Decoherence of quantum kinematical correlations: Elastic scattering of identical particles

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We include the dynamics of the angular straggling process in the angular distributions of Mott scattering of heavy ions. We model the passage of an incoming nucleus through a target as a diffusion process. It is then possible to derive a simple and physically transparent expression for the angular dispersion due to the straggling. The angular dispersion should be folded with the theoretical Mott cross section to see its effect on the amplitude of the Mott oscillations. Our results agree very well with data of ^{208}Pb + ^{208}Pb scattering. We define the "classical" limit as the limit when the angular dispersion due to straggling becomes comparable with the Mott oscillation period and get the disappearance of quantum interference occurring at the limit $0.050\sqrt{\xi}\frac{Z^4}{E^{3/2}} \geqslant 1$, where ξ stands for the target thickness, Z is the system's charge, and E is the center-of-mass energy. The experiments on lead are very close to this limit. We show that the kinematical correlations due to the identity of the particles is maintained, as it should be, and the action of the environment is to reduce the fringe visibility.

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I. INTRODUCTION

In the past few decades, given the impressive technological advances in the areas of quantum optics, superconducting devices, trapped ions, and nanosystems, among others, much has been learned about the process which leads to the disappearance of essentially quantum phenomena [1,2]. Time scales for coherence loss have been derived, and several of the dynamical processes leading to decoherence have been identified and even controlled in some cases. To our knowledge, up to now, kinematical correlations imposed, e.g., by the Pauli principle have received much less attention in specific dynamical contexts. Studies of such kinematical correlations have been carried out in the area of quantum information [3] in the absence of dynamical processes. The purpose of the present investigation is to verify how robust quantum kinematical correlations are to the deleterious action of an environment in a realistic experimental situation. For this purpose we choose the problem of elastic scattering of identical particles where the Pauli principle plays a decisive role.

Coherence phenomena appear in the scattering of identical particles due to the indistinguishability between target and projectile. For identical particles, the scattering must be invariant under the exchange of the relative coordinates of the projectile and target, and as a consequence, the scattering amplitude must be symmetrized, which leads to the appearance of strong oscillations in the angular distributions. Those oscillations are not present in the nonsymmetrized cross section and arise as a pure quantum effect of interference between amplitudes.

The main source of coherence loss in this situation is the angular straggling, which inevitably occurs when the projectile traverses the target. Experimentalists take the effect into consideration through numerical calculations [4], which renders the dependence of decoherence on specific physical parameters such as energy, atomic number, and target thickness very inconspicuous. We therefore study an analytical model for the angular straggling which is shown to give a precise account of the data, both qualitatively as well as quantitatively. The dynamics of the projectile straggling process in the target is modeled as a simple diffusion process. We analytically obtain a limit for the possibility of observing the oscillations in Mott scattering as a function of the relevant physical parameters such as target thickness, projectile charge, and incoming energy. The purpose of the present contribution is to model elastic scattering with a realistic target, i.e., taking into account the incoherent multiple scattering and investigating the experimental visibility limits as a function of the relevant physical parameters.

II. THE SCATTERING OF IDENTICAL PARTICLES

The invariance under exchange of projectile and target coordinates in the scattering of identical particles implies that the scattering amplitude must be symmetric (for bosons) or antisymmetric (for fermions). For spherical potentials the amplitude depends only on the scattering angle θ and is written as $f_\pm(\theta) = f(\theta) \pm f(\pi - \theta)$, where the signs $+$ and $-$ refer to scattering of bosons and fermions, respectively. The observable is the differential cross section, which is given by $\frac{d\sigma}{d\Omega} = |f_\pm(\theta)|^2$. In the case of pure Rutherford scattering the nonsymmetric amplitude is [5]

$$f(\theta) = \frac{\eta}{2k\sin(\theta/2)}\exp[-i\eta\ln\sin^2(\theta/2) + 2i\sigma_0], \tag{1}$$

where $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$ is the Sommerfeld parameter, $\sigma_0 = \arg\Gamma(1 + i\eta)$ is the $l = 0$ Coulomb phase shift, and θ is the scattering angle in the center-of-mass system.

Using Eq. (1) for the unsymmetrized amplitudes, the Mott cross section for scattering of spin-zero particles is easily obtained (see, for instance, [5]):

$$\frac{d\sigma}{d\Omega}^{\mathrm{Mott}} = \frac{Z^4 e^4}{16E^2}\bigg\{\csc^4(\theta/2) + \sec^4(\theta/2) + \frac{8}{\sin^2(\theta)}\cos[\eta\ln\tan^2(\theta/2)]\bigg\}. \tag{2}$$

The first two terms inside the braces describe the scattering at θ and $\pi - \theta$, respectively, and present no oscillations.

The third term comes from the interference between the two amplitudes and is strongly oscillatory. The larger the Sommerfeld parameter η is, the more rapid the oscillations become, and in the limit $\eta \to \infty$, the oscillations become so fast that any experimental angular resolution would wash out the interference effect, giving rise to decoherence. Thus, decoherence in Mott scattering would be any effect that causes a damping in the amplitude of the oscillations of the angular distributions.

III. DECOHERENCE AND THE ANGULAR STRAGGLING

One possible effect that causes decoherence is the angular straggling. Angular straggling is the fluctuation in the scattering angle due to the multiple atomic scattering processes, which occur as the projectile goes through the target. The angular straggling can be modeled as a diffusion process [6], where the square of the final angular dispersion is obtained as the mean-square angle at each collision multiplied by the number of collisions.

$$\sigma_\theta{}^2 = \langle\theta^2\rangle N_{\rm col}, \tag{3}$$

$$\langle\theta^2\rangle = \frac{\int \theta^2 d\sigma}{\int d\sigma}. \tag{4}$$

The scattering angle is taken from the classical Rutherford deflection function for small angles $\theta = Z_1 Z_2 e^2/Eb$, where E and θ are both in the center-of-mass system and $d\sigma = 2\pi b db = 2\pi b(\theta) db$. Substituting these formulas in Eq. (4) and integrating from $b_{\rm min}$ to $b_{\rm max}$, we get for $b_{\rm min} \ll b_{\rm max}$ (this condition always holds for heavy ions of few MeV)

$$\sigma_\theta{}^2 = 2N_{\rm col}\left(\frac{b_{\rm min}}{b_{\rm max}}\right)^2 \ln\left(\frac{b_{\rm max}}{b_{\rm min}}\right), \tag{5}$$

where the number of collisions $N_{\rm col} = \pi b_{\rm max}^2 Nt$ and Nt is the areal density of the target given by the number of atoms per square centimeter. The quantities $b_{\rm max}$ and $b_{\rm min}$ stand for maximum and minimum impact parameters relevant to the atomic collisions. Note that the product of the first two factors on the right-hand side of Eq. (5) does not depend on $b_{\rm max}$, and the effective screening parameter $b_{\rm max}$ enters only in the third factor. This factor is a very slowly varying function of the ratio $b_{\rm max}/b_{\rm min}$, which can be seen as a "decoherence" constant and contains the average effect of the interaction with the "environment," i.e., the target. The quantities $b_{\rm max}$ and $b_{\rm min}$ can be estimated by the formulas $b_{\rm min} = Z_1 Z_2 e^2/E$, the distance of the closest approach, and $b_{\rm max} = a$, the effective atomic screening parameter $a = 0.885 a_0/(Z_1^{2/3} + Z_2^{2/3})^{1/2}$, with $a_0 = 0.529 \times 10^{-8}$ cm [7]. We estimated the factor $\sqrt{\ln(b_{\rm max}/b_{\rm min})} \approx 2.5$ for several systems and energies, and using this factor, the angular straggling dispersion equation (5) can be rewritten in a simpler way as

$$\sigma_\theta = 0.7 \frac{Z_1 Z_2}{E}\sqrt{\xi/A_2}, \tag{6}$$

where ξ stands for the target thickness in g/cm^2 and A_2 is the target mass number. More sophisticated models for the angular straggling process exist [8]; however, as we shall see, the essential physics for the understanding of the decoherence process is contained in the present one.

TABLE I. Angular straggling variance σ_θ using a target thickness of 20 μg/cm^2. The energies and angles are in the center-of-mass system.

System (20 μg/cm^2)	$E_{\rm cm}$ (MeV)	Formula (5) (deg)	STOPX [9] (deg)	SRIM [10] (deg)
^{12}C + ^{12}C	5.0	0.41	0.29	0.25
^{16}O + ^{16}O	8.8	0.35	0.25	0.16
^{28}Si + ^{28}Si	20.0	0.35	0.25	0.17
^{208}Pb + ^{208}Pb	564.5	0.14	0.10	0.09

In Table I we show the calculated values of the angular dispersion σ_θ in the laboratory system using Eq. (5). We compare our results with calculations using the programs VAXPAK and SRIM [9,10]. Given the simplicity of the model presented here, we believe that the agreement is quite reasonable.

IV. APPLICATION TO EXPERIMENTAL DATA AND DISCUSSION

To obtain the effect of the angular straggling in the oscillations of Mott angular distribution, we perform the folding of the theoretical Mott cross section with the angular straggling Gaussian distribution as

$$\frac{d\sigma}{d\Omega}^{\rm expt} = \frac{1}{(2\pi\sigma_\theta^2)^{1/2}}\int \exp\left(-\frac{(\theta-\theta')^2}{2\sigma_\theta^2}\right)\frac{d\sigma(\theta')}{d\Omega}^{\rm Mott} d\theta'. \tag{7}$$

It is very difficult to find experimental data which display the effect of decoherence in the scattering of identical particles. In most cases the angular spread due to the angular straggling is much smaller than the period of the Mott oscillations, and its effect on the amplitude of the oscillations is not visible. In Fig. 1 we show the elastic scattering angular distribution for the ^{28}Si + ^{28}Si [11] system at $E_{\rm cm} = 20.0$ MeV and the predicted damping of the oscillations due to the angular straggling. We see that the effect is very small in this case, as expected, due

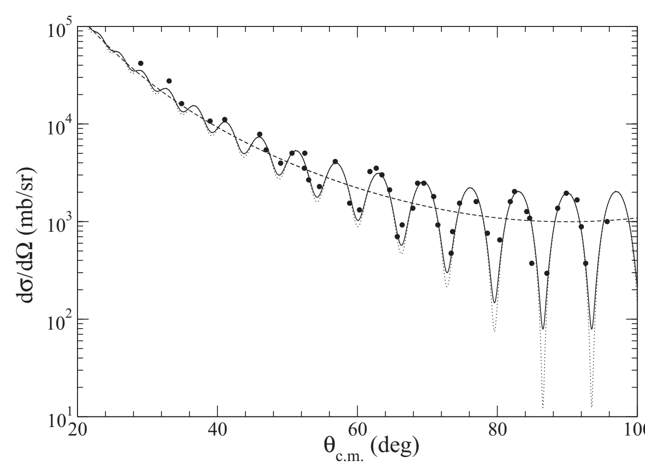

FIG. 1. Elastic scattering ^{28}Si +^{28}Si at $E_{\rm cm} = 20.0$ MeV. The points are the experimental data [11], the dotted line is the pure Mott cross section, and the dashed line is the classical Rutherford prediction. The solid line is the result using Eq. (7) with a target thickness of 100 μg/cm^2.

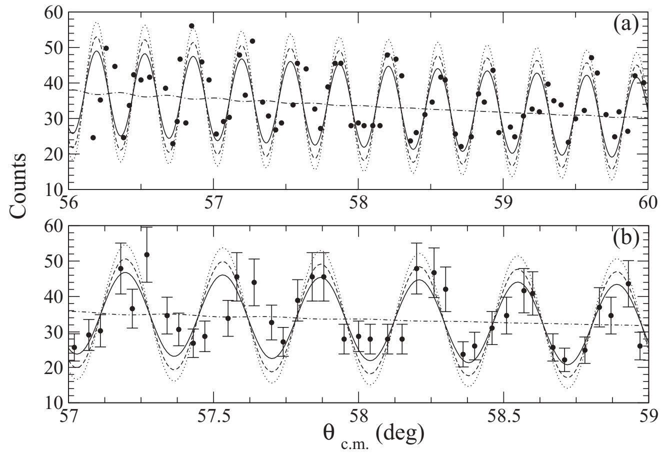

FIG. 2. (a) Elastic scattering angular distribution ^{208}Pb $+^{208}$Pb at $E_{\text{cm}} = 572.2$ MeV. (b) A close-up of (a). The dotted line represents pure Mott scattering ($\chi^2_{\text{red}} = 4.2$), the solid line represents the present model ($\chi^2_{\text{red}} = 1.6$), the dashed line uses the angular dispersion calculated by STOPX ($\chi^2_{\text{red}} = 2.7$), and the dot-dashed line is the classical limit ($\chi^2_{\text{red}} = 2.5$).

to the smallness of the angular straggling dispersion compared to the period of the Mott oscillations.

The ^{208}Pb + ^{208}Pb [12] data are the only set of data to our knowledge which clearly displays the effect of decoherence. In this case the period of the Mott oscillations is rather small, $T^{\text{lab}}_{\text{Mott}} \approx 0.18^\circ$, and is comparable to the angular dispersion due to the straggling. The target used in this experiment had a 6 μg/cm^2 layer of ^{208}Pb, facing the beam, and evaporated on a 15 μg/cm^2 carbon backing. Using formula (5), we derived $\sigma^{\text{lab}}_\theta = 0.041^\circ$ for the Pb target and $\sigma^{\text{lab}}_\theta = 0.020^\circ$ for the carbon target, both in the laboratory system. The total dispersion is obtained with the quadratic sum of these two factors as $\sigma^{\text{total-lab}}_\theta = 0.046^\circ$. The angular resolution of the detectors is of the order of 0.01° [12] and has little influence in the resulting angular dispersion. In Fig. 2 we present the experimental data compared with our calculations. The Mott cross section (dotted line) clearly overestimates the amplitude of the oscillations. Here a slightly different energy is used in the Mott calculations, 572.2 MeV instead of 564.5 MeV, to take into account the presence of non-Coulombic effects [12], such as relativistic effects, electron screening, and vacuum polarizability in the ^{208}Pb + ^{208}Pb scattering. The result obtained with the present model, Eq. (7), and using the above dispersion is shown as a solid line. The agreement is excellent, although the model is rather simple (χ^2 values are listed in the caption). We also plot the result using $\sigma^{\text{total-lab}}_\theta = 0.030^\circ$ (dashed line) predicted by the program STOPX [9].

The classical limit is shown as the dot-dashed line in Fig. 2 and is attained when the angular dispersion due to the straggling becomes comparable to the period of Mott oscillations $T^{\text{Mott}}_\theta = \pi/\eta$ for $\theta_{\text{cm}} = 90^\circ$, where η is Sommerfeld parameter. The condition $\sigma_\theta \geqslant T^{\text{Mott}}_\theta/2$ provides the limit above which the quantum interference effects would be washed out. This condition can be written as

$$0.050\sqrt{\xi}\,\frac{Z^4}{E^{3/2}} \geqslant 1. \tag{8}$$

$\xi = 35$ μg/cm^2 is obtained from the above formula using $Z = 82$ and $E = 564.5$ MeV.

As one can see, for a fixed energy the classical limit is strongly dependent on the atomic number of the particles. The larger the sizes of the projectile and target are, the less visible the interference fringes become. Also, for a fixed projectile and target, as the energy becomes smaller, one gets a blurred angular distribution. This energy dependence is a consequence of two effects that are concurring in the same direction: the characteristic pure Mott scattering for which the period of the oscillations becomes smaller as the energy decreases and the angular dispersion due to the straggling which increases with decreasing energy.

Of course, if the condition given by Eq. (8) is surpassed, one would not be able to observe quantum effects, even if one had sufficient experimental resolution in the detection system. From this point of view one can think of the classical limit as only partially due to the experimental resolution of the detection system but mainly determined by the effect of the environment. The detection accuracy can, in principle, play a major role in detecting the effect discussed here since if it were not precise enough, the interference fringes would not be visible and one would see the line characteristic of the classical scattering. We believe it is fair to say that the measuring process is also an essential ingredient to be considered in discussing the classical limit. However, in the present case, the angular precision due to the position resolution of the detectors and the angular resolution of the entire setup was at least one tenth lower than that necessary to observe the period of the Mott oscillations [12], so it is not affecting our results.

V. CONCLUSIONS

In summary we derive a simple analytic expression for the angular straggling of a heavy ion passing through a target which depends on parameters that can be easily calculated from the atomic number of the nuclei involved, the projectile energy, and the target thickness. The angular straggling is identified as the main process which causes the decoherence of the Mott oscillations in the ^{208}Pb + ^{208}Pb scattering. The effect of decoherence is visible as a damping in the amplitude of the Mott oscillations. We derive a simple inequality, depending only on the energy, the atomic number of the nuclei involved, and the target thickness, which displays the limit for the experimental observation of the Mott oscillations for a general case.

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