Detection efficiency in the loophole-free violation of Svetlichny's inequality

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(Received 22 July 2012; published 17 September 2012)

Svetlichny's inequality (SI) is a Bell-like inequality, the violation of which can be used to confirm the existence of genuine multipartite correlations. However, poor detector efficiency can possibly cause a so-called detection loophole in actual Svetlichny experiments. We derive an alternative SI to deal with this loophole. If the experimental data can violate this SI, it must result in loophole-free violation of the original SI. We show that the minimum detection efficiency needed for a loophole-free violation of the tripartite SI is about 0.97. For the general case of n particles, we give the analytic expression of the needed detection efficiency, and find that its value monotonically and rapidly approaches 1 as the number of particles increases.

DOI: 10.1103/PhysRevA.86.034102 PACS number(s): 03.65.Ud, 03.65.Ta, 03.67.−a

I. INTRODUCTION

A ubiquitous problem in physics is to understand the correlation which is observed among different events, and quantum theory opened a new world of nonclassical correlations. Bell [1] demonstrated that quantum theory predicts that separated systems can produce outcomes whose correlation cannot be explained by any local-hidden-variable theory. A more general version of the Bell inequality for two qubits was given by Clauser, Horne, Shimony, and Holt [2] (the CHSH inequality). Since then the experimental confirmation of Bell's prediction has been one of the fundamental challenges of modern physics [3–7]. In addition to causation and reality assumptions, the derivation of Bell's inequality requires three additional assumptions (or conditions): (i) The observer's measurement choices are not correlated with each other or with hidden variables, and these choices are random [8–10]; (ii) different observers' measurement events are spacelike separated; (iii) fair sampling is assumed, i.e., the detected data are representative of all those emitted from the source. So an incontrovertible experimental confirmation must simultaneously satisfy these three conditions, or one cannot say that the experimental violation of Bell's inequality demonstrates the downfall of local realistic theories. If any of the above three conditions is not satisfied, there exist loopholes in the experimental confirmation. In particular, the third condition of fair sampling is unsatisfactory; why should not the data recorded by detectors be special? Actually soon after the discovery of Bell's inequality, Pearle [11] pointed out that if the detectors' efficiency is not perfect then it is possible to devise a local-hidden-variable model which can also produce violation of the Bell inequality. So if we abandon the fair sampling assumption and at the same time the detection efficiency is too low, a so-called detection loophole arises.

The detection efficiency is defined as the ratio between the numbers of detected particles and the particles actually emitted by the source. The minimum detection efficiency which is required to close the detection loophole is called the threshold efficiency η_{crit}. It is an interesting question as to what the threshold efficiency η_{crit} is for any given scenario. The value of η_{crit} is now known for many scenarios [12–25]. The value of η_{crit} for a loophole-free experiment based on Hardy's approach has also been deduced [26,27]. In practice, one usually adopts photons for Bell experiments. Although photon experiments are able to close the locality loophole [28], the optical detection efficiencies are still too low to close the detection loophole. Since the detection efficiency is the product of the transmission efficiency and the detector efficiency, the loss of photons in the transmission from the source to the observers' locations can greatly reduce the detection efficiency. Recently, a precertification technique has been proposed [29,30]; the aim of the technique is to boost the transmission efficiency to 1. There are some other proposals for closing the experimental detection loophole in Refs. [31,32].

This paper focuses on the issue of the threshold detection efficiency in actual Svetlichny experiments. Svetlichny's inequality (SI) [33,34] is a Bell-like inequality, the violation of which can be used to confirm the existence of genuine multipartite correlations. We will derive an alternative SI to deal with the detection loophole in Svetlichny experiments. If the experimental data can violate this SI, it must result in the loophole-free violation of the original SI. At the same time, we will give the threshold efficiency which is required for a loophole-free violation of SI. The general case of n particles is also addressed.

II. THRESHOLD DETECTION EFFICIENCY FOR THREE-PARTICLE SI

The three-particle SI [33] can be used to confirm the existence of genuine three-particle correlations which are essentially different from two-particle correlations. This means that one can find a violation of SI only if there exist genuine three-particle correlations in a three-particle setting. Consider three observers Alice, Bob, and Carol, who share three entangled qubits. Each of them can choose to measure one of two dichotomous observables. We denote by A_1 and A_2 Alice's measurement results when she performs

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measurements a_1 and a_2, respectively, and similarly B_2, B_2, b_1, and b_2 (C_1, C_2, c_1, and c_2) for Bob's (Carol's) results, and the measurement results of all observables can be -1 or $+1$. Then SI is expressed as [33]

$$\begin{aligned}|E(A_1B_1C_1) + E(A_1B_1C_2) + E(A_1B_2C_1)\\ + E(A_2B_1C_1) - E(A_1B_2C_2) - E(A_2B_1C_2)\\ - E(A_2B_2C_1) - E(A_2B_2C_2)| \leqslant 4,\end{aligned} \tag{1}$$

where the $E(A_iB_jC_k)$'s represent the expectation values of the product of the measurement outcomes of the observables. The SI of Eq. (1) applies to the ideal case in which all experimental settings of the three observers give results, and only in the ideal case can one assert that the violation of SI of Eq. (1) confirms the existence of genuine three-particle correlations. So in order to obtain a loophole-free violation of SI of Eq. (1), we should get a violation of the following inequality:

$$\begin{aligned}|E(A_1B_1C_1|\Lambda_0) + E(A_1B_1C_2|\Lambda_0)\\ + E(A_1B_2C_1|\Lambda_0) + E(A_2B_1C_1|\Lambda_0)\\ - E(A_1B_2C_2|\Lambda_0) - E(A_2B_1C_2|\Lambda_0)\\ - E(A_2B_2C_1|\Lambda_0) - E(A_2B_2C_2|\Lambda_0)| \leqslant 4,\end{aligned} \tag{2}$$

where Λ_0 represents the ensemble where all measurements successfully give results, and $E(A_iB_jC_k|\Lambda_0)$ denotes the expectation value of the product of $A_iB_jC_k$ in the ensemble Λ_0. The violation of Eq. (2) is a (detection) loophole-free confirmation of the existence of genuine three-particle correlations.

However, $E(A_iB_jC_k|\Lambda_0)$ is inaccessible in actual experiments. The usual approach is to disregard these "not detected" events: only the coincident events contribute to the estimation of $E(A_iB_jC_k)$. A coincident event is one where Alice, Bob, and Carol all successfully obtain a measurement result in one trial. This approach is the same as saying that if a measurement fails one will get the value 0 as the measurement result. So in actual experiments one essentially calculates the conditional correlations $E(A_iB_jC_k|\Lambda_{A_iB_jC_k})$, where $\Lambda_{A_iB_jC_k}$ represents the ensemble where all measurements a_i, b_j, and c_k successfully give results -1 or 1.

Here for convenience we define all notations which will be used frequently in the following text; these notations are taken from Ref. [24]. We use Λ_{A_i} to denote the ensemble where Alice's measurement setting a_i successfully gives the result -1 or 1, and similarly for Λ_{B_j} and Λ_{C_k}; we use $\Lambda_{A_iB_j}$ to denote the ensemble where measurements setting a_i and b_j both give results -1 or1, and similarly for $\Lambda_{A_iB_jC_k}$, etc. Following these notations, the ensemble Λ_0 can be expressed as $\Lambda_{A_1A_2B_1B_2C_1C_2}$. $P(\Lambda_{A_i})$ denotes the probability of $A_i \neq 0$, and $P(\Lambda_{A_iB_j})$ denotes the probability that both A_i and B_j are nonzero, and similarly for $P(\Lambda_{A_iB_jC_k})$, etc.; $P(\Lambda_{A_i}|\Lambda_{B_j})$ denotes the conditional probability that $A_i \neq 0$ given that $B_j \neq 0$.

Proposition 1. If we define

$$\delta_3 = \min_{ijk} P\left(\Lambda_0\middle|\Lambda_{A_iB_jC_k}\right), \tag{3}$$

where $\min_{ijk}$ is taken over all measurements settings from Eqs. (2) and (3) we can obtain the inequality

$$\begin{aligned}\big|E\left(A_1B_1C_1\middle|\Lambda_{A_1B_1C_1}\right) + E\left(A_1B_1C_2\middle|\Lambda_{A_1B_1C_2}\right)\\ + E\left(A_1B_2C_1\middle|\Lambda_{A_1B_2C_1}\right) + E\left(A_2B_1C_1\middle|\Lambda_{A_2B_1C_1}\right)\\ - E\left(A_1B_2C_2\middle|\Lambda_{A_1B_2C_2}\right) - E\left(A_2B_1C_2\middle|\Lambda_{A_2B_1C_2}\right)\\ - E\left(A_2B_2C_1\middle|\Lambda_{A_2B_2C_1}\right) - E\left(A_2B_2C_2\middle|\Lambda_{A_2B_2C_2}\right)\big|\\ \leqslant 4(2-\delta_3).\end{aligned} \tag{4}$$

Proof. It is obvious that $\Lambda_0 \subset \Lambda_{A_iB_jC_k}$, and the ensemble $\Lambda_{A_iB_jC_k}$ can be split into two disjointed subensembles Λ_0 and its complement $\Lambda_0^c = \Lambda_{A_iB_jC_k} \backslash \Lambda_0$. We get

$$\begin{aligned}&E\left(A_iB_jC_k\middle|\Lambda_{A_iB_jC_k}\right) - \delta_3 E(A_iB_jC_k|\Lambda_0)\\ &\leqslant \left|P\left(\Lambda_0^c\middle|\Lambda_{A_iB_jC_k}\right)E\left(A_iB_jC_k\middle|\Lambda_0^c\right)\right|\\ &\quad + \left|P\left(\Lambda_0\middle|\Lambda_{A_iB_jC_k}\right)E(A_iB_jC_k|\Lambda_0) - \delta_3 E(A_iB_jC_k|\Lambda_0)\right|\\ &= P\left(\Lambda_0^c\middle|\Lambda_{A_iB_jC_k}\right)\left|E\left(A_iB_jC_k\middle|\Lambda_0^c\right)\right|\\ &\quad + \left[P\left(\Lambda_0\middle|\Lambda_{A_iB_jC_k}\right) - \delta_3\right]|E(A_iB_jC_k|\Lambda_0)|\\ &\leqslant P\left(\Lambda_0^c\middle|\Lambda_{A_iB_jC_k}\right)E\left(|A_iB_jC_k|\middle|\Lambda_0^c\right)\\ &\quad + \left[P\left(\Lambda_0\middle|\Lambda_{A_iB_jC_k}\right) - \delta_3\right]E(|A_iB_jC_k||\Lambda_0) = 1-\delta_3.\end{aligned} \tag{5}$$

Combining Eqs. (2) and (5), we obtain

$$\begin{aligned}&[\text{Left-hand side of Eq.(4)}]\\ &\quad\leqslant \delta_3 \times [\text{left-hand side of Eq. (2)}] + 8(1-\delta_3)\\ &\quad\leqslant 4\delta_3 + 8(1-\delta_3)\\ &\quad\leqslant 4(2-\delta_3).\end{aligned} \tag{6}$$

Hence the proof. ■

In order to obtain the threshold detection efficiency required for loophole-free violation of SI, we must get the relation between δ_3 and the detection efficiency. For simplicity, we assume that the detection efficiencies of the three observers are equal and independent of each other; this means that $P(\Lambda_{A_i}) = P(\Lambda_{B_j}) = P(\Lambda_{A_i}|\Lambda_{B_j}) = \eta$.

Proposition 2. $\delta_3 \geqslant 13 - \frac{12}{\eta}$.

Proof. First, it is obvious that $P(\Lambda_{A_i}|\Lambda_{A_iB_jC_k}) = P(\Lambda_{B_j}|\Lambda_{A_iB_jC_k}) = P(\Lambda_{C_k}|\Lambda_{A_iB_jC_k}) = 1$. For $i' \neq i$,

$$\begin{aligned}P\left(\Lambda_{A_{i'}}\middle|\Lambda_{A_iB_jC_k}\right) &= \frac{P\left(\Lambda_{A_{i'}A_i}\middle|\Lambda_{B_jC_k}\right)}{P\left(\Lambda_{A_i}\middle|\Lambda_{B_jC_k}\right)}\\ &= \frac{P\left(\Lambda_{A_{i'}}\middle|\Lambda_{B_jC_k}\right) + P\left(\Lambda_{A_i}\middle|\Lambda_{B_jC_k}\right)}{P\left(\Lambda_{A_i}\middle|\Lambda_{B_jC_k}\right)}\\ &\quad - \frac{P\left(\Lambda_{A_i} \cup \Lambda_{A_{i'}}\middle|\Lambda_{B_jC_k}\right)}{P\left(\Lambda_{A_i}\middle|\Lambda_{B_jC_k}\right)}\\ &\geqslant \frac{2\eta-1}{\eta}.\end{aligned} \tag{7}$$

In Eq. (7) we used that $P(\Lambda_{A_{i'}}|\Lambda_{B_jC_k}) = P(\Lambda_{A_i}|\Lambda_{B_jC_k}) = \eta$.

Now consider the general $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k})$. We find that if all equations $i' = i$, $j' = j$, $k' = k$ are available, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) = 1$. If two of the three equations are available, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geqslant \frac{2\eta-1}{\eta}$. For example,

$$\begin{aligned}&P\left(\Lambda_{A_2B_2C_3}\middle|\Lambda_{A_1B_2C_3}\right)\\ &\quad= P\left(\Lambda_{A_2}\middle|\Lambda_{A_1B_2C_3}\right) + P\left(\Lambda_{B_2C_3}\middle|\Lambda_{A_1B_2C_3}\right)\\ &\qquad - P\left(\Lambda_{A_2} \cup \Lambda_{B_2C_3}\middle|\Lambda_{A_1B_2C_3}\right)\\ &\quad\geqslant P\left(\Lambda_{A_2}\middle|\Lambda_{A_1B_2C_3}\right) \geqslant \frac{2\eta-1}{\eta}.\end{aligned} \tag{8}$$

If one of the three equations of $i^{'}=i$, $j^{'}=j$, $k^{'}=k$ is available, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geqslant 3-\frac{2}{\eta}$. For example,

$$\begin{aligned}
&P(\Lambda_{A_2B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&\quad = P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&\qquad - P(\Lambda_{A_2}\cup\Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) \\
&\quad \geqslant P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1C_3}|\Lambda_{A_1B_2C_3}) - 1 \\
&\quad \geqslant P(\Lambda_{A_2}|\Lambda_{A_1B_2C_3}) + P(\Lambda_{B_1}|\Lambda_{A_1B_2C_3}) - 1 \geqslant 3-\frac{2}{\eta}. \qquad (9)
\end{aligned}$$

In the case of $i^{'}\neq i$, $j^{'}\neq j$, and $k^{'}\neq k$, $P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \geqslant 4-\frac{3}{\eta}$, since

$$\begin{aligned}
&P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) \\
&\quad \geqslant P(\Lambda_{A_{i'}}|\Lambda_{A_iB_jC_k}) + P(\Lambda_{B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) - 1 \\
&\quad \geqslant 3\frac{2\eta-1}{\eta} - 2 = 4-\frac{3}{\eta}. \qquad (10)
\end{aligned}$$

Finally we calculate $P(\Lambda_0|\Lambda_{A_iB_jC_k})$:

$$\begin{aligned}
&P(\Lambda_0|\Lambda_{A_iB_jC_k}) \\
&\quad = P\left(\cap_{\{i'j'k'\}}\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}\right) \\
&\quad \geqslant \sum_{\{i'j'k'\}} P(\Lambda_{A_{i'}B_{j'}C_{k'}}|\Lambda_{A_iB_jC_k}) - 7 \\
&\quad \geqslant \left[1+3\left(2-\frac{1}{\eta}\right)+3\left(3-\frac{2}{\eta}\right)+\left(4-\frac{3}{\eta}\right)\right]-7 \\
&\quad = 13-\frac{12}{\eta}. \qquad (11)
\end{aligned}$$

From Eqs. (3) and (11), we prove the proposition. ■

Combining Proposition 1 with Proposition 2, we finally get a SI which can be directly compared with experimental data:

$$\begin{aligned}
&|E(A_1B_1C_1|\Lambda_{A_1B_1C_1}) + E(A_1B_1C_2y|\Lambda_{A_1B_1C_2}) \\
&\quad + E(A_1B_2C_1|\Lambda_{A_1B_2C_1}) + E(A_2B_1C_1|\Lambda_{A_2B_1C_1}) \\
&\quad - E(A_1B_2C_2|\Lambda_{A_1B_2C_2}) - E(A_2B_1C_2|\Lambda_{A_2B_1C_2}) \\
&\quad - E(A_2B_2C_1|\Lambda_{A_2B_2C_1}) - E(A_2B_2C_2|\Lambda_{A_2B_2C_2})| \\
&\quad \leqslant 4\left(\frac{12}{\eta}-11\right). \qquad (12)
\end{aligned}$$

From the derivation of Eq. (12), we know that if the experimental data can violate this SI, it must result in the violation of the SI of Eq. (2), which is a loophole-free violation of the original SI. It was shown by Svetlichny [33] that the maximum value of the left-hand side of Eq. (12) allowed in quantum mechanics is $4\sqrt{2}$. So in order to get the violation of Eq. (12) the detection efficiency must satisfy $\eta \geqslant \frac{12}{11+\sqrt{2}} \approx 0.9666$. We call this minimum efficiency (0.9666) required for the violation the threshold efficiency $\eta_{\rm crit}$.

III. THRESHOLD DETECTION EFFICIENCY FOR *n*-PARTICLE SI

Suppose n players share n particles, and each one of the players performs dichotomous measurements on each of the n particles. The measurement settings are represented by $x_1,x_2,\ldots,x_n$, with possible values 0 and 1, and the corresponding measurement results are represented by $A_{x_1},A_{x_2}\ldots,A_{x_n}$, respectively, with possible values -1 and 1. Then the n-particle SI can be expressed as [34]

$$\left|\sum_{\{x_i\}} v(x_1,x_2,\ldots,x_n)E(A_{x_1}A_{x_2}\cdots A_{x_n})\right| \leqslant 2^{n-1}, \qquad (13)$$

where $\{x_i\}$ stands for an n-tuple $x_1,\ldots,x_n$, $E(A_{x_1}A_{x_2}\ldots A_{x_n})$ represents the expectation value of the product of the measurement outcomes of observables $x_1,x_2,\ldots,x_n$, and $v(x_1,x_2,\ldots,x_n)$ is a sign function given by

$$v(x_1,x_2,\ldots,x_n) = (-1)^{[k(k-1)/2]}, \qquad (14)$$

where k is the number of times the index 1 appears in $(x_1,x_2,\ldots,x_n)$.

Similarly to the case of three particles, only in the ideal case can we assert that the violation of Eq. (13) is a confirmation of the existence of genuine n-particle correlations. So in order to obtain a loophole-free violation of the SI of Eq. (13), we should get a violation of the following inequality:

$$\left|\sum_{\{x_i\}} v(x_1,x_2,\ldots,x_n)E(A_{x_1}A_{x_2}\ldots A_{x_n}|\Lambda_0)\right| \leqslant 2^{n-1}, \qquad (15)$$

where Λ_0 represents the ensemble where all measurements successfully give results. The violation of Eq. (15) is a loophole-free confirmation of the existence of genuine n-particle correlations.

Similarly to Propositions 1 and 2, we can obtain the following two propositions for the case of n particles.

Proposition 3. If we define

$$\delta_n = \min_{\{x_i\}} P(\Lambda_0|\Lambda_{A_{x_1}A_{x_2}\ldots A_{x_n}}), \qquad (16)$$

where $\min_{\{x_i\}}$ is taken over all measurement settings $(x_1,x_2,\ldots,x_n)$, from Eqs. (15) and (16) we can obtain the inequality

$$\begin{aligned}
&\left|\sum_{\{x_i\}} v(x_1,x_2,\ldots,x_n)E(A_{x_1}A_{x_2}\ldots A_{x_n}|\Lambda_{A_{x_1}A_{x_2}\ldots A_{x_n}})\right| \\
&\quad \leqslant 2^{n-1}(2-\delta_n). \qquad (17)
\end{aligned}$$

Proof. Similar to the proof of Proposition 1.

Proposition 4. $\delta_n \geqslant 1+n(2-\frac{1}{\eta})+\sum_{k=2}^{n}\binom{n}{k}[k(2-\frac{1}{\eta})-(k-1)]-(2^n-1)$.

Proof. Similar to the proof of Proposition 2:

$$\begin{aligned}
&P(\Lambda_0|\Lambda_{A_{x_1}A_{x_2}\ldots A_{x_n}}) \\
&\quad = P\left(\cap_{\{x_i'\}}\Lambda_{A_{x_1'}A_{x_2'}\ldots A_{x_n'}}\Big|\Lambda_{A_{x_1}A_{x_2}\ldots A_{x_n}}\right) \\
&\quad \geqslant \sum_{\{x_i'\}} P\left(\Lambda_{A_{x_1'}A_{x_2'}\ldots A_{x_n'}}\Big|\Lambda_{A_{x_1}A_{x_2}\ldots A_{x_n}}\right) - (2^n-1) \\
&\quad \geqslant 1+n\left(2-\frac{1}{\eta}\right) \\
&\qquad + \sum_{k=2}^{n}\binom{n}{k}\left[k\left(2-\frac{1}{\eta}\right)-(k-1)\right]-(2^n-1). \qquad (18)
\end{aligned}$$

From Eqs. (16) and (18), we prove the proposition. ■

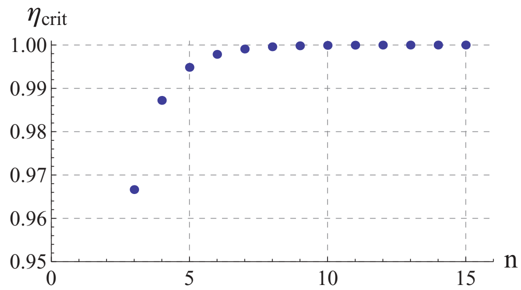

FIG. 1. (Color online) The threshold efficiency η_{crit} for the cases of n ranging from 3 to 15. The value of η_{crit} monotonically and rapidly approaches 1 as the value of n increases.

The maximum value of the left-hand side of Eq. (17) allowed in quantum mecahnics is $2^{n-1}\sqrt{2}$ [34]. Combining this with Proposition 4 we can obtain the threshold efficiency η_{crit} for the loophole-free confirmation of the existence of genuine n-particle correlations for any n. We depict η_{crit} for the cases of n ranging from 3 to 15 in Fig. 1. We find that as the value of n increases the value of η_{crit} monotonically and rapidly approaches 1.

IV. CONCLUSION

The imperfection of detector efficiency possibly causes the so-called detection loophole in actual Svetlichny experiments. We derive an alternative SI to deal with this detection loophole. If the experimental data can violate this SI, it must result in the loophole-free violation of the original SI. We give the threshold detection efficiency which is required for a loophole-free violation of SI for the general case of n particles. There is a remarkable contrast between our result for the SI and the case of Mermin inequalities, where the threshold detection efficiency is 0.75 for three parties and decreases to 0.5 as the number of parties tends to infinity [23]. The reason, we think, is that quantum mechanics allows a violation of Mermin inequalities that grows exponentially as the number of parties increases, while the increase of the number of parties does not contribute to magnifying the violation of SI.

ACKNOWLEDGMENTS

Y.X. is supported by the National Natural Science Foundation of China under Grant No. 11005031. F.-Y.H. is supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. Y6110314.

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