

Vacuum-state truncation via the quantum Zeno effect

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In the context of quantum-state engineering we analyze the effect of observation on nonlinear optical n -photon Fock-state generation. We show that it is possible to truncate the vacuum component from an arbitrary photon number superposition without modifying its remaining parts. In the course of the full dynamical analysis of the effect of observation, it is also found that the Zeno and the anti-Zeno effects repeat periodically. We discuss the close relationship between vacuum-state truncation and so-called “interaction-free” measurement. The proposed scheme may also be considered a way to generate extremely nonclassical states.

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The engineering of quantum states of an electromagnetic field is essential to fundamental studies of quantum optics [1,2] and is also important for the development of optical quantum-information technologies [3–5]. The quantum-optical-state engineering method we are interested in here is optical-state truncation. Pegg, Phillips, and Barnett [6] presented a scheme to prepare any required superposition of the vacuum and one-photon states by the choice of a proper measurement after a beam splitter. Because their scheme is effectively the same as a physical truncation of the photon number superposition making up the coherent state involved during the measurement, they called the device “quantum scissors” [6,7].

In this Brief Report, we present a scheme of quantum scissors in which only the vacuum component is truncated from any given photon number superposition state without corrupting the remaining parts: i.e., the transformation $\sum_{m=0}^{\infty} \alpha_m |m\rangle \rightarrow \sum_{m=1}^{\infty} \alpha_m |m\rangle$, apart from normalization. The vacuum state has a unique property different from other photon number states, in that it represents the “absence” of a photon while the other states (or their superpositions) represent the “presence” of photons. Considering a physical measurement using the electromagnetic field as a probe of the measurement, the absence of a photon may in a sense correspond to the absence of measurement. Therefore, one can expect that the presence or absence of photons may lead to totally different physical effects: “measurement” or “no measurement.” As a specific example, we consider the quantum Zeno effect, which refers to the suppression of the free dynamical evolution of a system when the system is subject to repeated measurements [8–12]. In the limit of infinitely frequent or continuous measurements, the observed system would be locked into its initial state. If the initial probe state is a superposition of the presence and absence of photons, then we may expect the total output state of the system and probe to be a superposition of two different physical effects: the occurrence and nonoccurrence of the Zeno effect. Thus, we show that it is possible to effectively truncate the vacuum component of the probe state by projecting the output state onto the state of occurrence of the Zeno effect.

The present scheme is also closely related to the so-called “interaction-free” measurement (IFM) (also known as “quantum interrogation”), which refers to the determination of the presence of an object without any photons being scattered by the object, thereby excluding any disturbance of it [13–20].

If we consider a coherent superposition of the presence and absence (the vacuum) of a quantum object, the IFM of the object becomes equivalent to vacuum-state truncation. Thus, our scheme may be considered a scheme of IFM in which the inferred object is an electromagnetic field with an IFM efficiency approaching 100%. On the other hand, Ref. [21] shows that any state can be converted into a nonclassical one with maximum nonclassical depth [22], simply by removing its vacuum component. Therefore the present scheme may also be considered a way to generate extremely nonclassical states.

We give a detailed description of the vacuum-state truncation in the following. We first consider the Kilin and Horoshko (KH) scheme for the generation of a pure Fock state of a single-mode field via a single-pass parametric interaction in a nonlinear medium [23]. In the KH scheme, the nonlinear medium is pumped by a classical coherent field to amplify the signal mode from the vacuum state into the n -photon Fock state by the $(n+2)$ th-order nonlinear interaction. The interaction Hamiltonian describing this parametric process is

$$H_n = \frac{\hbar g}{\sqrt{n!}} \left[(a^n + a^{\dagger n}) - \frac{1}{n} (a^\dagger a^{n+1} + a^{\dagger n+1} a) \right], \quad (1)$$

where a^\dagger (a) is the photon creation (annihilation) operator of the signal mode and g is a real coupling parameter that is related to the nonlinear susceptibilities satisfying the condition $\chi^{(n)} = -n\chi^{(n+2)}$. By means of the interaction Hamiltonian, Eq. (1), the initial vacuum state evolves as

$$\begin{aligned} U_{\text{KH}}(\tau)|0\rangle_a &= \exp(iH_n\tau/\hbar)|0\rangle_a \\ &= \cos(g\tau)|0\rangle_a + i\sin(g\tau)|n\rangle_a. \end{aligned} \quad (2)$$

If the interaction time is chosen as $\tau = \pi/2g$, the output state becomes a pure n -photon Fock state. (Note also that $U_{\text{KH}}(\tau)|n\rangle_a = i\sin(g\tau)|0\rangle_a + \cos(g\tau)|n\rangle_a$.)

We now discuss the manifestation of the Zeno effect in the KH scheme. Suppose the system is subject to repeated measurements at each very short time interval $\Delta\tau = \tau/N$ with large N . After the first measurement, the probability of finding the signal mode in the n -photon state is $\sin^2(g\Delta\tau) \approx (g\Delta\tau)^2$, which is very small for large N . The probability of remaining in the initial vacuum state is $\cos^2(g\Delta\tau) \approx 1 - (g\Delta\tau)^2$. The signal field is found in the vacuum state with relatively large probability at each measurement stage. Thus, the probability that the signal mode is found in the initial state after all N

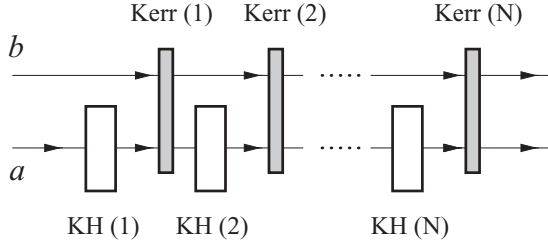


FIG. 1. Proposed scheme of vacuum-state truncation.

repeated measurements is

$$[\cos^2(g\Delta\tau)]^N \approx 1 - (g\tau)^2/N. \quad (3)$$

The signal mode would be locked to the initial vacuum state with certainty in the limit $N \rightarrow \infty$; i.e., each measurement reduces the signal field into $|0\rangle_a$ and the Zeno effect occurs.

For a full dynamical analysis of the Zeno effect, we consider a modification of the KH scheme as depicted in Fig. 1. The KH scheme's nonlinear medium is now divided into N equal pieces with the interaction time $\Delta\tau = \tau/N = \pi/2gN$ within each part. Assume that if the pieces are perfectly aligned, and if the field evolution is not disturbed after each piece, then the input vacuum state $|0\rangle_a$ in mode a would coherently evolve into the n -photon Fock state $|n\rangle_a$ as in the original KH scheme. However, in Fig. 1, we employ the optical Kerr effect to implement quantum nondemolition (QND) measurement of the photon number in mode a [24–26]. Modes a and b are coupled with each other by a series of N nonlinear Kerr mediums. As is well known in the QND scheme using the optical Kerr effect [25], the field of two coupled modes experiences a phase shift proportional to the photon number of another field. One can determine the photon number without destroying photons by the measurement of a phase shift imposed on another field. Thus, a field evolution of the KH type in mode a would be modified by a dispersive coupling with mode b .

The unitary operator describing the state evolution within each of the Kerr mediums is given by

$$U_K = \exp(i\kappa a^\dagger a b^\dagger b), \quad (4)$$

where a^\dagger and b^\dagger (a and b) are the creation (annihilation) operators of the corresponding modes and κ is a coupling parameter characterizing the Kerr interaction strength. For convenience, we assume the same interaction strength for each Kerr medium, and the self-phase modulation effect is neglected. We suppose the input field state in mode b is an arbitrary state $|\Phi\rangle_b$, which can be expanded in terms of the number states as

$$|\Phi\rangle_b = \sum_{m=0}^{\infty} \alpha_m |m\rangle_b. \quad (5)$$

Then, the total initial state of modes a and b is

$$|\Psi^{(0)}\rangle = |0\rangle_a |\Phi\rangle_b. \quad (6)$$

After the first KH and Kerr mediums, the state evolves as

$$\begin{aligned} |\Psi^{(1)}\rangle &= U_K U_{KH}(\Delta\tau) |\Psi^{(0)}\rangle \\ &= [\cos(g\Delta\tau)|0\rangle_a + i e^{i\kappa n b^\dagger b} \sin(g\Delta\tau)|n\rangle_a] |\Phi\rangle_b. \end{aligned} \quad (7)$$

We note that when the input state in mode b is a number state, $|\Phi\rangle_b = |m\rangle_b$, the phase factor becomes $e^{i\kappa n m}$. This phase factor will vanish for $\kappa = 2\pi l/nm$ ($l = 0, 1, 2, \dots$), which means that the optical Kerr effect is effectively removed in this case and the original KH process is recovered. Because our aim is to find the effect of any possible disturbance imposed by the QND measurement, we discard this case. That is, we suppose the coupling parameter is chosen appropriately so that $\kappa \neq 2\pi l/nm$.

Thus, this process is repeated $N - 1$ more times as shown in Fig. 1. The final output state has the form

$$\begin{aligned} |\Psi^{(N)}\rangle &= U_K U_{KH}(\Delta\tau) |\Psi^{(N-1)}\rangle \\ &= [v(b^\dagger b)|0\rangle_a + w(b^\dagger b)|n\rangle_a] |\Phi\rangle_b, \end{aligned} \quad (8)$$

where $v(b^\dagger b)$ and $w(b^\dagger b)$ are given by

$$\begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} \cos(g\Delta\tau) & i \sin(g\Delta\tau) \\ i e^{i\kappa n b^\dagger b} \sin(g\Delta\tau) & e^{i\kappa n b^\dagger b} \cos(g\Delta\tau) \end{pmatrix}^N \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (9)$$

After some calculation, we obtain that

$$\begin{aligned} v(b^\dagger b) &= e^{iN\delta/2} \left[-i \left(\frac{\cos(g\Delta\tau) \sin(\delta/2)}{\sin(\eta)} \right) \sin(N\eta) + \cos(N\eta) \right], \end{aligned} \quad (10)$$

$$w(b^\dagger b) = i e^{i(N+1)\delta/2} \left(\frac{\sin(g\Delta\tau)}{\sin(\eta)} \right) \sin(N\eta), \quad (11)$$

where $\delta = \kappa n b^\dagger b$ and $\eta = \cos^{-1}[\cos(g\Delta\tau) \cos(\delta/2)]$. We can easily check that $|v(b^\dagger b)|^2 + |w(b^\dagger b)|^2 = 1$. Using Eq. (5), the output state is written as

$$\begin{aligned} |\Psi^{(N)}\rangle &= \alpha_0 [\cos(Ng\Delta\tau)|0\rangle_a + i \sin(Ng\Delta\tau)|n\rangle_a] |0\rangle_b \\ &\quad + [v(b^\dagger b)|0\rangle_a + w(b^\dagger b)|n\rangle_a] |\Phi'\rangle_b, \end{aligned} \quad (12)$$

where the state $|\Phi'\rangle_b$ does not include the vacuum component, i.e.,

$$|\Phi'\rangle_b = \sum_{m=1}^{\infty} \alpha_m |m\rangle_b = |\Phi\rangle_b - \alpha_0 |0\rangle_b. \quad (13)$$

Since $g\Delta\tau = \pi/2N$, the first term of Eq. (12) becomes $i\alpha_0 |n\rangle_a |0\rangle_b$, and it corresponds to the isolated dynamics of the KH procedure. The other term exhibits some modifications of the KH process induced by the photons in mode b .

In the limit of large N , $v(b^\dagger b)$ and $w(b^\dagger b)$ become

$$v(b^\dagger b) = 1 - \left(\frac{i\pi^2}{8 \tan(\delta/2)} \right) \frac{1}{N} + O(N^{-2}), \quad (14)$$

$$w(b^\dagger b) = i e^{i(N+1)\delta/2} \left(\frac{\pi \sin(N\delta/2)}{2 \sin(\delta/2)} \right) \frac{1}{N} + O(N^{-2}). \quad (15)$$

Therefore, we obtain

$$|\Psi^{(N)}\rangle \rightarrow i\alpha_0 |n\rangle_a |0\rangle_b + |0\rangle_a |\Phi'\rangle_b \quad \text{as } N \rightarrow \infty. \quad (16)$$

This entangled state represents the desired correlation between modes a and b . For a finite number N , the state $|\Phi'\rangle_b$ is disturbed as a result of the measurements, although the photon number is conserved in the QND measurements. However, Eq. (16) indicates that the disturbance imposed

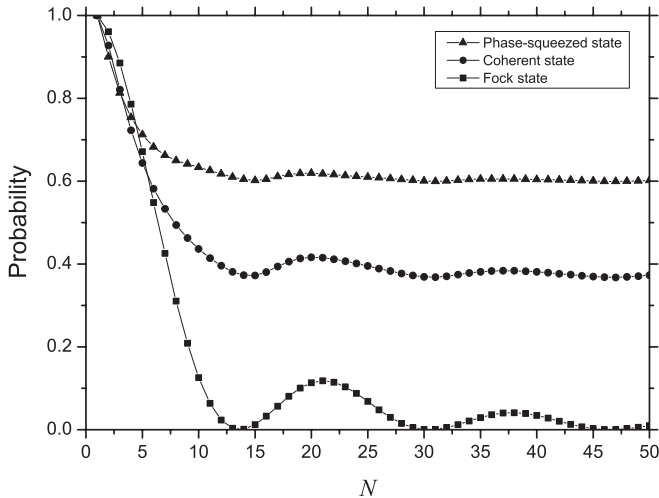


FIG. 2. Photon emission probabilities ($P_{n=2}$) as a function of N . The input state of mode b is a one-photon Fock state (squares), a coherent state (circles), and a phase-squeezed state (triangles). The probabilities approach the value $|\alpha_0|^2$ as N becomes large [Eq. (16)].

on $|\Phi'\rangle_b$ becomes arbitrarily small in the limit of very frequent measurements. That is, the quantum state of the $|\Phi'\rangle_b$ component in mode b remains in its initial state while mode b is correlated with mode a . In terms of quantum scissors, by projecting the output state, Eq. (16), onto the state $|0\rangle_a$, one can truncate only the vacuum component $|0\rangle_b$ in mode b without corrupting the remaining parts $|\Phi'\rangle_b$. Note also that if mode b is initially in the pure $|\Phi'\rangle_b$ state (i.e., $\alpha_0 = 0$), then Fock-state generation in mode a is always inhibited in the limit $N \rightarrow \infty$. Both modes a and b are locked into their initial states in this limit, and the Zeno effect occurs.

In Fig. 2 we verify our results by representing the probability P_n of n -photon Fock state generation in mode a as a function of N . For an arbitrary state $|\Phi\rangle_b$, the probability becomes

$$P_n = |\alpha_0|^2 + |w(b^\dagger b)|^2 |\Phi'\rangle_b|^2 = |\alpha_0|^2 + \sum_{m=1}^{\infty} |\alpha_m w(m)|^2, \quad (17)$$

which is always less than or equal to 1. We plot the probability using a numerical calculation of Eq. (17) for when the input state of mode b is a one-photon Fock state, a coherent state, and a phase-squeezed state, respectively. We suppose the states have the same mean photon number of $\langle m \rangle_b = 1$. For the case of the phase-squeezed state

$$|\Phi\rangle_b = e^{\alpha b^\dagger - \alpha^* b} e^{\frac{1}{2}\varepsilon^* b^2 - \frac{1}{2}\varepsilon b^2} |0\rangle_b, \quad (18)$$

we choose $\varepsilon = -0.5$ and $\alpha = 0.853498$ to make $\langle m \rangle_b = 1$. We also suppose $n = 2$, i.e., two-photon Fock state generation in mode a , and the Kerr interaction strength $\kappa = 0.2$.

As expected, Fig. 2 shows that $P_{n=2} \rightarrow |\alpha_0|^2$ in each case as $N \rightarrow \infty$. In addition, interestingly, it shows oscillation; i.e., the inhibition and enhancement of photon emission in mode a is repeated. The enhancement of photon emission by measurement is known as the inverse or anti-Zeno effect [27] and has been discussed in parametric down-conversion [11,28]. Using the arguments in Ref. [28], the oscillatory behavior is attributed to the interference of photon emission probability amplitudes originating from each part of the KH scheme's nonlinear medium. This interference becomes partially destructive (inhibition) or constructive (enhancement) according to the phase mismatch caused by the Kerr interaction. We also note that the Mandel Q parameter is $Q = (\langle \Delta^2 m \rangle_b - \langle m \rangle_b) / \langle m \rangle_b = -1$ for the one-photon Fock state, $Q = 0$ for the coherent state, and $Q = 1.67071$ for the super-Poissonian phase-squeezed state. Obviously, the amplitude of the oscillation is reduced as the Q -parameter increases. This is because the photon number fluctuation in mode b results in the fluctuation of phase mismatch. Thus, in Fig. 2, the one-photon Fock-state mode b shows the greatest oscillation. From Eq. (15), it can be seen that, if the input state in mode b is a pure m -photon Fock state, the period of oscillation is $N \approx 2\pi/\delta_0$, with δ_0 defined by $\delta_0 = \kappa n m - 2\pi l$ ($0 \leq \delta_0 < 2\pi$). On the other hand, the coherence between probability amplitudes decreases as N increases; i.e., the photon emission events in each part of the nonlinear medium become more distinguishable as the accuracy of the measurement increases. Therefore, the overall behavior is a gradual suppression of photon emission.

In summary, we have presented a scheme of vacuum-state truncation from an arbitrary photon number superposition without modifying its remaining parts. The scheme is a direct consequence of the quantum Zeno effect in nonlinear optical n -photon Fock-state generation. During the full dynamical analysis of the effect, we have also found the observation-induced oscillation in photon emission probabilities. The proposed scheme can be considered an IFM in which the inferred object is a light field without the vacuum component (i.e., $\sum_{m=1}^{\infty} \alpha_m |m\rangle_b$). It is interesting to note that one cannot obtain any further information about the object details by vacuum-state truncation (or IFM): all one knows is that some state orthogonal to the vacuum is present. We point out that the proposed scheme can also be considered a way to generate extremely nonclassical states.

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- [1] C. C. Gerry and P. L. Knight, *Introductory Quantum Optics* (Cambridge University Press, Cambridge, UK, 2005).
 [2] F. Dell'Anno, S. De Siena, and F. Illuminati, *Phys. Rep.* **428**, 53 (2006).

- [3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 [4] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, *Rev. Mod. Phys.* **74**, 145 (2002).

- [5] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, *Rev. Mod. Phys.* **79**, 135 (2007).
- [6] D. T. Pegg, L. S. Phillips, and S. M. Barnett, *Phys. Rev. Lett.* **81**, 1604 (1998).
- [7] W. Leoński and A. Kowalewska-Kudłażyk, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 2011), Vol. 56, pp. 131–185.
- [8] B. Misra and E. C. G. Sudarshan, *J. Math. Phys.* **18**, 756 (1977).
- [9] A. Peres, *Am. J. Phys.* **48**, 931 (1980).
- [10] A. Luis and J. Peřina, *Phys. Rev. Lett.* **76**, 4340 (1996).
- [11] J. Řeháček, J. Peřina, P. Facchi, S. Pascazio, and L. Miřta, Jr., *Phys. Rev. A* **62**, 013804 (2000).
- [12] P. Facchi and S. Pascazio, *J. Phys. A* **41**, 493001 (2008).
- [13] M. Renninger, *Z. Phys.* **158**, 417 (1960).
- [14] R. H. Dicke, *Am. J. Phys.* **49**, 925 (1981).
- [15] A. C. Elitzur and L. Vaidman, *Found. Phys.* **23**, 987 (1993).
- [16] P. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger, and M. A. Kasevich, *Phys. Rev. Lett.* **74**, 4763 (1995).
- [17] R. Jozsa, in *Lecture Notes in Computer Science*, edited by C. P. Williams (Springer-Verlag, Berlin, 1999), Vol. 1509, pp. 103–112.
- [18] O. Hosten, M. T. Rakher, J. T. Barreiro, N. A. Peters, and P. G. Kwiat, *Nature (London)* **439**, 949 (2006).
- [19] L. Vaidman, *Phys. Rev. Lett.* **98**, 160403 (2007).
- [20] T.-G. Noh, *Phys. Rev. Lett.* **103**, 230501 (2009).
- [21] C. T. Lee, *Phys. Rev. A* **52**, 3374 (1995).
- [22] C. T. Lee, *Phys. Rev. A* **44**, 2775R (1991).
- [23] S. Ya. Kilin and D. B. Horoshko, *Phys. Rev. Lett.* **74**, 5206 (1995).
- [24] V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, *Science* **209**, 547 (1980).
- [25] N. Imoto, H. A. Haus, and Y. Yamamoto, *Phys. Rev. A* **32**, 2287 (1985).
- [26] C. C. Gerry and T. Bui, *Phys. Lett. A* **372**, 7101 (2008).
- [27] W. C. Schieve, L. P. Horwitz, and J. Levitan, *Phys. Lett. A* **136**, 264 (1989).
- [28] A. Luis and L. L. Sánchez-Soto, *Phys. Rev. A* **57**, 781 (1998).