(\rho,\varphi)+\gamma^0E\,\psi_0(\rho,\varphi)=0. \quad (22)$$

From Ref. [41], by contrast, we can write Eq. (22) in the form

$$\begin{aligned} i\gamma^a e^{\mu}{}_{a}(x)\,\partial_\mu\psi_0(\rho,\varphi) &= i\gamma^a\left(\delta^\mu_a+X^\mu_a\right)\partial_\mu\psi_0(\rho,\varphi)\\ &= i\gamma^\nu\partial_\nu\left[\exp\left(\int dx^\beta\,X^\mu_\beta\,\partial_\mu\right)\right]\psi_0(\rho,\varphi), \end{aligned} \quad (23)$$

where X^μ_β corresponds to the perturbation proportional to the Burgers vector. Therefore, from Eqs. (22) and (23), we can recover the holonomy transformation obtained in Ref. [41] which is given by

$$U=\exp\left(\int dx^\nu\,X^\mu_\nu\,\partial_\mu\right)=e^{\vec{b}\cdot(-i\vec{\nabla})}. \quad (24)$$

Equation (24) is the holonomy experimented by the quasiparticle in graphene with an edge dislocation in the continuum limit. As observed by Mesaros *et al.* [41], the operator $-i\vec{\nabla}$ corresponds to the continuum translation generator; then, for the lattice case, the operator $\vec{b}\cdot(-i\vec{\nabla})$ can be replaced by $\vec{K}\tau_3$, that is, the translation generator of the lattice. Thereby, the holonomy (24) is given by

$$U=e^{\vec{b}\cdot\vec{K}\tau_3}=e^{2\pi\chi\hat{b}\cdot\vec{K}\tau_3}. \quad (25)$$

The phase shift given in Eq. (25) is a non-Abelian geometric phase and corresponds to that obtained by Mesaros *et al.* [41]. This holonomy angle is an effect of the translation on the wave function due to the Burgers vector that characterizes the nature of the topological defect and also produces a change in the local frame. In contrast, the phase shift (18) stems from the flux of the torsion of the defect; therefore this phase shift is similar to the Aharonov-Bohm effect [1], where the torsion of defect plays the role of the magnetic field. Hence, we have obtained two distinct contributions to the geometric phases in graphene, where one of them is an Abelian geometric phase (18) that stems from the nonminimal coupling with torsion (10) and one of them is a non-Abelian geometric phase (24) that stems from the change of local frames. The existence of this new phase (Abelian phase) in graphene can be verified through studies of scattering and other transport properties in graphene with an edge dislocation. Based on this fact, we argue that the geometric phase obtained in Eq. (18) can be used to investigate the appearance of this phase for massless fermionic quasiparticles in a graphene layer with an edge dislocation.

V. NONRELATIVISTIC ABELIAN QUANTUM PHASE

In this section, we investigate the nonrelativistic dynamics of the neutral particle in the presence of the topological defect (1). The nonrelativistic limit is studied here through the Foldy-Wouthuysen approach [57]. In this approach, we need first to write the Dirac equation in the form

$$i\frac{\partial\psi}{\partial t}=\hat{H}\psi, \quad (26)$$

where the Hamiltonian of the system must be written as a linear combination of even terms $\hat{\epsilon}$ and odd terms $\hat{O}$ as follows:

$$\hat{H}=\hat{\beta}\,m+\hat{O}+\hat{\epsilon}. \quad (27)$$

The aim of the Foldy-Wouthuysen approach [46,57] is to remove the operators from the Dirac equation that couple the "large" to the "small" components of the Dirac spinors by applying a unitary transformation. Basically, the even operators do not couple the "large" to the "small" components of the Dirac spinors, while the odd operators do couple them. Therefore, the even and the odd operators need to be Hermitian operators. Moreover, the even operators must satisfy the commutation relation $[\hat{\epsilon},\hat{\beta}]=0$, whereas the odd operators must satisfy the anticommutation relation $\{\hat{O},\hat{\beta}\}=0$. By considering just terms up to the order of m^{-1}, the Hamiltonian of the system in the low-energy limit becomes [46,57]

$$\hat{H}_{\rm NR}=\hat{\beta}\,m+\frac{\hat{\beta}}{2m}\hat{O}^2+\hat{\epsilon}. \quad (28)$$

Returning to the Dirac equation (11), we can write it in the form

$$i\frac{\partial\psi}{\partial t}=m\hat{\beta}\psi+\vec{\alpha}\cdot(\vec{p}-i\vec{\xi}-\nu\vec{T})\psi, \quad (29)$$

where the matrices $\hat{\alpha}^i$ and $\hat{\beta}$ are defined in Eq. (5), and $p_k = -i\, e^{\lambda}{}_k(x)\, \partial_\lambda$ and $\xi_k = \frac{i}{4}\, e^{\lambda}{}_k(x)\, \omega_{\lambda ab}(x)\, \Sigma^{ab}$. Thus, the odd terms of Eq. (29) are

$$\hat{O} = \vec{\alpha} \cdot (\vec{p} - i\vec{\xi} - \nu \vec{T}), \tag{30}$$

and there are no even terms in the Dirac equation (29). Then, substituting the odd terms (30) into the expression for the nonrelativistic Hamiltonian (28), we obtain the following nonrelativistic equation:

$$i\frac{\partial \psi}{\partial t} = m\hat{\beta}\,\psi + \frac{\hat{\beta}}{2m}[\vec{p} - i\vec{\xi} - \nu\vec{T}]^2\psi. \tag{31}$$

The nonrelativistic geometric phase can be also obtained by using the same procedure from Eqs. (12) to (17). Considering a two-component spinor, $\psi_0(\rho,\varphi)$ is the solution of the Schrödinger equation

$$i\frac{\partial \psi_0}{\partial t} = \frac{1}{2m}(\vec{p} - i\vec{\xi})^2\psi_0. \tag{32}$$

The nonrelativistic geometric phase is given by the topology of the defect through the presence of the trace vector in expression (31), that is,

$$\phi_{\rm NR} = \nu \int T_\varphi\, d\varphi = \nu \int\!\!\int \frac{\partial T_\varphi}{\partial x^\mu}\sqrt{-g}\, dx^\mu \wedge d\varphi = 2\pi\nu\chi, \tag{33}$$

where this contribution is independent of the velocity of the neutral particle; thus, it is a nondispersive geometric phase [9,10]. Note that the nonrelativistic topological phase (33) is identical to the relativistic topological phase (18). This occurs due to the nondispersive nature of the quantum phase, and the independence of the path developed by the neutral particle around the edge dislocation. Again, we can see that the presence of the topological defect yields an *Abelian geometric phase* in the nonrelativistic dynamics of the quantum particle.

VI. CONCLUSIONS

We have studied the appearance of an Abelian topological phase in the wave function of a neutral particle due to the presence of an edge dislocation. Due to the nature of the edge dislocation, we have introduced a nonminimal coupling into the Dirac equation in order to describe the coupling of torsion with fermions. We have seen, in both relativistic and nonrelativistic dynamics of the quantum particle, that the wave function acquires an Abelian quantum phase due to the structure of the topological defect. We have obtained this Abelian quantum phase by using the adiabatic approximation [2,15] and shown that the quantum phase acquired by the wave function of a neutral particle in the presence of the edge dislocation is a topological phase. The topological nature of the Abelian quantum phase obtained in Eqs. (18) and (33) comes from the fact that this quantum phase is nondispersive and does not depend on the path developed by the neutral particle around the edge dislocation. We have also seen that this quantum phase is nondispersive because it is independent of the velocity of the neutral particle. Comparing with the results obtained in Refs. [45,58], where the deficit of angle and the axial vector of the torsion field generate a non-Abelian geometric phase, we can see that the structure of this specific topological defect yields an Abelian geometric phase. Hence, the way of building the topological defect provides the kind of topological phase which the wave function will acquire in the dynamics of the spin-1/2 neutral particle. It is worth mentioning that the same procedure used in this work has been adopted in Ref. [59] in order to investigate Berry phases for relativistic charged particles in the presence of topological defects, for instance, a screw dislocation. In Sec. IV, we have discussed a possible application of this geometric phase in a condensed matter system in $(2+1)$ dimensions, namely, a graphene layer with an edge dislocation. We have obtained a contribution to the geometric quantum phase in this system. We claim that these results can be investigated in transport properties of a graphene layer with an edge dislocation, whose effects of quantum interference on the wave function of quasiparticles are significant and can be verified.

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