Acceleration of adiabatic transport of interacting particles and rapid manipulations of a dilute Bose gas in the ground state

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(Received 24 August 2012; published 19 December 2012)

We show a method to accelerate quantum adiabatic transport of identical spinless particles interacting with each other by developing the preceding fast-forward scaling theory formed for one-particle systems [Masuda and Nakamura, Proc. R. Soc. A **466**, 1135 (2010)]. We derive a driving potential which accelerates adiabatic dynamics of quantum systems composed of identical particles in order to obtain the final adiabatic states in any desired short time. We also exhibit an ideal rapid manipulation of dilute Bose gas in the ground state without energy excitation by using the fast-forward scaling theory.

DOI: 10.1103/PhysRevA.86.063624 PACS number(s): 67.85.−d, 37.90.+j, 81.16.Ta

I. INTRODUCTION

Technology to manipulate quantum states is rapidly evolving, and various methods to control quantum states have been reported in Bose-Einstein condensates (BECs) [1–4], in quantum computing [5], and in many other fields of applied physics. It would be very important to consider the acceleration of such manipulations of quantum states for manufacturing purposes and for innovation of technologies. Earlier we proposed the acceleration of quantum dynamics [6] and quantum adiabatic dynamics [7,8]. The theory is called "fast-forward scaling theory" or "fast-forward theory." We derived a driving potential which accelerates given quantum dynamics and generates exactly a target state in any desired short time, where the target state is defined as the final state in the given original dynamics.

The acceleration of quantum adiabatic dynamics is very important for many current and future technologies. Adiabatic manipulations seem to be an ideal method for the control of quantum systems because of the adiabatic theorem [9]. However, adiabatic dynamics can take too long time compared with the lifetime or coherent time of the system [10]. Acceleration of adiabatic dynamics overcomes the difficulty. Various methods of acceleration of adiabatic dynamics or shortcut to adiabaticity have been proposed: counterdiabatic protocol [11] and frictionless quantum driving [12], invariant-based inverse engineering [13], and fast-forward scaling theory [7,8]. Recently, applications of these methods to the control of BEC have been proposed theoretically [7,14–18] and been demonstrated experimentally [19–21]. However, the range of applications of the methods is still limited in simple cases. Construction of the theory for many-body systems and for more general controls is important and useful for various kinds of manipulations of quantum systems.

In this paper we extend the previous scheme of the acceleration of adiabatic dynamics to many-body systems. Our theory combines opposite ideas—the infinitely fast acceleration and ultimately slow adiabatic dynamics—and uses a space-dependent additional phase to give the driving potential. We exhibit acceleration of adiabatic transport of identical spinless particles interacting with each other. We show a driving potential which conveys the interacting particles without energy excitation.

We also propose an ideal rapid manipulation of dilute Bose gas in the ground state. By using the method, the final state of the Bose gas in the original adiabatic dynamics is generated in any short time without energy excitation. Driving potential is analytically derived in the case that wave function has spherical symmetry. In Sec. II we derive a formula of the driving potential for many-body system composed of identical spinless particles. In Sec. III we exhibit acceleration of adiabatic transport of interacting particles and rapid manipulations of dilute Bose gas by using fast-forward scaling theory. Section IV is devoted to conclusions and discussion.

II. FAST-FORWARD THEORY IN MANY-BODY SYSTEMS

We extend the framework of the fast-forward theory formed for one-particle systems to many-body systems composed of identical spinless particles. We derive a driving potential which realizes the final state of a given original dynamics from its initial state. First we derive a formula of the driving potential for (nonadiabatic) standard dynamics as a preparation for the acceleration of adiabatic dynamics. The formula is used in the derivation of a driving potential for adiabatic dynamics.

A. Standard fast forward

We consider a system composed of N identical spinless particles interacting with each other. The Hamiltonian is given by

$$H_0 = \sum_{j=1}^{N} \frac{\mathbf{p}_j^2}{2m} + V_e(\{\mathbf{r}\},t) + V_I(\{\mathbf{r}\}), \tag{1}$$

where j denotes the particles and m is mass. $\{\mathbf{r}\} = (\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)$ denotes a set of coordinates of all the particles. V_e is an external potential expressed by one-particle operators $v_e(\mathbf{r},t)$ as

$$V_e(\{\mathbf{r}\},t) = \sum_{j=1}^{N} v_e(\mathbf{r}_j,t). \tag{2}$$

V_I is a time-independent interaction potential which is a function of the relative positions of the particles.

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$\Psi_0 = \Psi_0(\{\mathbf{r}\},t)$ is a many-body wave function ruled by H_0. We call Ψ_0 standard state. Instead of a simply accelerated state of Ψ_0 we consider the fast-forwarded state Ψ_{FF} defined by

$$\Psi_{FF}(\{\mathbf{r}\},t) = \Psi_0(\{\mathbf{r}\},\Lambda(t))e^{if(\{\mathbf{r}\},t)}, \tag{3}$$

with the additional phase $f(\{\mathbf{r}\},t) \in R$, because it is not possible to realize the simply accelerated state $\Psi_0(\{\mathbf{r}\},\Lambda(t))$ [6]. $\Lambda(t)$ is defined as

$$\Lambda(t) = \int_0^t \alpha(t')dt', \tag{4}$$

where $\alpha(t) \in R$ is the magnification factor of the fast forward which characterizes the intensity of the acceleration. Time dependence of α is tuned so that the additional phase f disappears at the initial and final time of the acceleration. (The detail is shown later.) We define Ψ_0 at time T as the target state. Arbitrary time $T_F > 0$ is the final time of the acceleration. α relates T_F and T through

$$T = \int_0^{T_F} \alpha(t)dt. \tag{5}$$

The driving Hamiltonian for Ψ_{FF} is assumed as

$$H_{FF}(\{\mathbf{r}\},t) = \sum_{j=1}^{N} \frac{\mathbf{p}_j^2}{2m} + V_e(\{\mathbf{r}\},\Lambda(t)) + V_I(\{\mathbf{r}\}) + \mathcal{V}_{FF}(\{\mathbf{r}\},t), \tag{6}$$

where $\mathcal{V}_{FF}$ is called the driving potential. The Schrödinger equation is represented as

$$i\hbar\frac{\partial\Psi_{FF}}{\partial t} = H_{FF}\Psi_{FF}. \tag{7}$$

By using Eq. (3) and Schrödinger equations for Ψ_0 and Ψ_{FF} we can derive the equation

$$-\hbar\frac{\partial f}{\partial t}|\Psi_0(\Lambda(t))|^2 + [\alpha(t)-1]\left[\sum_{j=1}^{N} -\frac{\hbar^2}{2m}\Psi_0^*\big(\Lambda(t)\big)\nabla_j^2\Psi_0(\Lambda(t)\big) + (V_e(\Lambda(t)) + V_I)|\Psi_0(\Lambda(t))|^2\right]$$
$$+\frac{\hbar^2}{2m}\sum_{j=1}^{N}\left\{2i\nabla_j f\cdot\Psi_0^*(\Lambda(t))\nabla_j\Psi_0(\Lambda(t)) + i\big(\nabla_j^2 f\big)|\Psi_0|^2 - (\nabla_j f)^2|\Psi_0(\Lambda(t))|^2\right\} = \mathcal{V}_{FF}(t)|\Psi_0(\Lambda(t))|^2, \tag{8}$$

where $f(\{\mathbf{r}\},t)$ is abbreviated by f. By decomposing Eq. (8) into real and imaginary parts, we can obtain the driving potential and the additional phase as

$$\frac{\mathcal{V}_{FF}}{\hbar} = -\frac{\partial f}{\partial t} - \sum_{j=1}^{N}\left\{(\alpha-1)\frac{\hbar}{2m}\mathrm{Re}\big[\nabla_j^2\Psi_0/\Psi_0\big] + \frac{\hbar}{m}\nabla_j f\cdot\mathrm{Im}[\nabla_j\Psi_0/\Psi_0] + \frac{\hbar}{2m}(\nabla_j f)^2\right\} + (\alpha-1)\frac{V_e+V_I}{\hbar} \tag{9}$$

and

$$f(\{\mathbf{r}\},t) = \big(\alpha(t)-1\big)\eta(\{\mathbf{r}\},\Lambda(t)), \tag{10}$$

respectively. In Eq. (9) f, α, Ψ_0, V_e abbreviate $f(\{\mathbf{r}\},t)$, $\alpha(t)$, $\Psi_0(\{\mathbf{r}\},\Lambda(t))$, and $V_e(\{\mathbf{r}\},\Lambda(t))$, respectively. We suppose $\eta(\{\mathbf{r}\},t) \in R$ is the phase of $\Psi_0(\{\mathbf{r}\},t)$, that is, $\Psi_0(\{\mathbf{r}\},t) = \tilde{\Psi}_0(\{\mathbf{r}\},t)\exp[i\eta(\{\mathbf{r}\},t)]$, where $\tilde{\Psi}_0$ is the real amplitude of Ψ_0. Because f in Eq. (10) includes the factor $\alpha(t)-1$, the additional phase disappears everywhere when $\alpha = 1$. Therefore, we tune α so that α becomes unity at the initial and the final time of the acceleration and Eq. (5) is satisfied. In Eq. (9), a space-independent term was neglected, because it is concerned only with the space-independent phase on Ψ_{FF} and we are not concerned about it.

We should note that, although $\mathcal{V}_{FF}$ accelerates the dynamics, we cannot generate $\mathcal{V}_{FF}$ in general because we cannot control general many-body potentials. This scheme is basically applicable when $\mathcal{V}_{FF}$ in Eq. (9) is expressed in terms of one-body operators v_{FF} as

$$\mathcal{V}_{FF} = \sum_{j=1}^{N} v_{FF}(\mathbf{r}_j,t). \tag{11}$$

In the next section such accelerations are shown after extending the above formula for acceleration of adiabatic dynamics in the following section.

B. Acceleration of adiabatic dynamics

We derive a driving potential which accelerates adiabatic dynamics by using the formula in the previous section. We consider adiabatic dynamics of a system composed of N spinless particles. The Hamiltonian is given by

$$H_0 = \sum_{j=1}^{N}\frac{\mathbf{p}_j^2}{2m} + V_e(\{\mathbf{r}\},R(t)) + V_I(\{\mathbf{r}\}). \tag{12}$$

External field V_e is a function of an adiabatic parameter R defined by

$$R(t) = R_0 + \varepsilon t. \tag{13}$$

R_0 is the initial value of R. The constant value ε is the rate of adiabatic change in $R(t)$. ε is infinitesimally small, $\varepsilon \ll 1$. V_I is a time-independent interaction potential. We suppose that the system is in an nth energy eigenstate of an instantaneous Hamiltonian $H(R)$ in the adiabatic dynamics. The wave function is represented as

$$\Psi_0 = \Psi_0(\{\mathbf{r}\},t) = \phi_n(\{\mathbf{r}\},R(t))e^{-\frac{i}{\hbar}\int_0^t E_n(R(t'))dt'}e^{i\gamma(t)}, \tag{14}$$

where $\phi_n(\{\mathbf{r}\},R)$ is the nth energy eigenstate of the instantaneous Hamiltonian. E_n is the eigenenergy. $\gamma(t)$ is an adiabatic phase. ϕ_n satisfies

$$H_0(R)\phi_n(R) = E_n(R)\phi_n(R). \tag{15}$$

Now we consider the acceleration of the adiabatic dynamics. The wave function and Hamiltonian should be regularized [7], because we need a standard state which satisfies Schrödinger equation up to $O(\varepsilon)$ to apply the theory developed in the previous section. Standard wave function is modified with a phase $\varepsilon\theta(\{\mathbf{r}\},t)$ as

$$\Psi_0^{(\mathrm{reg})}(\{\mathbf{r}\},R,t) = \phi_n(\{\mathbf{r}\},R)e^{-\frac{i}{\hbar}\int_0^t E_n(R(t'))dt'}e^{i\varepsilon\theta}. \tag{16}$$

The regularized standard Hamiltonian is defined as

$$H_0^{(\mathrm{reg})} = \sum_{j=1}^{N}\frac{\mathbf{p}_j^2}{2m} + V_e(\{\mathbf{r}\},R(t)) + V_I(\{\mathbf{r}\}) + \varepsilon\tilde{V}(\{\mathbf{r}\},t). \tag{17}$$

$\theta = \theta(\{\mathbf{r}\},t)$ and $\tilde{V} = (\{\mathbf{r}\},t)$ are introduced so that the Schrödinger equation,

$$i\hbar\frac{\partial\Psi_0^{(\mathrm{reg})}}{\partial t} = H_0^{(\mathrm{reg})}\Psi_0^{(\mathrm{reg})}, \tag{18}$$

is satisfied up to $O(\varepsilon)$. Then θ should satisfy

$$\sum_{j=1}^{N}\frac{\hbar}{2m}\left[\nabla_j^2\theta + 2\mathrm{Re}[\nabla_j\phi_n/\phi_n]\cdot\nabla_j\theta\right] + \mathrm{Re}\left[\frac{\partial\phi_n}{\partial R}\Big/\phi_n\right] = 0. \tag{19}$$

$\tilde{V}$ is given by

$$\frac{\tilde{V}}{\hbar} = -\mathrm{Im}\left[\frac{\partial\phi_n}{\partial R}\Big/\phi_n\right] - \sum_{j=1}^{N}\frac{\hbar}{m}[\nabla_j\theta\cdot\mathrm{Im}[\nabla_j\phi_n/\phi_n]]. \tag{20}$$

We define a fast-forwarded state and driving Hamiltonian with $\Psi_0^{(\mathrm{reg})}$ as

$$\Psi_{FF}(\{\mathbf{r}\},t) = \Psi_0^{(\mathrm{reg})}(\{\mathbf{r}\},\Lambda(t))e^{if(\{\mathbf{r}\},t)}, \tag{21}$$

$$H_{FF}(t) = \sum_{j=1}^{N}\frac{\mathbf{p}_j^2}{2m} + V_e(\{\mathbf{r}\},\Lambda(t)) + V_I(\{\mathbf{r}\}) + \mathcal{V}_{FF}(\{\mathbf{r}\},t), \tag{22}$$

where f is the additional phase. $\Lambda(t)$ is defined by Eq. (4). We take the limit $\varepsilon\to 0, \alpha = O(1/\varepsilon)\to\infty$ and $\varepsilon\alpha = O(1)$; that is, we consider the infinitely fast acceleration of the ultimately slow dynamics. α relates the initial (R_0) and final (R_1) values of the adiabatic parameter as

$$R_1 - R_0 = \varepsilon\int_0^{T_F}\alpha(t)dt, \tag{23}$$

where T_F is the arbitrary final time of the acceleration. By using Eq. (15) in the Schrödinger equation of Ψ_{FF} we can obtain the driving potential and the additional phase in an analogous manner to [7] as

$$\frac{\mathcal{V}_{FF}}{\hbar} = -\frac{d\alpha}{dt}\varepsilon\theta - \alpha^2\varepsilon^2\frac{\partial\theta}{\partial R} - \sum_{j=1}^{N}\frac{\hbar}{2m}\alpha^2\varepsilon^2(\nabla_j\theta)^2 - \alpha\varepsilon\mathrm{Im}\left[\frac{\partial\phi_n}{\partial R}\Big/\phi_n\right] - \sum_{j=1}^{N}\alpha\varepsilon\frac{\hbar}{m}\mathrm{Im}\left[\frac{\nabla_j\phi_n}{\phi_n}\right]\cdot\nabla_j\theta, \tag{24}$$

and

$$f(\{\mathbf{r}\},t) = (\alpha-1)\varepsilon\theta, \tag{25}$$

respectively. In Eqs. (24) and (25) $\alpha(t)$, $\theta(\{\mathbf{r}\},R(\Lambda(t)))$, and $\phi_n(\{\mathbf{r}\},R(\Lambda(t)))$ are abbreviated by α, θ, and ϕ_n, respectively. In Eq. (24) we omit the terms which are space independent because they concern only the space-independent phase. $\varepsilon\alpha$ is tuned to be zero at the initial and final time of the acceleration so that the additional phase in Eq. (25) vanishes. It should be emphasized that the driving potential should be one-particle potential, because we cannot control general many-body potentials. Thus, the driving potential should be written as Eq. (11). Moreover, the additional phase must not change the statistics of the system. In general, the driving potential in Eq. (24) cannot be represented as Eq. (11) because it can contain an unrealizable many-body potential. However, in some specific but important manipulations, the driving potential becomes a one-particle potential. Such manipulations are exhibited in the following section.

III. APPLICATIONS

A. Acceleration of adiabatic transport of identical particles

Recently, several experimental investigations have been devoted to fast-atomic transport [22,23]. Theoretical investigations on the acceleration of adiabatic transport avoiding the energy excitation at the final time of the manipulation have been reported with the use of fast-forward scaling theory [7,8] and inverse engineering technique based on Lewis-Riesenfeld invariants [16,18,24]. These protocols can derive the same driving potential for transport of a particle. (The relation between fast-forward scaling theory and inverse engineering technique based on Lewis-Riesenfeld invariants was clarified in [25].) However, the range of applications of the theoretical protocols is still limited in the control of one-body systems or in the cases where the many-body state is governed by a time-dependent Gross-Pitaevskii (GP) equation. Here we consider more general many-body systems consisting of identical particles interacting with each other and show the rapid transport without energy excitation at the final time of the control by using the result of the previous section.

We consider adiabatic transport of identical particles interacting with each other in a trapping potential. The particles are conveyed into the x direction by a driving potential. We assume that the system is in the nth energy eigenstate of the instantaneous Hamiltonian. ϕ_n in Eq. (16) is represented with ϕ_n' as

$$\phi_n(\{\mathbf{r}\},R(t)) = \phi_n'(\{x - R(t),y,z\}), \tag{26}$$

where $\phi_n'(\{\mathbf{r}\})$ is a stationary wave function of the nth energy eigenstate trapped by a stationary potential $V_e'(\{\mathbf{r}\})$. External

potential V_e in Eq. (17) is written by V_e' as

$$V_e(\{\mathbf{r}\},R(t)) = V_e'(\{x - R(t),y,z\}). \tag{27}$$

The fast-forwarded state is defined by Eq. (21) with ϕ_n in Eq. (26). The phase $\theta(\{\mathbf{r}\},t)$ of the wave function in Eq. (16) is a solution of Eq. (19). Noting that

$$\frac{\partial \phi_n}{\partial R} = -\sum_{j=1}^{N} \frac{\partial \phi_n}{\partial x_j}, \tag{28}$$

we obtain the solution as

$$\theta = \sum_{j=1}^{N} \frac{m}{\hbar} x_j. \tag{29}$$

On the other hand, it turns out that $\tilde{V}$ in Eq. (20) vanishes everywhere. Substituting Eq. (29) into Eq. (24) we obtain the driving potential as

$$\mathcal{V}_{FF} = -\frac{d\alpha}{dt}\varepsilon m \sum_{j=1}^{N} x_j. \tag{30}$$

Therefore, by applying the driving potential,

$$v_{FF}(\mathbf{r}) = -\frac{d\alpha}{dt}\varepsilon m x, \tag{31}$$

with the translation of the original trapping potential as Eq. (27) we can accelerate trapped particles without energy excitation at the final time of the transport. The driving potential does not depend on the detail of the interaction among particles. The wave function of fast-forwarded state has the additional phase in Eq. (25) during the transport. The additional phase does not change the statistics of the system because θ is given by Eq. (29). $\alpha\varepsilon$ is chosen to be zero at the initial and final time of the transport so that the additional phase vanishes and the fast-forwarded state coincides with the target state. The distance of the transport is given by $\varepsilon \int_0^{T_F} \alpha(t)dt$. Moreover, although we have considered energy eigenstates so far, it turns out from Eq. (31) that the driving potential can transport not only energy eigenstates but also general states which is a superposition of energy eigenstates because the driving potential does not depend on energy levels. The form of the driving potential in Eq. (31) is identical to one for the transport of one particle or a wave packet of BEC [7,16]. Our result insists that, even in the case with interaction among the particles, we can transport them by tuning a spatially linear potential accompanied with the translation of the original trapping potential in Eq. (27) without inconvenient energy excitation and disturbance of the quantum system.

B. Rapid manipulation of dilute Bose gas in ground state

We propose an ideal rapid manipulation of dilute Bose gas in the ground state by using fast-forward scaling theory. Let us suppose that a system is composed of N identical Bose particles with mass m. We assume that the wave function of the ground state is represented as

$$\phi_0(\{\mathbf{r}\},R) = \varphi(\mathbf{r}_1,R)\varphi(\mathbf{r}_2,R)\cdots\varphi(\mathbf{r}_N,R) \tag{32}$$

for an instantaneous Hamiltonian $H(R)$, where φ is a one-particle wave function parametrized by R (mean-field approximation [26]). In general φ is not the wave function of the ground state of an noninteracting boson. We consider acceleration of the adiabatic dynamics of the Bose gas. Naive controls of external field without fast-forward theory would make the state much more complex than that in Eq. (32) with energy excitation. By using fast-forward theory we can transform the state from $\phi_0(\{\mathbf{r}\},R_0)$ to $\phi_0(\{\mathbf{r}\},R_1)$ in any short time, where R_0 and R_1 are the initial and final values of the adiabatic parameter.

Equation (19) is rewritten by using Eq. (32) as

$$\sum_{j=1}^{N}\left\{\frac{\hbar}{2m}\left(2\nabla_j\theta\cdot\mathrm{Re}[\nabla_j\varphi_j/\varphi_j] + \nabla_j^2\theta\right) + \mathrm{Re}\left[\frac{\partial\varphi_j}{\partial R}\Big/\varphi_j\right]\right\} = 0, \tag{33}$$

where φ_j denotes $\varphi(\mathbf{r}_j,R)$. In the derivation of Eq. (33) we used

$$\frac{\partial\phi_0}{\partial R}\Big/\phi_0 = \sum_{j=1}^{N}\frac{\partial\varphi_j}{\partial R}\Big/\varphi_j, \tag{34}$$

$$\nabla_j\phi_0/\phi_0 = \nabla_j\varphi_j/\varphi_j. \tag{35}$$

We assume the form of θ as

$$\theta = \sum_{j=1}^{N} a(\mathbf{r}_j,R), \tag{36}$$

where $a(\mathbf{r}_j,R)$ is real. Substituting Eq. (36) in Eq. (33) it turns out that a_j is obtained by solving

$$\frac{\hbar}{2m}\left(2\nabla_j a_j\cdot\mathrm{Re}[\nabla_j\varphi_j/\varphi_j] + \nabla_j^2 a_j\right) + \mathrm{Re}\left[\frac{\partial\varphi_j}{\partial R}\Big/\varphi_j\right] = 0. \tag{37}$$

Let us suppose that $\xi(\mathbf{r},R)$ is the phase of $\varphi(\mathbf{r},R)$. Then the phase $\eta(\{\mathbf{r}\},R)$ of $\phi_0(\{\mathbf{r}\},R)$ is written as

$$\eta(\{\mathbf{r}\},R) = \sum_{j=1}^{N}\xi(\mathbf{r}_j,R). \tag{38}$$

By using Eqs. (34)–(38) in Eq. (24), we obtain the driving potential as

$$\frac{V_{FF}}{\hbar} = \sum_{j=1}^{N}\frac{v_{FF}(\mathbf{r}_j,t)}{\hbar}, \tag{39}$$

with

$$\frac{v_{FF}}{\hbar}(\mathbf{r}_j,t) = -\frac{\partial\alpha}{\partial t}\varepsilon a_j - \alpha^2\varepsilon^2\frac{\partial a_j}{\partial R} - \alpha\varepsilon\mathrm{Im}\left[\frac{\partial\varphi_j}{\partial R}\Big/\varphi_j\right] - \frac{\hbar}{2m}(2\alpha\varepsilon\nabla_j a_j\cdot\nabla_j\xi_j + \alpha^2\varepsilon^2(\nabla_j a_j)^2). \tag{40}$$

In Eq. (40) α, a_j, φ_j, and ξ_j stand for $\alpha(t)$, $a(\mathbf{r}_j,R(\Lambda(t)))$, $\varphi(\mathbf{r}_j,R(\Lambda(t)))$, and $\xi(\mathbf{r}_j,R(\Lambda(t)))$, respectively. The driving potential in Eq. (39) is composed of one-body operators. These results insist that if the ground state of dilute Bose gas is represented by Eq. (32), we can transform the Bose gas from a ground state to another in any desired short time.

The form of the driving potential coincides with one obtained under the assumption that the dynamics of the dilute Bose gas is governed by the time-dependent GP equation [7].

However, in general, it is not guaranteed that the dynamics of the dilute Bose gas is well described by time-dependent GP equation in the case that the external potential is very rapidly changed, because rapid controls of external field can make the states much more complex than states represented by Eq. (32). Our theory insists that the existence of the ground state expressed by Eq. (32) is essential for the derivation of the driving potential and the assumption of the validity of the time-dependent GP equation is not necessary. The resultant driving potential in Eq. (40) can transform the state from one ground state to another rapidly.

1. Control of wave function with spherical symmetry

We show the way to obtain the driving potential for the case of spherically symmetric wave functions. We assume that the wave function is spherically symmetric in the adiabatic dynamics. $\varphi_j = \varphi(r_j, R)$ in Eq. (32) is represented as a function of $r_j = |\mathbf{r}_j|$ and R as

$$\varphi(r_j, R) = \tilde{\varphi}(r_j, R) e^{i\eta(r_j, R)}, \tag{41}$$

where $\tilde{\varphi}$ and η, which are real functions of r_j and R, are the amplitude and phase of φ_j, respectively. Now we derive a driving potential which transforms the system from $\phi_0(\{\mathbf{r}\}, R_0)$ to $\phi_0(\{\mathbf{r}\}, R_1)$. By using Eq. (41), Eq. (37) is rewritten as

$$\nabla \cdot (\tilde{\varphi}^2(r, R) \nabla a(r, R)) = -g(r, R), \tag{42}$$

where

$$g(r, R) \equiv \frac{2m}{\hbar} \tilde{\varphi}(r, R) \frac{\partial \tilde{\varphi}(r, R)}{\partial R}. \tag{43}$$

In the above equations we omit the subscript j of r_j for the simplicity of the notation. We assume the spherical symmetry of $a(r, R)$ and define a vector $\mathbf{E}(r, R)$ as

$$\mathbf{E}(r, R) \equiv \tilde{\varphi}^2(r, R) \nabla a(r, R) = \tilde{\varphi}^2(r, R) \frac{\partial a(r, R)}{\partial r} \mathbf{e}_r. \tag{44}$$

$\mathbf{e}_r$ is a radial unit vector. We consider the volume integral of Eq. (42) in a sphere with radius r as

$$\begin{aligned} \int \nabla \cdot \mathbf{E}(r, R) dV &= -\int g(r, R) dV \\ &= -\int_0^r 4\pi r'^2 g(r', R) dr'. \end{aligned} \tag{45}$$

With the use of Gauss's theorem $\int \nabla \cdot \mathbf{E} dV = \int \mathbf{E} \cdot d\mathbf{S}$, Eq. (45) leads to

$$|\mathbf{E}(r, R)| = E(r, R) = -\frac{1}{r^2} \int_0^r r'^2 g(r', R) dr'. \tag{46}$$

Combining Eqs. (44) and (46) we have

$$\frac{\partial a}{\partial r} = -\frac{1}{r^2 \tilde{\varphi}^2(r, R)} \int_0^r \frac{2m}{\hbar} r'^2 \tilde{\varphi}(r', R) \frac{\partial \tilde{\varphi}(r', R)}{\partial R} dr'. \tag{47}$$

a is obtained by integrating Eq. (47): $a(r, R) = a(r = 0, R) + \int_0^r [\partial a(r', R)/\partial r'] dr'$ for each R. It is seen that there is no singularity in $\partial_r a$ in Eq. (47) at $r = 0$ by substituting $\tilde{\varphi}(r, R) = \tilde{\varphi}(0, R) + \frac{\partial \tilde{\varphi}}{\partial r}(0, R) r + \cdots$ in the right-hand side. The value of a at the origin, $a(r = 0, R)$, is arbitrary because it is concerned only with the space-independent phase. Substituting $a(r, R)$ into Eq. (40) we can obtain the driving potential for the acceleration of adiabatic dynamics in which the wave function has spherical symmetry. We should choose $\alpha(t)$ so that α relates initial (R_0) and final (R_1) values of the adiabatic parameter through Eq. (23). Time dependence of $\varepsilon\alpha(t)$ determines the final time T_F through Eq. (23) for given R_0 and R_1. Note that although the time dependence of $\varepsilon\alpha(t)$ is arbitrary if Eq. (23) is satisfied, the driving potential depends on $\varepsilon\alpha(t)$. In the above manner we can obtain the driving

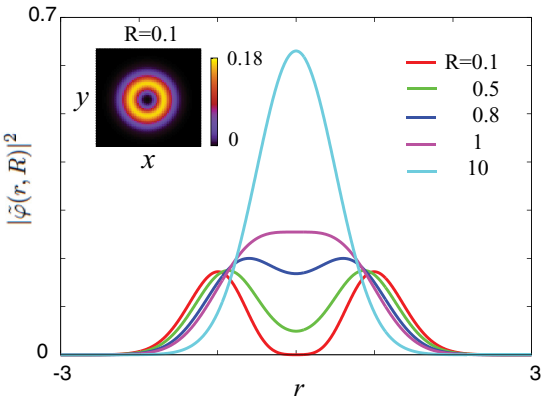

FIG. 1. (Color online) r dependence of $|\tilde{\varphi}(r, R)|^2$ for $R = 0.1, 0.5, 0.8, 1, 10\,(\times L)$ (curves shown from lowest to highest) and $A = 1(\times L^{-2})$, where L is a typical space scale like $L = 10^{-2}\times$ the linear dimension of a device. The inset shows profile of $|\tilde{\varphi}(r, R)|^2$ in xy plane for $R = 0.1$.

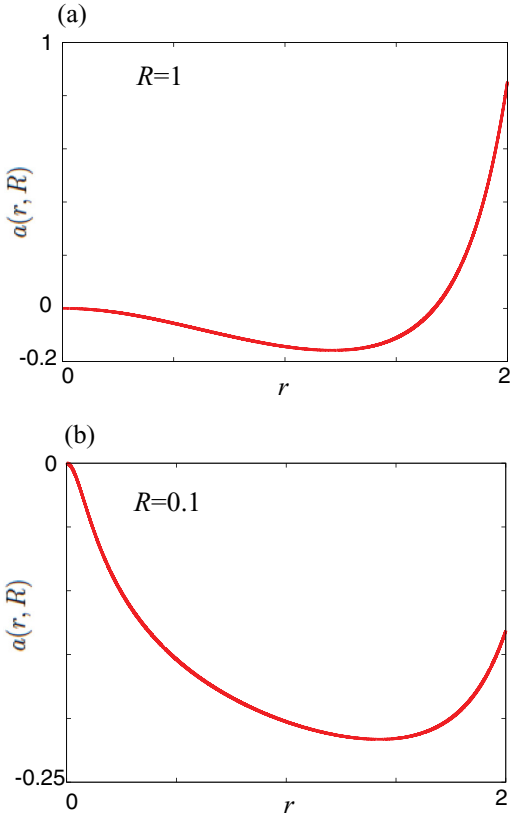

FIG. 2. (Color online) r dependence of $a(r, R)$ for (a) $R = 1$ and (b) $R = 0.1$ with $A = 1$. $a(r, R)$ is scaled by $L^{-1}\tau$, where L and τ are typical space and time scales like $L = 10^{-2}\times$ the linear dimension of a device and $\tau = 10^{-2}\times$ the phase coherent time, respectively.

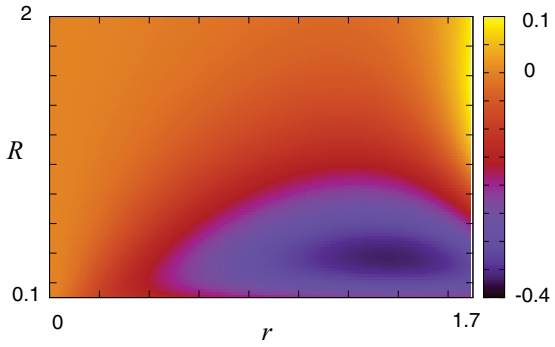

FIG. 3. (Color online) $r - R$ dependence of $a(r,R)$ for $0 < R < 2$, $0 < r < 1.7$, and $A = 1$. The color indicates $a(r,R)$.

potential which generates $\phi_0(\{\mathbf{r}\},R_1)$ from $\phi_0(\{\mathbf{r}\},R_0)$ in short time T_F when we have the wave function of the ground state.

As an example we show a behavior of function $a(r,R)$ in two dimensions for a particular $\tilde{\varphi}(r,R)$:

$$\tilde{\varphi}(r,R) = C(R)e^{-Ar^2}(r^2 + R^2), \tag{48}$$

where $r = \sqrt{x^2 + y^2}$ and $C(R)$ is a normalization factor given by

$$C(R) = \left\{\pi\left(\frac{1}{4A^3} + \frac{R^2}{2A^2} + \frac{R^4}{2A}\right)\right\}^{-1/2}. \tag{49}$$

We assume that the wave function of the ground state is represented by Eq. (32) with $\tilde{\varphi}$ in Eq. (48). With use of a typical space scale like $L = 10^{-2}\times$ the linear dimension of a device, we take $A = 1(\times L^{-2})$. Figure 1 shows r dependence of $|\tilde{\varphi}(r,R)|^2$ for various values of R. The inset shows a profile of $|\tilde{\varphi}(r,R)|^2$ in xy plane for $R = 0.1(\times L)$. $\tilde{\varphi}(r,R)$ is well approximated by a Gaussian function for $R > 10$. $\tilde{\varphi}(r,R)$ decreases around the origin with the decrease of R as seen in Fig. 1. We calculate r dependence of $a(r,R)$ by using Eq. (47) with a boundary condition $a(0,R) = 0$ for $R = 1$ and $R = 0.1$ as Figs. 2(a) and 2(b), respectively. $a(r,R)$ is scaled by $L^{-1}\tau$, where τ is a typical time scale like $\tau = 10^{-2}\times$ the phase coherent time. A behavior of $a(r,R)$ in rR space is exhibited in Fig. 3 for $0 < R < 2$ and $0 < r < 1.7\,(\times L)$. The driving potential can sharply increase for $r > 2$ because $a(r,R)$ increases sharply with r. In actual manipulations it may make the control of a driving potential difficult. However, it is expected that the influence of a deviation of a driving potential from the exact one at $r > 2$ does not cause a crucial problem because in this region the wave function is very small, as seen in Fig. 1.

IV. CONCLUSION

We have presented the acceleration of adiabatic dynamics of quantum systems composed of identical spinless particles interacting with each other. We have derived a driving potential, which accelerates the adiabatic dynamics, by extending the preceding fast-forward scaling theory formed for single-particle systems. The driving potential produces the final state of an original adiabatic dynamics from the initial state of the adiabatic dynamics in any desired short time. There is no energy excitation at the final time of the manipulation. Although the driving potential is a many-body potential in general, it becomes a one-body potential for particular manipulations: transport of interacting identical particles and control of dilute Bose gas in the ground state.

We have showed a driving potential for acceleration of adiabatic transport of interacting particles. The driving potential transports particles without energy excitation. The driving potential does not depend on the detail of the interaction among particles. It was also proved that the driving potential conveys not only energy eigenstates but also general states, which is a superposition of energy eigenstates because the driving potential does not depend on energy levels. Interestingly, the form of the driving potential is identical to one for one particle or a wave packet of BEC [7,16]. We can transport interacting particles by tuning the spatially linear potential combining with the translation of the original trapping potential even in the case that there is interaction among particles without inconvenient energy excitation at the final time of the manipulation.

We have also showed an ideal rapid manipulation of a dilute Bose gas in the ground state. We have derived a formula of the driving potential which accelerates adiabatic dynamics of the Bose gas with the mean-field approximation as Eq. (32). Naive rapid control of external fields without fast-forward theory would deform the states into more complex features. However, our results insist that we can transform a dilute Bose gas from an initial ground state to another without leaving disturbances in the wave function in any short time if the ground state is well represented by the mean-field approximation. As an example, we have applied our theory to the case that the wave function has spherical symmetry. The way to derive the driving potential was shown explicitly.

In actual experiments we cannot tune exactly the driving potential. In general, the deviation of the potential from exact driving potential can cause inconvenient disturbance or energy excitation on the quantum systems. In this paper the stability of the acceleration against the noise or deformation of the potential is not discussed as investigated for one-body system or a BEC wave packet [18,24]. It is expected that the stability of this protocol will be investigated in the future.

ACKNOWLEDGMENTS

We thank global COE program "Weaving Science Web Beyond Particle-Matter Hierarchy" for its financial support. We thank K. Nakamura for useful discussions and comments.

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