Wavelength scaling of high-order-harmonic-generation efficiency by few-cycle laser pulses: Influence of carrier-envelope phase

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We investigate the wavelength scaling of high-order-harmonic-generation (HHG) efficiency by few-cycle laser pulses where the influence of the carrier-envelope phase (CEP) becomes prominent. Based on the numerical solution of time-dependent Schrödinger equation, in the infrared region of λ (0.8–1.8 μm), we observe that the power law of the form λ^{-x} ($x \approx 5$–6) is still present; but for constant intensity and a fixed energy interval, it differs markedly in the CEP. We find that for a specific value of CEP the wavelength dependence follows a $\lambda^{-4.6}$ scaling in high-harmonic yield. We also perform classical trajectory calculations to gain some understanding of the CEP dependence of the wavelength scaling.

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I. INTRODUCTION

For nearly two decades, high-order harmonic generation (HHG) has been considered to be central to generating attosecond pulses in the extreme ultraviolet, which unveiled the probing of electronic processes in the attosecond time scale with high precision [1–3]. Many ideas and applications [1–3] thought to be crucial in science have been enabled thanks to the circumstances and vast developments in HHG. The general characteristics of the HHG process can be explained by the three-step model [4] in which the process happens by (i) ionization of the atom under the action of the laser field, (ii) propagation in the continuum, gaining kinetic energy, and finally (iii) returning back to the parent ion and emission of photons upon recombination. The maximum photon energy that can be generated during the harmonic emission is determined by the cutoff law $\omega_c = |E_b| + 3.17U_p$, where $|E_b|$ is the binding energy of the electron [4]. The maximum kinetic energy acquired from the laser field is $3.17U_p$, and U_p is linked to λ by $U_p \sim I\lambda^2$. *Prima facie*, using a longer wavelength has the advantage that many more energetic photons and as a result much shorter pulses can be generated, but the efficiency (denoted by ΔI) of the process suffers from λ^{-x} scaling [5]. Early studies [5] suggested a λ^{-3} scaling in the high-harmonic yield due to wave-packet spreading, but recent theoretical calculations [6–8] and experimental observations [9,10] have revealed a much faster decrease in the efficiency, i.e., on the order of $\sim\lambda^{-5}$–λ^{-6}. However, a number of studies suggest that the decrease could be compensated for by multicolor driving [11,12] in which they predict that $\sim\lambda^{-3}$–λ^{-4} law in wavelength scaling is possible. The classical description states that using a fixed energy interval in the harmonic spectrum brings out an additional factor of λ^{-2} [6,7] since the size of the plateau in the harmonic spectrum scales as $I\lambda^2$. Even then, experiments exhibit a much faster decrease contrary to theoretical predictions. In Ref. [9], it was observed that for a laser field of $\lambda = 2\ \mu$m the size of the plateau could be extended as opposed to $\lambda = 0.8\ \mu$m but the harmonic yield is six times less than theoretical predictions [6]. More dramatically, an overall $x \approx 6.3$–6.5 dependence in ΔI is overt [10]. Yet, so far no well-established description has been asserted. Due to its potential applications, wavelength scaling in the high-harmonic yield has attracted much attention over the years.

In this paper, we address the concept that the carrier-envelope phase (CEP) of the laser field plays a crucial role in wavelength scaling when few-cycle laser pulses are taken into account. The variation of the *profile* of the harmonic spectrum—generated by few-cycle laser pulses—due to the CEP is not surprising [13–15], since the temporal shape of the laser field changes dramatically as the CEP changes. For instance, it has been shown that for few-cycle laser fields, the effect of the CEP on a high-order above-threshold ionization process is straightforward and a critical parameter to control the field-matter interaction [14]. On the other hand, our calculations show that an additional factor of $\sim\lambda^{-1}$ may emerge in the wavelength scaling as a function of the CEP. The selection of the CEP is central to the xuv continuum, as well as isolated attosecond pulse generation via HHG. A laser field with longer wavelengths, and with a critically chosen CEP, has indeed the potential to yield a spectral interval with a much broader band of continuum. Atomic units (a.u.) are used throughout, unless otherwise stated.

II. RESULTS AND DISCUSSIONS

The HHG resulting from the interaction of the atom with a linearly polarized strong laser field is modeled by the numerical solution of the time-dependent Schrödinger equation in the length gauge,

$$i\frac{\partial\psi(\vec{r},t)}{\partial t} = \left[-\frac{1}{2}\nabla^2 + V(r) + zE(t)\right]\psi(\vec{r},t). \qquad (1)$$

The target atom is chosen to be hydrogen, so $V(r) = -1/r$. Here the time dependence of the electric field $E(t)$ is chosen to be $E(t) = E_0 \exp[-(4\ln 2)t^2/\tau^2]\cos(\omega_0 t + \phi_{\text{CEP}})$, where E_0 is the peak field amplitude and ω_0 is the laser frequency. τ is the field duration at FWHM. For τ we use a pulse duration of two cycles at FWHM in each calculation. The numerical solution of the time-dependent Schrödinger equation (TDSE) is carried

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TABLE I. x values in $\Delta I \propto \lambda^{-x}$ as found by fitting to ΔI values.

I (TW/cm^2)	ΔE (eV)	[17]	[8]	This work
160	20–50	5.3	5.3	5.7
320	20–50		5.0	4.8
320	20–70		5.0	4.9
320	20–90		4.9	4.7

out by the alternating direction implicit method [16], where for the solution of the time-dependent wave function, the ansatz of the form $\psi(\vec{r},t) = \sum_l R_l(r,t)Y_l^0(\theta)/r$ is considered. The harmonic spectrum is calculated by the Fourier transform of the dipole acceleration. The HHG yield, on the other hand, is obtained from the integration of the $|a(\omega)|^2$ [where, $a(\omega)$ is the Fourier transform of the dipole acceleration] over a fixed energy interval ΔE [6,7].

Before we demonstrate the CEP dependence of the wavelength scaling in HHG yield ΔI, we begin by calculating the wavelength dependence in multicycle laser pulses. The main reason for doing this is to provide a benchmark study. To make a direct comparison, an eight-cycle flat-top laser pulse with half-cycle ramp up and down is used. The intensity of the laser field is chosen to be 160 and 320 TW/cm^2. The wavelength dependence is considered between the values $\lambda = 0.8$ and 1.8 μm with a relatively coarse mesh of $\Delta\lambda = 0.1$ μm and the results are then fitted to a power law of the form $\Delta I \propto \lambda^{-x}$. The results are collected in Table I. As can be seen from the table, although there are small discrepancies, our results for x are in good agreement with those in [8,17] within 4% and the overall results are within $x \approx 5$–6. It should be noted that the predictions of Frolov *et al.* are based on the time-dependent effective range (TDER) method [8].

Figure 1 shows the harmonic spectrum from a hydrogen atom for two different wavelengths of the laser field, $\lambda = 0.8$ and $\lambda = 1.8$ μm. Hereafter, a Gaussian-type laser pulse with two cycles at FWHM is used. As seen in Fig. 1, for a longer wavelength of $\lambda = 1.8$ μm, an extended harmonic emission

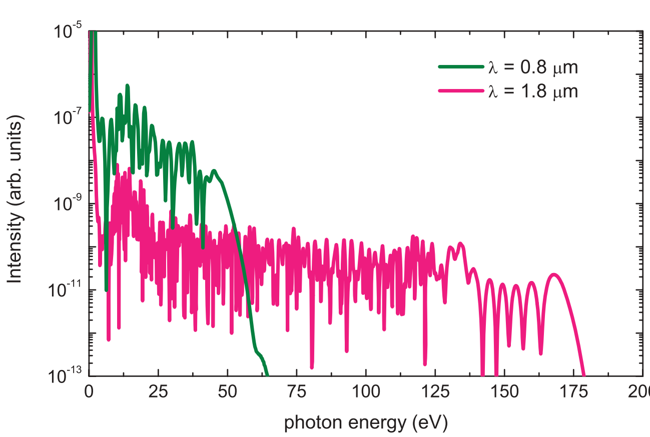

FIG. 1. (Color online) HHG from hydrogen atom for $\lambda = 0.8$ and $\lambda = 1.8$ μm. For the laser pulse, a Gaussian envelope function with two cycles at FWHM and $I = 160$ TW/cm^2 of intensity, $\phi_{\rm CEP} = 0$ is used. Note that when $\lambda = 1.8$ μm is used, although the cutoff is extended by ~125 eV its efficiency is lowered by roughly two orders of magnitude.

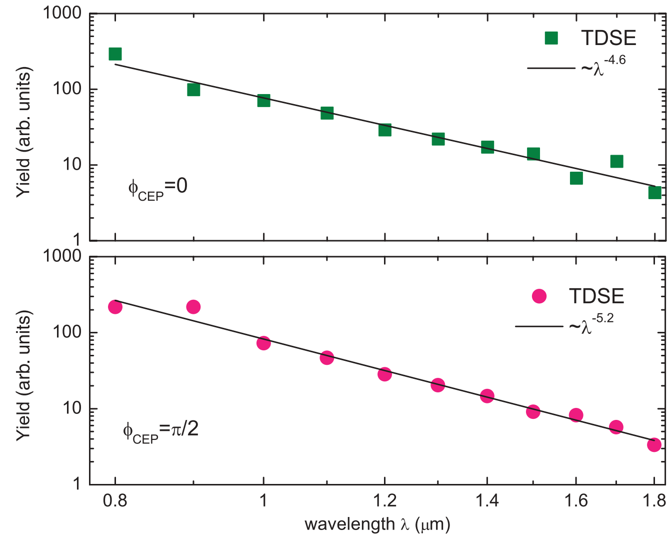

FIG. 2. (Color online) Wavelength dependence of the high-harmonic yield ΔI for two different CEP values. The intensity of the pulse is $I = 320$ TW/cm^2. The harmonic yield is taken over 20–50 eV energies.

could be generated, in which the cutoff position is ~125 eV longer than that of $\lambda = 0.8$ μm. This is because $\omega_c \sim I\lambda^2$. Another feature is the apparent double-plateau structure of the harmonic spectrum for $\lambda = 1.8$ μm. The first cutoff is located at 135 eV whereas the second cutoff is at 170 eV. On the other hand, beyond the first cutoff, the spectral profile of the harmonics are smooth. For the $\phi_{\rm CEP} = 0$ case of the temporal profile of the laser field, the highest peak of the laser field coincides with the peak of the laser pulse, whereas this peak does not repeat itself. Thus, the emission of the cutoff harmonics takes place once and an xuv continuum is formed [13,18]. However, the efficiency of the harmonic spectrum is substantially lower (see Fig. 1) than that for $\lambda = 0.8$ μm, due in part to the wave-packet spreading [5]. Another feature is that the harmonic spectrum exhibits a gradual cutoff at 44 eV for $\lambda = 0.8$ μm, whereas it is sharper at 170 eV for $\lambda = 1.8$ μm, in agreement with [6].

In Fig. 2, we investigated the wavelength scaling for $\phi_{\rm CEP} = 0$ as well as for $\phi_{\rm CEP} = \pi/2$ within the values $\lambda = 0.8$–1.8 μm with a relatively coarse mesh of $\Delta\lambda = 0.1$ μm. The intensity of the laser field is $I = 320$ TW/cm^2. For $\phi_{\rm CEP} = 0$ the wavelength scaling occurs with a power law $\sim\lambda^{-4.6}$, which is somewhat lower than previous predictions [6–8]. This suggests a relatively slower decrease in yield comparing with $\lambda^{-5},\lambda^{-6}$ scaling. For $\phi_{\rm CEP} = \pi/2$, however, the scaling is $\sim\lambda^{-5.2}$ and has a faster decrease than for $\phi_{\rm CEP} = 0$. Although it is quite troublesome to identify using this set of data, we may say that the oscillatory behavior (emphasized in Refs. [7,18]) does not manifest itself *entirely*, since ultra-short pulses are used in our case and thus the high-order returns generating the oscillatory behavior of the wavelength scaling are less likely [18]. We also perform wavelength scaling analysis for different intensities (160 and 320 TW/cm^2) and different energy intervals (20–50, 20–70, and 20–90 eV) for the hydrogen atom. We collect our x values of λ^{-x} dependence in Table II. In all cases the x values are higher for $\phi_{\rm CEP} = \pi/2$, which give a faster decrease in high-harmonic yield as a function of wavelength.

TABLE II. x values in $\Delta I \propto \lambda^{-x}$ for two different CEPs.

I (TW/cm^2)	ΔE (eV)	$\phi_{\rm CEP} = 0$	$\phi_{\rm CEP} = \pi/2$
160	20–50	4.3	4.4
320	20–50	4.6	5.2
320	20–70	4.6	5.2
320	20–90	4.5	5.1

However, for 160 TW/cm^2 the difference is negligible and for 320 TW/cm^2 intensity the difference in the CEP brings out roughly an additional factor of $\lambda^{-0.6}$ to wavelength scaling.

Next, we considered the CEP dependence of the high-harmonic yield for $\lambda = 0.8$ and $\lambda = 1.8$ μm between $\phi_{\rm CEP} = 0$ and $\pi/2$ and for a fixed energy interval. Note that due to the inversion symmetry of the laser field the CEP dependence would repeat itself in every $\pi/2$. Although all plots seen in Fig. 3 have their own genre with respect to CEP dependence, the general behavior is that it starts with a maximum at $\phi_{\rm CEP} = 0$ and have a minimum in between. Except for $\lambda = 1.8$ μm (empty circles) in Fig. 3(a), in all cases the high-harmonic yield is higher for $\phi_{\rm CEP} = 0$ than for $\pi/2$. On the other hand, in both Figs. 3(a) and 3(b), the high-harmonic yields exhibits a smoother ascent and descent for $\lambda = 0.8$ μm, whereas for $\lambda = 1.8$ μm they are somewhat diffused. The vague oscillatory behavior might be the effect of the quantum-path interference [6,7,17]. For Fig. 3(a) the intensity of the laser field is 160 TW/cm^2 and for $\lambda = 0.8$ μm the cutoff is at 44 eV. This means that the integration interval (20–50 eV) of the high-harmonic yield is around the cutoff position. For $\phi_{\rm CEP} = 0$ the cutoff is emitted once and only two returns (long and short) contribute to harmonic yield. For $\phi_{\rm CEP} = 0$–$\pi/2$ the cutoff is emitted twice, and four returns contribute to harmonic yield. For $\lambda = 1.8$ μm, however, the integration interval is well below the cutoff position, which suggests that several returns are possible. As pointed out in Refs. [7,17] high-order returns give rise to fine-scale oscillations in yield.

To gain a better understanding of the CEP dependence of the wavelength scaling in high-harmonic yield, we performed

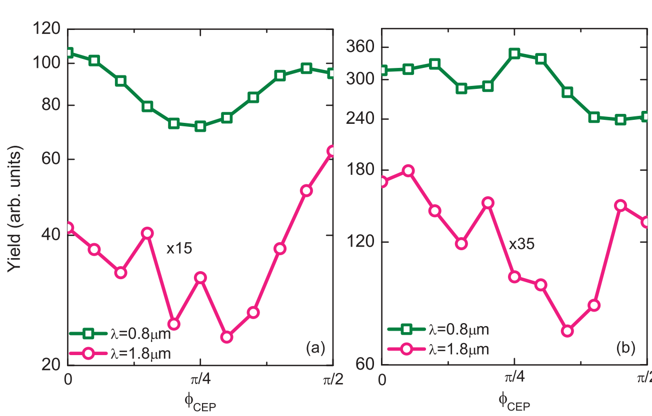

FIG. 3. (Color online) High-harmonic yield ΔI for $\lambda = 0.8$ and $\lambda = 1.8$ μm as a function of the CEP of the laser pulse. The intensity I of the pulse is (a) 160 and (b) 320 TW/cm^2. The high-harmonic yield is taken over 20–50 eV energies. ΔI values are multiplied by (a) 15 and (b) 35 for $\lambda = 1.8$ μm for a better view.

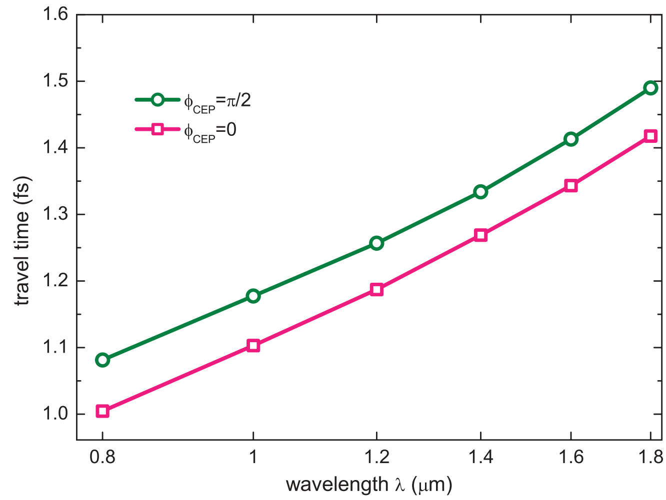

FIG. 4. (Color online) Wavelength dependence of the travel time of the classical electron trajectories that give 35 eV of energy upon recombination.

a series of classical trajectory calculations using a three-step model [4]. A classical travel time of a continuum electron trajectory may provide a deeper insight since the travel time is directly proportional to wave-packet spread, which in turn affects the efficiency of the harmonic emission. Due to the degeneracy of the quantum paths contributing to the harmonic emission, we decided to focus on the electron trajectories around the peak of the laser field, which provide the maximum contribution to the harmonic emission. For few-cycle laser pulses, the harmonic emission that forms the size of the plateau occurs once for $\phi_{\rm CEP} = 0$ and twice for $\phi_{\rm CEP} = \pi/2$. In addition to considering electron trajectories around the maximum, we focused on the short electron trajectory. As shown in Fig. 4, for a fixed photon energy of 35 eV, the travel time of the electron trajectories are systematically lower for $\phi_{\rm CEP} = 0$ at each value of λ. Since a short travel time suggests a higher efficiency in harmonic emission, the high-harmonic yield for $\phi_{\rm CEP} = 0$ should be higher than that for $\phi_{\rm CEP} = \pi/2$. Moreover, in both cases practically the travel time $\tau \propto \sqrt{\lambda}$. This qualitatively explains the decreasing efficiency in wavelength scaling in high-harmonic yield.

III. CONCLUSION

In conclusion, we addressed the CEP dependence of the wavelength scaling in high-harmonic yield for few-cycle laser pulses. It turns out that wavelength scaling in high-harmonic yield evidently differs in the CEP. We found that for $\phi_{\rm CEP} = \pi/2$ the wavelength scaling is $\lambda^{-5.2}$ whereas for $\phi_{\rm CEP} = 0$ it is $\lambda^{-4.6}$. As revealed by classical trajectories, the travel time of the electron trajectories are systematically lower for $\phi_{\rm CEP} = 0$ than for $\phi_{\rm CEP} = \pi/2$.

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