Covariant photon wave mechanics of evanescent fields

Ole Keller and Dann S. Olesen
Institute of Physics and Nanotechnology, Aalborg University, DK-9220 Aalborg Øst, Denmark
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A photon wave mechanical theory describing evanescent electromagnetic fields, as these appear in p-polarized total internal reflection from a flat dielectric-to-vacuum interface, is presented. In this first-quantized field theory, the Lorenz gauge four-potential relates to the wave functions of the transverse (T), longitudinal (L), and scalar (S) photons. For a homogeneous medium, the source domain (rim zone) of the L photons is shown to coincide with the longitudinal vector-field parts of an electron sheet current density located at the interface. The identical rim zones of the T and L photons exhibit one-dimensional (1D) exponential confinement with a spatial decay constant equal to the magnitude of the incident field's wave vector along the interface plane. The S-photon 1D localization is complete (has δ-function character). Dynamical equations are established for the L- and S-photon variables in the wave-vector–time domain. These Hamiltonian-like equations are easily upgraded to the second-quantized level. The link to the wave mechanics of near-field photons is made. When extended to the quantum electrodynamic level, the present theory is an alternative to the pioneering Carniglia-Mandel (CM) triplet-photon description. It is argued that our theory provides one with an improved physical understanding of the basic role of T photons in evanescent fields. Wave mechanics of evanescent fields give additional insight in the T-photon localizability problem. The particular difficulties arising for s-polarized external fields are addressed referring to the rim zone of quantum wells.

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I. INTRODUCTION

The evanescent field concept originates in a particular mode representation of the spatial part of a monochromatic (ω) free electromagnetic field. An introduction to this so-called angular spectrum representation can be found in various books, e.g., in [1–4]. The concept may easily be extended to media regarding which linear optical properties, to a good approximation, can be characterized by a real dielectric constant, $\varepsilon(\omega)$. This is not surprising since such ideal model media electromagnetically behave like a vacuum, just with renormalized speed of light, $c/\varepsilon^{1/2}(\omega)$. In the angular spectrum representation, a two-dimensional plane-wave (wave vector $\mathbf{q}_\parallel$) expansion of the field is made, and the resulting spatial mode spectrum consists of waves that propagate without damping $[q_\parallel < (\omega/c)\varepsilon^{1/2}(\omega)]$ and decay exponentially $[q_\parallel > (\omega/c)\varepsilon^{1/2}(\omega)]$ perpendicular to the plane spanned by the $\mathbf{q}_\parallel$ vectors. The last type are known as evanescent waves. Over the years evanescent fields have proved their importance in studies of, e.g., (i) diffraction, image formation, and spatial resolution problems in classical optics; (ii) propagation of electromagnetic surface and interface waves; (iii) surface-dipole interactions; (iv) phase conjugation; (v) spatial dispersion; (vi) total internal reflection; and (vii) optical tunneling. In recent years, numerous studies in the fields broadly characterized as near-field optics, nano-optics, and plasmonics, have invoked the evanescent field concept in one way or another. Many references to the above-mentioned applications of evanescent fields can be found in [1–6]. Readers interested in the history of optics may notice that evanescent fields have played a prominent role in the development of near-field optics [7] and optical tunneling [8,9]. The earliest known observation of light tunneling (between two glass prisms) is due to Newton [10].

Evanescent fields are perhaps most familiar in connection to total internal reflection (TIR) of light at a dielectric-to-vacuum interface, and it is this configuration which will be in focus in our covariant photon wave mechanical description. The first quantitative experimental studies were carried out by Quincke in 1866 [11] and Hall in 1902 [12]. Although quantum electrodynamics entered physics in the period 1925–1927 [13,14], in all theoretical studies of evanescent fields up to 1971 the electromagnetic field was treated as a classical quantity. In that year, Carniglia and Mandel [15] presented for the TIR configuration a quantization scheme for fields having evanescent parts. Related work was published by Agarwal [16] in 1975. Today, the pioneering work of Carniglia and Mandel stands as *the* theory for quantization of evanescent fields.

Notwithstanding the pioneering status of the Carniglia-Mandel paper, it appears that it leaves us with a number of unanswered physical questions and that the scheme used may not easily (cannot, perhaps) be generalized beyond its present scope. Furthermore, the Carniglia-Mandel approach has apparently not been applied in studies in near-field optics, nano-optics, and plasmonics, despite the fact that quantum optical experiments start to emerge in these rapidly evolving fields. The reason for this might be due to the rather complicated expressions one reaches for important quantities using their approach.

Partly motivated by recent developments in the theory of covariant photon wave mechanics (based on the four-vector potential) and this theory's extension to the quantum electrodynamic level, *and* the insight these developments have given us in fundamental near-field electrodynamics [4,17], we here present a covariant photon wave mechanical (first-quantized) description of the evanescent field in the TIR configuration. In a forthcoming paper the theory's extension to the second-quantized level is established. It seems that the covariant quantization scheme for evanescent fields also provides us with a better understanding of the tunneling process for single-photon wave packets, as this appears in the frustrated total internal reflection (FTIR) configuration [17].

Furthermore, the scheme is quite easily brought in contact with single-photon tunneling studied on the basis of a Green's function formalism for the transverse and longitudinal parts of the electric field [18]. The fundamental difference between the Carniglia-Mandel (CM) theory and ours can be highlighted by the following three [I-III] introductory remarks.

(I) In the CM approach, the medium has a space-independent (bulk) refractive index, $n(\mathbf{r}) = n_0$, and the authors claim that therefore there are no sources for the photons in the medium half space, and that the electromagnetic field hence effectively is a free field. Although we also adopt the $n(\mathbf{r}) = n_0$ approximation, we show that there always is an electronic source present but that it is located in the interface (surface) region. Since the thickness of this interface layer is much smaller than relevant optical wavelengths, it appears that the entire TIR problem becomes equivalent to a much simpler problem in which the incident field excites a current density sheet (located at the interface). Subsequently, the sheet current gives rise to the emergence of the reflected evanescent field. We can go beyond the $n(\mathbf{r}) = n_0$ model and treat inhomogeneous media, because the covariant approach operates with transverse (T, two types), longitudinal (L), and scalar (S) photons. The L and S photons are only present if the refractive index is space dependent, and they always are source-attached photons (virtual in a certain sense). In our approach the Carniglia-Mandel triplet-photon picture is *not* used.

(II) Within the framework of the triplet-photon model, Carniglia and Mandel establish [what they themselves (and we) characterize as] rather complicated expressions for the electric field commutator at two space-time points in the vacuum half space, in usual notation $[\hat{\mathbf{E}}(\mathbf{r},t),\hat{\mathbf{E}}(\mathbf{r}',t')]$. They find that the term in the commutator which refers to evanescent waves is nonzero off the light cone connecting two events at $\mathbf{r},t$ and $\mathbf{r}',t'$. Carniglia and Mandel claim that this spacelike lack of Einsteinian causality originates in the fact that the frequency dependence of the dielectric constant in the region of anomalous dispersion was neglected. It is correct that neglect of the Hilbert transform coupling between the real and imaginary parts of $\varepsilon(\omega)$ in itself will break the strict Einsteinian causality, but we do not believe that this fact is the main reason for the appearance of spacelike coupling in the evanescent tail. As argued previously on general grounds [4] and discussed for evanescent fields in Secs. II C and III A, the spacelike coupling appearing in the analysis of evanescent fields originates in the inability to localize transverse photons beyond the spatial extension of the longitudinal part of the sheet current density. No break of Einsteinian causality is associated with this interpretation [4,17].

(III) In part VII of the CM paper, Mandel's earlier-introduced configuration-space photon absorption operator $\hat{\mathbf{V}}(\mathbf{r},t)$ [19] is extended to cover the triplet mode case. In the Mandel theory, the integral of $\hat{\mathbf{V}}^{\dagger}(\mathbf{r},t) \cdot \hat{\mathbf{V}}(\mathbf{r},t)$ over a finite volume (V) *of linear dimensions, large compared with the wavelengths of all modes contributing to* $\hat{\mathbf{V}}(\mathbf{r},t)$, is given physical significance, representing the configuration-space photon number operator for the volume V. In the case of evanescent fields, where the tail field often is much smaller than the typical wavelengths, the Mandel theory is insufficient for a fundamental understanding in our opinion. In fact, this insufficiency already appears indirectly from the conclusion of the CM paper. Thus, after having obtained the integral of $\hat{\mathbf{V}}^{\dagger}(\mathbf{r},t) \cdot \hat{\mathbf{V}}(\mathbf{r},t)$ over V, Carniglia and Mandel conclude that this (cit.) "integral over a finite volume is less readily interpreted." In our fundamental four-potential description of evanescent fields, this kind of T-photon localization problem does not appear (cf. [4]).

After a brief review of the standard theory of p-polarized TIR, supplemented by a calculation and discussion of the transverse and longitudinal parts of the transmitted electric field and the associated parts of the field momentum (densities), we determine the source domain (rim zone) of the T photons, starting from the microscopic Maxwell-Lorentz equations. The photon wave mechanical description of evanescent fields in the Lorenz gauge is established next. Special emphasis is devoted to an analysis of the scalar and longitudinal photon dynamics, and dynamical equations are set up for these two photon types in the wave-vector−time domain. The Hamiltonian form of these equations readily allows one to extend the approach to the second-quantized level. Previously, one of the present authors (O.K.) has introduced a near-field photon concept [20]. We establish the dynamical equation for the near-field photon variable and show that a divergence, present in the T and L dynamics when the photon wave number coincides with the vacuum wave number ω/c, disappears in the near-field photon description. Finally, remarks on the s-polarized case are given on the basis of a quantum-well picture of the surface response.

II. EVANESCENT FIELDS: TRANSVERSE AND LONGITUDINAL ELECTRODYNAMICS

A. Total internal reflection of p-polarized light

We begin with a summary of some key formulas describing the total internal reflection of a monochromatic plane electromagnetic wave from a flat interface between the vacuum and a homogeneous medium with a real relative dielectric constant ε (>1). Although assuming that ε is real and the medium homogeneous are approximations, these will do for our purposes. We focus on the case where the electric fields are polarized in the scattering plane (p polarization), which we take as the xz plane in a Cartesian coordinate system where the z axis is directed perpendicular to the interface. Remarks on the s-polarized case are given in Sec. IV. The scattering geometry (wave-vector diagram) is shown in Fig. 1 for an angle of incidence larger than the critical angle. In complex notation all fields have the common form

$$F(\mathbf{r},t) = F(z;\mathbf{q}_{\parallel},\omega)e^{i(\mathbf{q}_{\parallel}\cdot\mathbf{r}-\omega t)}, \tag{1}$$

where ω and $\mathbf{q}_{\parallel}$ are the angular frequency and the vectorial wave-vector component parallel ($\parallel$) to the interface. It is the system's translational invariance along the boundary plane which makes $\mathbf{q}_{\parallel}$ common to all the fields. The z dependence of the incident (i) and reflected (r) electric fields are given by

$$\mathbf{E}_i(z;\mathbf{q}_{\parallel},\omega) = \mathbf{E}_i^0(\mathbf{q}_{\parallel},\omega)e^{iq_{\perp}z}, \tag{2}$$

$$\mathbf{E}_r(z;\mathbf{q}_{\parallel},\omega) = \mathbf{E}_r^0(\mathbf{q}_{\parallel},\omega)e^{-iq_{\perp}z}, \tag{3}$$

where

$$q_{\perp} = \left[\left(\frac{\omega}{c}\right)^2 \varepsilon - q_{\parallel}^2\right]^{\frac{1}{2}} \tag{4}$$

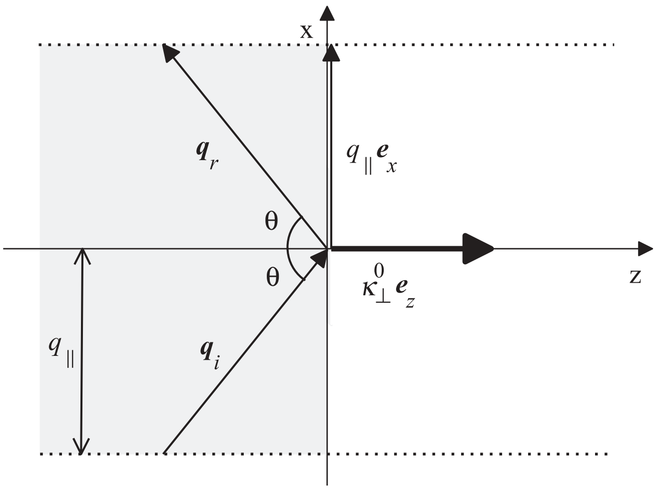

FIG. 1. Wave-vector diagram in TIR. The wave vectors of the incident ($\mathbf{q}_i$) and reflected ($\mathbf{q}_r$) homogeneous waves are real, have equal components ($q_\| \mathbf{e}_x$) parallel to the interface plane (xy plane, here), and the same magnitude ($|\mathbf{q}_i| = |\mathbf{q}_r|$). The wave vector of the transmitted inhomogeneous wave is complex, with a real part ($q_\| \mathbf{e}_x$) along the interface and an imaginary part ($\kappa_\perp^0 \mathbf{e}_z$) normal to it. In TIR, $\omega/c < q_\| < (\omega/c)\varepsilon^{1/2}$.

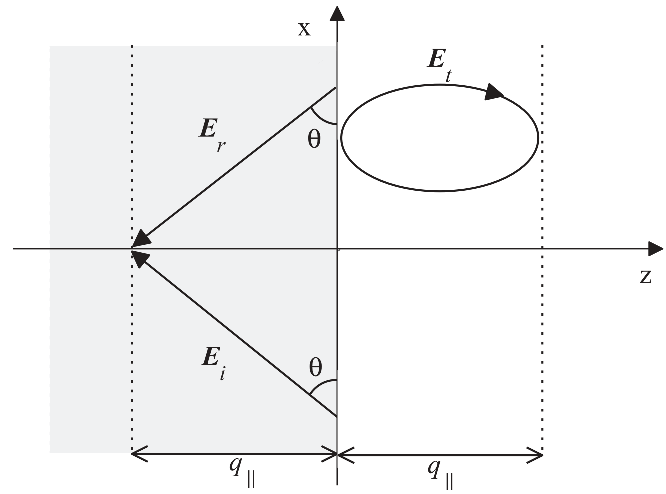

FIG. 2. Diagram indicating the linear polarization states of the incident ($\mathbf{E}_i$) and reflected ($\mathbf{E}_r$) electric fields and the clockwise elliptical polarization of the transmitted electric field ($\mathbf{E}_t$) in TIR. The polarization states are normalized so that they have the same z component, $q_\|$ [corresponding to a $\pi/2$ rotation of the related (complex) wave vectors].

is the component of the incident wave vector perpendicular ($\perp$) to the interface, c being the speed of light in vacuum. The z dependence of the transmitted (t) electric field has the form

$$\mathbf{E}_t(z;\mathbf{q}_\|,\omega) = \mathbf{E}_t^0(\mathbf{q}_\|,\omega)e^{iq_\perp^0 z}, \tag{5}$$

$$q_\perp^0 = \left[\left(\frac{\omega}{c}\right)^2 - q_\|^2\right]^{\frac{1}{2}} \tag{6}$$

being the z component of the wave vector of the transmitted field.

To determine the vectorial amplitudes of the reflected ($\mathbf{E}_r^0$) and transmitted ($\mathbf{E}_t^0$) fields for a given incident field amplitude ($\mathbf{E}_i^0$) one needs an appropriate set of boundary conditions. In general, microscopic surface current and charge densities are created in the scattering process and the choice of "correct" boundary conditions requires a study of a complicated self-consistency problem [21,22]. For the present purpose (where ε is assumed to be real), we may assume that the surface is passive and hence use the standard boundary conditions for nonmagnetic media: the electric and magnetic fields parallel to the surface are continuous. With the sign convention indicated in Fig. 2, these conditions give $E_{i,x}^0 + E_{r,x}^0 = E_{t,x}^0$ and $(E_{i,x}^0 - E_{r,x}^0)\varepsilon = (q_\perp/q_\perp^0)E_{t,x}^0$, respectively. In obtaining the last relation the Maxwell equation $\mathbf{B} = (i\omega)^{-1}\nabla \times \mathbf{E}$ was used to eliminate $\mathbf{B}$ in favor of $\mathbf{E}$. By eliminating $E_{r,x}^0$ between these equations, one obtains $E_{t,x}^0 = [2\varepsilon q_\perp^0/(q_\perp + \varepsilon q_\perp^0)]E_{i,x}^0$, and with $E_{t,z}^0 = (-q_\|/q_\perp^0)E_{t,x}^0$ the transmitted electric field becomes

$$\mathbf{E}_t(x,z;\omega) = \frac{2\varepsilon}{q_\perp + \varepsilon q_\perp^0}(q_\perp^0 \mathbf{e}_x - q_\| \mathbf{e}_z)E_{i,x}^0 e^{i(q_\| x + q_\perp^0 z)} \tag{7}$$

in the space-frequency domain. The quantity $E_{i,x}^0$ is the x component of $\mathbf{E}_i^0$. Unit vectors along the Cartesian axes are denoted by $\mathbf{e}_\alpha$, $\alpha = x,y,z$. Using the Maxwell equation $\nabla \times \mathbf{E} = i\omega\mathbf{B}$, the transmitted magnetic field ($\mathbf{B}_t$) is readily determined from Eq. (7). Thus,

$$\mathbf{B}_t(x,z;\omega) = \frac{2\varepsilon\omega c^{-2}}{q_\perp + \varepsilon q_\perp^0}\mathbf{e}_y E_{i,x}^0 e^{i(q_\| x + q_\perp^0 z)}. \tag{8}$$

Being interested in understanding the role of the photons in evanescent fields, we turn our attention towards angles of incidence above the critical angle where $q_\| > \omega/c$ [and, of course, $q_\| < (\omega/c)\varepsilon^{1/2}$]. In the evanescent regime the transmitted wave becomes inhomogeneous (Fig. 1): it propagates without damping along the surface and decays exponentially away from the surface. The complex wave vector thus is given by $q_\| \mathbf{e}_x + i\kappa_\perp^0 \mathbf{e}_z$, where

$$\kappa_\perp^0 = \left[q_\|^2 - \left(\frac{\omega}{c}\right)^2\right]^{\frac{1}{2}} \tag{9}$$

is the decay constant in the z direction ($\kappa_\perp^0 > 0$). The evanescent fields are obtained by making the substitution $q_\perp^0 \to i\kappa_\perp^0$ in Eqs. (7) and (8). Hence,

$$\mathbf{E}_t(x,z;\omega) = \frac{2\varepsilon}{q_\perp + i\varepsilon\kappa_\perp^0}(i\kappa_\perp^0 \mathbf{e}_x - q_\| \mathbf{e}_z)E_{i,x}^0 e^{iq_\| x}e^{-\kappa_\perp^0 z}, \tag{10}$$

$$\mathbf{B}_t(x,z;\omega) = \frac{2\varepsilon\omega c^{-2}}{q_\perp + i\varepsilon\kappa_\perp^0}\mathbf{e}_y E_{i,x}^0 e^{iq_\| x}e^{-\kappa_\perp^0 z}. \tag{11}$$

As indicated in Fig. 2, the transmitted electric field is elliptically polarized. The reader may recall [or prove by means of Eqs. (10) and (11)] that the z component of the cycle-averaged ($\langle\cdots\rangle$) transmitted Poynting vector $\mathbf{S}_t$ vanishes in the evanescent regime, i.e.,

$$\mathbf{e}_z\cdot\langle\mathbf{S}_t\rangle(z;\mathbf{q}_\|,\omega) = \frac{1}{2\mu_0}\mathbf{e}_z \cdot \mathrm{Re}[\mathbf{E}_t(x,z;\omega) \times \mathbf{B}_t^*(x,z;\omega)] = 0,$$
$$q_\| > \frac{\omega}{c}, \tag{12}$$

where Re means the real part of the subsequent quantity. Therefore, in the evanescent tail there is no net energy flow perpendicular to the surface.

B. Transverse and longitudinal fields

It is known that classical electrodynamics can be reformulated in such a manner that it appears as a first-quantized theory for photons [23,24]. When reinterpreted properly, the Maxwell-Lorentz theory is called photon wave mechanics, and upon second quantization one obtains the standard theory for photons [4,7,23,24]. We consequently expect that traces of photon dynamics in evanescent fields may emerge upon a deeper analysis of the standard theory for TIR.

In a sense the photon concept belongs to free space, and the light quanta here are called transverse photons (T photons). In the evanescent tail there is no (charged) matter, and at first sight one might expect the presence of free T photons here. Although this expectation turns out to be incorrect, let us see where it leads to. T photons always belong solely to the divergence-free (here called transverse) part of the electromagnetic field and they therefore propagate with the vacuum speed of light [4]. The electric field in Eq. (10) has both divergence-free [transverse (T)] [$\mathbf{E}_t^T$] and rotational-free [longitudinal (L)] [$\mathbf{E}_t^L$] parts. The longitudinal part of $\mathbf{E}_t$ is obtained letting $c \to \infty$, formally [4]. Hence,

$$\mathbf{E}_t^L(x,z;\omega) = \mathbf{E}_t(x,z;\omega|c \to \infty) = \frac{2\varepsilon}{1+\varepsilon}(\mathbf{e}_x + i\mathbf{e}_z)E_{i,x}^0 e^{q_\parallel(ix-z)}. \quad (13)$$

From the Maxwell equation $\nabla \cdot \mathbf{B} = 0$, one knows that the magnetic field never has a longitudinal part. [It also follows immediately from Eq. (11) that $\mathbf{B}_t(x,z;\omega|c \to \infty) = \mathbf{0}$, as required.] In Sec. II C, we shall comment upon the macroscopic result in Eq. (13). The transverse part of the evanescent electric field is obtained by subtraction:

$$\mathbf{E}_t^T(x,z;\omega) = \mathbf{E}_t(x,z;\omega) - \mathbf{E}_t^L(x,z;\omega). \quad (14)$$

Let us now consider the momentum density $\mathbf{g}_t$ of the total electromagnetic field [25] in the vacuum half space. Its cycle-averaged value, which is given by

$$\langle \mathbf{g}_t \rangle(z;\mathbf{q}_\parallel,\omega) = \frac{\varepsilon_0}{2}\mathrm{Re}[\mathbf{E}_t(x,z;\omega) \times \mathbf{B}_t^*(x,z;\omega)], \quad (15)$$

equals the corresponding Poynting vector divided by c^2. The division of $\mathbf{E}_t$ into its T and L parts splits $\langle \mathbf{g}_t \rangle$ into

$$\langle \mathbf{g}_t \rangle(z;\mathbf{q}_\parallel,\omega) = \langle \mathbf{g}_t^{TT} \rangle(z;\mathbf{q}_\parallel,\omega) + \langle \mathbf{g}_t^{LT} \rangle(z;\mathbf{q}_\parallel,\omega), \quad (16)$$

where

$$\langle \mathbf{g}_t^{TT} \rangle(z;\mathbf{q}_\parallel,\omega) = \frac{\varepsilon_0}{2}\mathrm{Re}\left[\mathbf{E}_t^T(x,z;\omega) \times \mathbf{B}_t^*(x,z;\omega)\right] \quad (17)$$

and

$$\langle \mathbf{g}_t^{LT} \rangle(z;\mathbf{q}_\parallel,\omega) = \frac{\varepsilon_0}{2}\mathrm{Re}\left[\mathbf{E}_t^L(x,z;\omega) \times \mathbf{B}_t^*(x,z;\omega)\right]. \quad (18)$$

The superscripts TT and LT refer to the vector-field character of the electric and magnetic fields entering the cross-products. The integral of $\mathbf{g}_t^{TT}(\mathbf{r},t)$ over the vacuum domain relates to the momentum of the transmitted transverse field [25]. By inserting Eq. (15) into Eq. (12) it appears that

$$\mathbf{e}_z \cdot \langle \mathbf{g}_t \rangle(z;\mathbf{q}_\parallel,\omega) = 0, \quad q_\parallel > \frac{\omega}{c}, \quad (19)$$

in the evanescent regime. This implies that

$$\mathbf{e}_z \cdot \langle \mathbf{g}_t^{LT} \rangle(z;\mathbf{q}_\parallel,\omega) = -\mathbf{e}_z \cdot \langle \mathbf{g}_t^{TT} \rangle(z;\mathbf{q}_\parallel,\omega) = \frac{2\varepsilon_0\varepsilon^2\omega}{c^2(1+\varepsilon)}\frac{q_\perp}{q_\perp^2 + (\varepsilon\kappa_\perp^0)^2}\left|E_{i,x}^0\right|^2 e^{-(q_\parallel+\kappa_\perp^0)z}, \quad (20)$$

where the last expression is obtained by inserting Eqs. (11) and (13) into Eq. (18). A schematic illustration of the various cycle-averaged momentum densities is shown in Fig. 3. The integral

$$\int_0^\infty \mathbf{e}_z \cdot \langle \mathbf{g}_t^{TT} \rangle(z;\mathbf{q}_\parallel,\omega)dz = -\frac{2\varepsilon_0\varepsilon^2\omega}{c^2(1+\varepsilon)}\frac{q_\perp\left|E_{i,x}^0\right|^2}{[q_\perp^2 + (\varepsilon\kappa_\perp^0)^2](q_\parallel + \kappa_\perp^0)} \quad (21)$$

indicates that the transverse field even in the evanescent regime ($q_\parallel > \omega/c$) possesses a nonvanishing momentum in the direction perpendicular to the surface. From the point of view of T photons, this result is interesting. Naively, one might

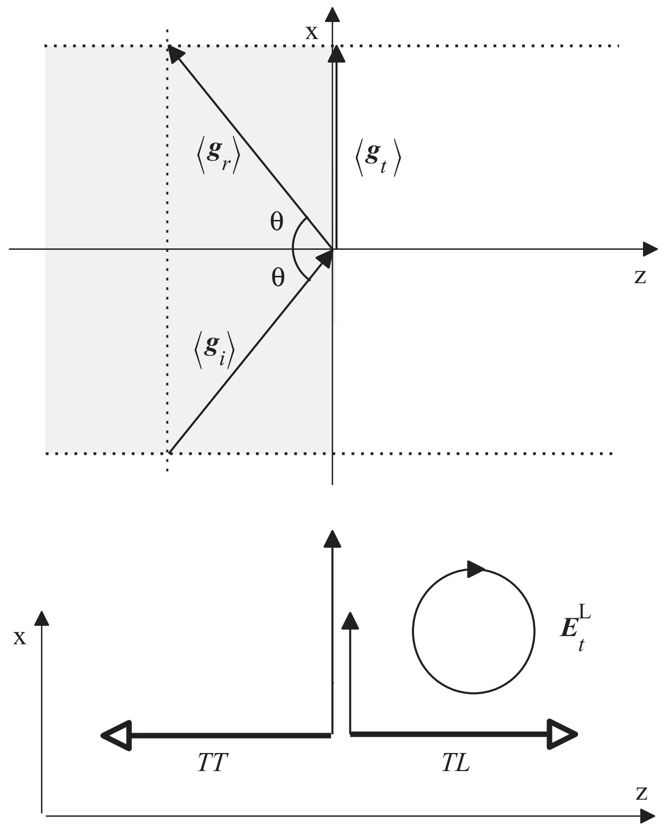

FIG. 3. Top: Cycle-averaged momentum densities in TIR. Bottom: Vectorial components of the TT and LT parts of $\langle \mathbf{g}_t \rangle$. The oppositely directed z components have the same magnitude. The transmitted longitudinal electric field ($\mathbf{E}_t^L$) lies in the scattering plane and is clockwise circular polarized, as indicated.

expect a transmitted T photon once generated will escape from the surface into the vacuum half space, even if the vacuum field is evanescent. Furthermore, one might perhaps anticipate that a monochromatic (nonlocalized) photon's momentum density would be constant in space. The results in Eqs. (20) and (21) do not live up to these expectations. Although at least the time-averaged transverse momentum is space independent, its z component has the "wrong" sign. (The z component of the momentum is directed towards the surface, not away from it.) So if one insists that T photons exist in the evanescent field, how can they be pulled towards the surface when there is no current density (from charged particles) in the vacuum? This is the question to be studied in the next subsection.

C. Rim zone: Source domain of T photons

To address the question above, we generalize the framework for our treatment from macroscopic to microscopic classical electrodynamics. In the microscopic approach the electromagnetic field satisfies the Maxwell-Lorentz equations, and in these the role of matter enters via the microscopic charge [$\rho(\mathbf{r},t)$] and current [$\mathbf{J}(\mathbf{r},t)$] densities [25,26]. The source domain of the divergence-free (called transverse in what follows) part of the electric field $\mathbf{E}^T(\mathbf{r},t)$ is known to be the region in space where the transverse part of the current density is nonvanishing [4]. Without losing the central point of the argumentation, we may assume that the microscopic current density exhibits infinitesimal translation invariance along the surface plane, i.e., has the form given in Eq. (1) for monochromatic excitations. The division of a vector field with the spatial form given in Eq. (1) into its transverse and longitudinal parts is obtained on the basis of a proper split of $\mathbf{F}(z;\mathbf{q}_\parallel,\omega)$, namely,

$$\mathbf{F}(z;\mathbf{q}_\parallel,\omega) = \mathbf{F}^T(z;\mathbf{q}_\parallel,\omega) + \mathbf{F}^L(z;\mathbf{q}_\parallel,\omega), \tag{22}$$

where

$$\mathbf{F}^K(z;\mathbf{q}_\parallel,\omega) = \int_{-\infty}^{\infty} \boldsymbol{\delta}^K(z-z';\mathbf{q}_\parallel)\cdot\mathbf{F}(z';\mathbf{q}_\parallel,\omega)dz', \quad K = T,L, \tag{23}$$

$\boldsymbol{\delta}^K$ being the transverse ($K = T$) and longitudinal ($K = L$) δ functions in disk contraction. The sum of the two dyadic δ functions equals the Dirac δ function multiplied by the 3×3 unit tensor ($\mathbf{U}$), that is,

$$\boldsymbol{\delta}^L(z-z';\mathbf{q}_\parallel) + \boldsymbol{\delta}^T(z-z';\mathbf{q}_\parallel) = \mathbf{U}\delta(z-z'). \tag{24}$$

With $\mathbf{q}_\parallel = q_\parallel \mathbf{e}_x$ the explicit expression for the longitudinal *delta* function is given by [18]

$$\boldsymbol{\delta}^L(z-z';q_\parallel\mathbf{e}_x) = \mathbf{e}_z\mathbf{e}_z\delta(z-z') + \frac{q_\parallel}{2}e^{-q_\parallel|z-z'|}[\mathbf{e}_x\mathbf{e}_x - \mathbf{e}_z\mathbf{e}_z + i(\mathbf{e}_x\mathbf{e}_z+\mathbf{e}_z\mathbf{e}_x)\mathrm{sgn}(z-z')], \tag{25}$$

where $\mathrm{sgn}(z-z') = +1$ for $z > z'$ and -1 for $z < z'$. The expression for $\boldsymbol{\delta}^T$ is obtained by combining Eqs. (24) and (25). Given the explicit expressions for $\boldsymbol{\delta}^K$ ($K = T,L$), the T and L parts of the microscopic current density are determined by the *spatially nonlocal* connection

$$\mathbf{J}^K(z;q_\parallel\mathbf{e}_x,\omega) = \int_{-\infty}^{\infty} \boldsymbol{\delta}^K(z-z';q_\parallel\mathbf{e}_x)\cdot\mathbf{J}(z';\mathbf{q}_\parallel,\omega)dz'. \tag{26}$$

Let us assume that the current density is confined to the half space $z' < 0$, i.e.,

$$\mathbf{J}(z';q_\parallel\mathbf{e}_x,\omega) \equiv \mathbf{J}(z') = \mathbf{J}_B(z')\theta(-z'), \tag{27}$$

where θ is the Heaviside unit step function, and $\mathbf{J}_B(z')$ may be called the bulk (B) current density. With the (model) current density in Eq. (27), the longitudinal and transverse parts of $\mathbf{J}(z)$ are given by [leaving the reference to $\mathbf{q}_\parallel$ and ω implicit]

$$\mathbf{J}^L(z) = -\mathbf{J}^T(z) = \frac{q_\parallel}{2}e^{-q_\parallel z}(\mathbf{e}_x+i\mathbf{e}_z)(\mathbf{e}_x+i\mathbf{e}_z) \cdot \int_{-\infty}^{0}\mathbf{J}_B(z')e^{q_\parallel z'}dz', \quad z > 0 \tag{28}$$

in the vacuum half space (Fig. 4, upper part). Although the total current density $\mathbf{J}(z)$ [Eq. (27)] is zero in the vacuum (of course), its T and L parts do not vanish here because of the spatial nonlocality in the connection between $\mathbf{J}^K$ and $\mathbf{J}$. The region outside matter where $\mathbf{J}^L = -\mathbf{J}^T \neq \mathbf{0}$ has been called the rim zone [20], and here its extension is given by $\exp(-q_\parallel z)$ [see Eq. (28)].

It follows from the considerations above that the source domain for the transverse part of the electromagnetic field, and thus also for the T photons [see Sec. III A], extends into the vacuum half space. We can elaborate on this point by considering the longitudinal part of the microscopic electric field $\mathbf{E}^L(z)$. It appears from the Maxwell-Lorentz equations

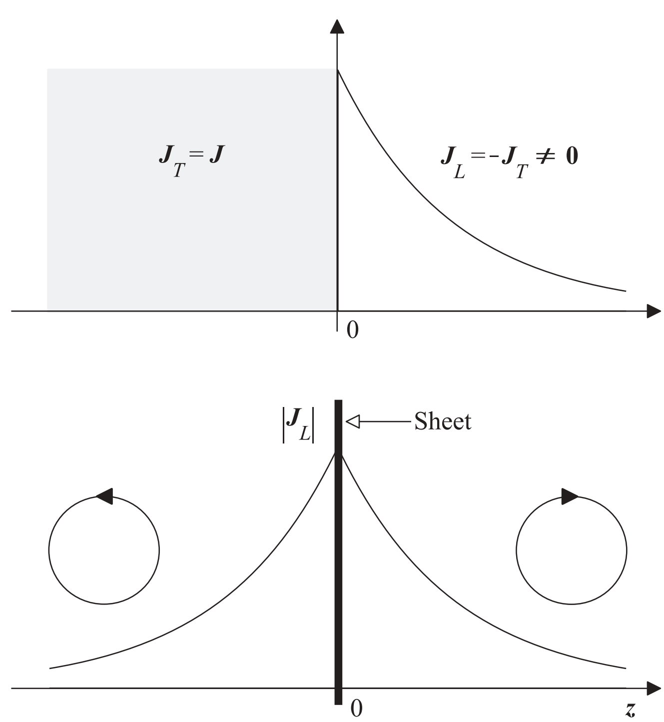

FIG. 4. Top: In our model of p-polarized TIR, the bulk current density is divergence free ($\mathbf{J} = \mathbf{J}_T$). In the vacuum half space, $\mathbf{J}_T + \mathbf{J}_L = \mathbf{J} = \mathbf{0}$. The T-photon source domain is exponentially [$\sim\exp(-iq_\parallel z)$] confined, as indicated. Bottom: Exponentially confined source domain ($\sim|\mathbf{J}_L|$) of T photons originating in a current density sheet located at $z = 0$.

that one *always* has

$$\mathbf{E}^L(z;\mathbf{q}_\parallel,\omega) = (i\varepsilon_0\omega)^{-1}\mathbf{J}^L(z;\mathbf{q}_\parallel,\omega). \tag{29}$$

The presence of a longitudinal electric field in the rim zone thus means that the T photons are *not free* in this zone. The longitudinal electric field originates in the instantaneous Coulomb interaction between charges, and this field appears in the Coulomb gauge formulation of the quantum theory solely via the dynamical particle variables [4,25].

By combining Eqs. (28) and (29), it follows that the longitudinal electric field in the vacuum half space ($z > 0$), $\mathbf{E}^L(z;\mathbf{q}_\parallel,\omega) \equiv \mathbf{E}_t^L(z;\mathbf{q}_\parallel,\omega) \equiv \mathbf{E}_t^L(z)$, is given by

$$\mathbf{E}_t^L(z) = \frac{q_\parallel}{2i\varepsilon_0\omega}e^{-q_\parallel z}(\mathbf{e}_x + i\mathbf{e}_z)(\mathbf{e}_x + i\mathbf{e}_z) \cdot \int_{-\infty}^{0} \mathbf{J}_B(z')e^{q_\parallel z'}dz'. \tag{30}$$

A comparison of Eqs. (13) and (30) [multiplied by $\exp(iq_\parallel x)$] shows that the field in both cases has the form

$$\mathbf{E}_t^L(x,z,\omega) = A(\mathbf{e}_x + i\mathbf{e}_z)e^{q_\parallel(ix-z)}. \tag{31}$$

We cannot expect, on the basis of the different frameworks used, that the amplitude A is the same in the macroscopic and microscopic approaches. We may conclude, however, that the perhaps surprising result obtained for the transverse-field (T-photon) momentum in Sec. II B can be traced back to the rim-zone coupling between the T field and matter. In reaching the result in Eq. (13), it was assumed that the macroscopic medium was homogeneous. Homogeneity implies that the source region for the T field is not the entire half space $z < 0$ plus the rim zone, as one might believe. To illustrate the consequence of homogeneity, let us return to the microscopic result in Eq. (30). As it stands, it seems that that the amplitude of $\mathbf{E}^L(z)$ contains contributions related to $\mathbf{J}_B(z)$ in the entire half space $z' < 0$. However, if one assumes that the medium, even in the microscopic sense, is (approximately) homogeneous, the current density induced by $\mathbf{E}_i + \mathbf{E}_r$ in the p-polarized case necessarily must be divergence free, that is,

$$iq_\parallel J_{B,x}(z) + \frac{d}{dz}J_{B,z}(z) = 0. \tag{32}$$

Here the reader may recall that Gauss' law in the microscopic Maxwell-Lorentz equations is $\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$, where ρ is the microscopic charge density [4,25,26]. The condition in Eq. (32) allows one to simplify the expression for $\mathbf{E}_t^L$. A partial integration of the term containing $J_{B,z}(z')$ in the first member of the equation below thus gives

$$\begin{aligned}\mathbf{E}_t^L(z) &= \frac{q_\parallel}{2i\varepsilon_0\omega}e^{-q_\parallel z}(\mathbf{e}_x + i\mathbf{e}_z) \times \int_{-\infty}^{0}(J_{B,x}(z') + iJ_{B,z}(z'))e^{q_\parallel z'}dz' \\ &= \frac{1}{2\varepsilon_0\omega}e^{-q_\parallel z}(\mathbf{e}_x + i\mathbf{e}_z)\bigg\{J_{B,z}(0) - \int_{-\infty}^{0}\left[iq_\parallel J_{B,x}(z') + \frac{d}{dz'}J_{B,z}(z')\right]e^{q_\parallel z'}dz'\bigg\}.\end{aligned} \tag{33}$$

The integral in Eq. (33) vanishes because of Eq. (32). From our final expression for $\mathbf{E}_t^L$, viz.,

$$\mathbf{E}_t^L(z) = \frac{J_{B,z}(0)}{2\varepsilon_0\omega}e^{-q_\parallel z}(\mathbf{e}_x + i\mathbf{e}_z), \quad z > 0, \tag{34}$$

it thus appears that the longitudinal field in the rim zone has the bulk current density at the interface $J_{B,z}(0)$ as source. The longitudinal electric field in the entire space is given by [18]

$$\mathbf{E}^L(z) = \frac{J_{B,z}(0)}{2\varepsilon_0\omega}e^{-q_\parallel|z|}[\mathbf{e}_x + i\mathbf{e}_z\,\mathrm{sgn}(z)] \tag{35}$$

in the homogeneous case, as the reader may show by combining Eqs. (25)–(27), and remembering the presence of the contact term $\mathbf{e}_z\mathbf{e}_z\delta(z - z')$ for $z < 0$. Note that circular polarization directions of $\mathbf{E}^L(z)$ are opposite in the two half spaces. It thus appears from Eqs. (25), (26), (29), and (35) that the rim zone is that of a current density sheet located at $z = 0$. See also Fig. 4, lower part.

III. PHOTON WAVE MECHANICS IN EVANESCENT FIELDS

A proper reinterpretation of the microscopic Maxwell-Lorentz equations allows one to consider these as a wave mechanical (first-quantized) theory for photons. In the covariant description, three types of photons appear, viz., transverse (T), longitudinal (L), and scalar (S) photons. The wave functions of these photons species relate to the transverse, longitudinal, and scalar parts of the four-potential. From a fundamental point of view, the covariant photon theory has a number of advantages compared the other first-quantized theories [17], and below we shall see how the theory may deepen our understanding of the physics of evanescent fields. In Part III, references to T, L, and S fields are moved from superscripts to subscripts.

A. Evanescent four-potential in the Lorenz gauge

The potential formulation of the Maxwell-Lorentz theory takes a particularly simple form in the Lorenz gauge [4,25], and for vectors of the form given in Eq. (1), the components ($\mu = 0-3$) of the (covariant) four-current density $\{J_\mu(z;\mathbf{q}_\parallel,\omega)\} \equiv \{J_\mu(z)\}$ and the (covariant) four-potential $\{A_\mu(z;\mathbf{q}_\parallel,\omega)\} \equiv \{A_\mu(z)\}$ are connected by the integral relation

$$A_\mu(z) = \mu_0\int_{-\infty}^{\infty} g(z - z')J_\mu(z')dz', \tag{36}$$

where

$$g(z - z') = \frac{i}{2q_\perp^0}e^{iq_\perp^0|z-z'|} \to \frac{1}{2\kappa_\perp^0}e^{-\kappa_\perp^0|z-z'|} \tag{37}$$

is the scalar (Huygens) propagator. The expression after the arrow gives the form of $g(z - z')$ in the evanescent ($q_\perp^0 \to i\kappa_\perp^0$) regime. In the absence of a current density, the net effect of the longitudinal and scalar photons vanishes, whereas the transverse photon can "exist" in free space. The L- and S-photon dynamics is closely connected to the physics in the rim zone, and let us therefore first consider the related vector and scalar potentials.

The longitudinal vector potential [$\mathbf{A}_L(z)$], which satisfies the integral relation

$$\mathbf{A}_L(z) = \mu_0 \int_{-\infty}^{\infty} g(z-z')\mathbf{J}_L(z')dz', \quad (38)$$

can be determined from a knowledge of the longitudinal current density distribution [$\mathbf{J}_L(z)$]. From Eqs. (29) and (35) it appears that

$$\mathbf{J}_L(z) = \frac{i}{2} J_{B,z}(0) e^{-q_\parallel |z|}[\mathbf{e}_x + i\mathbf{e}_z \operatorname{sgn}(z)]. \quad (39)$$

In the evanescent ($q_\parallel > \omega/c$) region, we hence have

$$\mathbf{A}_L(z) = \frac{i\mu_0}{4\kappa_\perp^0} J_{B,z}(0) \int_{-\infty}^{\infty} e^{-\kappa_\perp^0 |z-z'|} e^{-q_\parallel |z'|}[\mathbf{e}_x + i\mathbf{e}_z \operatorname{sgn}(z')]dz'. \quad (40)$$

Although a bit tedious, it is a straightforward matter to calculate the integral for all z. The final result can be written as follows:

$$\mathbf{A}_L(z) = \frac{i\mu_0}{2\kappa_\perp^0} J_{B,z}(0)[\theta(z)\mathbf{I}^>(z) + \theta(-z)\mathbf{I}^<(z)], \quad (41)$$

where [with $q_0 = \omega/c$, $\kappa_\perp^0 = (q_\parallel^2 - q_0^2)^{1/2}$]

$$\mathbf{I}^>(z) = q_0^{-2}\left[(q_\parallel \mathbf{e}_x + i\kappa_\perp^0 \mathbf{e}_z)e^{-\kappa_\perp^0 z} - \kappa_\perp^0(\mathbf{e}_x + i\mathbf{e}_z)e^{-q_\parallel z}\right] \quad (42)$$

and

$$\mathbf{I}^<(z) = q_0^{-2}\left[(q_\parallel \mathbf{e}_x - i\kappa_\perp^0 \mathbf{e}_z)e^{\kappa_\perp^0 z} - \kappa_\perp^0(\mathbf{e}_x - i\mathbf{e}_z)e^{q_\parallel z}\right]. \quad (43)$$

In a manner to be described in Sec. III B, the longitudinal vector potential $\mathbf{A}_L(z)\exp[i(\mathbf{q}_\parallel \cdot \mathbf{r} - \omega t)]$ relates to the spatial spectrum of longitudinal photons at a given frequency (ω). The longitudinal current density distribution is the source of the L photons, and it appears from Eq. (39) that $\mathbf{J}_L(z)$ decays exponentially away from the surface ($z = 0$) plane to both sides, the decay constant being $q_\parallel$. The expressions for $\mathbf{I}^>(z)$ and $\mathbf{I}^<(z)$, given in Eqs. (42) and (43), tell us that also the longitudinal vector potential $\mathbf{A}_L(z)$ decreases with the distance from the $z = 0$ plane. The falloff is characterized by a superposition of two exponential functions $\exp(-q_\parallel|z|)$ and $\exp(-\kappa_\perp^0|z|)$. The polarization states of the two contributions are circular ($\mathbf{e}_x \pm i\mathbf{e}_z$) and elliptical ($q_\parallel \mathbf{e}_x \pm i\kappa_\perp^0 \mathbf{e}_z$), respectively, and the rotation directions are opposite in the two half spaces. Let us denote the part of $\mathbf{A}_L(z)$ which relates to the $\exp(-q_\parallel|z|)$ term by $\mathbf{A}_L(z;[q_\parallel])$ and the remaining part by $\mathbf{A}_L(z;[\kappa_\perp^0])$, i.e.,

$$\mathbf{A}_L(z) = \mathbf{A}_L(z;[q_\parallel]) + \mathbf{A}_L(z;[\kappa_\perp^0]). \quad (44)$$

A comparison of Eqs. (39) and (41)–(43) now shows that

$$i\omega\mathbf{A}_L(z;[q_\parallel]) = (i\varepsilon_0\omega)^{-1}\mathbf{J}_L(z) = \mathbf{E}_L(z), \quad (45)$$

the last member coming from Eq. (29).

In order to understand the reason behind the simple connection in Eq. (45), we first calculate the scalar part of the four-potential, $A_0(z;\mathbf{q}_\parallel,\omega) \equiv A_0(z)$. It appears from Eq. (36) [for $\mu = 0$] that

$$A_0(z) = \mu_0 \int_{-\infty}^{\infty} g(z-z')J_0(z')dz'. \quad (46)$$

Remembering that $J_0(z) = c\rho(z)$, where $\rho(z) \equiv \rho(z;\mathbf{q}_\parallel,\omega)$ is the charge density, it is easy to obtain the scalar potential. The charge density is calculated from the equation of continuity, which here amounts to

$$i\omega\rho(z) = \left(i\mathbf{q}_\parallel + \mathbf{e}_z\frac{d}{dz}\right)\cdot[\mathbf{J}_B(z)\theta(-z)], \quad (47)$$

since the current density is confined to the half plane $z < 0$ [see Eq. (27)]. For our divergence-free bulk current density, Eq. (47) leads to

$$\rho(z) = \frac{i}{\omega} J_{B,z}(0)\delta(z) \quad (48)$$

in view of Eq. (32), and because $d\theta(-z)/dz = -\delta(z)$. The only present charge hence is a surface charge, with a singular density given by Eq. (48). The source of the S photon, therefore, is confined to the surface plane. By inserting Eq. (48) into Eq. (46), and using the explicit form of the scalar propagator, one finds the following result for the scalar potential in the evanescent regime:

$$A_0(z) = \frac{i\mu_0}{2\kappa_\perp^0 q_0} J_{B,z}(0)\left[\theta(z)e^{-\kappa_\perp^0 z} + \theta(-z)e^{\kappa_\perp^0 z}\right]. \quad (49)$$

The result in Eq. (45) now can be explained. From the general relation between the longitudinal electric field and the four-potential, namely,

$$\mathbf{E}_L(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}_L(\mathbf{r},t) - c\nabla A_0(\mathbf{r},t), \quad (50)$$

one obtains the connection

$$\mathbf{E}_L(z) = i\omega\mathbf{A}_L(z) - c\left(iq_\parallel \mathbf{e}_x + \mathbf{e}_z\frac{d}{dz}\right)A_0(z). \quad (51)$$

By inserting here the expressions for $\mathbf{A}_L(z)$ [Eqs. (41)–(43)] and $A_0(z)$ [Eq. (49)], it appears that

$$\mathbf{E}_L(z) = i\omega\mathbf{A}_L(z;[q_\parallel]). \quad (52)$$

The circular polarized contribution, which decays according to $\exp(-q_\parallel|z|)$, thus relates to the longitudinal electric field in the rim zone. Furthermore, light can be thrown on this result by studying the regime ($q_\parallel > q_0$). The longitudinal and scalar potentials in this regime are readily written down by making the substitution $\kappa_\perp^0 \to -iq_\perp^0$ in Eqs. (41)–(43) and Eq. (49). The nonpropagating part

$$\mathbf{A}_L(z;[q_\parallel]) = -\frac{i}{2\varepsilon_0\omega^2} J_{B,z}(0)[\theta(z)(\mathbf{e}_x + i\mathbf{e}_z)e^{-q_\parallel z} + \theta(-z)(\mathbf{e}_x - i\mathbf{e}_z)e^{q_\parallel z}] \quad (53)$$

is unchanged by the substitution, and the propagating parts $\mathbf{A}_L(z;[\kappa_\perp^0 \to -iq_\perp^0])$ and $A_0(z;[\kappa_\perp^0 \to -iq_\perp^0])$, which *both* propagate to infinity ($\pm\infty$), *together* give no longitudinal electric field outside the rim zone. This cancellation of course also occurs in the evanescent regime. A schematic illustration of the results obtained for the L and S potentials is presented in Figs. 5 and 6.

The transverse vector potential is given generally by

$$\mathbf{A}_T(z) = \mu_0 \int_{-\infty}^{\infty} g(z-z')\mathbf{J}_T(z')dz', \quad (54)$$

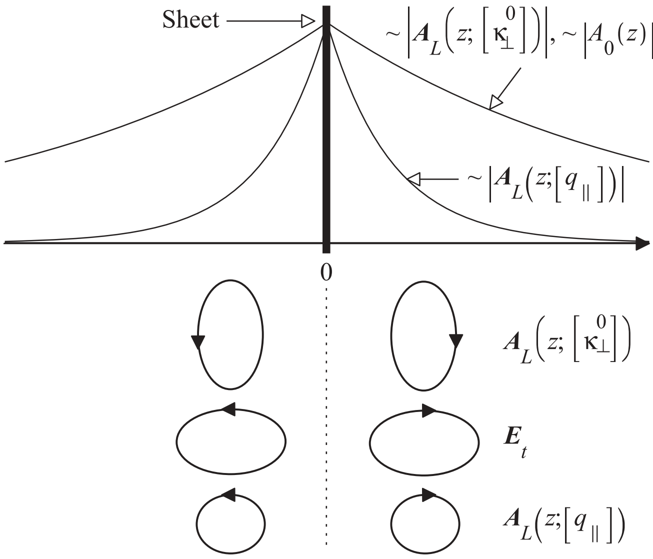

FIG. 5. Schematic illustration showing that the sheet fields $\mathbf{A}_L(z;[\kappa_\perp^0])$ and $A_0(z)$ have the same spatial decay constant ($\kappa_\perp^0$). The decay constant ($q_\parallel$) for $\mathbf{A}_L(z;[q_\parallel])$ is larger. The elliptical polarizations of $\mathbf{A}_L(z;[\kappa_\perp^0])$ and $\mathbf{E}_t(z)$, and the circular polarization of $\mathbf{A}_L(z;[q_\parallel])$, in the two half spaces ($z<0$, $z>0$), are indicated in the lower part of the figure.

and in the vacuum half space this gives in the evanescent regime

$$\mathbf{A}_T(z) = \frac{\mu_0}{2\kappa_\perp^0} e^{-\kappa_\perp^0 z} \int_{-\infty}^{0} e^{\kappa_\perp^0 z'} \mathbf{J}_B(z') dz' - \mathbf{A}_L(z). \quad (55)$$

Outside the rim zone, where $\mathbf{E}_L(z) = \mathbf{0}$, one obtains

$$\mathbf{A}_T(z) = (i\omega)^{-1}\mathbf{E}(z), \quad (56)$$

and for current densities with finite support in space-time the T photons related to $\mathbf{A}_T$ are free once the source has stopped its activity.

B. Scalar-photon dynamics

In the covariant theory of photon wave mechanics, the dynamical variable of the scalar photon is identical to the positive-frequency part of the scalar potential [4], properly normalized. In the present context the S-photon dynamical variable is given by

$$A_0(\mathbf{r},t) = A_0(z)e^{i(Q_\parallel x - \omega t)}, \quad (57)$$

where

$$A_0(z) = \frac{i\mu_0}{2\kappa_\perp^0 q_0} J_{B,z}(0) e^{-\kappa_\perp^0 |z|}. \quad (58)$$

For notational uniformity (see below), we have made the replacement $q_\parallel \to Q_\parallel$. To go from the space representation above to the wave-vector (∼momentum) representation, we need the Fourier-integral transform of $A_0(z)$, viz.,

$$A_0(Q_\perp) = \int_{-\infty}^{\infty} A_0(z) e^{-iQ_\perp z} dz = \frac{i\mu_0 J_{B,z}(0)}{q_0[Q_\perp^2 + (\kappa_\perp^0)^2]}. \quad (59)$$

Introducing the wave number

$$Q = (Q_\parallel^2 + Q_\perp^2)^{\frac{1}{2}} \quad (60)$$

belonging to the plane-wave mode $\exp[i(Q_\parallel x + Q_\perp z)]$, the scalar photon variable takes the form

$$A_0(\mathbf{Q};t) = \frac{i\mu_0 J_{B,z}(0)}{q_0\left(Q^2 - q_0^2\right)} e^{-i\omega t} \quad (61)$$

in the wave-vector representation. The expression for $A_0(\mathbf{Q};t)$ can be rewritten in a useful manner which relates to the Fourier-integral transform of the sheet charge density in Eq. (48), namely,

$$\rho(Q_\perp) = \int_{-\infty}^{\infty} \rho(z) e^{-iQ_\perp z} dz = \frac{i}{\omega} J_{B,z}(0) \equiv \tilde{\rho}, \quad (62)$$

a wave-number ($Q_\perp$) independent quantity. In terms of the associated current density

$$\mathfrak{J}_0 = c\tilde{\rho}, \quad (63)$$

we finally obtain

$$A_0(\mathbf{Q};t) = \frac{\mu_0 \mathfrak{J}_0}{Q^2 - q_0^2} e^{-i\omega t}. \quad (64)$$

The scalar photon variable in Eq. (64) satisfies the dynamical equation

$$\left(cQ - i\frac{\partial}{\partial t}\right) A_0(\mathbf{Q};t) = \frac{\mathfrak{J}_0}{\varepsilon_0(\omega + cQ)} e^{-i\omega t}. \quad (65)$$

In free space ($\mathfrak{J}_0 = 0$), Eq. (65), after multiplication by Planck's constant (divided by 2π) $\hbar$, is reduced to the quantum-mechanical-wave equation for the scalar photon, written in Hamiltonian form, namely,

$$\hat{H} A_0(\mathbf{Q};t) = i\hbar \frac{\partial}{\partial t} A_0(\mathbf{q};t), \quad (66)$$

where $\hat{H} = c\hbar Q$ is the Hamilton operator in the wave-vector (Q) representation. The results in Eq. (65) are in agreement with the wave mechanical theory for scalar photon wave packets composed of positive-frequency components in the monochromatic ($\omega > 0$) limit [with regularization at $t = -\infty$]; see Eq. (5.11) in [20].

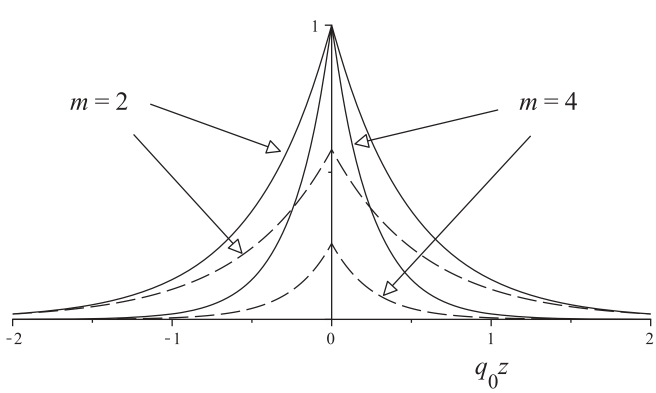

FIG. 6. Using a normalization constant $N = \mu_0|J_{B,z}(0)|/(q_0^2\sqrt{2})$, the quantities $|\mathbf{A}_L(z;[q_\parallel])|/N = \exp(-mq_0|z|)$ [full line], and $\sqrt{2}|A_0(z)|/N = (m^2-1)^{-1/2}\exp[-(m^2-1)^{1/2}q_0|z|]$ [dashed line] are plotted as a function of $q_0 z$ for two values of $m = q_\parallel/q_0$.

C. Longitudinal-photon dynamics

The dynamical variable of the longitudinal photon in the covariant theory of photon wave mechanics is obtained from the positive-frequency part of the longitudinal vector potential. For our evanescent field we thus have

$$\mathbf{A}_L(\mathbf{r},t) = [\mathbf{A}_L(z;[Q_\parallel]) + \mathbf{A}_L(z;[\kappa_\perp^0])]e^{i(q_\parallel x-\omega t)}, \quad (67)$$

where [from Eqs. (41)–(43)]

$$\mathbf{A}_L(z;[Q_\parallel]) = -\frac{i\mu_0}{2q_0^2}J_{B,z}(0)[\mathbf{e}_x + i\mathbf{e}_z\,\mathrm{sgn}(z)]e^{-q_\parallel|z|} \quad (68)$$

and

$$\mathbf{A}_L(z;[\kappa_\perp^0]) = \frac{i\mu_0}{2\kappa_\perp^0 q_0^2}J_{B,z}(0)[Q_\parallel\mathbf{e}_x + i\kappa_\perp^0\mathbf{e}_z\,\mathrm{sgn}(z)]e^{-\kappa_\perp^0|z|}. \quad (69)$$

The Fourier-integral transforms of the expressions in Eqs. (68) and (69) are given by

$$\mathbf{A}_L(Q_\perp;[Q_\parallel]) = -\frac{i\mu_0 J_{B,z}(0)}{q_0^2 Q^2}(Q_\parallel\mathbf{e}_x + Q_\perp\mathbf{e}_z) \quad (70)$$

and

$$\mathbf{A}_L(Q_\perp;[\kappa_\perp^0]) = \frac{i\mu_0 J_{B,z}(0)}{q_0^2(Q^2 - q_0^2)}(Q_\parallel\mathbf{e}_x + Q_\perp\mathbf{e}_z). \quad (71)$$

The sum of the contributions in Eqs. (70) and (71) may be written in the form

$$\mathbf{A}_L(\mathbf{Q}) = \frac{\mu_0 q_0 \mathfrak{J}_0}{Q^2(Q^2 - q_0^2)}(Q_\parallel\mathbf{e}_x + Q_\perp\mathbf{e}_z). \quad (72)$$

The longitudinal photon variable in $\mathbf{Q}$ space $A_L(\mathbf{Q};t)$, calculated from

$$A_L(\mathbf{Q};t) = \mathbf{e}_\mathbf{Q}\cdot\mathbf{A}_L(\mathbf{Q})e^{-i\omega t} \quad (73)$$

where

$$\mathbf{e}_\mathbf{Q} = \frac{1}{Q}(Q_\parallel\mathbf{e}_x + Q_\perp\mathbf{e}_z), \quad (74)$$

is a unit vector in the $\mathbf{Q}$ direction and hence becomes

$$A_L(\mathbf{Q};t) = \frac{\mu_0 q_0 \mathfrak{J}_0}{Q(Q^2 - q_0^2)}e^{-i\omega t}. \quad (75)$$

The longitudinal photon variable in Eq. (75) satisfies the dynamical equation

$$\left(cQ - i\frac{\partial}{\partial t}\right)A_L(\mathbf{Q};t) = \frac{\mathfrak{J}_0}{\varepsilon_0(\omega + cQ)}\frac{q_0}{Q}e^{-i\omega t}, \quad (76)$$

which in free space reduces to the Hamiltonian form of the wave equation in $\mathbf{Q}$ space for the longitudinal photon, viz.,

$$\hat{H}A_L(\mathbf{Q};t) = i\hbar\frac{\partial}{\partial t}A_L(\mathbf{q};t). \quad (77)$$

The result in Eq. (76) agrees with that of the general theory for L-photon wave packets in the monochromatic limit (with regularization at $t = -\infty$) [see [20], and combine herein Eqs. (5.10), (6.1), (6.6), and (6.7)].

It may be useful to remind the reader that the energy(H)-momentum(p) relation for a free massless relativistic particle (here the photon) is $H = cp = c\hbar Q$. In operator form one thus obtains $\hat{H} = c\hbar Q$, since $\hat{Q} = Q$ in the wave-vector representation. The Hamilton operator is the same for the S and L photons [Eqs. (66) and (77), respectively].

D. Near-field photon dynamics

The potential formulation of photon wave mechanics given above refers to the Lorenz gauge, in which the S and L wave functions are coupled by the gauge condition

$$icQA_L(\mathbf{Q};t) + \frac{\partial}{\partial t}A_0(\mathbf{Q};t) = 0. \quad (78)$$

The reader may check that the wave functions in Eqs. (61) and (75) do satisfy the Lorenz gauge condition. Although the S and L wave functions do not vanish in free space, the gauge condition in Eq. (78) implies that the two wave functions are identical in free space:

$$A_L^{\mathrm{free}}(\mathbf{Q};t) = A_0^{\mathrm{free}}(\mathbf{Q};t). \quad (79)$$

At this point one must remember that the wave equations in Eqs. (65) and (76) have the complete solutions

$$A_I(\mathbf{Q};t) = A_I^{\mathrm{free}}(\mathbf{Q};t) + A_I^{inh}(\mathbf{Q};t), \quad I = S,L. \quad (80)$$

It is the inhomogeneous (inh) solutions that relate to the surface current density $\mathfrak{J}_0$, which we have discussed in Secs. III A and III B.

In near-field electrodynamics in general [4] and for the evanescent field case studied here, Eq. (79) makes it useful to introduce a so-called near-field (NF) photon dynamical variable $A_{\mathrm{NF}}(\mathbf{Q};t)$ by the definition [20]

$$A_{\mathrm{NF}}(\mathbf{Q};t) = \frac{i}{\sqrt{2}}[A_L(\mathbf{Q};t) - A_0(\mathbf{Q};t)]. \quad (81)$$

In free space the near-field photon wave function $A_{\mathrm{NF}}^{\mathrm{free}}(\mathbf{Q};t)$ vanishes. Furthermore, $A_{\mathrm{NF}}(\mathbf{Q};t)$ is invariant against gauge transformations *within* the Lorenz gauge [20]. By combining Eqs. (64), (75), and (81) one obtains

$$A_{\mathrm{NF}}(\mathbf{Q};t) = \frac{\mu_0\mathfrak{J}_0}{i\sqrt{2}Q(Q + q_0)}e^{-i\omega t}. \quad (82)$$

The Q dependence of A_0, A_L, and A_{NF} are compared in Fig. 7. The $A_{\mathrm{NF}}(\mathbf{Q};t)$ variable satisfies a dynamical equation of the form

$$\left(cQ - i\frac{\partial}{\partial t}\right)A_{\mathrm{NF}}(\mathbf{Q};t) = \frac{\mathfrak{J}_0}{i\sqrt{2}\varepsilon_0 cQ}e^{-i\omega t} \quad (83)$$

with the constraint that $A_{\mathrm{NF}}^{\mathrm{free}}(\mathbf{Q};t) = 0$. The importance of the near-field photon concept for studies of evanescent fields (and near fields as such) is closely related to the fact that the right-hand side of Eq. (83) (multiplied by $\sqrt{2}$) equals the projection of the longitudinal electric field on the $\mathbf{Q}$ direction. Thus,

$$E_L(\mathbf{Q};t) \equiv \mathbf{e}_\mathbf{Q}\cdot\mathbf{E}_L(Q_\perp)e^{i\omega t}$$
$$= i\omega\mathbf{e}_\mathbf{Q}\cdot\mathbf{A}_L(Q_\perp;[Q_\parallel])e^{-i\omega t} = \frac{\mathfrak{J}_0}{i\varepsilon_0 cQ}e^{-i\omega t}, \quad (84)$$

as the reader may verify using Eqs. (52), (62), (63), and (70), and hence

$$\left(cQ + i\frac{\partial}{\partial t}\right)A_{\mathrm{NF}}(\mathbf{Q};t) = \frac{1}{\sqrt{2}}E_L(\mathbf{Q};t), \quad (85)$$

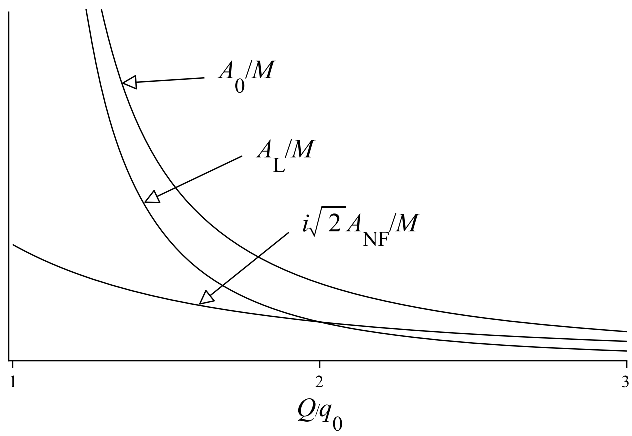

FIG. 7. Normalized (with $M = \mu_0 \mathcal{J}_0/q_0^2$) plots of the scalar ($A_0$), longitudinal ($A_L$), and near-field ($i\sqrt{2}A_{\rm NF}$) photon variables as functions of Q/q_0. Note that the NF variable is not singular at the vacuum wave number, i.e., for $Q = q_0$.

as anticipated [20]. In near-field optics the coupling between the transverse and longitudinal electric fields in the rim zone plays a crucial role, and Eq. (85) therefore forms a good starting point for field-quantized studies in this subfield of quantum electrodynamics. The plus sign between the operators cQ and $i\partial/\partial t$ in Eq. (85) ensures that near-field photons do not exist in free space [4,20].

IV. OUTLOOK: QUANTUM-WELL RIM ZONE

It appears from the foregoing analysis that the electron dynamics near the surface plays an important role in the photon wave mechanics of the evanescent field in TIR, at least in the p-polarized case. It is also clear that the conclusions reached were based on a model for the induced current density, which in many aspects is hopelessly oversimplified. However, the model, a divergence-free bulk current density terminated abruptly at the surface, indicates that a deeper understanding of the photon wave mechanics of evanescent fields in TIR requires some sort of self-consistent microscopic description of the electrodynamic surface-bulk interaction. To avoid the many complications one is confronted with in such a study, we suggest that one turns from the TIR configuration to quantum-well (QW) systems. Usually, the thickness of a QW will be orders of magnitude smaller than the characteristic wavelength(s) of the external electromagnetic field exciting the electrons in the well. In relation to photon wave mechanics, the QW therefore may often be considered as an electric-dipole (ED) receiver and emitter [22].

Some of the key problems one needs to address in the photon wave mechanics of QWs may be illustrated by considering a QW ED current density of the following simple form:

$$\mathbf{J}(\mathbf{r};\omega) = I\mathbf{e}_y e^{i(q_\parallel + G_x)x} e^{iG_y y}\delta(z). \qquad (86)$$

The current density in Eq. (86) is the one induced in the direction perpendicular to the scattering (xz) plane by an arbitrarily polarized external field. The amplitude I which relates to the microscopic conductivity of the QW must be calculated quantum mechanically. In the framework of linear response theory, the structure of I will contain the (many-body) transition current densities between various bound states in the well, the eigenenergies of these states, and the probability that the given state is occupied. In the TIR analysis it was assumed that the system exhibits translational invariance in all directions parallel to the surface plane. A corresponding assumption has not been made in Eq. (86). Thus, the $\mathbf{J}(\mathbf{r},\omega)$ above relates to a given spatial 2D Fourier component $\mathbf{G} = (G_x, G_y, 0)$ of the QW structure. The amplitude I depends on the selected $\mathbf{G}$, of course. To determine the source region for T photons emitted from the QW, the spatial Fourier transform of Eq. (86), viz.,

$$\mathbf{J}(\mathbf{Q},\omega) = (2\pi)^2 I\mathbf{e}_y \delta(q_\parallel + G_x - Q_x)\delta(G_y - Q_y), \qquad (87)$$

is multiplied by the $\mathbf{Q}$-space longitudinal δ function $\mathbf{QQ}/Q^2$, and the product then taken back to direct space by an inverse Fourier transformation. The two δ functions in Eq. (87) immediately allow one to obtain the following 1D integral expression for the longitudinal part of the QW current density:

$$\mathbf{J}_L(\mathbf{r};\omega) = I e^{i(q_\parallel + G_x)x} e^{iG_y y}\mathbf{e}_y \cdot \left[\int_{-\infty}^{\infty} \mathbf{e}\mathbf{e}\, e^{iQ_z z}\frac{dQ_z}{2\pi}\right], \qquad (88)$$

where

$$\mathbf{e} = \frac{(q_\parallel + G_x)\mathbf{e}_x + G_y\mathbf{e}_y + Q_z\mathbf{e}_z}{\left[(q_\parallel + G_x)^2 + G_y^2 + Q_z^2\right]^{\frac{1}{2}}}. \qquad (89)$$

Since

$$\mathbf{e}_y \cdot \mathbf{e}\mathbf{e} = \frac{G_y[(q_\parallel + G_x)\mathbf{e}_x + G_y\mathbf{e}_y + Q_z\mathbf{e}_z]}{(q_\parallel + G_x)^2 + G_y^2 + Q_z^2}, \qquad (90)$$

the integral over Q_z is carried out easily, giving finally

$$\mathbf{J}_L(\mathbf{r};\omega) = \frac{I}{2a} G_y[(q_\parallel + G_x)\mathbf{e}_x + G_y\mathbf{e}_y + ia\mathbf{e}_z\,\mathrm{sgn}(z)] \times e^{-a|z|} e^{i[(q_\parallel + G_x)x + G_y y]}, \qquad (91)$$

where

$$a = \left[(q_\parallel + G_x)^2 + G_y^2\right]^{\frac{1}{2}}. \qquad (92)$$

It appears from Eq. (91) that the source domain of the T photon is exponentially confined in the direction perpendicular to the QW plane, the decay constant being a. For $\mathbf{G} \to \mathbf{0}$ [$I(\mathbf{G} \to \mathbf{0}) \equiv I_0$] one obtains asymptotically,

$$\mathbf{J}_L(\mathbf{r};\omega) = I_0 G_y[\mathbf{e}_x + i\mathbf{e}_z\,\mathrm{sgn}(z)]e^{q_\parallel(ix - |z|)}. \qquad (93)$$

The result in Eq. (93) puts the TIR analysis in perspective. Using the simple model current density in Eq. (27), with a divergence-free bulk current density [see Eq. (32)], we concluded that the source region for the transverse photon, $\mathbf{J}_T(z) = -\mathbf{J}_L(z)$, had the form given by Eq. (39). The presence of an evanescent tail, $\sim\exp(-q_\parallel z)$, in the vacuum half space is necessary to explain the T-photon tunneling process in p-polarized FTIR [18]. It is known [9] that photon tunneling also occurs with s-polarized fields. In a classical perspective this is in agreement with the fact that evanescent fields also occur in s-polarized TIR. The photon-wave mechanical connection is a bit more subtle in the s-polarized case, however. A standard classical calculation for s-polarized scattering analogous to the one summarized in Sec. II A shows that the transmitted electric field decays as $\sim\exp(-\kappa_\perp^0 z)$ in the evanescent regime, but in this case the model current density,

$\mathbf{J}(z) = J_B(z)\mathbf{e}_y\theta(-z)$, has no longitudinal part [cf. Eq. (28)]. Extended to the photon wave mechanical level, the source domain of the T photon hence does not extend into the vacuum. Is this conclusion erasing the picture of T-photon tunneling as due to lack of spatial photon localization? No. It tells us that the naive model which works for p-polarized fields cannot be employed in the s-polarized fields case. The reason originates in the fact that the model for p-polarized gives us a necessary charge density at the surface [Eq. (48)]. With the model current density $\mathbf{J}(x,z) = J_B(z)\mathbf{e}_y\theta(-z)e^{iq_\parallel x}$ comes no (surface) charge density. From a fundamental microscopic point of view, an s-polarized external field inevitably gives rise to an induced movement of the electrons also normal to the surface region. The simple extension of the s-polarized model current density to the form in Eq. (86), asymptotically ($\mathbf{G} \to \mathbf{0}$), provides us with the needed nonvanishing longitudinal current density [Eq. (93)]. The (circular) polarization and spatial dependence of the longitudinal current densities now are in agreement for the p-polarized [Eq. (44) multiplied by $\exp(iq_\parallel x)$] and s-polarized [Eq. (93)] cases. A comparison of the model amplitudes $(i/2)J_{B,z}(0)$ and $I_0 G_y$ is meaningless, of course.

[1] P. C. Clemmow, *The Plane Wave Spectrum Representation of Electromagnetic Fields* (Pergamon, New York, 1966).
[2] M. Nieto-Vesperinas, *Scattering and Diffraction in Physical Optics* (Wiley-Interscience, New York, 1991).
[3] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
[4] O. Keller, *Quamtum Theory of Near-Field Electrodynamics* (Springer, Berlin, 2011).
[5] D. Courjon, *Near-Field Microscopy and Near-Field Optics* (Imperial College Press, London, 2003).
[6] L. Novotny and B. Hecht, *Principles of Nano-Optics* (Cambridge University Press, Cambridge, 2006).
[7] L. Novotny, in *Progress in Optics*, edited by E. Wolf, Vol. 50 (Elsevier, Amsterdam, 2007), p. 137.
[8] H. M. Nussenzveig, in *Progress in Optics*, edited by E. Wolf, Vol. 50 (Elsevier, Amsterdam, 2007), p. 185.
[9] R. Y. Chiao and A. M. Steinberg, in *Progress in Optics*, edited by E. Wolf, Vol. 37 (North-Holland, Amsterdam, 1997), p. 345.
[10] I. Newton, *Opticks* (Dover, New York, 1952).
[11] G. Quincke, Ann. Phys. Chem. **5**, 1 (1866).
[12] E. E. Hall, Phys. Rev. **15**, 73 (1902).
[13] M. Born, W. Heisenberg, and P. Jordan, Z. Phys. **35**, 557 (1925).
[14] P. A. M. Dirac, Proc. Roy. Soc. A **112**, 661 (1926); **114**, 243 (1927).
[15] C. K. Carniglia and L. Mandel, Phys. Rev. D **3**, 280 (1971).
[16] G. S. Agarwal, Phys. Rev. A **11**, 230 (1975).
[17] O. Keller, *Light: The Physics of the Photons* (Francis and Taylor, London, to appear in 2013).
[18] O. Keller, Phys. Rev. A **60**, 1652 (1999).
[19] L. Mandel, Phys. Rev. **144**, 1071 (1966).
[20] O. Keller, Phys. Rev. A **76**, 062110 (2007).
[21] P. J. Feibelman, Prog. Surf. Sci. **12**, 287 (1982).
[22] O. Keller, J. Opt. Soc. Amer. **12**, 987 (1995); **12**, 997 (1995).
[23] I. Bialynicki-Birula, in *Progress in Optics*, edited by E. Wolf, Vol. 36 (Elsevier, North-Holland, Amsterdam, 1996), p. 245.
[24] O. Keller, Phys. Rep. **411**, 1 (2005).
[25] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms, Introduction to Quantum Electrodynamics* (Wiley-Interscience, New York, 1989).
[26] S. R. DeGroot, *The Maxwell Equations* (North-Holland, Amsterdam, 1969).