

Pulse propagation in a medium of Λ -type atoms

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The propagation of a weak-field pulse through a medium of three-level atoms is considered. Each atom has a Λ -type level scheme in which the two lower levels are stable. A strong control field drives one of the electronic transitions while the signal field drives the coupled transition under conditions where EIT (electromagnetically induced transparency) is usually operative. The input pulse is slowed and compressed as it enters the medium, adiabatically following the EIT solution. However, if the control field is changed suddenly when the pulse is in the medium, the signal field is transformed into two pulses, one of which propagates as a normal EIT pulse and the other with a different speed and an amplitude that oscillates in time. The temporal oscillations are transformed into both spatial and temporal oscillations as the pulse exits the medium. An analytic expression is derived for the pulse intensity which provides a good approximation to the exact result at all times. It is shown that the oscillating component of the exiting pulse can be spatially compressed in comparison with the input pulse.

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I. INTRODUCTION

In a recent article [1], we derived a rather peculiar result. We showed that if a weak, off-resonant pulse is launched from within a low-density medium of two-level atoms having index of refraction approximately equal to unity, there are always two possible propagation speeds, symmetrically centered about $v_g = c/2$, neither of which corresponds to the normal group velocity associated with the propagation of an off-resonant pulse in the medium. However, things somehow return to “normal” for a pulse that is sent *into* the medium from vacuum. In this limit, the atoms and the field remain adiabatically in a dressed state of the atom-field system that propagates with the normal group velocity.

The propagation of off resonant light with reduced group velocity in a medium of two-level atoms is but one example of the way in which a medium can alter the propagation of light. Over the past twenty years or so, there has been increased interest in light-matter interactions that lead to significantly reduced group velocities for the light (slow light) [2]. This work was stimulated in large part by the demonstration of EIT (electromagnetically induced transparency) [3] in atomic vapors [4,5], but since has also led to the observation of slow light in a variety of condensed-matter systems [6], including pure crystals [7], doped crystals [8], optical fibers [9], semiconductor quantum wells [10], photonic crystal waveguides [11], fiber gratings [12], and coupled resonators [13]. The question then arises as to whether or not the propagation effects predicted in Ref. [1] will also manifest themselves in these systems. In this article, we consider pulse propagation in a medium of three-level atoms whose levels are in a Λ configuration. Such a medium serves as a prototypical example in which both EIT and slow light can be observed. We show that effects related to, but not identical with, those in Ref. [1] can be observed in this EIT geometry. Moreover there is a simple way to observe the modified pulse dynamics by changing the amplitude or phase of the control field when the pulse is in the medium. We believe that similar effects should be observable in other systems exhibiting slow light, but each system must be considered on a case by case basis. For example, we

expect the modified pulse dynamics to be somewhat different in systems where the slow light is produced by preparing a medium in which a spectral hole has been burned at the central frequency of the incoming pulse [14,15].

In the conventional EIT geometry that we consider, a weak or signal pulsed field and a cw control field drive coupled transitions in a three-level Λ scheme (see Fig. 1) [3]. The signal field is resonant with the 1-2 transition and the control field with the 2-3 transition. The two lower states of the Λ configuration do not decay. In standard treatments of this problem [3], there is an adiabatic approximation invoked that results in the signal pulse propagating with a group velocity equal to

$$v_{g1} = \frac{c}{1 + \frac{\alpha^2}{|\chi'|^2}}, \quad (1)$$

where χ' is one-half the Rabi frequency associated with the control field and

$$\alpha^2 = \frac{\mathcal{N}\omega_{21}}{2\hbar\epsilon_0} |\mu_{21}|^2 \quad (2)$$

is related to a so-called cooperativity parameter. The quantity \mathcal{N} is the atomic density and μ_{21} is a dipole matrix element. For $\alpha \gg |\chi'|$, the signal corresponds to “slow light,” since $v_{g1} \ll c$.

We will show that pulse propagation in this medium generally consists of two components, owing to the fact that the adiabatic approximation fails as soon as the group velocity of the EIT component deviates significantly from the speed of light. One component propagates with the usual EIT propagation speed v_{g1} , and the other with speed

$$v_{g2} = \frac{1}{2} \frac{c}{1 + \frac{|\chi'|^2}{\alpha^2}}. \quad (3)$$

Moreover, the field amplitude of the pulse having group velocity v_{g2} oscillates as a function of time with frequency $(|\chi'|^2 + \alpha^2)^{1/2}/2$ as it propagates in the medium.

Although two propagation speeds are possible, it turns out that under typical EIT conditions a signal pulse sent *into* the medium from vacuum adiabatically stays in the

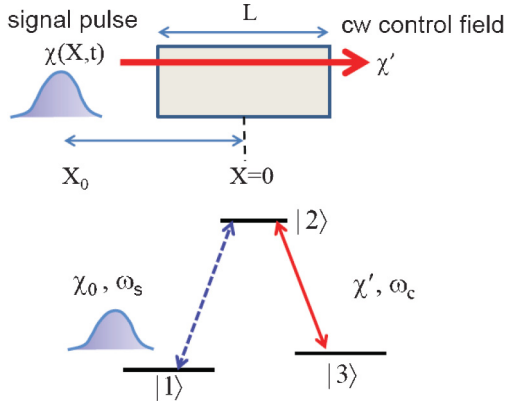


FIG. 1. (Color online) A signal pulse is sent into a medium of three level atoms, each of whose level schemes is shown in the figure. A cw control field is also present. The quantity χ_0 is one half the maximum Rabi frequency associated with the signal field $\chi(X,t)$, whose central frequency is $\omega_s = \omega_{21}$. The quantity χ' is one-half the Rabi frequency associated with the control field, whose frequency is $\omega_c = \omega_{23}$.

state associated with normal EIT propagation. However, by suddenly varying the control field amplitude or phase when the pulse is in the medium, we can convert the EIT pulse into a superposition of pulses propagating at the two speeds v_{g1} and v_{g2} . In fact, for a phase shift of π (sudden change in the sign of the control field amplitude) and for $\chi' = \alpha$, the signal pulse is *entirely* converted from an EIT pulse to the temporally oscillating pulse propagating with speed v_{g2} . When this pulse exits the medium, the temporal oscillations are converted into both spatial and temporal oscillations of the pulse intensity. Moreover, for a proper choice of system parameters, the spatial envelope of the emerging pulse associated with propagation at speed v_{g2} in the medium can be *narrower* than that of the initial pulse. If the initial pulse is a single-photon pulse, we have the possibility of creating a somewhat unusual single-photon output pulse. We should note that Matsko *et al.* [16] discussed the problem of nonadiabatic switching in EIT. Their emphasis was on information storage and retrieval related to the EIT component of the pulse, rather than on the pulse dynamics of the component propagating with speed v_{g2} .

We first derive equations characterizing the system in the slowly varying amplitude and phase approximation. We then follow the pulse as it enters the medium, propagates inside the medium, and leaves the medium. When the pulse enters the medium, it adiabatically stays in the state associated with normal EIT. As such, it is spatially compressed by v_{g1}/c inside the medium and propagates at speed v_{g1} with negligible dispersion. If we left this pulse on its own it would decompress as it exits the medium and reproduce the initial pulse profile. However, when the pulse is in the medium, we suddenly change the (complex) control field amplitude, breaking the adiabatic following that is associated with EIT. As a consequence, the EIT pulse is split into two components, one propagating with the EIT speed v_{g1} and the other with speed v_{g2} . These components emerge from the medium and are decompressed by factors of c/v_{g1} and c/v_{g2} , respectively. The temporal oscillations of the pulse having group velocity v_{g2} are converted into both spatial and temporal oscillations of the outgoing pulse

intensity. For an initial pulse having a Gaussian spatial profile, we derive a simple analytic expression for the pulse amplitude that accurately follows the exact solution at all times.

II. GENERAL FORMALISM

We consider the propagation of a weak signal pulse into a medium of stationary, three-level atoms (see Fig. 1) that are uniformly distributed with density \mathcal{N} in a cylinder of length L (centered at $X = 0$) having cross sectional area σ . The signal pulse drives the 1-2 transition having frequency ω_{21} while a cw control drives the 2-3 transition having frequency ω_{23} . The pulsed signal and cw control electric-field vectors are given by

$$\mathbf{E}_s(X,t) = \frac{1}{2}\hat{\epsilon}_s A(X,t)e^{ik_s(X-ct)} + \text{c.c.}, \quad (4a)$$

$$\mathbf{E}_c(X,t) = \frac{1}{2}\hat{\epsilon}_c E_c e^{ik_c(X-ct)} + \text{c.c.}, \quad (4b)$$

where the signal field pulse amplitude $A(X,t)$ is a slowly varying function of X and t compared with $e^{ik_s(X-ct)}$, the control field amplitude E_c is constant, $\omega_s = k_s c$ is the signal field carrier frequency, $\omega_c = k_c c$ is the control field frequency, and the $\hat{\epsilon}$'s are the field polarizations. We neglect all modes of the radiation field having propagation vectors in other than the \hat{x} direction; that is, we limit our discussion to an effective one-dimensional problem in which scattering into transverse modes of the field is not significant. Although the control field amplitude is taken to be a constant, we will allow for a sudden change in its (complex) amplitude at a time when the signal pulse is in the medium. The index of refraction of the medium is assumed to be equal to unity for the range of frequencies in the input pulse, which is a good approximation under typical EIT conditions.

In a field interaction representation in which density matrix elements are written as [17]

$$\rho_{12}(X,t) = \tilde{\rho}_{12}(X,t)e^{-ik_s(X-ct)}, \quad (5a)$$

$$\rho_{32}(X,t) = \tilde{\rho}_{32}(X,t)e^{-ik_c(X-ct)}, \quad (5b)$$

$$\rho_{13}(X,t) = \tilde{\rho}_{32}(X,t)e^{-i(k_s - k_c)(X-ct)}, \quad (5c)$$

and in the dipole and rotating wave approximations, the full evolution equations for density-matrix elements and the signal field amplitude are given in the Appendix. We are interested here in a more restrictive problem in which the signal field is weak, justifying a perturbation theory approach. In this limit $\rho_{22}(X,t) \approx \rho_{33}(X,t) \approx \tilde{\rho}_{23}(X,t) \approx 0$; $\rho_{11}(X,t) \approx 1$. Spontaneous decay can be neglected as well, provided $\gamma_2 L/c < 1$, where γ_2 is the excited state decay rate. With these approximations and, in addition, assuming that the fields are resonant with their respective transitions, $\omega_s = \omega_{21}$ and $\omega_c = \omega_{23}$, the full equations reduce to

$$\partial \tilde{\rho}_{31}(X,t)/\partial t = -i\chi'^* \tilde{\rho}_{21}(X,t), \quad (6a)$$

$$\partial \tilde{\rho}_{21}(X,t)/\partial t = -i\chi' \tilde{\rho}_{31}(X,t) - i\chi(X,t), \quad (6b)$$

$$\frac{\partial \chi(X,t)}{\partial t} + c \frac{\partial \chi(X,t)}{\partial X} = -i\alpha^2 \tilde{\rho}_{21}(X,t)G(X), \quad (6c)$$

where

$$\chi(X,t) = -\frac{\mu_{21}A(X,t)}{2\hbar}, \quad (7a)$$

$$\chi' = -\frac{\mu_{23}E_c}{2\hbar} \quad (7b)$$

are one-half the Rabi frequencies associated with the signal field and control field transitions, respectively, μ_{23} is a dipole matrix element,

$$G(X) = \Theta(L/2 + X)\Theta(L/2 - X) \quad (8)$$

restricts the atom-field interaction to the volume of the medium [$\Theta(X)$ is a Heaviside function], and α^2 is given by Eq. (2). Equation (6c) is the propagation equation for the field amplitude, obtained in a slowly varying amplitude and phase approximation [17]. Although not included in Eqs. (6), we will allow for a sudden change in the (complex) amplitude of the control field or the introduction of a phase shift ϕ on $\tilde{\rho}_{13}$ at a time when the pulse is in the medium. It should be noted that the density-matrix elements appearing in Eqs. (II) are *single-particle* density-matrix elements for an atom located at position X .

Equations (6) can be solved numerically for the signal field. The initial conditions are chosen such that the signal pulse is centered at $X = -X_0 \ll -L/2$, that is, to the left of the medium, and all atoms are in state $|1\rangle$. Correspondingly, the initial conditions are

$$\tilde{\rho}_{21}(X,0) = \tilde{\rho}_{31}(X,0) = 0, \quad (9)$$

along with some initial pulse profile $\chi(X,0)$ centered at $X = -X_0$. For example, we can choose the initial pulse amplitude to be the Gaussian

$$\chi(X,0) = \pi^{-1/4} \chi_0 e^{-(X+X_0)^2(\Delta k)^2/2}, \quad (10)$$

where χ_0 (assumed real) is one-half the maximum Rabi frequency associated with the signal field transition and $c\Delta k$ is the pulse bandwidth. Consistent with conventional EIT, we assume that the bandwidth is small compared with the control field Rabi frequency,

$$c\Delta k \ll |\chi'|, \quad (11)$$

and that the signal field is weak,

$$\chi_0 \ll c\Delta k. \quad (12)$$

It is sometimes convenient to re-express Eqs. (6) as a single-partial differential equation. Taking two partial time derivatives of Eq. (6c), and using Eqs. (6a) and (6b), we find

$$\begin{aligned} \frac{\partial^3 \chi(X,t)}{\partial t^3} + c \frac{\partial^3 \chi(X,t)}{\partial t^2 \partial X} + |\chi'|^2 \frac{\partial \chi(X,t)}{\partial t} + c |\chi'|^2 \frac{\partial \chi(X,t)}{\partial X} \\ + \alpha^2 \frac{\partial \chi(X,t)}{\partial t} G(X) = 0, \end{aligned} \quad (13)$$

subject to the initial conditions, obtained from Eqs. (6) and (9),

$$\frac{\partial \chi(X,0)}{\partial t} = -c \frac{\partial \chi(X,0)}{\partial X}; \quad (14a)$$

$$\frac{\partial^2 \chi(X,0)}{\partial t^2} = -\alpha^2 \chi(X,0) G(X) + c^2 \frac{\partial^2 \chi(X,0)}{\partial X^2}. \quad (14b)$$

Although we have an exact equation of motion describing pulse propagation into and out of the medium, this equation must be solved numerically. Moreover, the numerical solution becomes time intensive with increasing α and χ' . It turns out, however, that we can derive a relatively simple analytic expression for the pulse amplitude that is in excellent agreement with the exact solution. The method for obtaining this solution is

discussed in the following sections. The analytic expression enables us to understand in a simple manner all the features of pulse propagation in this problem if the control field amplitude is changed suddenly when the pulse is in the medium.

It will prove convenient to introduce the following dimensionless variables:

$$\tau = c\Delta kt, \quad (15a)$$

$$\tilde{X} = \Delta kX, \quad (15b)$$

$$\tilde{X}_0 = \Delta kX_0, \quad (15c)$$

$$\tilde{L} = \Delta kL, \quad (15d)$$

$$\tilde{\alpha} = \alpha / (c\Delta k), \quad (15e)$$

$$\tilde{\chi}' = \chi' / (c\Delta k), \quad (15f)$$

$$\tilde{\chi}(X,t) = \chi(\tilde{X},\tau) / (c\Delta k), \quad (15g)$$

$$\tilde{\chi}_0 = \chi_0 / (c\Delta k), \quad (15h)$$

$$\beta_1 = v_{g_1}/c. \quad (15i)$$

It is assumed that

$$\tilde{\chi}' \gg 1, \quad (16)$$

consistent with the need for a transparency window in EIT that is large compared with the pulse bandwidth. In terms of dimensionless variables, condition (12) is

$$\tilde{\chi}_0 \ll 1. \quad (17)$$

III. THE PULSE ENTERS THE MEDIUM—EIT SOLUTION

When the pulse enters the medium, it stays adiabatically in the state associated with EIT. To prove this, we start from Eq. (13),

$$\begin{aligned} \frac{\partial^3 \tilde{\chi}(\tilde{X},\tau)}{\partial \tau^3} + \frac{\partial^3 \tilde{\chi}(\tilde{X},\tau)}{\partial \tau^2 \partial \tilde{X}} + |\tilde{\chi}'|^2 \frac{\partial \tilde{\chi}(\tilde{X},\tau)}{\partial \tau} + |\tilde{\chi}'|^2 \frac{\partial \tilde{\chi}(\tilde{X},\tau)}{\partial \tilde{X}} \\ + \tilde{\alpha}^2 \frac{\partial \tilde{\chi}(\tilde{X},\tau)}{\partial \tau} G(\tilde{X}) = 0 \end{aligned} \quad (18)$$

with the initial conditions given by Eqs. (14). The solution outside the medium, when $G(\tilde{X}) = 0$ is that of a pulse propagating in vacuum. Inside the medium, where $G(\tilde{X}) = 1$, the adiabatic approximation corresponds to the neglect of the $\frac{\partial^3 \tilde{\chi}(\tilde{X},\tau)}{\partial \tau^3} + \frac{\partial^3 \tilde{\chi}(\tilde{X},\tau)}{\partial \tau^2 \partial \tilde{X}}$ terms in the equation, since these are smaller than the other terms by a factor of $1/|\tilde{\chi}'|^2 \ll 1$. In this adiabatic limit Eq. (18) reduces to

$$\frac{\partial \tilde{\chi}(\tilde{X},\tau)}{\partial \tilde{X}} + \frac{1}{\beta_1} \frac{\partial \tilde{\chi}(\tilde{X},\tau)}{\partial \tau} = 0, \quad (19)$$

where $\beta_1 = v_{g_1}/c$.

In this section, we limit the discussion to propagation into the medium and consider the two regions

$$X < -L/2; \quad \frac{\partial \tilde{\chi}_1(\tilde{X},\tau)}{\partial \tau} + \frac{\partial \tilde{\chi}_1(\tilde{X},\tau)}{\partial \tau} = 0, \quad (20a)$$

$$-L/2 \leq X \leq L/2; \quad \frac{\partial \tilde{\chi}_2(\tilde{X},\tau)}{\partial \tilde{X}} + \frac{1}{\beta_1} \frac{\partial \tilde{\chi}_2(\tilde{X},\tau)}{\partial \tau} = 0. \quad (20b)$$

The solution of these equations must be formulated such that $\tilde{\chi}_2(-\tilde{L}/2,\tau) = \tilde{\chi}_1(-\tilde{L}/2,\tau)$. With this boundary

condition, the solution is

$$\tilde{\chi}_1(\tilde{X}, \tau) = \tilde{\chi}(\tilde{X} - \tau, 0); \quad X < -L/2, \quad (21a)$$

$$\tilde{\chi}_2(\tilde{X}, \tau) = \tilde{\chi} \left[(\tilde{X} - \beta_1 \tau) / \beta_1 + \frac{\beta_0 \tilde{L}}{2\beta_1}, 0 \right]; \quad -L/2 \leq X \leq L/2, \quad (21b)$$

where

$$\beta_0 = 1 - \beta_1 = \frac{\tilde{\alpha}^2}{\tilde{\alpha}^2 + |\chi'|^2}. \quad (22)$$

Equations (21) corresponds to a pulse that is compressed spatially by an amount β_1 as it enters the medium and then propagates in the medium with group velocity v_{g1} .

It is convenient to have the pulse maximum at the center of the medium when $\tau = 0$, since we want to change the control field amplitude at this time. To do so we change the initial time from $\tau = 0$ to $\tau = \tau_0$ with

$$\tau_0 = - \left[\left(\tilde{X}_0 - \frac{\tilde{L}}{2} \right) + \frac{\tilde{L}}{2\beta_1} \right] = - \left(\tilde{X}_0 + \frac{\beta_0 \tilde{L}}{2\beta_1} \right). \quad (23)$$

The incident pulse is centered at $-\tilde{X}_0 \ll -L/2$ at $\tau = \tau_0$. The net effect of the shift of the time origin is that the solution (21) is changed to

$$\tilde{\chi}_1(\tilde{X}, \tau) = \tilde{\chi}[\tilde{X} - (\tau - \tau_0), \tau_0]; \quad X < -L/2, \quad (24a)$$

$$\tilde{\chi}_2(\tilde{X}, \tau) = \tilde{\chi} \left[[\tilde{X} - \beta_1(\tau - \tau_0)] / \beta_1 + \frac{\beta_0 \tilde{L}}{2\beta_1}, \tau_0 \right]; \quad -L/2 \leq X \leq L/2. \quad (24b)$$

We assume that the pulse is entirely in the medium at $\tau = 0$, but the theory can be modified easily to allow for the more general case where the pulse compression is not sufficient to guarantee this possibility.

Using Eq. (23), we find that inside the medium

$$\tilde{\chi}_2(\tilde{X}, \tau) = \tilde{\chi} \left(\frac{\tilde{X}}{\beta_1} - \tau - \tilde{X}_0, \tau_0 \right). \quad (25)$$

For the initial Gaussian pulse

$$\tilde{\chi}(\tilde{X}, \tau_0) = \tilde{\chi}_0 \pi^{-1/4} e^{-(\tilde{X} + \tilde{X}_0)^2 / 2}, \quad (26)$$

centered at $\tilde{X} = -\tilde{X}_0 < -\tilde{L}/2$ at $\tau = \tau_0$, we obtain

$$\tilde{\chi}_2(\tilde{X}, 0) = \tilde{\chi}_0 \pi^{-1/4} e^{-\tilde{X}^2 / (2\beta_1^2)}. \quad (27)$$

As desired, the pulse is centered at $\tilde{X} = 0$ when $\tau = 0$. For $\tilde{\chi}' \gtrsim 10$, the difference between the exact solution, obtained from Eq. (13) and the approximate solution (25) is negligible.

Moreover, it follows from Eqs. (25), (6c), and (6b) that

$$\tilde{\rho}_{21}(\tilde{X}, 0) \sim -i\pi^{-1/4} \frac{\tilde{\chi}_0}{|\tilde{\chi}'|^2 \beta_1} e^{-\tilde{X}^2 / (2\beta_1^2)} \approx 0; \quad (28)$$

$$\tilde{\rho}_{31}(\tilde{X}, 0) \approx -\frac{\tilde{\chi}_2(\tilde{X}, 0)}{\tilde{\chi}'},$$

the standard EIT results. In the following section, Eqs. (26) and (28), with some modification, are used as initial conditions for a pulse centered at $X = 0$ in the medium.

IV. PULSE INSIDE AND EXITING THE MEDIUM—CHANGING THE CONTROL FIELD AMPLITUDE

Now that the pulse has entered the medium, it propagates as a normal EIT pulse. If we leave it on its own, it will decompress by an amount $1/\beta_1$ as it exits the medium and reproduce the initial pulse profile. However, if we modify the control field or the value of $\tilde{\rho}_{31}(\tilde{X}, \tau)$ suddenly (in a time less than $1/\tilde{\alpha}$) at $\tau = 0$, dramatic changes can occur in the subsequent evolution of the pulse. For example, suppose we change the (complex) amplitude of the control field amplitude at $\tau = 0$ such that, for times $\tau > 0$, (one-half) its Rabi frequency is given by

$$\tilde{\chi}'_+ = \tilde{\chi}'(\tau > 0) = \xi \tilde{\chi}'(\tau < 0) \equiv \xi \tilde{\chi}', \quad (29)$$

where the complex parameter ξ reflects the sudden change in the control field and $\tilde{\chi}'$ always refers to the control field amplitude for times $\tau < 0$. The ground-state coherence does *not* change when the control field is changed suddenly, but when expressed in terms of the *new* control field amplitude $\tilde{\chi}'(\tau > 0)$, $\tilde{\rho}_{31}(\tilde{X}, 0)$ no longer corresponds to an EIT solution. In other words, Eqs. (28) become

$$\tilde{\rho}_{21}(\tilde{X}, 0) \approx 0; \quad (30a)$$

$$\tilde{\rho}_{31}(\tilde{X}, 0) \approx -\frac{\tilde{\chi}(\tilde{X}, 0)}{\tilde{\chi}'} = -\frac{\xi \tilde{\chi}(\tilde{X}, 0)}{\tilde{\chi}'_+}. \quad (30b)$$

Unless $\xi = 1$, the initial conditions no longer correspond to EIT conditions for $\tau > 0$. Alternatively, we could actually change the phase of the ground-state coherence by applying a short off-resonant pulse that acts only on level 3, producing an ac Stark induced phase shift ϕ of the state 3 amplitude. In that case, we would take as our new initial conditions

$$\tilde{\rho}_{21}(\tilde{X}, 0) \approx 0; \quad (31a)$$

$$\tilde{\rho}_{31}(\tilde{X}, 0^+) \approx e^{i\phi} \tilde{\rho}_{31}(\tilde{X}, 0^-) = -e^{i\phi} \frac{\tilde{\chi}(\tilde{X}, 0)}{\tilde{\chi}'}. \quad (31b)$$

For the sake of definiteness, we will assume that it is the control field amplitude which is changed such that $\tilde{\chi}'(\tau > 0) = \xi \tilde{\chi}'$ and the initial conditions are given by Eqs. (30).

To follow the dynamics of the pulse within the medium, we set $G(\tilde{X}) = 1$ in Eq. (6c) and replace $\tilde{\chi}'$ by $\xi \tilde{\chi}'$. Defining

$$\tilde{\chi}(\tilde{X}, \tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\tilde{\delta}_\kappa f(\tilde{\delta}_\kappa, \tau) e^{-i\tilde{\delta}_\kappa(\tilde{X} - \tau)}, \quad (32)$$

$$\tilde{\rho}_{ij}(\tilde{X}, \tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\tilde{\delta}_\kappa \tilde{\rho}_{ij}(\tilde{\delta}_\kappa, \tau) e^{-i\tilde{\delta}_\kappa \tilde{X}}, \quad (33)$$

where

$$\tilde{\delta}_\kappa = \frac{\omega_{21} - \omega_k}{c \Delta k}, \quad (34)$$

we find that Eqs. (6) can be transformed into

$$\partial \tilde{\rho}_{31}(\tilde{\delta}_\kappa, \tau) / \partial \tau = -i (\xi \tilde{\chi}')^* \tilde{\rho}_{21}(\tilde{\delta}_\kappa, \tau), \quad (35a)$$

$$\partial \tilde{\rho}_{21}(\tilde{\delta}_\kappa, \tau) / \partial \tau = -i \xi \tilde{\chi}' \tilde{\rho}_{31}(\tilde{\delta}_\kappa, \tau) - i f(\tilde{\delta}_\kappa, \tau) e^{i\tilde{\delta}_\kappa \tau}, \quad (35b)$$

$$\frac{\partial f(\tilde{\delta}_\kappa, \tau)}{\partial \tau} = -i \tilde{\alpha}^2 \tilde{\rho}_{21}(\tilde{\delta}_\kappa, \tau) e^{-i\tilde{\delta}_\kappa \tau}, \quad (35c)$$

which, in turn, can be reduced to the ordinary differential equation for $f(\tilde{\delta}_\kappa, \tau)$,

$$\frac{\partial^3 f(\tilde{\delta}_\kappa, \tau)}{\partial \tau^3} + 2i\tilde{\delta}_\kappa \frac{\partial^2 f(\tilde{\delta}_\kappa, \tau)}{\partial \tau^2} - (\tilde{\delta}_\kappa^2 - |\xi \chi'|^2 - \tilde{\alpha}^2) \frac{\partial f(\tilde{\delta}_\kappa, \tau)}{\partial \tau} + i\tilde{\delta}_\kappa \tilde{\alpha}^2 f(\tilde{\delta}_\kappa, \tau) = 0, \quad (36)$$

with initial conditions

$$f(\tilde{\delta}_\kappa, 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\tilde{X} \tilde{\chi}(\tilde{X}, 0) e^{i\tilde{\delta}_\kappa \tilde{X}}, \quad (37a)$$

$$\partial f(\tilde{\delta}_\kappa, 0)/\partial \tau = 0, \quad (37b)$$

$$\partial^2 f(\tilde{\delta}_\kappa, 0)/\partial \tau^2 = -\tilde{\alpha}^2 (1 - \xi) f(\tilde{\delta}_\kappa, 0), \quad (37c)$$

where Eqs. (30) were used.

With a trial solution

$$f(\tilde{\delta}_\kappa, \tau) = \sum_{j=1}^3 f_j e^{i\mu_j \tau}, \quad (38)$$

we are led to the characteristic equation

$$\mu^3 + 2\tilde{\delta}_\kappa \mu^2 + (\tilde{\delta}_\kappa^2 - \tilde{\alpha}^2 - |\xi \chi'|^2) \mu - \tilde{\delta}_\kappa \tilde{\alpha}^2 = 0. \quad (39)$$

The roots can be calculated exactly but are not written explicitly here, owing to their complexity. Once the roots are determined, the expansion coefficients f_j can be determined from Eqs. (38) and (37) as solutions of

$$\sum_{j=1}^3 f_j = f(\tilde{\delta}_\kappa, 0), \quad (40a)$$

$$\sum_{j=1}^3 \mu_j f_j = 0, \quad (40b)$$

$$\sum_{j=1}^3 \mu_j^2 f_j = \tilde{\alpha}^2 (1 - \xi) f(\tilde{\delta}_\kappa, 0). \quad (40c)$$

The field is then obtained using Eq. (32). Our results are expressed in terms of a dimensionless intensity defined by

$$I(\tilde{X}, \tau) = |\tilde{\chi}(\tilde{X}, \tau)/\tilde{\chi}_0|^2. \quad (41)$$

Thus, the exact solution $I_{\text{in}}(\tilde{X}, \tau)$ inside the medium is given by Eqs. (41), (32), (38), (39), and (40) with $f(\tilde{\delta}_\kappa, 0)$ obtained as the Fourier transform of the EIT solution (25) at $\tau = 0$.

We can use the fact that, consistent with Eq. (11), the detuning $\tilde{\delta}_\kappa$ is small compared with $\tilde{\chi}'$ to obtain a very good approximation to the exact solution. We expand the roots about $\tilde{\delta}_\kappa = 0$ and find

$$\mu_1 \approx \frac{\tilde{\Omega}}{2} - \frac{\tilde{\alpha}^2 + 2|\xi \chi'|^2}{2(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)} \tilde{\delta}_\kappa + \frac{\tilde{\alpha}^2(\tilde{\alpha}^2 + 4|\xi \chi'|^2)}{8(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)^{5/2}} \tilde{\delta}_\kappa^2, \quad (42a)$$

$$\mu_2 \approx -\frac{\tilde{\Omega}}{2} - \frac{\tilde{\alpha}^2 + 2|\xi \chi'|^2}{2(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)} \tilde{\delta}_\kappa - \frac{\tilde{\alpha}^2(\tilde{\alpha}^2 + 4|\xi \chi'|^2)}{8(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)^{5/2}} \tilde{\delta}_\kappa^2, \quad (42b)$$

$$\mu_3 \approx -\frac{4\tilde{\alpha}^2}{\tilde{\Omega}^2} \tilde{\delta}_\kappa, \quad (42c)$$

where

$$\tilde{\Omega} = 2\sqrt{\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2}. \quad (43)$$

Terms of order $\tilde{\delta}_\kappa^2$ in Eqs. (42) have been retained to allow for corrections resulting from dispersion. Given the initial condition (37a), with $\tilde{\chi}(\tilde{X}, 0)$ given by Eq. (27), values of $\tilde{\delta}_\kappa$ less than or of order $1/\beta_1$ contribute to the Fourier integrals. When $\tilde{\alpha} > |\xi \tilde{\chi}'|$, the dispersion terms in Eqs. (42a) and (42b) are of order $\tilde{\alpha}^3/|\tilde{\chi}'|^4$ when $\tilde{\delta}_\kappa = 1/\beta_1$; these terms become significant for times $\tau \sim |\tilde{\chi}'|^4/\tilde{\alpha}^3$. On the other hand, the terms of third order in $\tilde{\delta}_\kappa$ are of order $1/|\tilde{\chi}'|^2$ and contribute only if the pulse remains in the medium for times $\tau \gtrsim |\tilde{\chi}'|^2$. The major corrections to dispersion arise from the terms of order $\tilde{\delta}_\kappa^4$ in roots μ_1 and μ_2 .

The approximate solution, obtained from Eqs. (38), (40), and (42) is then

$$f_{\text{ap}}(\tilde{\delta}_\kappa, \tau) = \exp\left(-2i\frac{\tilde{\alpha}^2 + 2|\xi \chi'|^2}{\tilde{\Omega}^2} \tilde{\delta}_\kappa \tau\right) (f_1^{\text{ap}} e^{ip\tilde{\delta}_\kappa^2 \tau/2} e^{i\tilde{\Omega}\tau/2} + f_2^{\text{ap}} e^{-ip\tilde{\delta}_\kappa^2 \tau/2} e^{-i\tilde{\Omega}\tau/2}) + f_3^{\text{ap}} \exp\left[-\frac{4i\tilde{\alpha}^2}{\tilde{\Omega}^2} \tilde{\delta}_\kappa \tau\right], \quad (44)$$

where

$$p = \tilde{\alpha}^2 \frac{(\tilde{\alpha}^2 + 4|\xi \chi'|^2)}{4(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)^{5/2}}, \quad (45)$$

and, to zeroth order in $\tilde{\delta}_\kappa$, the amplitudes f_j , obtained from the solution of Eqs. (40), are found to be

$$f_1^{\text{ap}} = f_2^{\text{ap}} = \frac{\tilde{\alpha}^2(1 - \xi)}{2(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)} f(\tilde{\delta}_\kappa, 0), \quad (46a)$$

$$f_3^{\text{ap}} = \frac{(|\xi \tilde{\chi}'|^2 + \tilde{\alpha}^2 \xi)}{(\tilde{\alpha}^2 + |\xi \tilde{\chi}'|^2)} f(\tilde{\delta}_\kappa, 0). \quad (46b)$$

The corrections to f_1^{ap} and f_2^{ap} are linear in $\tilde{\delta}_\kappa$, while the corrections to f_3^{ap} are of order $\tilde{\delta}_\kappa^2$; neither are important for the range of parameters considered in this work. It is seen that, unless $\xi = 1$, the non-EIT component of the signal field is comparable to the EIT component whenever $\tilde{\alpha} > \tilde{\chi}'$, that is, whenever the condition for slow light is satisfied.

The approximate field amplitude is given as the Fourier transform (32) of Eq. (44). For other than a Gaussian pulse, the field amplitude must be calculated numerically. However, for the Gaussian input field amplitude given by Eq. (27) the corresponding amplitude in $\tilde{\delta}_\kappa$ space is also Gaussian,

$$f(\tilde{\delta}_\kappa, 0) = \tilde{\chi}_0 \beta_1 \pi^{-1/4} e^{-\tilde{\delta}_\kappa^2 \beta_1^2/2}. \quad (47)$$

Using Eqs. (41), (32), and (44)–(47), we then find

$$I_{\text{in}}^{\text{ap}}(\tilde{X}, \tau) = \left| \pi^{-1/4} \frac{e^{i\tilde{\Omega}\tau/2}}{w_1} \frac{\tilde{\alpha}^2(1-\xi)}{2(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \exp\left[-\frac{(\tilde{X} - \tilde{\beta}_2\tau)^2}{2w_1^2\beta_1^2}\right] + \pi^{-1/4} \frac{e^{-i\tilde{\Omega}\tau/2}}{w_2} \frac{\tilde{\alpha}^2(1-\xi)}{2(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \exp\left[-\frac{(\tilde{X} - \tilde{\beta}_2\tau)^2}{2w_2^2\beta_1^2}\right] + \pi^{-1/4} \frac{(|\xi\tilde{\chi}'|^2 + \tilde{\alpha}^2\xi)}{(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \exp\left[-\frac{(\tilde{X} - \tilde{\beta}_1\tau)^2}{2\beta_1^2}\right] \right|^2, \quad (48)$$

where

$$\tilde{\beta}_1 = \frac{|\xi\tilde{\chi}'|^2}{\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2}, \quad (49a)$$

$$\tilde{\beta}_2 = \frac{\tilde{\alpha}^2}{2(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \quad (49b)$$

are the two propagation speeds and

$$w_1 = \sqrt{1 - i p \tau / \beta_1^2}, \quad (50a)$$

$$w_2 = \sqrt{1 + i p \tau / \beta_1^2}. \quad (50b)$$

The quantity $\beta_1|w_1| = \beta_1|w_2|$ gives the spatial width of the non-EIT component at time τ . Notice that the EIT component in Eq. (48) vanishes if $\tilde{\alpha} = |\tilde{\chi}'|$ and $\xi = -1$.

If we neglect dispersion by setting $p = 0$, we find that Eq. (48) reduces to

$$I_{\text{in}}^{\text{nd}}(\tilde{X}, \tau) = \pi^{-1/2} \left| \frac{(|\xi\tilde{\chi}'|^2 + \tilde{\alpha}^2\xi)}{(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} e^{-(\tilde{X} - \tilde{\beta}_1\tau)^2/2\beta_1^2} + \frac{\tilde{\alpha}^2(1-\xi)\cos(\tilde{\Omega}\tau/2)}{(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} e^{-(\tilde{X} - \tilde{\beta}_2\tau)^2/2\beta_1^2} \right|^2, \quad (51)$$

where the *nd* superscript indicates that dispersion is neglected. The corresponding solution for an arbitrary initial pulse shape is

$$I_{\text{in}}^{\text{nd}}(\tilde{X}, \tau) = \left| \frac{(|\xi\tilde{\chi}'|^2 + \tilde{\alpha}^2\xi)}{(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \frac{\tilde{\chi}(\tilde{X} - \tilde{\beta}_1\tau, 0)}{\tilde{\chi}_0} + \frac{\tilde{\alpha}^2(1-\xi)\cos(\tilde{\Omega}\tau/2)}{(\tilde{\alpha}^2 + |\xi\tilde{\chi}'|^2)} \frac{\tilde{\chi}(\tilde{X} - \tilde{\beta}_2\tau, 0)}{\tilde{\chi}_0} \right|^2. \quad (52)$$

The solution consists of the absolute square of the sum of two terms, one that propagates at the normal EIT speed and

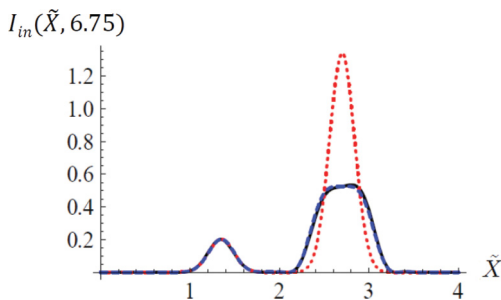


FIG. 2. (Color online) Field intensity in the medium for $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, $\xi = -1$, and $\tau = 6.75$. Solution including dispersion (dashed blue line), solution neglecting dispersion (dotted red line), and the exact solution (solid black line).

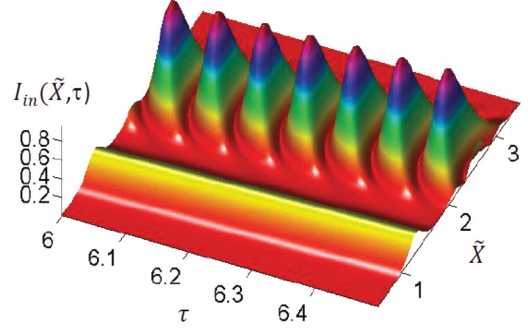


FIG. 3. (Color online) Field intensity in the medium for $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, $\xi = -1$, and τ in the range 6.0–6.5. The EIT component is present, as is the oscillating component.

the other that propagates with speed $\tilde{\beta}_2$ and oscillates with frequency $\tilde{\Omega}/2$. There can be interference between the two components at early times, but the two components separate with increasing τ , owing to their different propagation speeds. The oscillation represents an exchange between the signal field and the control field; any decrease in the signal field is accompanied by gain in the control field and a decrease in the state 3 population.

In Fig. 2 we show the results for $\xi = -1$ (realized by a sudden change in the phase of the control field by π at $\tau = 0$), $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, and $\tau = 6.75$. For these values $\tilde{\beta}_1 = \beta_1 = 0.2$ and $\tilde{\beta}_2 = 0.4$. The solution including dispersion (dashed line), solution neglecting dispersion (dotted line), and the exact solution (solid line) (obtained by using the exact solutions for the roots and amplitudes f_j , and then numerically taking the Fourier transform) are shown. Both components of the pulse are seen, with the EIT component centered at $\tilde{X} = \tilde{\beta}_1\tau = 0.135$ and the other component at $\tilde{X} = \tilde{\beta}_2\tau = 0.27$. The solution including dispersion is approximately equal to the exact solution, but the solution neglecting dispersion departs from the exact solution for the non-EIT component. The solution neglecting dispersion would approach the exact solution if we were to increase $\tilde{\chi}'$ to 1000, while keeping the ratio $\tilde{\chi}'/\tilde{\alpha}$ constant. The oscillation in time of the second component is indicated in Fig. 3 where the solution is shown

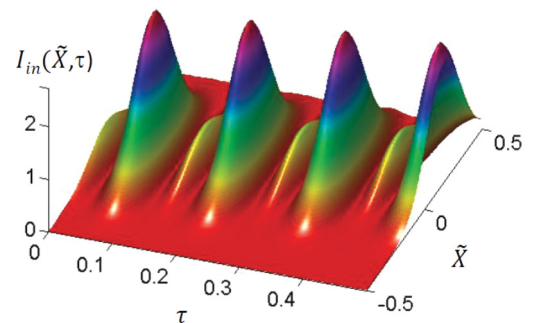


FIG. 4. (Color online) Field intensity in the medium for $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, $\xi = -1$, and τ in the range 0–0.5. Interference between the two pulse components is seen.

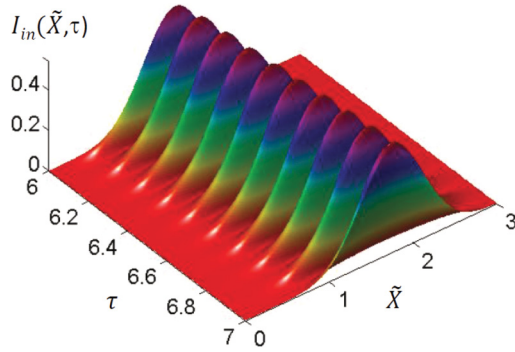


FIG. 5. (Color online) Field intensity in the medium for $\tilde{\chi}' = \tilde{\alpha} = 20$, $\xi = -1$, and τ in the range 6.0–7.0. The EIT component is no longer present.

for $\xi = -1$, $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, and τ in the range 6.0–6.5. The broadening and distortion of the non-EIT pulse wave fronts is

$$I(\tilde{X}, \tau) = \begin{cases} I_{\text{EIT}}(\tilde{X}, \tau); & \tau < 0, \\ I_{\text{in}}(\tilde{X}, \tau); & -L/2 \leq X \leq L/2; \tau \geq 0, \\ I_{\text{in}}[\tilde{L}/2, \tau - (\tilde{X} - \tilde{L}/2)]; & X \geq L/2; \tau \geq 0, \end{cases} \quad (53)$$

where

$$I_{\text{EIT}}(\tilde{X}, \tau) = \begin{cases} |\tilde{\chi}[\tilde{X} - (\tau - \tau_0), \tau_0]/\tilde{\chi}_0|^2; & X < -L/2; \tau < 0, \\ |\tilde{\chi}(\frac{\tilde{X}}{\tilde{\beta}_1} - \tau - \tilde{X}_0, \tau_0)/\tilde{\chi}_0|^2; & -L/2 \leq X \leq L/2; \tau < 0, \end{cases} \quad (54)$$

and τ_0 is given by Eq. (23). The corresponding equations for the approximate solution including dispersion and neglecting dispersion are obtained by changing I_{in} to $I_{\text{in}}^{\text{ap}}$ and $I_{\text{in}}^{\text{hd}}$, respectively. The graphs shown below are for $I_{\text{in}}^{\text{ap}}$ which is very nearly equal to the exact results for the range of parameters considered.

In Fig. 6 we graph the solution (53) for $\xi = -1$, $\tilde{\chi}' = 20$, $\tilde{\alpha} = 20$, $\tilde{L} = 4$, and $\tau = 16$, that is, for a time when the pulse has emerged totally from the medium; for these values $\tilde{\beta}_1 = \beta_1 = 0.5$ and $\tilde{\beta}_2 = 0.25$. The EIT component is not present, and the temporal oscillations of the remaining component have been converted to spatial oscillations. Moreover, the pulse

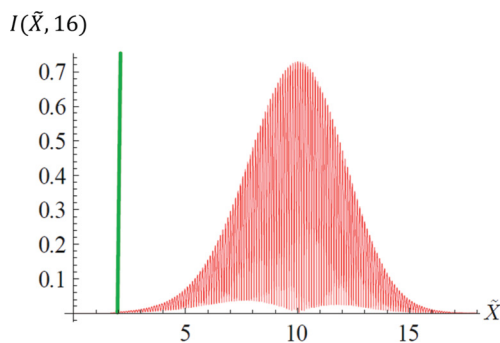


FIG. 6. (Color online) Field intensity exiting the medium with $\tilde{\chi}' = 20$, $\tilde{\alpha} = 20$, $\xi = -1$, $\tilde{L} = 4$, and $\tau = 16$. The green vertical line is the end of the medium. The EIT component is absent and the other component is spatially broadened.

a result of dispersion. The interference of the two components is illustrated in Fig. 4 with $\xi = -1$, $\tilde{\chi}' = 20$, $\tilde{\alpha} = 40$, and τ in the range 0–0.5. Finally, in Fig. 5 we show the solution for $\xi = -1$, $\tilde{\chi}' = \tilde{\alpha} = 20$, and τ in the range 6.0–7.0. The EIT component is no longer present, having been converted totally to the oscillating component. For these parameters, $\tilde{\beta}_1 = \beta_1 = 0.5$ and $\tilde{\beta}_2 = 0.25$ such that the “fast component” actually propagates with a speed $\tilde{\beta}_2 = 0.25$ that is half that of the EIT component (were it present). Dispersion plays a less important role in Figs. 4 and 5 than it does in Figs. 2 and 3.

A. Exiting the medium

When the pulse exits the medium, it propagates as a free radiation pulse. Since the index of refraction is equal to unity, there is no reflection. We can write the entire solution as

is decompressed by a factor $1/\tilde{\beta}_2$ on leaving the medium, which implies that its spatial width is $\tilde{\beta}_1/\tilde{\beta}_2 = 2$ times that of the pulse *initially* sent into the medium. In other words, the pulse is spatially *broadened* by a factor of 2 compared with the initial pulse. In Fig. 7 the solution is shown for $\xi = -1$, $\tilde{\chi}' = 40$, $\tilde{\alpha} = 160$, $\tilde{L} = 4$, and $\tau = 40$ ($\tilde{\beta}_1 = \beta_1 = 0.059$ and $\tilde{\beta}_2 = 0.47$). Both pulse components are now present. The width of the EIT component equals that of the incident pulse. Had dispersion been negligible, the width of the oscillating component would have been $\tilde{\beta}_2/\tilde{\beta}_1 = 8$ times smaller than that of the incident pulse; however dispersion results in a broadening of this component by a factor of approximately 2.2,

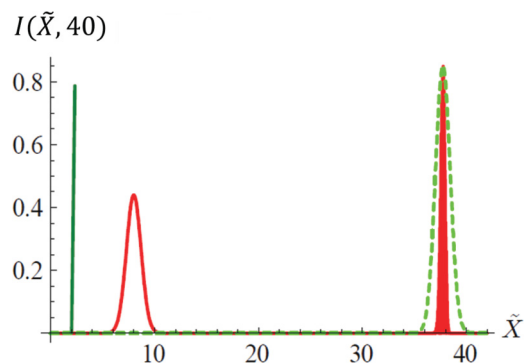


FIG. 7. (Color online) Field intensity exiting the medium with $\tilde{\chi}' = 40$, $\tilde{\alpha} = 160$, $\xi = -1$, $\tilde{L} = 4$, and $\tau = 40$. The EIT component is now present and the other component is spatially narrowed (the dashed green curve represents the original pulse width).

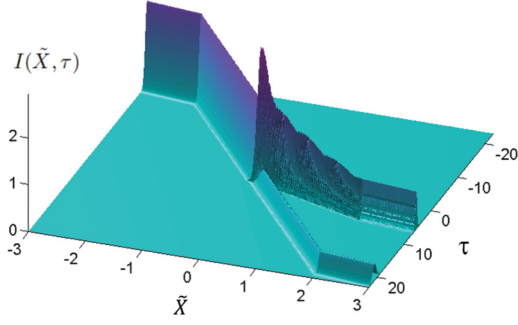


FIG. 8. (Color online) Field intensity dynamics with $\tilde{\chi}' = 20$, $\tilde{\alpha} = 60$, $\xi = -1$, and $\tilde{L} = 4$.

although it is still narrower than the incident pulse (drawn for convenience in the figure). The EIT component is centered at $\tilde{X} = \tilde{L}/2 + \tau - \tilde{L}/(2\tilde{\beta}_1) = 8.1$ and the oscillating component at $\tilde{X} = \tilde{L}/2 + \tau - \tilde{L}/(2\tilde{\beta}_2) = 37.75$. Finally in Fig. 8 we show the entire evolution for $\xi = -1$, $\tilde{\chi}' = 20$, $\tilde{\alpha} = 60$, and $\tilde{L} = 4$ ($\beta_1 = \beta_1 = 0.1$; $\beta_2 = 0.45$). The oscillating component emerges much earlier from the medium than the EIT component. The large peaks near $\tau = 0$ correspond to constructive interference between the EIT and oscillating component.

V. DISCUSSION

Normally, in applications involving slow light, one wants large values of α/χ' , since the speed of light in the medium is reduced by this factor. To observe the effects outlined in this paper, it is best to restrict α/χ' to a value of 10 or so, since dispersion reduces the amplitude of the oscillating component. We have shown that propagation of a weak probe field in a medium of Λ system atoms also subjected to a strong control field consists of two components. One component propagates with the “normal” EIT or slow light velocity, while the other component, which oscillates as a function of time as it propagates in the medium, has a maximum propagation velocity equal to $c/2$. When this component leaves the medium, the temporal oscillations are converted into both spatial and temporal oscillations. If a single photon pulse is used as the input, the output field can be in superposition of spatially separated components. Any losses from excited state decay have been neglected.

To observe these effects, it is necessary that $\alpha \gtrsim \chi' \gg \gamma_2$; moreover, to see the oscillations, one requires that $\alpha L/c \gtrsim 1$. Optimal conditions might involve atoms having $\gamma_2 \approx 10^7 \text{ s}^{-1}$, $\chi' \approx 10^{10} \text{ s}^{-1}$, $\alpha \approx 10^{10} \text{ s}^{-1}$ which would require a sample size on the order of a centimeter or so. Field switching would then have to be on a time scale of picoseconds. With larger sample sizes, some of these frequencies can be reduced. It appears that the system parameters in the experiment of Kasapi *et al.* [18] might be favorable. In that experiment a 10-cm-long Pb vapor cell was used with a (pulsed) control field Rabi frequency in excess of 10^{10} s^{-1} and a value of α that was roughly 13 times χ' at an atomic density of $2 \times 10^{14} \text{ atoms/cm}^3$.

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APPENDIX

The full three-level dynamics is governed by the equations [17]

$$\partial \rho_{11}(X,t)/\partial t = i [\chi(X,t)\tilde{\rho}_{12}(X,t) - \chi^*(X,t) \times \tilde{\rho}_{21}(X,t)] + \gamma_{2,1}\rho_{22}(X,t), \quad (\text{A1a})$$

$$\partial \tilde{\rho}_{33}(X,t)/\partial t = i [\chi' \tilde{\rho}_{32}(X,t) - \chi'^* \tilde{\rho}_{23}(X,t)] + \gamma_{2,3}\rho_{22}(X,t), \quad (\text{A1b})$$

$$\partial \rho_{22}(X,t)/\partial t = -i [\chi(X,t)\tilde{\rho}_{12}(X,t) - \chi^* \tilde{\rho}_{21}(X,t)] - i(\chi' \tilde{\rho}_{32} - \chi'^* \tilde{\rho}_{23}) - \gamma_2 \rho_{22}, \quad (\text{A1c})$$

$$\partial \tilde{\rho}_{13}(X,t)/\partial t = i \chi' \tilde{\rho}_{12}(X,t) - i \chi(X,t)^* \tilde{\rho}_{23}(X,t) - [\gamma_{13} + i(\delta - \delta')] \tilde{\rho}_{13}, \quad (\text{A1d})$$

$$\partial \tilde{\rho}_{12}(X,t)/\partial t = i \chi^* \tilde{\rho}_{13}(X,t) - i \chi(X,t)^* [\rho_{22}(X,t) - \rho_{11}(X,t)] - (\gamma_{12} - i\delta) \tilde{\rho}_{12}(X,t), \quad (\text{A1e})$$

$$\partial \tilde{\rho}_{32}(X,t)/\partial t = i \chi(X,t)^* \tilde{\rho}_{31}(X,t) - i \chi'^* [\rho_{22}(X,t) - \rho_{33}(X,t)] - (\gamma_{32} - i\delta') \tilde{\rho}_{32}(X,t), \quad (\text{A1f})$$

$$\tilde{\rho}_{ji}(X,t) = \tilde{\rho}_{ij}^*(X,t), \quad (\text{A1g})$$

$$\frac{\partial \chi(X,t)}{\partial X} + \frac{1}{c} \frac{\partial \chi(X,t)}{\partial t} = -i \alpha^2 \frac{\tilde{\rho}_{21}(X,t)}{c} G(X), \quad (\text{A1h})$$

where

$$\chi(X,t) = -\frac{\mu_{21}A(X,t)}{2\hbar}, \quad (\text{A2a})$$

$$\chi' = -\frac{\mu_{23}E_c}{2\hbar}, \quad (\text{A2b})$$

are one-half the Rabi frequencies associated with the signal field and control field transitions, respectively,

$$\delta = \omega_{21} - \omega_s, \quad (\text{A3a})$$

$$\delta' = \omega_{23} - \omega_c, \quad (\text{A3b})$$

are atom-field detunings, $\gamma_2 = \gamma_{2,3} + \gamma_{2,1}$ is the excited-state decay rate, $\gamma_{2,j}$ is the partial decay rate from level 2 to level j , and γ_{ij} is the decay rate associated with the ij atomic coherence, μ_{23} is a dipole matrix element,

$$G(X) = \Theta(L/2 + X)\Theta(L/2 - X) \quad (\text{A4})$$

restricts the atom-field interaction to the volume of the medium [$\Theta(X)$ is a Heaviside function], and α^2 is given by Eq. (2). Equation (A1h) is the propagation equation for the field amplitude, obtained in a slowly varying amplitude and phase approximation [17].

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