Transparency and slow light in a four-level quantum system near a plasmonic nanostructure

Sofia Evangelou, Vassilios Yannopapas, and Emmanuel Paspalakis*

Materials Science Department, School of Natural Sciences, University of Patras, 265 04 Patras, Greece

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We study theoretically the effects of a plasmonic nanostructure on the linear absorption and dispersion spectrum of a four-level double- V-type quantum system. In the quantum system under study one V-type transition is influenced by the interaction with surface plasmons while the other V-type transition interacts with free-space vacuum. As plasmonic nanostructure we consider a two-dimensional array of metal-coated dielectric nanospheres. We analyze the optical properties of a linearly polarized laser field that couples the lowest state with the upper states in the free-space transitions. We show that, due to the presence of the plasmonic nanostructure, effects of optical transparency are created. These effects are also complemented by the existence of slow light.

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I. INTRODUCTION

Recently, there has been significant interest in the interaction of light with quantum emitters, such as atoms, molecules, and quantum dots, near various plasmonic nanostructures. Examples of the phenomena studied include the modification of spontaneous emission [1–6], the alteration of resonance fluorescence [7–11], the creation of quantum interference effects in spontaneous emission [12–16], the creation and enhancement of entanglement [17–20], and resonance energy transfer [21–24] between distant quantum systems. In addition, the enhancement of several nonlinear optical phenomena, such as up-conversion processes [25], Kerr nonlinearity [26], nonlinear optical rectification [27], second-order harmonic generation [28], difference-frequency generation [29], fourwave mixing [30], and gain without inversion [31], has been studied in coupled quantum and plasmonic systems.

In this work we study the linear absorption and dispersion properties of a four-level double- V-type quantum system near a plasmonic nanostructure. In the system under study one V-type transition is influenced by the interaction with surface plasmons while the other V-type transition interacts with free-space vacuum. As plasmonic nanostructure we consider a two-dimensional array of metal-coated dielectric nanospheres wherein we calculate the relevant decay rates by a rigorous electromagnetic Green's tensor technique. The structure also interacts with a linearly polarized laser field, which couples the lowest state with the upper states in the free-space transitions. Using a density matrix methodology for the theoretical description of the absorption and dispersion properties, we show that, due to the presence of the plasmonic nanostructure, effects of optical transparency are created. These effects are also accompanied by the existence of slow light. In addition, we explore the dependence of these phenomena on the distance of the quantum system from the plasmonic nanostructure.

We note that recently Hatef and Singh studied the effects of spontaneous-emission interference in the absorption and dispersion properties of metallic photonic crystals doped with quantum dots in the V-type configuration interacting with either a single probe laser field or a probe and a coupling laser field [32]. In this work effects such as induced transparency and gain without inversion were identified. In addition, Singh and co-workers have shown that induced transparency can occur in the energy absorption spectra of quantum dots in the V-type configuration interacting with a probe and a coupling laser field near metallic nanoparticles [33]. Furthermore, Lu and Zhu investigated the probe absorption and dispersion of a quantum dot (modeled as a two-level system) in the presence of a pump field and showed that a hole induced by coherent population oscillation appears in the absorption spectrum of the probe field when excitons and plasmons interact [34]. They also pointed out that the system may exhibit slow light which depends strongly on the distance between the quantum dot and the metallic nanoparticle.

This article is organized as follows. In the next section we present the density matrix equations for the interaction of a double- V-type system with a laser field and a plasmonic nanostructure, and define the several terms that enter the density matrix equations. We outline the method for calculating the electromagnetic Green's tensor and give results for the spontaneous-emission decay rates as a function of distance from the plasmonic nanostructure. We also use the density matrix equations and calculate the linear susceptibility of the quantum system. Then, in Sec. III we present results for the absorption and dispersion spectra and the group velocity for different distances of the quantum system from the plasmonic nanostructure, with or without the free-space spontaneous decay rate. Finally, in Sec. IV we summarize our findings.

II. THEORETICAL MODEL AND CALCULATION OF THE LINEAR SUSCEPTIBILITY

The quantum system of interest is shown in Fig. 1. We consider a four-level system with two closely lying upper states $|2\rangle$ and $|3\rangle$, and two lower states $|0\rangle$ and $|1\rangle$. We will call this system a double- V-type system, in order to identify easily two different three-level V-type transitions in the structure. We note that the system is also topologically equivalent to the so-called double- Λ -type system. The quantum system is located in vacuum at distance *d* from the surface of the plasmonic nanostructure. We take states $|2\rangle$ and $|3\rangle$ to characterize two Zeeman sublevels. Then the dipole moment operator is taken

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^{*}paspalak@upatras.gr



FIG. 1. The quantum system under study is a double- V-type system where the two upper states $|2\rangle$ and $|3\rangle$ decay with spontaneous emission to the two lower states $|0\rangle$ and $|1\rangle$. The system interacts with a probe laser field that couples state $|0\rangle$ with states $|2\rangle$ and $|3\rangle$.

as $\vec{\mu} = \mu'(|2\rangle\langle 0|\hat{\varepsilon}_{-} + |3\rangle\langle 0|\hat{\varepsilon}_{+}) + \mu(|2\rangle\langle 1|\hat{\varepsilon}_{-} + |3\rangle\langle 1|\hat{\varepsilon}_{+}) +$ H.c., where $\hat{\varepsilon}_{\pm} = (\mathbf{e}_{z} \pm i\mathbf{e}_{x})/\sqrt{2}$ describe the right-rotating $(\hat{\varepsilon}_{+})$ and left-rotating $(\hat{\varepsilon}_{-})$ unit vectors and μ and μ' are taken to be real.

The quantum system interacts with a linearly polarized continuous-wave electromagnetic (laser) field, with electric field $\vec{E}(t) = \hat{z}E_0 \cos(\omega t)$, where E_0 is the electric field amplitude and ω its angular frequency. The laser field couples state $|0\rangle$ with states $|2\rangle$ and $|3\rangle$. The Hamiltonian that describes the interaction of the electromagnetic field with the quantum system, in the dipole and rotating-wave approximations, is given by

$$H = \hbar \left(-\delta - \frac{\omega_{32}}{2} \right) |2\rangle \langle 2| + \hbar \left(-\delta + \frac{\omega_{32}}{2} \right) |3\rangle \langle 3|$$
$$- \frac{\hbar \Omega}{2} (|0\rangle \langle 2| + |0\rangle \langle 3| + \text{H.c.}). \tag{1}$$

Here, $\delta = \omega - \tilde{\omega}$ is the detuning from resonance with the average transition energies of states $|2\rangle$ and $|3\rangle$ from state $|0\rangle$, with $\tilde{\omega} = (\omega_3 + \omega_2)/2 - \omega_0$, $\omega_{32} = (\omega_3 - \omega_2)/2$, and Ω is the Rabi frequency defined as $\Omega = \mu' E_0/(\sqrt{2\hbar})$. Also, $\hbar\omega_n$ with n = 0-3, is the energy of state $|n\rangle$.

Both excited states $|2\rangle$ and $|3\rangle$ decay spontaneously to state $|0\rangle$ with decay rates $2\gamma'_2$ and $2\gamma'_3$, respectively, and to state $|1\rangle$ with decay rates $2\gamma_2$ and $2\gamma_3$, respectively. We assume that the transitions $|2\rangle$, $|3\rangle$ to $|1\rangle$ lie within the surface-plasmon bands of the plasmonic nanostructure, whereas the transitions $|2\rangle$, $|3\rangle$ to $|0\rangle$ are spectrally far from the surface-plasmon bands and are not influenced by the plasmonic nanostructure. Therefore, in the transitions $|2\rangle$, $|3\rangle$ to $|0\rangle$ the spontaneous decay occurs due to the interaction of the quantum system with free-space vacuum modes, and we refer to this decay as free-space spontaneous decay.

We note that recently Gu *et al.* have studied the modification of the spontaneous emission spectrum of transitions influenced by surface plasmons [15] and the phenomenon of coherent population trapping [16] for a four-level double- Λ configuration (very similar to the one studied here) near a plasmonic nanostructure.

We choose the energy difference of states $|2\rangle$ and $|3\rangle$ to be rather small, i.e., ω_{32} , to be just a few Γ_0 , where Γ_0 is the decay rate of states $|2\rangle$ and $|3\rangle$ to state $|1\rangle$ in the vacuum. The latter is taken to be the same for both states. We can therefore assume that $\gamma_2 = \gamma_3 = \gamma$ and $\gamma'_2 = \gamma'_3 = \gamma'$ [13].

Using the Hamiltonian of Eq. (1) we obtain the following equations for the density matrix elements of the system, assuming a Markovian response:

$$\dot{\rho}_{00}(t) = 2\gamma' [\rho_{22}(t) + \rho_{33}(t)] - i\frac{\Omega}{2} [\rho_{02}(t) - \rho_{20}(t)] - i\frac{\Omega}{2} [\rho_{03}(t) - \rho_{30}(t)], \qquad (2)$$

$$\dot{\rho}_{22}(t) = -2(\gamma + \gamma')\rho_{22}(t) + i\frac{\Omega}{2}[\rho_{02}(t) - \rho_{20}(t)] - \kappa[\rho_{23}(t) + \rho_{32}(t)], \qquad (3)$$

$$\dot{\rho}_{33}(t) = -2(\gamma + \gamma')\rho_{33}(t) + i\frac{\Omega}{2}[\rho_{03}(t) - \rho_{30}(t)] -\kappa[\rho_{23}(t) + \rho_{32}(t)], \qquad (4)$$

$$\dot{\rho}_{20}(t) = \left(i\delta + i\frac{\omega_{32}}{2} - \gamma - \gamma'\right)\rho_{20}(t) - i\frac{\Omega}{2}\rho_{22}(t) - i\frac{\Omega}{2}\rho_{23}(t) + i\frac{\Omega}{2}\rho_{00}(t) - \kappa\rho_{30}(t),$$
(5)

$$\dot{\rho}_{30}(t) = \left(i\delta - i\frac{\omega_{32}}{2} - \gamma - \gamma'\right)\rho_{30}(t) - i\frac{\Omega}{2}\rho_{33}(t) - i\frac{\Omega}{2}\rho_{32}(t) + i\frac{\Omega}{2}\rho_{00}(t) - \kappa\rho_{20}(t),$$
(6)

$$\dot{\rho}_{23}(t) = (i\omega_{32} - 2\gamma - 2\gamma')\rho_{23}(t) + i\frac{\Omega}{2}\rho_{03}(t) - i\frac{\Omega}{2}\rho_{20}(t) - \kappa[\rho_{22}(t) + \rho_{33}(t)],$$
(7)

with $\rho_{00}(t) + \rho_{11}(t) + \rho_{22}(t) + \rho_{33}(t) = 1$ and $\rho_{nm}(t) = \rho_{mn}^{*}(t)$. Here, κ is the coupling coefficient between states $|2\rangle$ and $|3\rangle$ due to spontaneous emission in a modified anisotropic vacuum [35] and it is responsible for the appearance of quantum interference [36].

The values of γ and κ are obtained from [37–40]

$$\gamma = \frac{\mu_0 \mu^2 \bar{\omega}^2}{\hbar} \hat{\varepsilon}_- \cdot \operatorname{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}; \bar{\omega}) \cdot \hat{\varepsilon}_+, \qquad (8)$$

$$\kappa = \frac{\mu_0 \mu^2 \bar{\omega}^2}{\hbar} \hat{\varepsilon}_+ \cdot \operatorname{Im} \mathbf{G}(\mathbf{r}, \mathbf{r}; \bar{\omega}) \cdot \hat{\varepsilon}_+.$$
(9)

Here, $\mathbf{G}(\mathbf{r}, \mathbf{r}; \omega)$ is the electromagnetic Green's tensor, where **r** refers to the position of the quantum emitter, and μ_0 is the permeability of vacuum. Also, $\bar{\omega} = (\omega_3 + \omega_2)/2 - \omega_1$. From Eqs. (8) and (9) we obtain the values of γ and κ as [37–40]

$$\gamma = \frac{\mu_0 \mu^2 \bar{\omega}^2}{2\hbar} \operatorname{Im}[G_{\perp}(\mathbf{r}, \mathbf{r}; \bar{\omega}) + G_{\parallel}(\mathbf{r}, \mathbf{r}; \bar{\omega})] = \frac{1}{2} (\Gamma_{\perp} + \Gamma_{\parallel}),$$
(10)

$$\kappa = \frac{\mu_0 \mu^2 \bar{\omega}^2}{2\hbar} \operatorname{Im}[G_{\perp}(\mathbf{r}, \mathbf{r}; \bar{\omega}) - G_{\parallel}(\mathbf{r}, \mathbf{r}; \bar{\omega})] = \frac{1}{2} (\Gamma_{\perp} - \Gamma_{\parallel}).$$
(11)

Here, $G_{\perp}(\mathbf{r}, \mathbf{r}; \bar{\omega}) = G_{zz}(\mathbf{r}, \mathbf{r}; \bar{\omega})$ and $G_{\parallel}(\mathbf{r}, \mathbf{r}; \bar{\omega}) = G_{xx}(\mathbf{r}, \mathbf{r}; \bar{\omega})$ denote components of the electromagnetic Green's tensor where the symbol \perp (||) refers to a dipole oriented normal



FIG. 2. (Color online) (a) A metal-coated dielectric nanosphere and (b) a two-dimensional array of such spheres used in this work.

(along the *z* axis) [parallel (along the *x* axis)] to the surface of the nanostructure. Finally, we define the spontaneous emission rates normal and parallel to the surface as $\Gamma_{\perp,\parallel} = \mu_0 \mu^2 \bar{\omega}^2 \text{Im}[G_{\perp,\parallel}(\mathbf{r},\mathbf{r};\bar{\omega})]/\hbar$. The degree of quantum interference is defined as $p = (\Gamma_{\perp} - \Gamma_{\parallel})/(\Gamma_{\perp} + \Gamma_{\parallel})$. For p = 1 we have maximum quantum interference in spontaneous emission [36]. This can be achieved by placing the emitter close to a structure that completely quenches Γ_{\parallel} . We stress that when the emitter is placed in vacuum, $\Gamma_{\perp} = \Gamma_{\parallel}$ and $\kappa = 0$, so no quantum interference occurs in the system.

We note that the effects of optical transparency and slow light in a four-level system exhibiting quantum interference in spontaneous emission were studied by Paspalakis *et al.* using solutions of the coupled Maxwell-Schrödinger equations [41]. In addition, similar phenomena have been studied in three-level V-type systems with quantum interference in spontaneous emission using either solutions of the coupled Maxwell-Bloch equations [42] or calculations of the susceptibility using density matrix equations [43]. Here, such effects are studied when a four-level quantum system is placed near a plasmonic nanostructure that leads to quantum interference in spontaneous emission.

We next consider a two-dimensional lattice whose sites are occupied by electromagnetic scatterers (see Fig. 2). The corresponding electromagnetic Green's tensor which provides the corresponding spontaneous emission rates Γ_{\perp} and Γ_{\parallel} is given by [12,44,45]

$$G_{ii'}^{EE}(\mathbf{r},\mathbf{r}';\omega) = g_{ii'}^{EE}(\mathbf{r},\mathbf{r}';\omega) - \frac{i}{8\pi^2} \iint_{\text{SBZ}} d^2 \mathbf{k}_{\parallel} \sum_{\mathbf{g}} \frac{1}{c^2 K_{\mathbf{g};z}^+} \times v_{\mathbf{g}\mathbf{k}_{\parallel};i}(\mathbf{r}) \exp(-i\mathbf{K}_{\mathbf{g}}^+ \cdot \mathbf{r}) \hat{\mathbf{e}}_{i'}(\mathbf{K}_{\mathbf{g}}^+), \qquad (12)$$

with

$$\upsilon_{\mathbf{g}\mathbf{k}_{\parallel};i}(\mathbf{r}) = \sum_{\mathbf{g}'} R_{\mathbf{g}';\mathbf{g}}(\omega,\mathbf{k}_{\parallel}) \exp(-i\mathbf{K}_{\mathbf{g}'}^{-}\cdot\mathbf{r})\hat{\mathbf{e}}_{i}(\mathbf{K}_{\mathbf{g}'}^{-}) \quad (13)$$

and

$$\mathbf{K}_{\mathbf{g}}^{\pm} = \{ \mathbf{k}_{\parallel} + \mathbf{g}, \pm [q^2 - (\mathbf{k}_{\parallel} + \mathbf{g})^2]^{1/2} \}.$$
 (14)

The vectors \mathbf{g} denote the reciprocal-lattice vectors corresponding to the two-dimensional (2D) periodic lattice of the

PHYSICAL REVIEW A 86, 053811 (2012)

plane of scatterers and \mathbf{k}_{\parallel} is the reduced wave vector which lies within the surface Brillouin zone (SBZ) associated with the reciprocal lattice [46]. When $q^2 = \omega^2/c^2 < (\mathbf{k}_{\parallel} + \mathbf{g})^2$, $\mathbf{K}_{\mathbf{g}}^{\pm}$ defines an evanescent wave. The term $g_{ii'}^{EE}(\mathbf{r}, \mathbf{r}'; \omega)$ of Eq. (12) is the free-space Green's tensor and $\hat{\mathbf{e}}_i(\mathbf{K}_{\mathbf{g}}^{\pm})$ the polar unit vector normal to $\mathbf{K}_{\mathbf{g}}^{\pm}$. $R_{\mathbf{g}',\mathbf{g}}(\omega,\mathbf{k}_{\parallel})$ is the reflection matrix which provides the sum (over \mathbf{g} 's) of reflected beams generated by the incidence of a plane wave from the left of the plane of scatterers [46]. Also, in Eq. (12), the terms corresponding to *s*-polarized waves [those containing components with the azimuthal unit vector $\hat{\mathbf{e}}_i(\mathbf{K}_{\mathbf{g}}^{\pm})$ normal to $\mathbf{K}_{\mathbf{g}}^{\pm}$] have only a small contribution to the decay rates and have therefore been neglected.

The plasmonic nanostructure considered in this study is a two-dimensional array of touching metal-coated silica nanospheres (see Fig. 2). Ordered arrays of metallic nanoshells can be fabricated via self-assembly [47,48] and nanopatterning-nanolithographic [49,50] techniques. The dielectric function of the shell is provided by a Drude-type electric permittivity given by

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)},\tag{15}$$

where ω_p is the bulk plasma frequency and τ the relaxation time of the conduction-band electrons of the metal. The dielectric constant of SiO₂ is taken to be $\epsilon = 2.1$. In the calculations we have taken $\tau^{-1} = 0.05\omega_p$. The lattice constant of the square lattice is $a = 2c/\omega_p$ and the sphere radius $S = c/\omega_p$ with core radius $S_c = 0.7c/\omega_p$.

The results of Γ_{\perp} and Γ_{\parallel} as a function of the distance from the plasmonic nanostructure are presented in Fig. 3 for $\bar{\omega} = 0.632\omega_p$. We find that Γ_{\parallel} exhibits significant suppression and its actual value becomes significantly smaller than the free-space decay rate. Also, as the distance between the quantum system and the plasmonic nanostructure increases, the value of Γ_{\parallel} has an initial strong decrease up to a distance approximately $0.4c/\omega_p$, while for larger distances and up to c/ω_p the value of Γ_{\parallel} changes weakly and in an oscillating manner. In addition, the Γ_{\perp} value decreases with the increase of the distance between the quantum system and the plasmonic nanostructure. For distances close to the plasmonic nanostructure, Γ_{\perp} becomes considerably larger than the free-space decay rate. The value of Γ_{\perp} is larger than the free-space decay rate for distances up to $0.6c/\omega_p$, while for distances between $0.65c/\omega_p$ and c/ω_p the value of Γ_{\perp} becomes smaller than the free-space decay rate. As Γ_{\perp} is much larger than Γ_{\parallel} in the whole region of distances of Fig. 3, the corresponding quantum interference factor p is always above 0.95, and sometimes even larger than 0.99.

The absorption and dispersion properties of the system for a weak laser field are determined by the linear electric susceptibility, which is given by [51]

$$\chi^{(1)}(\omega) = \frac{\sqrt{2}N\mu'}{\varepsilon_0 E_0} \left(\rho_{20}^{(1)} + \rho_{30}^{(1)}\right) = \frac{N\mu'^2}{\varepsilon_0 \hbar} \frac{\rho_{20}^{(1)} + \rho_{30}^{(1)}}{\Omega}, \quad (16)$$

where $\rho_{20}^{(1)}$ and $\rho_{30}^{(1)}$ are calculated in steady state and in first order in terms of E_0 (or Ω). Here, ε_0 is the vacuum permittivity and N is the density of the quantum system.

Using perturbation theory, we obtain from Eqs. (2)–(7)

$$\rho_{20}^{(1)} = \frac{i\frac{\Omega}{2}\left(-i\delta + i\frac{\omega_{32}}{2} + \gamma + \gamma'\right) - i\kappa\frac{\Omega}{2}}{\left(-i\delta + i\frac{\omega_{32}}{2} + \gamma + \gamma'\right)\left(-i\delta - i\frac{\omega_{32}}{2} + \gamma + \gamma'\right) - \kappa^2},\tag{17}$$

$$\rho_{30}^{(1)} = \frac{i\frac{\Omega}{2}\left(-i\delta - i\frac{\omega_{32}}{2} + \gamma + \gamma'\right) - i\kappa\frac{\Omega}{2}}{\left(-i\delta + i\frac{\omega_{32}}{2} + \gamma + \gamma'\right)\left(-i\delta - i\frac{\omega_{32}}{2} + \gamma + \gamma'\right) - \kappa^2}.$$
(18)

We substitute $\rho_{20}^{(1)}$ and $\rho_{30}^{(1)}$ from Eqs. (17) and (18) into Eq. (16), use Eqs. (10) and (11), and obtain after some algebra

$$\chi^{(1)}(\omega) = \frac{N\mu^{\prime 2}}{\varepsilon_0 \hbar} \frac{\delta + i\gamma' + i\Gamma_{\parallel}}{-\delta^2 + \frac{\omega_{32}^2}{4} + (\Gamma_{\parallel} + \gamma')(\Gamma_{\perp} + \gamma') - i\delta(\Gamma_{\parallel} + \Gamma_{\perp} + 2\gamma')}.$$
(19)



FIG. 3. The spontaneous decay rate as a function of the distance from the plasmonic nanostructure with $(\omega_p \tau)^{-1} = 0.05$ and $\bar{\omega} = 0.632\omega_p$ for a dipole which is normally (Γ_{\perp} , top figure) and tangentially (Γ_{\parallel} , middle figure) oriented with respect to a plane of spheres. The corresponding quantum interference factor p is displayed in the bottom figure. Γ_0 is the decay rate in the vacuum.

III. FORMS OF THE ABSORPTION AND DISPERSION SPECTRA AND VALUES OF GROUP VELOCITY

In Fig. 4 we present the real and imaginary parts of $\chi^{(1)}$ as functions of the detuning δ for $\gamma' = 0$. In Fig. 4(a) we show the case where the quantum system is in vacuum, i.e., without the plasmonic nanostructure. There, the absorption spectrum is the sum of two Lorenzians with width Γ_0 that are close to each other as their centers differ by $\omega_{32} = 1.5\Gamma_0$. Also, there is negative dispersion at the center, i.e., at $\delta = 0$.

In Fig. 4(b) where the quantum system is near the plasmonic nanostructure we obtain a double-peaked absorption spectrum with a very pronounced minimum at $\delta = 0$ ($\omega = \tilde{\omega}$). The minimum in the absorption spectrum is given by

$$\operatorname{Im}[\chi^{(1)}(\tilde{\omega})] = \frac{N\mu'^2}{\varepsilon_0 \hbar} \frac{\gamma' + \Gamma_{\parallel}}{\frac{\omega_{22}^2}{4} + (\Gamma_{\parallel} + \gamma')(\Gamma_{\perp} + \gamma')}, \quad (20)$$

which for $\gamma' = 0$ reduces to

$$\operatorname{Im}[\chi^{(1)}(\tilde{\omega})] = \frac{N\mu^{\prime 2}}{\varepsilon_0 \hbar} \frac{\Gamma_{\parallel}}{\frac{\omega_{32}^2}{4} + \Gamma_{\parallel} \Gamma_{\perp}}.$$
 (21)

A similar behavior is obtained in the absorption spectra of Figs. 4(c) and 4(d) which correspond to larger distances of the quantum system from the plasmonic nanostructure. There, the whole of the absorption spectra are enhanced in comparison with Fig. 4(b) as Γ_{\perp} decreases with distance. Γ_{\parallel} also decreases with distance, as well as the minimum of absorption, given by Eq. (21). Moreover we obtain an almost complete cancellation of the absorption, i.e., optical transparency at $\delta = 0$. Also, the decrease of Γ_{\perp} , mainly, leads to decrease of the width of the absorption peaks. The strong decreases of absorption at $\delta = 0$ and of the width of the absorption peaks are also manifested in the spectra of Fig. 5 which are obtained for longer distances (than those of Fig. 4) between the quantum emitter and the nanostructure.

Another interesting feature in Figs. 4(b)–4(d) and 5 is the very steep positive dispersion obtained near $\delta = 0$. This can influence the group velocity of light at the transparency window. The group velocity at $\omega = \tilde{\omega}$ is given by [51,52]

$$v_g = \frac{c}{1 + \frac{1}{2} \operatorname{Re}[\chi^{(1)}(\tilde{\omega})] + \frac{\tilde{\omega}}{2} \operatorname{Re}\left[\frac{\partial \chi^{(1)}}{\partial \omega}\right]_{\omega = \tilde{\omega}}}, \qquad (22)$$

with *c* being the speed of light in vacuum. We find that $\operatorname{Re}[\chi^{(1)}(\tilde{\omega})] = 0$ and calculate $\operatorname{Re}[\partial \chi^{(1)}/\partial \omega]_{\omega = \tilde{\omega}}$, obtaining



FIG. 4. (Color online) The absorption spectrum [Im($\chi^{(1)}$), solid curve] and the dispersion spectrum [Re($\chi^{(1)}$), dashed curve] of the quantum system in units $N\mu'^2/(\hbar\epsilon_0\Gamma_0)$, as given by Eq. (19). (a) is in the absence of the plasmonic nanostructure, while (b), (c), and (d) are in the presence of the plasmonic nanostructure. We take $\omega_{32} = 1.5\Gamma_0$ and $\gamma' = 0$. In (a) the decay rate is Γ_0 , and in (b)–(d) $\bar{\omega} = 0.632\omega_p$. Finally, in (b) $d = 0.3c/\omega_p$, (c) $d = 0.4c/\omega_p$, and (d) $d = 0.5c/\omega_p$.

after some algebra

The group index [52] is defined as



FIG. 5. (Color online) The absorption spectrum [Im($\chi^{(1)}$), solid curve] and the dispersion spectrum [Re($\chi^{(1)}$), dashed curve] of the quantum system in units $N\mu'^2/(\hbar\epsilon_0\Gamma_0)$, as given by Eq. (19), in the presence of the plasmonic nanostructure. We take $\omega_{32} = 1.5\Gamma_0$, $\gamma' = 0$, and $\bar{\omega} = 0.632\omega_p$. Finally, in (a) $d = 0.6c/\omega_p$, (b) $d = 0.7c/\omega_p$, (c) $d = 0.8c/\omega_p$, and (d) $d = 0.9c/\omega_p$.



FIG. 6. (Color online) The group index, in units of $L = N\tilde{\omega}{\mu'}^2/(2\hbar\varepsilon_0\Gamma_0^2)$, as a function of the distance from the plasmonic nanostructure. The parameters that we use are $\omega_{32} = 1.5\Gamma_0$, $\bar{\omega} = 0.632\omega_p$, and $\gamma' = 0$.

As the group index is much larger than unity, Eq. (23) becomes

$$v_g \approx \frac{2\varepsilon_0 \hbar c \left[\frac{\omega_{32}^2}{4} + (\Gamma_{\parallel} + \gamma')(\Gamma_{\perp} + \gamma')\right]^2}{\tilde{\omega} N \mu'^2 \left[\frac{\omega_{32}^2}{4} - (\Gamma_{\parallel} + \gamma')^2\right]}.$$
 (25)

For $\gamma' = 0$ the above equation becomes

$$v_g \approx \frac{2\varepsilon_0 \hbar c \left(\frac{\omega_{32}^2}{4} + \Gamma_{\parallel} \Gamma_{\perp}\right)^2}{\tilde{\omega} N \mu'^2 \left(\frac{\omega_{32}^2}{4} - \Gamma_{\parallel}^2\right)} \,. \tag{26}$$

In Fig. 6 we show the group index as a function of the distance from the plasmonic nanostructure and for the same parameters as in Figs. 4 and 5. Evidently, slow light occurs in the system

under study [51–54], wherein the group velocity v_g decreases as the emitter moves away from the nanostructure.

In Figs. 7 and 8 we present results for the case of nonzero free-space spontaneous decay rate γ' . To begin with we choose a small value for this decay rate, i.e., $\gamma' = 0.03\Gamma_0$. We obtain a similar qualitative behavior for both the absorption and dispersion as for $\gamma' = 0$. However, the minimum of the absorption spectrum is larger for $\gamma' = 0.03\Gamma_0$, about one order of magnitude in comparison with Figs. 4(c) and 4(d), and about twice in comparison spectrum is larger by about one order of ragnitude for Figs. 8(a)–8(d) in comparison with Figs. 5(a)–5(d).

In Fig. 9(a) we show the group index as a function of the distance from the plasmonic nanostructure for the same parameters as in Figs. 7 and 8. Slow light is also evident in this case. The actual value of group velocity depends on the distance from the plasmonic nanostructure and is larger than for $\gamma' = 0$. Using typical values for the parameters, $\tilde{\omega} =$ $1.5\omega_p$, $\hbar\omega_p = 8.99$ eV (typical for gold), $\gamma' = 0.03\Gamma_0$, and $\Gamma_0 = 10^8 \text{ s}^{-1}$, we calculate from the formula for the free-space decay rate, $2\gamma' = \tilde{\omega}^3 \mu'^2 / (3\pi \varepsilon_0 \hbar c^3)$, the dipole matrix element $\mu' = 2.5 \times 10^{-12} \text{ e m}$. In Fig. 9(b), we show v_g for the above parameters and for $N = 10^{20} \text{ m}^{-3}$. We find that the group velocity is decreased by four orders of magnitude from the speed of light in vacuum.

For larger values of γ' (not shown here) we have found that the absorption minimum and the group velocity increase further. For the parameters used above we find that for values of γ' larger than approximately $0.8\Gamma_0$ the phenomena of absorption suppression and slow light are completely destroyed.



FIG. 7. (Color online) The absorption spectrum [Im($\chi^{(1)}$), solid curve] and the dispersion spectrum [Re($\chi^{(1)}$), dashed curve] of the quantum system in units $N\mu'^2/(\hbar\epsilon_0\Gamma_0)$, as given by Eq. (19). (a) is in the absence of the plasmonic nanostructure, while (b), (c), and (d) are in the presence of the plasmonic nanostructure. We take $\omega_{32} = 1.5\Gamma_0$ and $\gamma' = 0.03\Gamma_0$. In (a) the decay rate is Γ_0 , and in (b)–(d) $\bar{\omega} = 0.632\omega_p$. Finally, in (b) $d = 0.3c/\omega_p$, (c) $d = 0.4c/\omega_p$, and (d) $d = 0.5c/\omega_p$.



FIG. 8. (Color online) The absorption spectrum [Im($\chi^{(1)}$), solid curve] and the dispersion spectrum [Re($\chi^{(1)}$), dashed curve] of the quantum system in units $N\mu'^2/(\hbar\epsilon_0\Gamma_0)$, as given by Eq. (19), in the presence of the plasmonic nanostructure. We take $\omega_{32} = 1.5\Gamma_0$, $\gamma' = 0.03\Gamma_0$, and $\bar{\omega} = 0.632\omega_p$. Finally, in (a) $d = 0.6c/\omega_p$, (b) $d = 0.7c/\omega_p$, (c) $d = 0.8c/\omega_p$, and (d) $d = 0.9c/\omega_p$.

IV. SUMMARY

In summary, we have studied the effect of a plasmonic nanostructure on the linear absorption and dispersion properties of a double- V-type quantum system. In this system one V-type transition interacts with the surface-plasmon bands of the nanostructure while the other V-type transition interacts with the vacuum modes. The system also interacts with a probe laser field that couples to the free-space V-type transition. As a case study, we consider a two-dimensional array of metal-coated dielectric nanospheres that modifies the relevant decay rates of the quantum system which are calculated by a rigorous electromagnetic Green's tensor technique.

We have shown that both the absorption and dispersion spectra are strongly modified by the presence of the plasmonic nanostructure. We have found, in particular, that there is a strong suppression of absorption at a laser frequency on resonance with the average transition energies of states $|2\rangle$ and $|3\rangle$ from state $|0\rangle$. The absorption suppression is also accompanied by very steep positive dispersion which leads to a slow-light effect. For typical parameter values, the group velocity can be reduced by up to four orders of magnitudes with respect to the speed of light in vacuum. We have also shown that the absorption and dispersion spectra depend strongly on the distance of the quantum system from the plasmonic nanostructure, as well as on the free-space decay rates. Stronger absorption suppression and lower group velocities of light were found for distances between $0.5c/\omega_p$ and c/ω_p . We also found that large free-space decay rates are detrimental to the phenomena found here.



FIG. 9. (Color online) (a) The group index, in units of $L = N\tilde{\omega}{\mu'}^2/(2\hbar\varepsilon_0\Gamma_0^2)$, as a function of the distance from the plasmonic nanostructure. The parameters that we use are $\omega_{32} = 1.5\Gamma_0$, $\bar{\omega} = 0.632\omega_p$, and $\gamma' = 0.03\Gamma_0$. (b) The group velocity of light, in units of $c/10^5$, as a function of the distance from the plasmonic nanostructure using the parameters $\gamma' = 0.03\Gamma_0$, $\hbar\omega_p = 8.99 \text{ eV}$, $\tilde{\omega} = 1.5\omega_p$, $N = 10^{20} \text{ m}^{-3}$, $\mu' = 2.5 \times 10^{-12} e \text{ m}$, and $\Gamma_0 = 10^8 \text{ s}^{-1}$.

The present results show that effects similar to those obtained by electromagnetically induced transparency [51,52] can be obtained by placing a quantum system near a plasmonic nanostructure which induces quantum interference in spontaneous emission. Therefore, we expect that the results of this paper can be used in optoelectronics and quantum information processing as an alternative to electromagnetically induced transparency.

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TRANSPARENCY AND SLOW LIGHT IN A FOUR-LEVEL ...

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