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Decoherence suppression via non-Markovian coherent feedback control

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In this paper, we present a coherent feedback control scheme for non-Markovian bosonic systems, in which an engineered quantum control field is introduced to couple both the system and the noise bath. The closed-loop dynamics of the system is described by an exact non-Markovian quantum Langevin equation, where the spectral density functions of the noise and the quantum control field, as well as their coupling, are combined into a single memory kernel function. We show that the coupling between the quantum control field with the noise bath can be used as a feedback control to modulate the memory kernel function. As a result, the noise bath can be driven out of resonance with the system and the decoherence can be efficiently suppressed. The effectiveness of the controllability is demonstrated with a photonic circuit in photonic crystals.

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I. INTRODUCTION

Decoherence is the major obstacle that hinders the processing of quantum information in various physical implementations [1,2]. Due to the strong memory effect of the environment, non-Markovian decoherence is prevalent in nanoscale solid-state devices [3–6], e.g., the electron transport in semiconductors [7,8], the substrate defects induced decoherence [9], the trapped atom coupled to an engineered reservoir [10], and the dissipative light transport in photonic crystals [11,12], where the correlation time of the environment is comparable with the time scale of the system. The complete decoherence dynamics induced by the environment is encapsulated in a single spectral density which characterizes all the back-action memory effects between the system and its environment through the coupling and also the environmental density of states [13].

To enhance the coherence in various quantum devices, many schemes have been proposed for decoherence suppression. In the dynamical decoupling (DD) approach, the period DD [14,15] was presented, following which the concatenation [16] and Uhrig DDs [17,18] were proposed to improve the fault-tolerance threshold and reduce the number of control pulses. However, this method usually requires high-frequency hard pulses that are expensive in the laboratory. Optimization methods were also explored, which numerically search the control pulses to minimize some decoherence indices, including the distance between the state (gate operation) of the actual system and that of an ideal system [9,19–22] and the spectral overlap between the system and the environment [23,24]. The control optimization usually relies on a model of the system and the noise, and hence the control performance is dependent on the model precision. Since generally the interaction of the environment with the control is ignored, the structure of the environment is not changed.

Another class of decoherence control strategies was designed by engineering the environment. From a more general point of view, the environment (as a quantum object) with tunable coupling to a quantum system can affect the spectral density and hence behaves as a quantum controller [25,26]. When the back-action from the system is nonnegligible, the environment acts as a direct coherent feedback controller or a measurement device [25]. Such coherent feedback scheme can be extended to indirect coherent feedback schemes mediated by optical fields, In this regard, the environment engineering can be treated as a class of feedback control strategy. There have been many studies on feedback control of decoherence [25,27–29]. As an analog of classical feedback control, measurement-based feedback control was proposed to suppress the decoherence in Markovian atom-cavity systems, where the extracted classical information is processed to adjust the semiclassical control [28–31]. The measurement-based feedback is limited by its information processing speed that has to be kept up with the evolution of the system dynamics [32], which is almost impossible in most solid-state systems whose time scales range from picoseconds to nanoseconds. To overcome this difficulty, coherent quantum feedback control (CQFC) schemes were introduced via a quantum feedback channel [25,33,34] and experimentally demonstrated in Ref. [35,36]. It was also applied to many other interesting problems, e.g., spontaneous switching suppression [37], multipartite quantum entanglement generation [38], and cooling quantum oscillator [39].

Existing quantum coherent feedback strategies were mainly established for Markovian dissipative quantum systems via quantum input-output theory [36–38] and transfer function theory [40–43]. For non-Markovian systems, one can engineer the environments [10,44–47] by modulating non-Markovian dynamics via tunable system-environment interactions in various physical systems. With proper designs, the coherence time can be prolonged by modifying the noise spectrum of the environment, or more precisely speaking, modifying the spectral density. However, the engineering environment can only suppress the decoherence induced by the structured environment itself. To our knowledge, CQFC has not been systematically applied to open quantum systems with naturally existing non-Markovian environment, due to the difficulty of

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the modulation on such environments and the analysis on the underlying memory effects.

In this paper, we present a CQFC scheme for a class of non-Markovian open boson systems, and examine the coherence control mechanism via the non-Markovian quantum Langevin equation. The CQFC scheme is performed by introducing a quantum control field (quantum controller) that is coupled to both the system and the noise bath for feedback control. As we will see, the coupling of a quantum controller to the system usually induces additional decoherence. However, letting the quantum controller couple to the noise bath will change, as a feedback effect, the spectrum of the noise bath such that the memory kernel function can be efficiently modulated. As a result, the noise source can be significantly manipulated to suppress the decoherence phenomena of the system.

The rest of this paper is organized as follows. In Sec. II, the systems considered in this paper are modeled. The generalized non-Markovian Langevin equation for the coherent feedback control system is established in Sec. III. The mechanism of decoherence suppression via coherent feedback is analyzed in Sec. IV. Then, Sec. V applies the strategies to a photonic crystal system. Finally, conclusions are drawn in Sec. VI.

II. MODELING NON-MARKOVIAN COHERENT FEEDBACK CONTROL SYSTEMS

The coherent feedback control system is sketched in Fig. 1 for a non-Markovian open quantum system that is coupled to a noise bath. To be more specific, the open quantum system we consider here is a Bose system, such as an optical system or a bosonic atomic system. The noise bath could be an arbitrary thermal bath containing an infinite number of harmonic oscillators. A quantum control field [48] is introduced to control the decohering structure of the noise bath so that it should couple with both the system and the noise bath quantum mechanically. This design leads to the closed-loop system to be studied in this paper. Generally, the interconnection between these parts induces bidirectional

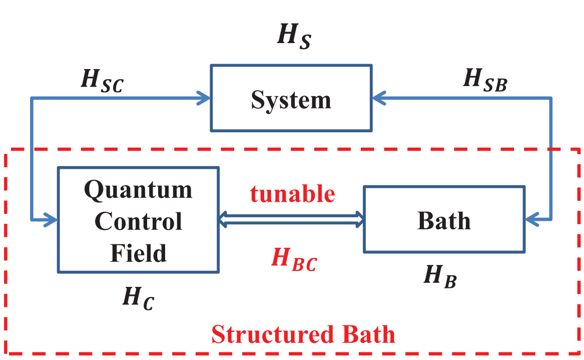

FIG. 1. (Color online) The structure diagram of the coherent feedback loop, where the quantum control field and the noise bath jointly form a structured bath. The Hamiltonians of the system, the noise bath, and the quantum control field are denoted as H_S, H_B, H_C, respectively, and their mutual interaction Hamiltonians are denoted as H_{SB}, H_{SC}, H_{BC}, respectively. The interaction Hamiltonian H_{BC} between the quantum control field and the noise bath is tunable. The feedback system consists of closed loops flowing in both clockwise and anticlockwise directions.

causal effects (i.e., two interconnected systems always affect each other) [25,49] as indicated by the bidirectional arrowed lines in Fig. 1. Thus, the design leads to a closed-loop control system with information flowing in both clockwise and anticlockwise directions.

Explicitly, the basic Hamiltonian of the Bose system, the noise bath, and the coupling between them can be written as

$$\begin{aligned} H_{S-B} &= H_S + H_B + H_{SB} \\ &= \omega_0 \hat{a}^\dagger \hat{a} + \sum_k \omega_{Bk} \hat{b}_k^\dagger \hat{b}_k + \sum_k (V_{Bk}^* \hat{a}^\dagger \hat{b}_k + V_{Bk} \hat{a} \hat{b}_k^\dagger). \end{aligned} \tag{1}$$

Here, $\hat{a}$ ($\hat{a}^\dagger$) in the system Hamiltonian H_S is the bosonic annihilation (creation) operator with the angular frequency ω_0. The noise bath is described by the Hamiltonian H_B with the bosonic annihilation (creation) operator $\hat{b}_k$ ($\hat{b}_k^\dagger$) and the frequency ω_{Bk} for each mode. The coupling strength between the system and each mode of the noise bath is denoted as V_{Bk}. For simplicity, the nonlinear boson-boson interactions are not considered here. A simple physical realization of Eq. (1) can be easily found in quantum optics, where the system can be a cavity, or more interestingly a micro or nano cavity, the noise bath is given by all the possible photon modes surrounding the cavity, and the cavity photon leakage is described by the coupling Hamiltonian H_{SB} [50].

To control the noise dynamics fully quantum mechanically, we introduce a quantum control field, such as a quantum electromagnetic field pulse. In terms of quantum electrodynamics (QED), the quantum control field is given by

$$H_C = \frac{1}{2} \int d^3x (\vec{E}^2 + \vec{B}^2) = \sum_k \omega_{Ck} \hat{c}_k^\dagger \hat{c}_k, \tag{2}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields distributed over the whole space of the environment; the quantized annihilation (creation) operator for each electromagnetic mode with the frequency ω_{Ck} is denoted as $\hat{c}_k$ ($\hat{c}_k^\dagger$). The control field acting on the system is described by the Hamiltonian

$$H_{SC} = \sum_k (V_{Ck}^* \hat{a}^\dagger \hat{c}_k + V_{Ck} \hat{a} \hat{c}_k^\dagger). \tag{3}$$

The coupling strength of the system with each mode of the quantum control field is given by V_{Ck}. Usually one treats the control field as a classical pulse; i.e., $\vec{E}$ and $\vec{B}$ are treated as a classical variable. Correspondingly, Eq. (3) is reduced to $H_{SC} \to K(t) = \kappa(t)\hat{a}^\dagger + \kappa^*(t)a$ and $\kappa(t)$ is proportional to the electric field strength $\vec{E}$.

As we will see later, to manipulate the decoherence structure of the noise bath, we must introduce a tunable coupling between the quantum control field and the noise bath:

$$H_{BC} = \sum_k \sum_{k'} (F_{kk'} \hat{b}_k \hat{c}_{k'}^\dagger + F_{kk'}^* \hat{b}_k^\dagger \hat{c}_{k'}), \tag{4}$$

where $F_{kk'}$ denotes the tunable coupling strength between the kth noise mode and the k'th quantum control field mode. The tenability of the coupling strengths can be simply realized using a beam splitter in quantum optics, for an example.

A tunable coupling between the control field and the noise bath is crucial since it can induce an effective quantum feedback control to directly modulate the noise bath. The importance of the tunable coupling can be seen as follows. The noise structure containing the complete information about the effect of the environment is encapsulated in a single spectral density, as pointed out by Leggett *et al.* many years ago [13]. The spectral density of an open quantum system is defined as a function of the density of states of its environment, multiplying this together with the coupling strength and its complex conjugate between the system and the environment; see Eq. (10) below. Here, the noise bath and the quantum control field jointly together form a structured bath for the system (as shown in Fig. 1). The tunable coupling strength $F_{kk'}$ between the control field and the noise bath can change the spectrum (the density of states) of this structured bath in a control way. Therefore it can significantly modulate the coherence dynamics of the system.

Thus, the total Hamiltonian of our coherent feedback control system is given by

$$H = H_{S-B} + H_C + H_{SC} + H_{BC}. \tag{5}$$

The detailed realization of such a coherent feedback control through the quantum control field is shown in the next sections.

III. NON-MARKOVIAN QUANTUM LANGEVIN EQUATIONS

A. Exact non-Markovian quantum Langevin equations

The dynamics of a system can be fully described by the equation of motion for the operator $\hat{a}(t)$, which is determined from the Heisenberg equations of motion by eliminating all the quantum degrees of freedom of the noise bath and the control field. The result is given by the following exact non-Markovian quantum Langevin equation (see the detailed derivation in Appendix A),

$$\dot{\hat{a}}(t) = -i\omega_0\hat{a}(t) - \int_0^t d\tau G(t-\tau)\hat{a}(\tau) - i\hat{\epsilon}_{\rm n}(t), \tag{6}$$

where the memory kernel function $G(t)$ is responsible for the dissipation process, and the equivalent noise term $\hat{\epsilon}_{\rm n}(t)$ corresponds to the noise generated from both the noise bath and the quantum control field.

In the absence of feedback couplings, the memory kernel function $G(t) = G_0(t)$ with

$$G_0(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega J(\omega)e^{-i\omega t}, \tag{7}$$

where the spectral function $J(\omega) = 2\pi\varrho(\omega)[|V_B(\omega)|^2 + |V_C(\omega)|^2]$ is the noise spectrum in the continuous limit (see Appendix A for more details). It only depends on the coupling strengthes of the quantum control field and the noise bath. The noise is expressed as

$$\hat{\epsilon}_{\rm n}(t) = \int_{-\infty}^{+\infty} d\omega\varrho(\omega)v^{\dagger}(\omega)\hat{\epsilon}(\omega,0)e^{-i\omega t}, \tag{8}$$

where

$$\hat{\epsilon}(\omega,0) = \begin{bmatrix}\hat{b}(\omega,0)\\ \hat{c}(\omega,0)\end{bmatrix}, \quad v(\omega) = \begin{bmatrix}V_B(\omega)\\ V_C(\omega)\end{bmatrix}$$

are the value of $[\hat{b}(\omega,t),\hat{c}(\omega,t)]^T$ at $t=0$ and the coupling strength in the continuous frequency form, respectively. In the above derivation, we have assumed that the quantum control field and the noise bath share the same state density function $\varrho(\omega)$.

When the feedback couplings are introduced, both $G(t)$ and $\hat{\epsilon}_{\rm n}(t)$ are affected. Assume $F_{kk'} = f_k\delta_{kk'}$ and further express f_k as $f(\omega) = r(\omega)e^{i\theta(\omega)}$ in the continuous limit and in the polar coordinate. The memory kernel function is seen to be split as $G(t) = G^+(t) + G^-(t)$ with

$$G^{\pm}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega J^{\pm}(\omega)e^{-i[\omega\pm r(\omega)]t}, \tag{9}$$

where the spectral functions given by

$$J^{\pm}(\omega) = \pi\varrho(\omega)|V_B(\omega) \pm V_C(\omega)e^{-i\theta(\omega)}|^2 \tag{10}$$

are modulated by feedback parameters $r(\omega)$ and $\theta(\omega)$. Since the $J^{\pm}(\omega)$ is frequency dependent, the dynamics is generally non-Markovian. Compared with the case without feedback, the split in the memory kernel function shows that the noises can be changed by interference with the quantum control field. The equivalent noise $\hat{\epsilon}_{\rm n}(t)$ in Eq. (6) is

$$\hat{\epsilon}_{\rm n}(t) = \int_{-\infty}^{+\infty} d\omega\varrho(\omega)v^{\dagger}(\omega)\Phi(\omega,t)\hat{\epsilon}(\omega,0)e^{-i\omega t}, \tag{11}$$

where the unitary matrix

$$\Phi(\omega,t) = \begin{bmatrix}\cos[r(\omega)t] & -ie^{-i\theta(\omega)}\sin[r(\omega)t]\\ -ie^{i\theta(\omega)}\sin[r(\omega)t] & \cos[r(\omega)t]\end{bmatrix} \tag{12}$$

is the feedback-induced modulation matrix for each mode. Note that the equivalent noises with or without feedback both satisfy the following commutation relationship:

$$[\hat{\epsilon}_{\rm n}(t),\hat{\epsilon}_{\rm n}^{\dagger}(t')] = G_0(t-t'), \tag{13}$$

which characterizes the statistics of the quantum fluctuation in the noise. The conservation property (13) is not altered by the coherent feedback.

Owing to the linearity of Eq. (6), the solution of $\hat{a}(t)$ is expressed as

$$\hat{a}(t) = u(t)\hat{a}(0) + \int_0^t d\tau u(t-\tau)\hat{\epsilon}_{\rm n}(\tau), \tag{14}$$

where the complex number valued coefficient $u(t)$ satisfies the following integral-differential equation:

$$\dot{u}(t) = -i\omega_0 u(t) - \int_0^t d\tau G(t-\tau)u(\tau), \quad u(0)=1. \tag{15}$$

The absolute value of $u(t)$ is called scaled amplitude of the system. The effect of the feedback Hamiltonian H_{BC} is embodied in both $u(t)$ and $\hat{\epsilon}_{\rm n}(t)$ in Eq. (14). The first term in Eq. (14) describes the system dissipative evolution from its initial state $\hat{a}(0)$ and the second term characterizes the dynamics excited by the noise $\hat{\epsilon}_{\rm n}(t)$.

The above Langevin equation provides the basis for analyzing the closed-loop feedback control dynamics. The feedback control parameters $r(\omega)$ and $\theta(\omega)$ are involved in the memory kernel function $G(t)$ in Eq. (6), which can be used to manipulate the system dynamics by modulating $G(t)$.

Note that when the control field is treated as a classical field, the noise structure of the system remains unchanged with a kernel similar to that of Eq. (7) for the open boson system given above (see Eqs. (62)–(65) in Ref. [12]). This is because the creation and annihilation operators of the system and the bath are only shifted by a constant. For the quantum control field without feedback, namely without H_{BC}, the non-Markovian memory kernel $G(t)$ is a summation of the respective memory kernels of the noise bath and the controller [see Eq. (7)]. Every memory kernel induces its own dissipation and decoherence to the systems. The above results show that only after including H_{BC} to form a closed-loop feedback, the spectral density of the noise bath can be changed, and the non-Markovian dynamics may become controllable.

B. The Markovian limit

As a special case, the coherent feedback control model in the Markovian limit can be directly obtained from (6). This corresponds to all the spectrum functions being flat; i.e.,

$$\varrho(\omega)v^{\dagger}(\omega)v(\omega) \equiv \frac{\gamma_0}{2\pi}, \quad \varrho(\omega)v(\omega) \equiv \frac{\gamma_{\rm n}}{\sqrt{2\pi}},$$

where the decay rate γ_0 and $\gamma_{\rm n}$ are constants. In this case, the memory kernel in Eq. (6) is reduced to

$$G(t) = \frac{\gamma_0}{2\pi}\delta(t), \tag{16}$$

where $\delta(t)$ is the Dirac function. Different from the non-Markovian case, $G(t)$ is independent of the feedback coupling amplitude $r(\omega)$ and phase $\theta(\omega)$. However, the noise $\hat{\epsilon}_{\rm n}(t)$ is still affected by the feedback. For example, under constant feedback strength $f(\omega) \equiv f_0 = r_0 e^{i\theta_0}$, the equivalent noise becomes

$$\hat{\epsilon}_{\rm n}(t) = \gamma_{\rm n}^{\dagger}\begin{bmatrix} \cos r_0 t & -ie^{-i\theta_0}\sin r_0 t \\ -ie^{i\theta_0}\sin r_0 t & \cos r_0 t \end{bmatrix}\hat{\epsilon}_{\rm in}(t), \tag{17}$$

which is driven by a standard quantum white noise

$$\hat{\epsilon}_{\rm in}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} d\omega \hat{\epsilon}(\omega,0)e^{-i\omega t}. \tag{18}$$

The resulting Markovian quantum Langevin equation reads

$$\dot{\hat{a}}(t) = -i\omega_0 \hat{a}(t) - \frac{\gamma_0}{2}\hat{a}(t) - i\hat{\epsilon}_{\rm n}(t), \tag{19}$$

where $\hat{\epsilon}_{\rm n}(t)$ satisfies

$$[\hat{\epsilon}_{\rm n}(t),\hat{\epsilon}_{\rm n}^{\dagger}(t')] = \frac{\gamma_0}{2\pi}\delta(t-t'). \tag{20}$$

In Eq. (19), the decay rate γ_0 and $\gamma_{\rm n}$ of the system mode $\hat{a}(t)$ and the input field only depend on the coupling strengths V_B and V_C. The feedback amplitude r_0 and phase θ_0 cannot affect the dissipation process of $\hat{a}(t)$ or the statistic characteristic of the equivalent noise (20). Although the noise-induced dynamics is still influenced by the feedback [it is shown in Eq. (17) that the noise $\hat{\epsilon}_{\rm n}(t)$ is modulated by the phase θ_0 and the amplitude r_0], feedback has minor influence on the decoherence on the Markovian systems. However, as will be seen below it plays a key role in the non-Markovian case where energy and entropy transfer between the bath and the system can be changed by the feedback.

IV. COHERENT FEEDBACK SUPPRESSION OF DECOHERENCE VIA SPECTRUM MODULATION

The components of the memory kernel function $G(t)$ in Eq. (9) are the key to manipulating the dissipation process in the system dynamics. The split of the kernel function into two parts shows that the closed-loop dynamics is changed by interference between the noise and the quantum control field, as each part is a phase-modulated superposition of their spectral density functions. As in Ref. [51], the interference can be utilized to reduce noises. In this section, we show that such effect can be used to suppress the decoherence effect by tuning the feedback coupling functions $r(\omega)$ and $\theta(\omega)$.

To see more clearly how the feedback modulates the memory kernel, we first transform Eq. (9) via the following variable substitution:

$$\nu^{\pm} = \omega \pm r(\omega). \tag{21}$$

Suppose that ω can be solved as a function of $\nu^{\pm}$, say $\omega = \Theta^{\pm}(\nu)$. We can replace it into (9), and obtain the following modified memory kernel:

$$G^{\pm}(t) = \frac{1}{2\pi}\int_{\nu^{\pm}(\mathbb{R})} d\nu \tilde{J}^{\pm}(\nu)e^{-i\nu t}, \tag{22}$$

where

$$\tilde{J}^{\pm}(\nu) = \frac{d\Theta^{\pm}(\nu)}{d\nu}J^{\pm}[\Theta^{\pm}(\nu)].$$

The resulting total memory kernel function is thus written as

$$G(t) = \frac{1}{2\pi}\int_{\nu^{+}(\mathbb{R})} d\nu \tilde{J}^{+}(\nu)e^{-i\nu t} + \frac{1}{2\pi}\int_{\nu^{-}(\mathbb{R})} d\nu \tilde{J}^{-}(\nu)e^{-i\nu t}, \tag{23}$$

which shows that the kernel function can be taken as the superposition of two parts in the frequency domain.

In the absence of feedback, we have $\tilde{J}^{+}(\nu) = \tilde{J}^{-}(\nu)$. However, when feedback is introduced, they are separated and distorted. Such property can be used to suppress the decoherence via careful design.

The amplitude of the feedback coupling $r(\omega)$ can have a very complex form in practice. For illustration, in the following we will analyze via a simple case how the noise spectrum is changed under a constant feedback coupling function $f(\omega) \equiv r_0 e^{i\theta_0}$, under which the decoherence can be reduced.

Suppose that the spectral functions are nonzero only on some finite interval $[\omega_L,\omega_U]$. When the feedback coupling strength is constant, i.e., $r(\omega) \equiv r_0$ for all $\omega \in \mathbb{R}$, Eq. (23) is transformed to

$$G(t) = \frac{1}{2\pi}\int_{\omega_L + r_0}^{\omega_U + r_0} d\omega \tilde{J}^{+}(\omega)e^{-i\omega t} + \frac{1}{2\pi}\int_{\omega_L - r_0}^{\omega_U - r_0} d\omega \tilde{J}^{-}(\omega)e^{-i\omega t}. \tag{24}$$

Here, the spectral functions are modified to be

$$\tilde{J}^{\pm}(\omega) = J^{\pm}(\omega \mp r_0),$$

which are displaced on the frequency axis by the amount of r_0 without being distorted.

Physically, decoherence originates from the resonance between the system's mode and the noise bath modes. The

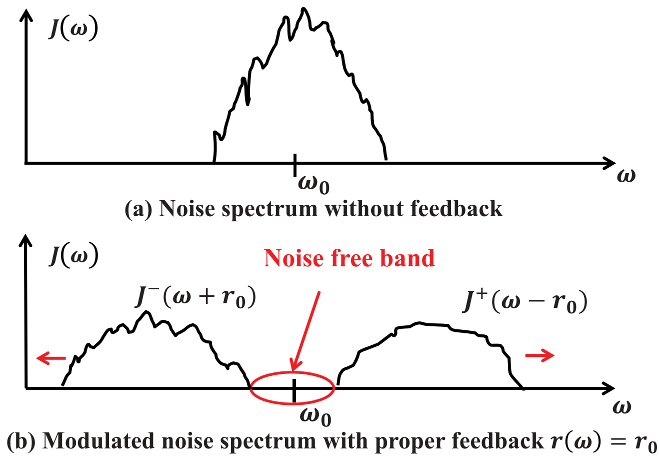

FIG. 2. (Color online) The shape of the noise spectrum without and with feedback $r(\omega) = r_0$. (a) The noise spectrum without feedback, where the system characteristic frequency ω_0 is in the noise band and its resonance with the noise leads to decoherence. (b) The noise spectrum under constant feedback coupling function $r(\omega) = r_0$, where the noise spectrum is split and a noise-free band appears under sufficient large feedback coupling strength. The resulting off-resonance with the noise can be used to suppress non-Markovian decoherence.

strength of the interaction near the characteristic frequency of the system determines how strong the decoherence will be. As shown in Fig. 2(a), when the system's characteristic frequency ω_0 is near the peak of the noise spectrum, the evolution of the system is remarkably affected by the noises, and consequently decoherence will occur. However, as shown in Fig. 2(b), the constant-strength feedback coupling pushes $\tilde{J}^{\pm}(\omega)$ to the opposite directions in the frequency domain. The amplitude of each part depends on how the quantum control field interferes with the noise bath, which can be seen in the example in Sec. V. Under strong feedback coupling, a noise-free band will appear, in which the amplitude of the noise spectrum is zero (or very small). When the the noise-free band is sufficiently broad, the system will be driven out of resonance with the bath and hence the decoherence can be reduced. The effectiveness of decoherence suppression depends on how close the working frequency is to the nearest boundary of the noise-free band. In the limiting case, for example, when the noise-free band is infinitely broad (i.e., in the absence of noise), the decoherence is completely suppressed.

The above analysis shows that decoherence can be suppressed by selecting proper feedback parameters such that the magnitude of the modulated spectral density function is as small as possible near the characteristic frequency of the system. This method makes it possible to design the system in the frequency domain with relatively simple algebraic calculations. Compared with existing methods, this is much simpler without having to do it in the time domain that requires calculation of time-dependent trajectories.

In practice, the ability of decoherence suppression with feedback is limited by the shape and magnitude of $f(\omega)$. The circumstance discussed here is idealized for simplifying the analysis. Realistic feedback coupling should have finite bandwidth, which restricts the width of the noise-free band. Nevertheless, they can be used to suppress the noise, although the corresponding design is more complicated.

We may also point out that there are other methods to suppress decoherence if spin systems are also considered. For example, for an environment-engineering approach, the noise spectrum can be modulated through the increase of the noise amplitude by changing the coupling strength between the system and the environment; see Ref. [47]. Such method does not actively remove the noise. For dynamical decoupling methods, the noise power is reduced via decreasing the time interval Δt between two continuous pulses. This method can only generate a noise-free band in the extreme case that Δt is to be zero [14]. For other methods, such as that given in Refs. [23,24], one reduces decoherence by minimizing the overlap spectrum between the system and the noise. Although decoherence suppression can be obtained via various methods, the corresponding noise modulation schemes are very different. At least to our knowledge, there is no way to obtain the same effects of noise modulation beyond the one we show here through feedback control.

V. EXAMPLE IN PHOTONIC CRYSTAL SYSTEMS

To demonstrate the above idea of decoherence suppression, we consider a physical implementation of the model with a photonic crystal structure [12,46,52,53]. As shown in Fig. 3, the system is realized by a cavity as a point defect, and the noise bath and quantum control field are represented by two linear defects coupled to the cavity. The two linear defects form waveguides for photon transport, which are coupled together at the other end.

As shown in Fig. 3, the coupler [i.e, the Hamiltonian in Eq. (4)] can be realized by overlapping the two waveguides, through which the photons can be exchanged at the interface [54–56]. The coupling strength $F_{kk'}$ is tuned by adjusting the length of the overlap regime and the shape of the defects, or by changing the materials in the defects of the coupler with different refractivities and directly coupling two waveguides

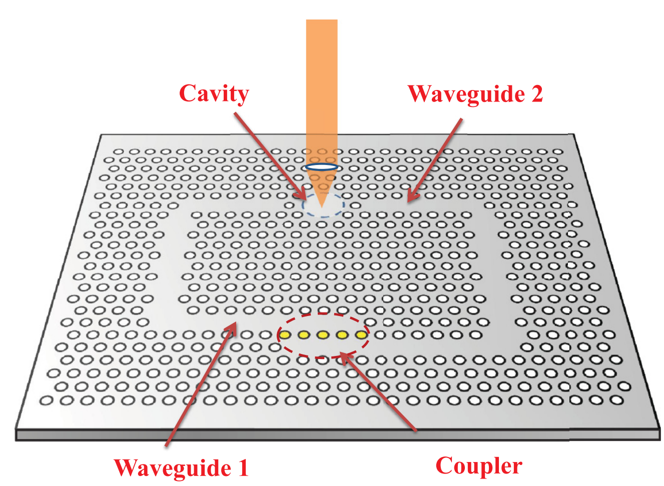

FIG. 3. (Color online) The schematic diagram of actual coherent feedback loop in photonic crystal system where the cavity (system) is a point defect and the waveguides are linear defects. The introduced quantum control field and its interaction with the noise bath are realized by a waveguide and a coupler, respectively.

with a designed impurity crystal. The coupler can also be implemented by placing some nonlinear medium (e.g., atoms, Kerr crystal) between the waveguides to couple the waveguides via matter-field interactions [57].

According to the tight-binding model [12], the frequency-dependent coupling strengths between the cavity and the waveguide are

$$V_{B(C)}(\omega) = \begin{cases} \frac{\eta}{\sqrt{2\pi}}\sqrt{4\xi_{B(C)}^2-(\omega-\omega_{B(C)})^2}, & |\omega-\omega_{B(C)}|\leqslant 2\xi_{B(C)}, \\ 0, & \text{otherwise}, \end{cases} \tag{25}$$

where $\omega_{B(C)}$ are the center frequencies of the two waveguides. The strength of $V_{B(C)}$ and the width of the noise band are determined by η and $\xi_{B(C)}$, respectively. The state densities $\varrho_{B(C)}(\omega)$ are given as

$$\varrho_{B(C)}(\omega) = \begin{cases} \frac{1}{\sqrt{4\xi_{B(C)}^2-(\omega-\omega_{B(C)})^2}}, & |\omega-\omega_{B(C)}|\leqslant 2\xi_{B(C)}, \\ 0, & \text{otherwise}. \end{cases} \tag{26}$$

For simplicity, we assume that the center frequencies of the two waveguides are identical with the system working frequency, i.e., $\omega_B = \omega_C = \omega_0$, and other parameters of the two waveguides are the same, i.e., $V_B(\omega) = V_C(\omega)$, $\varrho_B(\omega) = \varrho_C(\omega)$, and $\xi_B = \xi_C = \xi$. In addition, we assume that the width of the band gap is sufficiently broad so that the shifted noise bands are always contained in the band gap.

We assume that the feedback coupling is constant; i.e., $f(\omega) = r_0 e^{i\theta_0}$. Then, according to Eqs. (25) and (26), the modified memory kernel $G(t) = G^+(t) + G^-(t)$ with

$$G^{\pm}(t) = (1 \pm \cos\theta_0)\eta^2 \times \int_{\omega_L}^{\omega_U} d\omega\sqrt{4\xi^2-(\omega-\omega_0)^2}e^{-i[\omega\pm r_0]t}, \tag{27}$$

where the coupling strength r_0 adjusts their profiles and locations. The phase shift θ_0 modulates the heights of the branches in the frequency domain. Here, θ_0 can be restricted in $[0,\frac{\pi}{2}]$ due to the symmetry of Eq. (27).

In numerical simulations, we select parameters that can be realized through engineering [12,47] as follows: the characteristic frequency of the cavity $\omega_0 = 10$ GHz, $\xi = 0.3$ GHz, and the coupling strength $\eta = 0.3$. The change of $r(\omega)$ alters the decay process in the cavity mode, which is measured by the scaled cavity amplitude $|u(t)| = |\langle a(t)\rangle/\langle a(0)\rangle|$. When $|u(t)|$ is close to 1, the system is maintained coherent.

In Fig. 4, the affection of the feedback on the noise spectrum function and the corresponding evolution of $|u(t)|$ are plotted under constant feedback couplings. In all cases, the curves of $|u(t)|$ in absence of feedback ($r_0 = 0$) oscillatingly and quickly decay when the system's characteristic frequency $\omega_0 = 10$ GHz resides right at the center of the noise band.

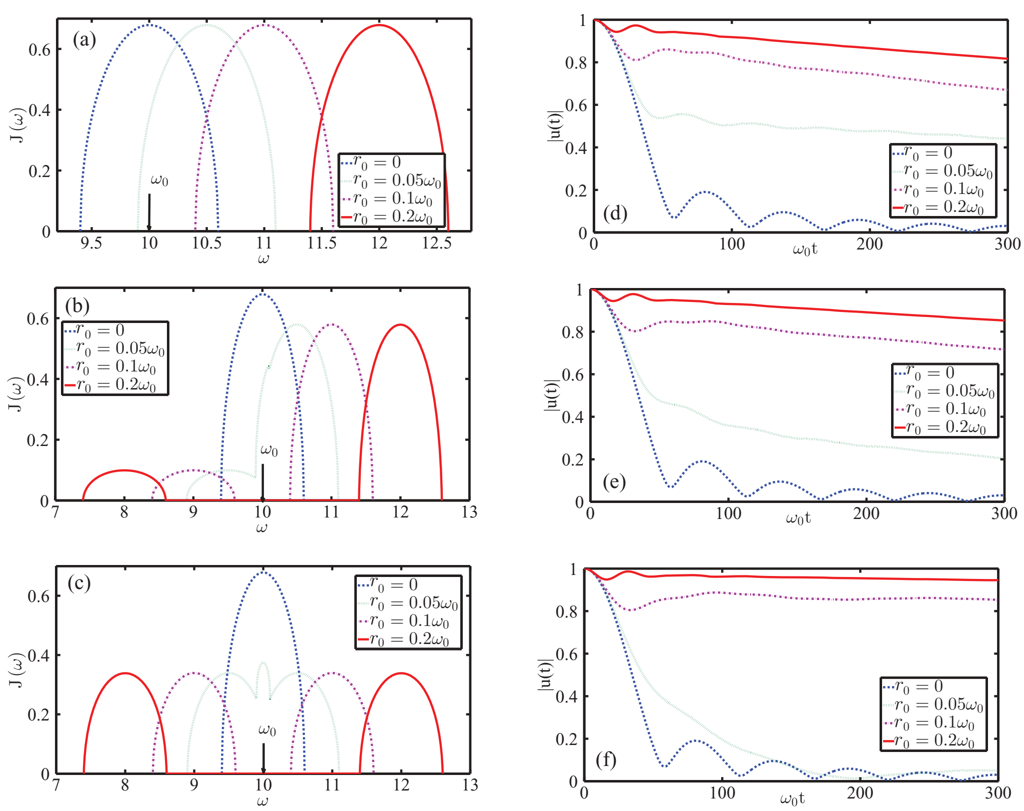

FIG. 4. (Color online) The feedback modulated spectral function in the memory kernel [(a)–(c)] and the corresponding dynamics of scaled cavity amplitude $|u(t)|$ [(d)–(f)] under the constant coupling strength $r(\omega) = r_0$, where ω_0 is the system's characteristic frequency. The figures in each row correspond to different values of the phase variant θ_0, i.e., $\theta_0 = 0$ in (a) and (d), $\theta_0 = \frac{\pi}{4}$ in (b) and (e), and $\theta_0 = \frac{\pi}{2}$ in (c) and (f), which modulate the amplitudes of the two branches. In each case, e.g., $\theta_0 = 0$, when the feedback coupling strength r_0 is increased, the separation of the noise bands induces noise-free bands and the decay of $|u(t)|$ is seen to be remarkably reduced.

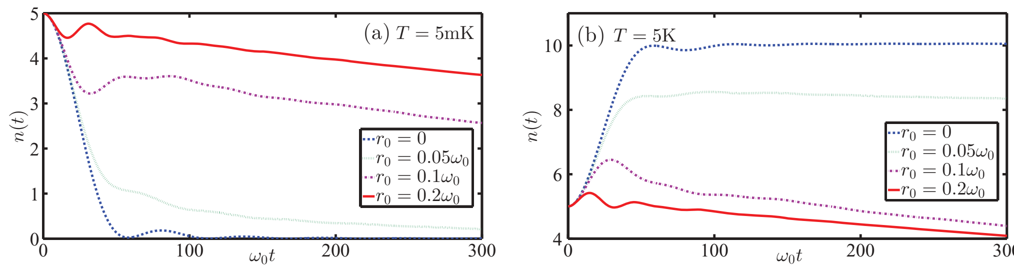

FIG. 5. (Color online) The time evolution curves of the photon number $n(t) = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle$ in the cavity under different values of the constant coupling strength r_0 in (a) the low-temperature $T = 5$ mK case and (b) the high-temperature $T = 5$ K case. The initial photon number is $n(0) = 5$. When the feedback coupling strength r_0 is increased, the processes of the injecting and absorbing photons are slowed down.

The suppression of decoherence can be observed when implementing the coherent feedback, which moves the noise bands away from the system's characteristic frequency. For sufficiently great values of r_0, the magnitude of $|u(t)|$ can be maintained at a high level for a long time. In the first case $\theta_0 \equiv 0$ as shown in Figs. 4(a) and 4(d), the branch $\tilde{J}^-(\omega) = 0$ is missing due to the completely destructive interference between the quantum control field and the noise bath, and it is only the branch $\tilde{J}^+(\omega)$ that affects the system dynamics. For other values of θ_0 shown in Figs. 4(b) and 4(c), the height of $\tilde{J}^\pm(\omega)$ is modulated by feedback. When r_0 is sufficiently large (e.g., $r_0 = 0.1\omega_0$ and $r_0 = 0.2\omega_0$), the spectral function is split into two separate parts between which a noise-free band appears, which contributes to the decoherence suppression that can be observed from the evolution of the corresponding $|u(t)|$.

To investigate the influence of the equivalent noise, we examine the variation of the average photon number $n(t) = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle$ (see Fig. 5) contained in the cavity, which is also dependent on the noise $\hat{\epsilon}_n(t)$. The computation formulas are given in Appendix B. Here, we consider two cases corresponding to the baths being initially in the thermal equilibrium state at a low temperature $T = 5$ mK and a high temperature $T = 5$ K. The initial average photon number $n(0)$ in the cavity is 5. The feedback scheme is chosen as the constant function coupling with the phase $\theta_0 = \frac{\pi}{4}$. In the low-temperature case (a), the baths tend to cool down the cavity by absorbing photons from it, and the fluctuation induced by $\hat{\epsilon}_n(t)$ in the photon number $n(t)$ is very small such that the dynamics can be nearly described by $n(t) \approx |u(t)|^2 n(0)$. The increase of the parameter r_0 can slow down the damping of the photon number $n(t)$, and hold it close to $n(0)$ (e.g., $r_0 = 0.2\omega_0$). In the high-temperature case (b), it can be seen that the baths tend to heat up the cavity by injecting photons from the noise source $\hat{\epsilon}_n(t)$. Hence, a sufficiently strong feedback coupling r_0 is able to prohibit the exchange of photons with the external environment, which exhibits the ability of feedback on decoupling the baths.

VI. CONCLUSION

In this paper, we have presented a coherent quantum feedback scheme for manipulating non-Markovian quantum dynamics, and applied it to the suppression of decoherence by tuning the coupling strength between the quantum control field and the noise bath. The derived non-Markovian Langevin equation shows that the feedback can reshape the memory kernel function, which may be used to suppress the decoherence by driving the noise bath out of resonance with the system. The performance can be improved by increasing the coupling strength. In the example a of photonic crystal, the coupling strength is bounded by the band gap of the photonic crystal, which needs to be carefully chosen in the applications. Moreover, the strategy allows for compact hardware implementations, which can be fabricated for scalable quantum devices on chip [58].

Although our discussion has focused on constant couplings, whose applicability has to be justified in practice, the idea of the feedback modulation is described in detail and can be extended to more general systems with more complicated system-bath couplings. More importantly, the idea of noise spectrum modulation shows that the control design can be conveniently done in the frequency domain, which can be extended to other quantum control problems.

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APPENDIX A: DERIVATION OF NON-MARKOVIAN LANGEVIN EQUATION

To facilitate the following derivation, we assume that $\omega_{Bk} = \omega_{Ck} \equiv \omega_k$ and $F_{kk'} = f_k\delta_{kk'}$; i.e., only resonating pairs of modes are effectively coupled and manipulated. According to the general Heisenberg equation in quantum mechanics

$$\dot{\hat{O}}(t) = -i[\hat{O}(t), H(t)]$$

for arbitrary operator $\hat{O}(t)$, where we let $\hbar = 1$ hereafter, we can obtain the following equations of motion for the system mode and bath modes:

$$\dot{\hat{a}}(t) = -i\omega_0\hat{a}(t) - i\sum_k [V_{Bk}^*\hat{b}_k(t) + V_{Ck}^*\hat{c}_k(t)], \quad \text{(A1)}$$

$$\dot{\hat{b}}_k(t) = -i\omega_k\hat{b}_k(t) - if_k^*\hat{c}_k(t) - iV_{Bk}\hat{a}(t), \quad \text{(A2)}$$

$$\dot{\hat{c}}_k(t) = -i\omega_k\hat{c}_k(t) - if_k\hat{b}_k(t) - iV_{Ck}\hat{a}(t). \quad \text{(A3)}$$

First, the modes of the noise bath and the quantum control field in Eqs. (A2) and (A3) can be paired and solved in terms of the system mode:

$$\hat{\epsilon}_k(t) = \Phi_k(t)e^{-i\omega_k t}\hat{\epsilon}_k(0) - i\int_0^t d\tau\,\Phi_k(t-\tau)v_k \times e^{-i\omega_k(t-\tau)}\hat{a}(\tau), \quad \text{(A4)}$$

where

$$\hat{\epsilon}_k(t) = \begin{bmatrix}\hat{b}_k(t)\\ \hat{c}_k(t)\end{bmatrix}, \quad v_k = \begin{bmatrix}V_{Bk}\\ V_{Ck}\end{bmatrix}.$$

Using the polar coordinates $f_k = r_k e^{i\theta_k}$ of the feedback parameters, we have

$$\Phi_k(t) = \exp\left[-it\begin{pmatrix}0 & f_k^*\\ f_k & 0\end{pmatrix}\right] = \begin{bmatrix}\cos[r_k t] & -ie^{-i\theta_k}\sin[r_k t]\\ -ie^{i\theta_k}\sin[r_k t] & \cos[r_k t]\end{bmatrix}. \quad \text{(A5)}$$

Then, substituting Eq. (A4) into Eq. (A1), we can get a closed-form dynamical equation of the system, i.e., the quantum Langevin equation as

$$\dot{\hat{a}}(t) = -i\omega_0\hat{a}(t) - \int_0^t d\tau\,G(t-\tau)\hat{a}(\tau) - i\hat{\epsilon}_{\rm n}(t), \quad \text{(A6)}$$

where the memory kernel function and the equivalent noise are defined as

$$G(t) = \sum_k v_k^\dagger\Phi_k(t)v_k e^{-i\omega_k t},$$
$$\hat{\epsilon}_{\rm n}(t) = \sum_k v_k^\dagger\Phi_k(t)\hat{\epsilon}_k(0)e^{-i\omega_k t}. \quad \text{(A7)}$$

The memory kernel function is further decomposed as

$$G(t) = G^+(t) + G^-(t) \quad \text{(A8)}$$

with

$$G^\pm(t) = \frac{1}{2}\sum_k |V_{Bk} \pm V_{Ck}e^{-i\theta_k}|^2 e^{-i(\omega_k \pm r_k)t}, \quad \text{(A9)}$$

which are modulated by r_k and θ_k.

Moreover, one can examine that the equivalent noise $\hat{\epsilon}_{\rm n}(t)$ satisfies the following commutative relationship:

$$[\hat{\epsilon}_{\rm n}(t),\hat{\epsilon}_{\rm n}^\dagger(t')] = \sum_k [|V_{Bk}|^2 + |V_{Ck}|^2]e^{-i\omega_k(t-t')}. \quad \text{(A10)}$$

In the continuous limits that the modes of the quantum control field and the noise bath are very densely distributed, one can approximate the sum in the above expressions by an integral over a continuous distribution of the angular frequency ω. Assume that the density of states of the quantum control field and the noise bath are both $\varrho(\omega)$. The feedback coupling strength is expressed in the polar coordinate as $f(\omega) = r(\omega)e^{i\theta(\omega)}$. We have

$$G^\pm(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\,J^\pm(\omega)e^{-i[\omega\pm r(\omega)]t}, \quad \text{(A11)}$$

where $J^\pm(\omega) = \pi\varrho(\omega)|V_B(\omega) \pm V_C(\omega)e^{-i\theta(\omega)}|^2$, and the noise term $\hat{\epsilon}_{\rm n}(t)$ in Eq. (6) is

$$\hat{\epsilon}_{\rm n}(t) = \int_{-\infty}^{+\infty} d\omega\varrho(\omega)v^\dagger(\omega)\Phi(\omega,t)\hat{\epsilon}(\omega,0)e^{-i\omega t}, \quad \text{(A12)}$$

where

$$\hat{\epsilon}(\omega,0) = \begin{bmatrix}\hat{b}(\omega,0)\\ \hat{c}(\omega,0)\end{bmatrix}, \quad v(\omega) = \begin{bmatrix}V_B(\omega)\\ V_C(\omega)\end{bmatrix},$$

and

$$\Phi(\omega,t) = \begin{bmatrix}\cos[r(\omega)t] & -ie^{-i\theta(\omega)}\sin[r(\omega)t]\\ -ie^{i\theta(\omega)}\sin[r(\omega)t] & \cos[r(\omega)t]\end{bmatrix}. \quad \text{(A13)}$$

The commutative relationship of the equivalent noise is rewritten as

$$[\hat{\epsilon}_{\rm n}(t),\hat{\epsilon}_{\rm n}^\dagger(t')] = \int_{-\infty}^{+\infty} d\omega\varrho(\omega)[|V_B(\omega)|^2 + |V_C(\omega)|^2]e^{-i\omega(t-t')}. \quad \text{(A14)}$$

APPENDIX B: CALCULATION OF PHOTON NUMBER $n(t)$

First, we assume the total system and two baths are initially uncorrelated. Then using Eq. (14), the average photon number of the system $n(t) = \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle$ can be calculated as

$$n(t) = \langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = |u(t)|^2 n(0) + \int_0^t d\tau\int_0^t d\tau' u(t-\tau')\widetilde{G}(\tau,\tau')u^*(t-\tau), \quad \text{(B1)}$$

where

$$\widetilde{G}(\tau,\tau') = \sum_k \frac{e^{i\omega_k(\tau-\tau')}}{4}[(V^+e^{-ir_k\tau} + V^-e^{ir_k\tau})^* \times (V^+e^{-ir_k\tau'} + V^-e^{ir_k\tau'})n_{Bk} + (V^+e^{-ir_k\tau} - V^-e^{ir_k\tau})^* \times (V^+e^{-ir_k\tau'} - V^-e^{ir_k\tau'})n_{Ck}], \quad \text{(B2)}$$

with $V^\pm = V_{Bk} \pm V_{Ck}e^{-i\theta_k}$, and the initial photon distribution function of the baths $n_{Bk} = \langle\hat{b}_k^\dagger(0)\hat{b}_k(0)\rangle$, $n_{Ck} = \langle\hat{c}_k^\dagger(0)\hat{c}_k(0)\rangle$, which are equal to $1/(e^{\beta\omega_k} - 1)$, where $\beta = 1/k_B T_{B(C)}$ and $T_{B(C)}$ are the temperature of the noise bath and the quantum control field, respectively.

In the continuous limit, $\widetilde{G}(\tau,\tau')$ is reexpressed in a continuous frequency form as

$$\widetilde{G}(\tau,\tau') = \int_{-\infty}^{+\infty} d\omega\varrho(\omega)[[V^+(\omega)e^{-ir(\omega)\tau} + V^-(\omega)e^{ir(\omega)\tau}]^* \times [V^+(\omega)e^{-ir(\omega)\tau'} + V^-(\omega)e^{ir(\omega)\tau'}]n_B(\omega,T) + [V^+(\omega)e^{-ir(\omega)\tau} - V^-(\omega)e^{ir(\omega)\tau}]^* \times [V^+(\omega)e^{-ir(\omega)\tau'} - V^-(\omega)e^{ir(\omega)\tau'}]n_C(\omega,T)] \times \frac{e^{i\omega(\tau-\tau')}}{4}, \quad \text{(B3)}$$

where $V^\pm(\omega) = V_B(\omega) \pm V_C(\omega)e^{-i\theta(\omega)}$.

When the temperature of the noise bath and the quantum control field are the same, where $n_B(\omega,T) = n_C(\omega,T) = 1/(e^{\beta\omega} - 1)$, Eq. (B3) can be simplified to $\widetilde{G}(\tau,\tau') =$

$\widetilde{G}^{+}(\tau,\tau') + \widetilde{G}^{-}(\tau,\tau')$ with

$$\widetilde{G}^{\pm}(\tau,\tau') = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega J^{\pm}(\omega)e^{-i[\omega\pm r(\omega)](\tau'-\tau)}, \quad \text{(B4)}$$

which is coincident with Eq. (A11). Hence, we have $\widetilde{G}(\tau,\tau') = G(\tau' - \tau)$ when the noise bath and the quantum control field are at the same temperature.

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