Dissipative preparation of multibody entanglement via quantum feedback control

Jie Song,[1,*] Yan Xia,[2] Xiu-Dong Sun,[1] and He-Shan Song[3]
[1]*Department of Physics, Harbin Institute of Technology, Harbin 150001, People's Republic of China*
[2]*Department of Physics, Fuzhou University, Fuzhou 350002, China*
[3]*School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, China*
(Received 9 July 2012; published 4 September 2012)

We investigate the generation of a multibody Dicke state in a coupled cavity system subject to environmental noise. Based on quantum feedback control, cavity decay may play a constructive role in obtaining the intended state. The required interaction time need not be accurately controlled. In addition, the feedback operations are only applied to a single atom in one cavity during the whole evolution process, and it is not necessary to change the control strategy as the number of atoms increases. Thus, our proposal can exploit the core advantage of coupled cavities to implement a scalable control scheme for preparing multibody entanglement.

DOI: 10.1103/PhysRevA.86.034303 PACS number(s): 03.67.Bg, 03.65.Ud

By now, it is well known that entanglement is a key resource for quantum computation and quantum communication. Therefore, many theoretical schemes have been proposed to generate entangled states [1–3], which lead to experimental realization of few-body entangled states [4,5]. Because multibody entanglement is very important for studying the further characterization of many-body physics, extensive experimental work has been performed in order to observe multiqubit Greenberger-Horne-Zeilinger (GHZ) states [6–9]. Different from the GHZ state, the Dicke state, which is more robust than the GHZ state against decoherence, has valuable applications in quantum-information processing. These two classes of entangled states cannot be converted into each other by local operations and classical communications [10]. Thus, it is also favorable to generate a Dicke state in different systems. For example, in Refs. [11–15], the authors have presented schemes to generate a multiqubit Dicke state by using a Raman adiabatic passage, a selective atom-cavity interaction, and a Rydberg blockade.

Unfortunately, a real quantum system interacts with the environmental noise inevitably so that the quantum coherence is destroyed. To reduce the detrimental effects of noise, several authors have presented different methods for generating entanglement between two atoms that are coupled to a cavity in the presence of decoherence [16–19]. In addition, the authors of Ref. [20] show that optical pumping in combination with spontaneous emission can be used to generate multibody entanglement, such as a GHZ state and a linear cluster state. On the other hand, via quantum feedback control, the future dynamics of a system is modified according to the measurement results in a noisy environment. Recently, an experiment [21] that generates a photon number state by real-time quantum feedback has been reported. Besides, several feedback schemes [22–25] for the creation of two-atom, three-atom, and four-atom entangled states have been presented in a single cavity. Altogether, quantum feedback can be a significant step towards implementing complex quantum-information operations. However, with increasing the qubit number, the dynamics, in general, becomes complex, and the entanglement is very fragile under the influence of noise. Thus, one of the remaining challenges in developing the feedback scheme for quantum-information processing is to devise methods to prepare different types of entanglement in large quantum systems.

In this Brief Report, we propose a feedback strategy to create a multibody Dicke state in a coupled cavity system. Via quantum feedback control, the system is deterministically driven to the intended steady state in the presence of cavity decay. The fluctuation in the cavity decay rate has almost no influence on the generation of steady-state entanglement. Compared with the control strategies in a single cavity, our scheme can be directly generalized to create multibody entanglement. The control strategy need not be changed as the atomic number increases. In addition, because only the single qubit feedback operation is applied to an atom, which is alone in a cavity, the operation has no chance of disturbing the other atoms in another cavity [26–28].

We consider that $(n+1)$ identical atoms interact with two spatially separated cavities, as shown in Fig. 1. The atom $(n+1)$ is trapped in cavity 2, and the other n atoms are trapped in cavity 1. The energy of level $|g\rangle$ is taken to be zero as the energy reference point. The lower-lying level $|f\rangle$ and the upper levels $|u\rangle$, $|e\rangle$ have energies ω_f, ω_u, and ω_e, respectively ($\hbar = 1$). In cavity 1, the transition $|g\rangle \leftrightarrow |e\rangle$ ($|f\rangle \leftrightarrow |e\rangle$) is coupled to the cavity mode with the coupling constant g_1 (g_2), whereas, the transition $|g\rangle \leftrightarrow |u\rangle$ is driven by a classical field. In cavity 2, the transitions $|f\rangle \leftrightarrow |e\rangle$ and $|g\rangle \leftrightarrow |e\rangle$ are driven through a classical field and a cavity mode with the coupling constants Ω_2 and g_3, respectively. The Hamiltonian for the whole system is written as

$$H = H_1 + H_2 + H_h, \tag{1}$$

where

$$\begin{aligned} H_1 &= \omega_0 c_A^\dagger c_A + \omega_a c_a^\dagger c_a + \sum_{k=1}^{n} [\omega_f |f\rangle_k\langle f| + \omega_e |e\rangle_k\langle e| \\ &\quad + \omega_u |u\rangle_k\langle u| + (g_1 c_A |e\rangle_k\langle g| + g_2 c_a |e\rangle_k\langle f| \\ &\quad + \Omega_1 e^{-i\omega_1 t} |u\rangle_k\langle g| + \text{H.c.})], \\ H_2 &= \omega_0 c_B^\dagger c_B + \omega_f |f\rangle_{n+1}\langle f| + \omega_e |e\rangle_{n+1}\langle e| \\ &\quad + (\Omega_2 e^{-i\omega_2 t} |e\rangle_{n+1}\langle f| + g_3 c_B |e\rangle_{n+1}\langle g| + \text{H.c.}), \\ H_h &= \nu (c_A^\dagger c_B + c_B^\dagger c_A), \end{aligned} \tag{2}$$

*jsong@hit.edu.cn

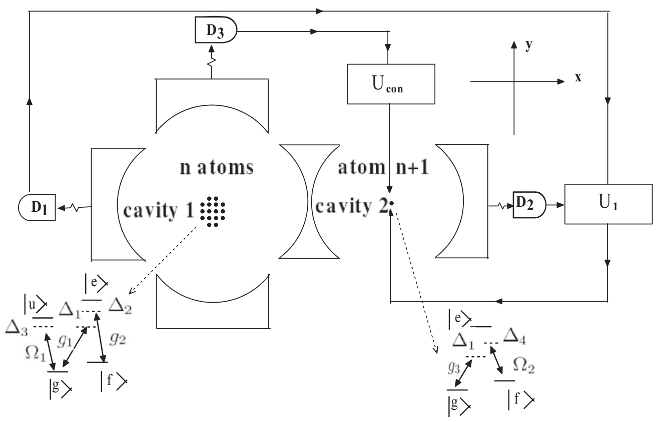

FIG. 1. (a) Schematic of the coupled cavity setup. Correspondingly, the atomic level configuration is given. Two cavities 1 and 2 are coupled by photon hopping along the x direction. The photons can been detected by detector D_3 (D_1 and D_2) in the y (x) direction.

where H_1 (H_2) corresponds to the system consisting of atoms $1,2,\ldots,n$ (atom $n+1$) and cavity field 1 (2). H_h denotes the interaction between cavities. Subscripts $(n+1)$ and k represent the $(n+1)$-th and kth atom. c_A and c_a are the annihilation operators for the modes of cavity 1, respectively, and c_B is the annihilation operator for the mode of cavity 2. ω_a (ω_0) is the frequency of cavity mode c_a (c_A or c_B). ν is the hopping rate of photons between two cavities. In the interaction picture, the Hamiltonian is

$$\begin{aligned} H_i = & \sum_{k=1}^{n} [(g_1 e^{i\Delta_1 t} c_A |e\rangle_k\langle g| + g_2 e^{i\Delta_2 t} c_a |e\rangle_j\langle f| \\ & + \Omega_1 e^{i\Delta_3 t}|u\rangle_j\langle g|)] + (g_3 e^{i\Delta_1 t} c_B |e\rangle_{n+1}\langle g| \\ & + \Omega_2 e^{i\Delta_4 t}|e\rangle_{n+1}\langle f|) + \nu c_A^\dagger c_B + \text{H.c.}, \end{aligned} \tag{3}$$

where $\Delta_1 = \omega_e - \omega_0$, $\Delta_2 = \omega_e - \omega_f - \omega_a$, $\Delta_3 = \omega_u - \omega_1$, and $\Delta_4 = \omega_e - \omega_f - \omega_2$. The dissipative dynamics is governed by the master equation,

$$\begin{aligned} \dot{\rho} = & -i[H_i,\rho] - \frac{\kappa_1}{2}(c_A^\dagger c_A \rho - 2U_1 c_A \rho c_A^\dagger U_1^\dagger + \rho c_A^\dagger c_A) \\ & - \frac{\kappa_2}{2}(c_B^\dagger c_B \rho - 2U_1 c_B \rho c_B^\dagger U_1^\dagger + \rho c_B^\dagger c_B) \\ & - \frac{\kappa_3}{2}(c_a^\dagger c_a \rho - 2U_{\text{con}} c_a \rho c_a^\dagger U_{\text{con}}^\dagger + \rho c_a^\dagger c_a), \end{aligned} \tag{4}$$

where $U_1 c_A \rho c_A^\dagger U_1^\dagger$, $U_1 c_B \rho c_B^\dagger U_1^\dagger$, and $U_{\text{con}} c_a \rho c_a^\dagger U_{\text{con}}^\dagger$ mean that the control operations are applied immediately after a quantum jump happens. (The corresponding operations are given in the following paragraph.) By introducing two new bosonic modes [29],

$$M_1 = \frac{c_A + c_B}{\sqrt{2}}, \quad M_2 = \frac{c_A - c_B}{\sqrt{2}}, \tag{5}$$

the Hamiltonian becomes

$$\begin{aligned} H_1 = & \sum_{k=1}^{n} \left[\left(\frac{g_1}{\sqrt{2}} (e^{i(\Delta_1-\nu)t} M_1 + e^{i(\Delta_1+\nu)t} M_2)|e\rangle_k\langle g| \right.\right. \\ & \left.\left. + g_2 e^{i\Delta_2 t} c_a |e\rangle_j\langle f| + \Omega_1 e^{i\Delta_3 t}|u\rangle_j\langle g| \right) \right] \\ & + \frac{g_3}{\sqrt{2}} (e^{i(\Delta_1-\nu)t} M_1 - e^{i(\Delta_1+\nu)t} M_2)|e\rangle_{n+1}\langle g| \\ & + \Omega_2 e^{i\Delta_4 t}|e\rangle_{n+1}\langle f| + \text{H.c.} \end{aligned} \tag{6}$$

If we set $\{(\Delta_1 - \nu), \Delta_1, \Delta_2, \Delta_4, \nu\} \gg \{g_1, g_2, g_3, \Omega_2\}$, $\Delta_3 \gg \Omega_1$, and $(\Delta_1 - \nu) = \Delta_2 = \Delta_4$, the excited states can be adiabatically eliminated. Moreover, normal mode M_2 is almost decoupled with mode M_1. Then, it is natural to describe the system by the following effective Hamiltonian:

$$\begin{aligned} H_2 = & -\left\{ \sum_{k=1}^{n} [\lambda_1 |g\rangle_k\langle g| M_1^\dagger M_1 + \lambda_2 |g\rangle_k\langle g| + \lambda_3 |f\rangle_k\langle f| c_a^\dagger c_a \right. \\ & + (\lambda_4 |g\rangle_k\langle f| M_1^\dagger c_a + \text{H.c.})] + \lambda_5 |g\rangle_{n+1}\langle g| M_1^\dagger M_1 \\ & \left. + \lambda_6 |f\rangle_{n+1}\langle f| + (\lambda_7 |g\rangle_{n+1}\langle f| M_1^\dagger + \text{H.c.}) \right\}, \end{aligned} \tag{7}$$

where $\lambda_1 = \frac{g_1^2}{2(\Delta_1-\nu)}$, $\lambda_2 = \frac{\Omega_1^2}{\Delta_3}$, $\lambda_3 = \frac{g_2^2}{\Delta_2}$, $\lambda_4 = \frac{g_1 g_2}{\sqrt{2}(\Delta_1-\nu)}$, $\lambda_5 = \frac{g_3^2}{2(\Delta_1-\nu)}$, $\lambda_6 = \frac{\Omega_2^2}{\Delta_4}$, and $\lambda_7 = \frac{g_3\Omega_2}{\sqrt{2}\Delta_4}$. Note that only one excited state in the two cavities is considered. Under the conditions $n\lambda_1 \approx -\lambda_2$ and $\lambda_5 = \lambda_6$, the Stark shift terms corresponding to parameters λ_1, λ_2, λ_5, and λ_6 can be eliminated. We have supposed that the conditions mentioned above are satisfied by adjusting classical fields and the detuning Δ_3. We define $S_z = \sum_{j=1}^{n} \frac{1}{\sqrt{2}}(|f\rangle_j\langle f| - |g\rangle_j\langle g|)$. If the average number of atoms in state $|f\rangle$ is much smaller than n, we have $\sum_{j=1}^{n} |f\rangle_j\langle f| = (n + 2S_z) \approx 0$. Here, $S_z \approx -n/2$. Then, the master equation reduces to the following form:

$$\begin{aligned} \dot{\rho} = & -i[H_3,\rho] - \frac{\kappa_1+\kappa_2}{2}(M_1^\dagger M_1 \rho - 2U_1 M_1 \rho M_1^\dagger U_1^\dagger + \rho M_1^\dagger M_1) \\ & - \frac{\kappa_3}{2}(c_a^\dagger c_a \rho - 2U_{\text{con}} c_a \rho c_a^\dagger U_{\text{con}}^\dagger + \rho c_a^\dagger c_a), \end{aligned} \tag{8}$$

with

$$\begin{aligned} H_3 = & -[\sqrt{n(m+1)}\lambda_4 |\Psi(n,m+1)\rangle\langle\Psi(n,m)| M_1 c_a^\dagger \\ & + \lambda_7 |g\rangle_{n+1}\langle f| M_1^\dagger + \text{H.c.}], \end{aligned} \tag{9}$$

where $|\Psi(n,m)\rangle = \frac{1}{\sqrt{C_n^m}} (\sum_k P_k |f\rangle_1 |f\rangle_2 \cdots |f\rangle_m |g\rangle_{m+1} |g\rangle_{m+2} \cdots |g\rangle_n)$. Here, $\{P_k\}$ represents the set of all distinct permutations of qubits.

To see how to choose the feedback operation, we first start with a simple steady-state analysis in the absence of feedback. At the initial time, all the atoms are in ground state $|\Psi(n,0)\rangle$, and the cavities are in vacuum states. Atom $(n+1)$ is pumped in state $|f\rangle_{n+1}$. Via off-resonance Raman transition, atom $(n+1)$ emits a photon and then is in state $|g\rangle_{n+1}$. In cavity 1, if the atoms absorb the photon, they emit another photon and are transferred to state $|\Psi(n,1)\rangle$. Because of the strong decoherence of the cavity field, the steady state can be expressed in the form $\rho = [\alpha|\Psi(n,0)\rangle\langle\Psi(n,0)| + \beta|\Psi(n,1)\rangle\langle\Psi(n,1)|]|g\rangle_{n+1}\langle g|$. Here, $\alpha \gg \beta$. As a result, the atom system evolves to state $|\Psi(n,1)\rangle$ with a very low probability. To obtain state $|\Psi(n,1)\rangle$ determinedly, feedback operations need to be implemented in the system. We give a schematic of the corresponding dynamics process in Fig. 2. Different from the no feedback case, the feedback operation $U_1 = \exp(i\pi\sigma_x/2)$ drives atom $(n+1)$ from $|g\rangle_{n+1}$ into $|f\rangle_{n+1}$ after detector D_1 or D_2 clicked ($\sigma_x = |g\rangle_{n+1}\langle f| + |f\rangle_{n+1}\langle g|$). The

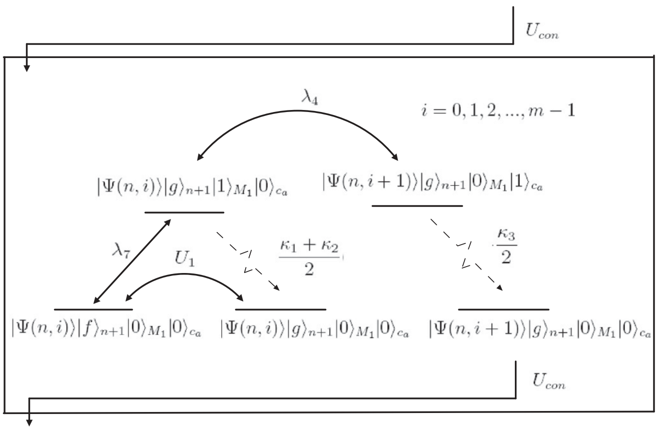

FIG. 2. Schematic of the dynamics process.

total excitation number N is conservative before the operation $U_{\rm con}$ is performed ($N = \langle M_1^\dagger M_1 + c_a^\dagger c_a + \sum_{j'=1}^{n+1} |f\rangle_{j'}\langle f|\rangle$). If the intended state is $|\Psi(n,1)\rangle$, no feedback operation is performed after detector D_3 clicked (the operator $U_{\rm con}$ is expressed by I, which is the identity operator on the entire system space). Then, the system has reached its steady state. If we aim to obtain state $|\Psi(n,m)\rangle$, the conditional operator $U_{\rm con}$ is $\exp(i\pi\sigma_x/2)$ before the mth time that detector D_3 clicks. When D_3 clicks for the mth time, $U_{\rm con}$ is I, and then the steady state of the atoms in cavity 1 will be $|\Psi(n,m)\rangle$. This result illustrates a possible mechanism, leading to a long-lived Dicke state by using feedback control.

In the following, we will confirm the validity of the above theoretical analysis and will address the influence of parameter variations on creating the intended Dicke state. Without loss of generality, we use the generation of an n-qubit W state as an example ($n = 100$). By solving Eq. (8) numerically, we calculate the dynamics evolution of the density matrix. For the sake of simplicity, we set $10\lambda_4 = \lambda_7 = \lambda$, $\kappa_1 = \kappa_2 = 0.5\kappa$, $g_1 = g_2 = g$, and $2\Omega_2^2 = g_3^2 = 10\sqrt{2}g$. In Fig. 3(a), the populations of different states are shown in the case that no feedback operation is implemented. The population of state $|\Psi(100,1)\rangle$ is about 0.0952. It is obvious that the population of the W state in the steady-state regime is very small. In Fig. 3(b), the time evolution of states $|\Psi(100,1)\rangle|g\rangle_{n+1}$ and $|\Psi(100,0)\rangle|f\rangle_{n+1}$ in the controlled case is given. We observe a perfect transfer between the populations of the ground state and the intended state. (The population of the W state is about 0.9999.) Thus, quantum feedback control is beneficial to obtain the entangled state in our system. We note that cavity decay plays a dominant role during the whole process. Since it is difficult to control the noise strength, the variations in the cavity decay rate should be unavoidable. In Fig. 3(c), we show the population of $|\Psi(100,1)\rangle$ versus the evolution time and the decay rate κ in the presence of feedback control. When κ is varied from 0 to 20λ, the interaction time required to reach the steady state increases gradually. This result can be easily understood. Because a large decay rate κ damps the population of the photon along the x direction, the transition from $|\Psi(100,0)\rangle$ to $|\Psi(100,1)\rangle$ is greatly suppressed. As for a small decay rate κ, the effect of feedback control on the system is reduced. However, a very small decay rate κ means that there is no dissipation along the x direction. Interestingly, the system is still driven to the wanted state because only the cavity decay along the y direction can also be used to efficiently create entanglement. In Fig. 3(d), when κ_3 is changed from 0.2λ to 0.8λ, the interaction time deceases due to the fact that a large decay rate corresponds to a fast decoherence process. Furthermore, it is also found that, as κ_3 becomes larger ($\kappa_3 > 0.8\lambda$), the interaction time increases. The interesting phenomenon can be interpreted as a consequence of quantum Zeno effect [30]. Because a strong decay rate will increase the interaction strength between states $|\Psi(100,1)\rangle|1\rangle_{c_a}$ and $|\Psi(100,1)\rangle|0\rangle_{c_a}$, the probability of the transition from $|\Psi(100,0)\rangle|1\rangle_{M_1}$ to $|\Psi(100,1)\rangle|1\rangle_{c_a}$ is decreased greatly. As a result, the interaction time is prolonged. Once the cavity field along the y direction collapses into a vacuum state, the system is driven into the intended steady state. The probability for obtaining the Dicke state will not be changed when the decay rate fluctuates within a certain range. Moreover, the fidelity will not be affected by the fluctuations of cavity decay rates because the steady state is the pure state $|\Psi(100,1)\rangle\langle\Psi(100,1)|$ instead of the mixed state $\rho = \alpha'|\Psi(100,1)\rangle\langle\Psi(100,1)| + \beta'|\Psi(100,0)\rangle\langle\Psi(100,0)|$ under the condition that detector D_3 clicks (α' and β' are real numbers). In reality, the coupling constants are likely to be different due to technical difficulties. The fluctuations

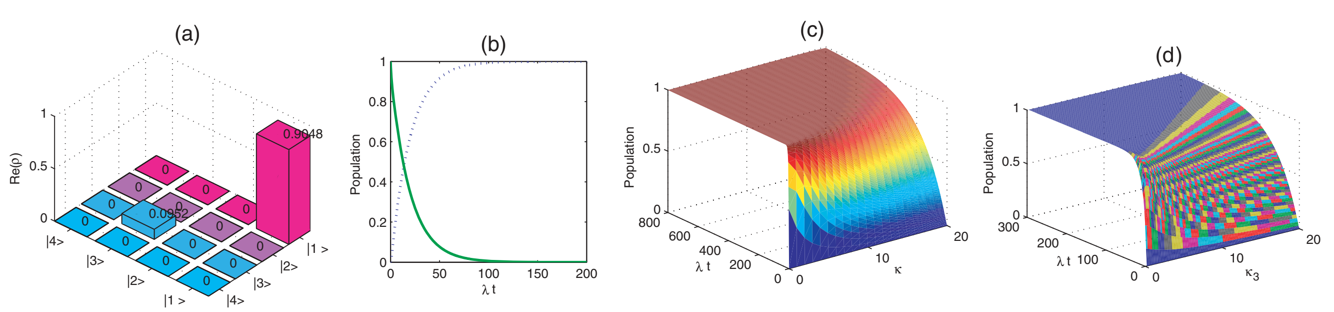

FIG. 3. (Color online) (a) A steady-state density matrix in the space spanned by the basis state vectors $\{|1\rangle = |\Psi(100,0)\rangle|g\rangle_{n+1}, |2\rangle = |\Psi(100,0)\rangle|f\rangle_{n+1}, |3\rangle = |\Psi(100,1)\rangle|g\rangle_{n+1}$, and $|4\rangle = |\Psi(100,1)\rangle|f\rangle_{n+1}\}$. Shown is the real part of the density matrix for the uncontrolled case. The system parameters are chosen as $\kappa = \kappa_3 = 6\lambda$. (b) Variation in the populations for system states under the feedback control. The parameters are the same as those in (a). Solid and dashed lines describe the populations of $|\Psi(100,0)\rangle|f\rangle_{n+1}$ and $|\Psi(100,1)\rangle|g\rangle_{n+1}$, respectively. Plot of the population of state $|\Psi(100,1)\rangle$ as a function of (c) κ and λt at $\kappa_3 = 6\lambda$; (d) κ_3 and λt at $\kappa = 6\lambda$. Other common parameters: $n = 100$, $\nu = 100g$, $\Delta_2 = 200g$, and $\Delta_4 = 100g$.

in coupling constants corresponding to those atoms in cavity 1 should reduce the fidelity of the wanted state. We set parameters δ_k as the fluctuations of the coupling constant g_1. Here, the subscript k corresponds to the atom k. Without loss of generality, we suppose that there are ten atoms whose coupling constants are not equal to g_1. It can be calculated that the fidelity of the intended state is higher than 99% for $\delta_m = 0.1g_1$ and $\delta_{m'} = 0$ [$(m = 1,2,\ldots,10)$, $(m' = 11,12,\ldots,100)$, and the other parameters are the same as in Fig. 3(a)]. Thus, the entanglement might be obtained with high fidelity when the coupling constants vary over a small range.

Let us give a brief analysis of the experimental implementation. There are some alternative atomic models [31–33] which are expected to perform the control scheme in the experiment. In our proposal, the process for generating a Dicke state might be disturbed by the decoherence. Because the effect of cavity decay is useful for obtaining the wanted state, we only discuss the influence of atomic spontaneous emission. The effective atomic-spontaneous-emission rate can be given by $\gamma_{\text{eff}} \sim \gamma g^2/(\Delta_1 - \nu)^2$ [34,35] (γ denotes the spontaneous emission rate, which is related to the decay channel from a higher level to a lower level). When the condition $(\Delta_1 - \nu) \gg g$ holds, we can omit atomic spontaneous emission because it affects the dynamics evolution much less than cavity decay κ or κ_3. Using the coupling parameters $g_1 = g_2 \sim 10^3$ MHz and $\gamma \sim 10$ MHz [33] for the case in Fig. 3, the required interaction time is on the order of 8 μs, which is much less than the effective excited-state lifetime $T_r \sim 1/\gamma_{\text{eff}} \approx 10^{-3}$ s. Hence, the scheme for generating multibody entanglement is effective. In addition, finite detection efficiency will degrade the performance of quantum feedback. If no detection happens, there is one probability that the detectors fail to detect in the photon, in which case, no corresponding feedback operation is implemented on atom $(n + 1)$. There is another probability that atomic spontaneous emission occurs, in which case, we cannot estimate the atomic state appropriately. Since the excitation number remains unchanged, the evolution of the system freezes into an unknown steady state. As a result, detector D_3 will not click. Then, we will drive the system back to the ground state and will restart the feedback iteration. As long as detector D_3 clicks, we can determinately obtain the intended state. Therefore, the proposed feedback scheme is feasible in the experiment.

In conclusion, we analyze the dynamics of a two-site coupled cavity model when the two cavities contain n atoms and an atom, respectively. The feedback operations can be performed on the single atom in the second cavity after the photons emitted by the system are detected. The n atoms in the first cavity are deterministically driven into a steady Dicke state. The contributions of this Brief Report are summarized as follows: (1) The idea can be extended to directly create an n-qubit Dicke state. The control strategy need not be changed in the process of evolution. Thus, the present scheme is a scalable scheme for the creation of a multibody Dicke state. (2) The feedback operation is implemented on a single atom, which will not affect the other atoms in another cavity. The feature makes the scheme more feasible in experimental implementation. (3) Cavity decay will be helpful for obtaining the intended entanglement. The feedback control may be performed effectively even when the decay rate varies over a wide range. (4) The system is driven into a wanted steady state, so the interaction time need not be controlled strictly. (5) At the end of dynamics evolution, the atoms are in a metastable state. Therefore, the Dicke state can be stored in the atoms for large time scales.

J.S. thanks Professor S. B. Zheng for valuable discussions. This work was supported by the MOST of China (973 Project No. 2007CB307001), National Natural Science Foundation of China under Grants No. 11205037 and No. 11105030, the Fundamental Research Funds for the Central Universities under Grant No. HIT.NSRIF. 2010010, the China Postdoctoral Science Foundation under Grant No. 2011M500648, and the Heilongjiang Postdoctoral Fund under Grant No. LBH-Z11126.

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