Production and diagnostics of spin-polarized heavy ions in sequential two-electron radiative recombination

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In the present work we investigate the sequential radiative recombination (RR) of initially bare ions colliding with two spatially separated electron targets. It is shown that magnetic sublevel population of the hydrogenlike ions, produced by the electron capture from the first target, depends on the emission direction of the (first) RR photon. This population, which can be expressed in terms of the polarization parameters, affects then the angular and polarization properties of the radiation emitted in the collision with the second target. The coincidence measurements of two subsequent RR photons may allow one to understand, therefore, the production and diagnostics of the ion spin polarization. In order to describe this polarization production and diagnostic scheme we derive the general expression for the γ-γ correlation function. Detailed calculations for the dependence of this function on the geometry of photon emission and collision energy are performed for the radiative recombination of bare uranium ions.

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I. INTRODUCTION

During the last decade radiative recombination of highly charged heavy ions remains the subject of intense theoretical and experimental research (see Ref. [1], and references therein). In this process a free (loosely bound) electron is captured into a bound ionic state and the excess energy is released as an x-ray photon. Being a time-reversed photoionization, radiative recombination (RR) allows one to study the fundamental electron-photon interaction in large-energy and strong-field regimes, which is not accessible otherwise in the direct (photoionization) channel. Moreover, the RR process attracts considerable attention due to its sensitivity to the spin, relativistic, and QED effects in the structure and dynamics of heavy atomic systems (see, e.g., Refs. [2–10]).

While most of the radiative recombination studies in the past have dealt with unpolarized ions and electron targets, much of today's interest is focused on collisions involving spin-polarized particles. For example, a number of theoretical proposals have been made recently to employ polarized ions (and/or targets) to the analysis of many-body, relativistic, and even parity nonconservation effects in heavy ions [11–13]. Moreover, the application of the RR as a probe process for measuring the polarization of hydrogenlike ions in storage rings was discussed in a number of papers [14–16]. Such a spin diagnostics is a stringent requirement for future experiments aiming at searching for an electric dipole moment of heavy nuclei and exploring parity nonconservation phenomena in a high-Z domain. Up to the present, however, no experimental verification of the ion-spin effects on the properties of RR radiation has been provided owing to the lack of techniques capable of production and preservation of heavy-ion polarization.

In this contribution, therefore, we propose a method that can be used to investigate electron capture into spin-polarized hydrogenlike ions. Even though the application of this method will not yield the polarization of the entire ion beam, it will allow one to select out of such a beam the ions with certain polarization and to operate with them. This selective operation can be achieved in experimental setup where electrons from two spatially separated targets are subsequently captured into the K shell of an initially bare ion and where two emitted K-RR photons are measured in coincidence. Since the characteristics of the first and second RR photons are correlated in this scheme via the spin states of "intermediate" hydrogenlike ions, the proposed approach may help to "emulate" not only the production but also the diagnostics of heavy-ion beams. To illustrate such diagnostics, below we will pay special attention to the linear polarization of the second photon when the first photon is recorded under a given direction but without its spin being observed. These "angular-polarization" correlations can be measured today at the GSI facility in Darmstadt and may provide an important tool for studying processes involving spin-polarized heavy ions.

The paper is organized as follows. In Sec. II we present the geometry under which the angular-polarization correlations in the sequential two-electron capture are to be considered. To describe consistently such correlations we use the density matrix theory. Based on this theory, which is the most natural tool for studying polarization and correlation phenomena in atomic collisions [17,18], we derive in Sec. III A the density matrix of a hydrogenlike ion following collision with the first target. We show that for the RR into the ground $1s_{1/2}$ state this matrix can be parametrized in terms of polarization parameters. This polarization can be modified if between two collisions the ion penetrates the region where the external magnetic and electric fields are presented. Since in the planned GSI experiment the beamline will likely be exposed to a weak magnetic field, we investigate its influence on the (intermediate) ion density matrix in Sec. III B. This matrix is then used in Sec. III C to calculate the linear polarization of the RR photons emitted in the collision with the second electron target. We will show that this polarization reflects the population of intermediate ion

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states and, hence, the emission direction of the first photon. In Sec. IV detailed calculations of such angular-polarization correlations are performed for subsequential electron capture into the ground states of (initially bare, finally heliumlike) uranium ions. Based on the calculations performed we also determine the setup geometry at which the correlation effects can be observed most clearly. A summary of the results is presented in Sec. V. Relativistic units $\hbar = c = m_e = 1$ are used throughout the paper.

II. GEOMETRY

In order to describe the process of the sequential two-electron capture by bare ion, we need to agree first on the experimental setup and geometry under which the emission of both photons is observed. We consider the electron capture from an unpolarized target by an initially bare ion with zero nuclear spin. It is convenient to consider the electron capture in the projectile frame, i.e., in the rest frame of the ion, where the process can be viewed as a collision of two spatially localized unpolarized electron bunches with the ion target. In this frame we choose the quantization axis (Z axis) along the momentum $\mathbf{p}_i$ of the incoming electron. After the capture of the first electron (from the first bunch) into the bound state of the hydrogenlike ion, a photon is emitted in the direction $\mathbf{k}_1$. Together with the electron momentum this wave vector defines the reaction plane (XZ plane). As one can see in Fig. 1, only one polar angle θ_1 is required to define the direction of the first photon emission. As we will show later, the relative magnetic sublevel population of the resulting hydrogenlike ion depends on angle θ_1. Therefore, the properties of the second photon, emitted in the direction $\mathbf{k}_2$ characterized by two angles (θ_2,φ_2), should also be dependent on θ_1. In order to investigate such correlations between two RR x rays, we consider the linear polarization of the second photon measured in a coincidence with the first photon emission direction. As usual, the polarization of RR photons (vector $\boldsymbol{\epsilon}_2$) is defined in the plane $X'Y'$ perpendicular to the photon momentum $\mathbf{k}_2$.

Polarization of the second photon uniquely depends on the emission angle of the first gamma quanta only when the population of the intermediate hydrogenlike ion will not change during the time between the collisions. However, this condition is not met if the ion moves in the presence of external electric and magnetic fields. Since the experiments in accelerators and storage rings are mostly held in the presence of external magnetic fields, we will also consider the influence of such a field on the outcome of the experiment. To perform this analysis we assume that the magnetic field $\mathbf{B}$ is homogeneous and directed along the X axis. Below we will take into account the influence of this field on the population of the intermediate hydrogenlike ion.

III. THEORY

The process of the sequential capture of two electrons by bare ion can be divided into three steps. In the first step, a free electron is captured by a bare ion and the recombination photon is recorded by a polarization-insensitive detector in the direction $\mathbf{k}_1$. The interaction of the resulting hydrogenlike ion with an external magnetic field constitutes the second step of the process. In the last step, the RR of the hydrogenlike ion with an electron from the second target leads to emission of the second photon whose linear polarization is assumed to be available for the analysis. In what follows we will discuss how these steps can be analyzed within the density matrix approach.

A. Electron capture into an initially bare ion (first step)

We start our analysis from the first step of the process: the radiative recombination of a free electron having the asymptotic four-momentum $p_i = (p_i^0, \mathbf{p}_i)$ and the spin projection μ_i with a bare ion. The incoming electron wave function is given by the partial wave expansion

$$|p_i\,\mu_i\rangle = \frac{1}{\sqrt{4\pi}}\frac{1}{\sqrt{p_i\varepsilon_i}}\sum_\kappa i^l \exp(i\Delta_\kappa)\sqrt{2l+1} \times C^{j\mu_i}_{l0,\,(1/2)\mu_i}\,|\varepsilon_i\kappa\mu_i\rangle, \quad (1)$$

where Δ_κ is the Coulomb phase shift, $C^{j\mu_i}_{l0,\,(1/2)\mu_i}$ is the Clebsch-Gordan coefficient, and $|\varepsilon_i\kappa\mu_i\rangle$ is the partial electron wave with the energy $\varepsilon_i = p_i^0$ and the Dirac quantum number κ. In order to find the population of the resulting ion, it is most natural to use the density matrix theory. Since the detailed description of the radiative recombination within the framework of this theory has been presented during the last few

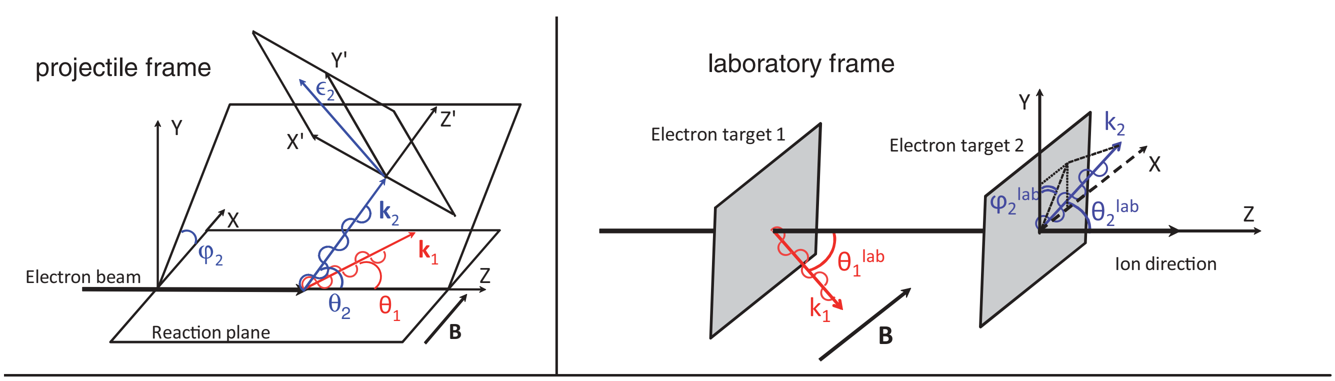

FIG. 1. (Color online) Geometry for the sequential two-electron radiative recombination of an initial bare (finally heliumlike) ion in the rest frame of an ion (left panel) and in the rest frame of an electron (right panel). Here $\mathbf{B}$ is the external magnetic field, and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of photons emitted in collisions with first and second electron targets, correspondingly.

years in several works (see, e.g., [19,20]), we will not repeat it here and just present the final expression for the density matrix of the residual ion:

$$\langle n_d\, \kappa_d\, \mu_d | \hat{\rho}_d | n_d\, \kappa_d\, \mu_d' \rangle = \frac{N_1}{2} \sum_{\mu_i} \langle n_d\, \kappa_d\, \mu_d | \hat{R}^\dagger(\mathbf{x};\mathbf{k},\boldsymbol{\epsilon}) | p_i\, \mu_i \rangle \times \langle p_i\, \mu_i | \hat{R}(\mathbf{x};\mathbf{k},\boldsymbol{\epsilon}) | n_d\, \kappa_d\, \mu_d' \rangle, \quad (2)$$

where $|n_d\, \kappa_d\, \mu_d\rangle$ denotes bound hydrogenic states, N_1 is a normalization factor, which is chosen in such a way that the trace of the density matrix is unity, and $\hat{R} = -e\boldsymbol{\alpha} \cdot \mathbf{A}$ is the transition operator characterizing the interaction of the electron with the radiation field. Moreover, $\boldsymbol{\alpha}$ is a vector of Dirac matrices and

$$\mathbf{A}(\mathbf{x};\mathbf{k},\boldsymbol{\epsilon}) = \frac{\boldsymbol{\epsilon}\, \exp(i\mathbf{k}\cdot\mathbf{x})}{\sqrt{2k^0(2\pi)^3}} \quad (3)$$

is the wave function of the photon with momentum $\mathbf{k}$, energy $k^0 = |\mathbf{k}|$, and polarization $\boldsymbol{\epsilon}$. A detailed description of the calculation method of the transition amplitudes $\langle n_d\, \kappa_d\, \mu_d | \hat{R}^\dagger(\mathbf{x};\mathbf{k},\boldsymbol{\epsilon}) | p_i\, \mu_i \rangle$ is presented in a number of papers (see, e.g., [1,19,21,22]).

Equation (2) presents the general expression of the density matrix of a hydrogenlike ion following radiative capture of an unpolarized electron. While, of course, this expression can be used to describe the capture into arbitrary state $|n_d\, \kappa_d\, \mu_d\rangle$, below we will focus on the K-shell RR. For relativistic ion-electron collisions such a ground-state recombination is the dominant process and has been well studied experimentally over the last decade [1,6,9]. For the K-RR, the total angular momentum of the bound electron is $j_d = 1/2$ and the density matrix (2) is a 2×2 matrix, which can be parametrized by three parameters [17,18]:

$$\langle n_d\, \kappa_d\, \mu_d | \hat{\rho}_d | n_d\, \kappa_d\, \mu_d' \rangle = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - i\,P_y \\ P_x + i\,P_y & 1 - P_z \end{pmatrix}. \quad (4)$$

These parameters can be interpreted as describing polarization vector $\mathbf{P} = (P_x, P_y, P_z)$, whose length $P = |\mathbf{P}|$ and polar angles define the degree and orientation of the ion beam polarization.

Further evaluation of the ion density matrix (2) and polarization parameters P_i $\{i = x,y,z\}$ requires multipole expansion of both the electron (1) and photon (3) wave functions, and use of the angular momentum algebra. Since such an analysis has been discussed in a number of previous works [1,19,20] we will not repeat it here. We will recall instead an important symmetry property of the matrix (4) which follows from this analysis: if the photons, emitted in the course of recombination of unpolarized electrons with bare ions, are observed in a particular setup, only single parameter P_y is nonzero in the right-hand side of Eq. (4). This prediction confirms previous studies [18] that suggest that the residual ion polarization should be perpendicular to the reaction plane, as follows immediately from simple symmetry considerations. The process under consideration is characterized by two vectors: the wave vector of the emitted photon and the initial electron momentum. The ion spin polarization vector $\mathbf{P}$ is an axial vector, which can be constructed from two vectors only as their vector product $[\mathbf{p}_i \times \mathbf{k}]$. Hence it can be directed only perpendicular to the reaction plane.

B. Influence of the magnetic field

In the previous section we have discussed that electron recombination leads to the polarization of the residual hydrogenlike ions in the direction perpendicular to the reaction plane. In the absence of external fields the ion polarization will remain unchanged until collision with the second electron's bunch and, hence, the second electron capture. In the planned experiment, however, hydrogenlike ions between two collisions will likely move in the presence of a weak magnetic field. In this section we shall study the influence of the B field on the polarization state of the ion. To simplify our analysis, we assume that all hydrogenlike ions, produced by the RR in the polarization state $\mathbf{P} = (0, P_y, 0)$ defined by Eq. (4), have the same velocity $\mathbf{v}$. It is well known that in the presence of a magnetic field the spin of the charged particle precesses due to the interaction of the magnetic moment with this field. In the relativistic case the spin motion is described by the Bargmann-Michel-Telegdi (BMT) equation [23–25]:

$$\frac{d\mathbf{P}}{dt} = -\mu_0 \mathbf{P} \times \left[\left(g - 2 + \frac{2}{\gamma} \right) \mathbf{B}_\perp + \frac{g}{\gamma} \mathbf{B}_\parallel + \left(g - 2 + \frac{2}{\gamma + 1} \right) [\mathbf{E} \times \mathbf{v}] \right], \quad (5)$$

where $\mu_0 = \frac{|e|}{2}$ is the Bohr magneton, g is the g factor, and γ is a Lorentz factor of the particle under consideration. $\mathbf{B}_\perp$ and $\mathbf{B}_\parallel$ are the components of the magnetic field $\mathbf{B}$, perpendicular and parallel to the velocity of particle $\mathbf{v}$, respectively, and $\mathbf{E}$ is the external electric field. It should be mentioned that in the BMT equation $\mathbf{E}$ and $\mathbf{B}$ are the fields in the laboratory frame, while $\mathbf{P}$ is given in the rest frame of the moving ion. Since in the scenario under consideration the external electric field is absent and the magnetic field is directed along the axis X (see Fig. 1), Eq. (5) takes the form

$$\frac{d\mathbf{P}}{dt} = -\mu_0 \left(g - 2 + \frac{2}{\gamma} \right) [\mathbf{P} \times \mathbf{B}]. \quad (6)$$

By solving this equation analytically we find polarization vector $\mathbf{P}' = (0, P_y', P_z')$ of the ion beam in the time t:

$$P_y' = |\mathbf{P}| \cos \omega t, \quad (7)$$

$$P_z' = |\mathbf{P}| \sin \omega t, \quad (8)$$

where $\omega = \mu_0 |\mathbf{B}|(g - 2 + \frac{2}{\gamma})$ depends on the Lorentz factor γ and, hence, on the absolute value of the velocity $\mathbf{v}$. As one can expect, Eqs. (7) and (8) describe the precession of the ion magnetic moment around the B-field direction, given by the X axis.

From Eqs. (7) and (8) one can find the density matrix of the hydrogenlike ions after passing the homogeneous magnetic field $\mathbf{B}$:

$$\langle n_d\, \kappa_d\, \mu_d | \hat{\rho}_d^{(mf)} | n_d\, \kappa_d\, \mu_d' \rangle = \frac{1}{2} \begin{pmatrix} 1 + P_z' & -i\,P_y' \\ i\,P_y' & 1 - P_z' \end{pmatrix}. \quad (9)$$

Of course, the same results can be obtained directly from solving the Liouville equation for the density matrix [17]. One can also see, that the absolute value of vector $\mathbf{P}$ is retained [26], and hence, the mixedness of the system is not changed by the magnetic field. In the next section, we will employ Eq. (9) to calculate the polarization properties of the photons emitted in the recombination of the second electron into the ground $(1s)^2$ state of the (finally) heliumlike ions.

C. Capture of the second electron

Equations (4) and (9) describe the polarization of the hydrogenlike ions produced by RR of electrons from the first target and having propagated some time t in the presence of an external magnetic field. When colliding with the second target, the ions may radiatively capture the (second) electron. In this capture process, the ion polarization can be partially transferred to the polarization of RR photons. In order to analyze this polarization transfer, let us first remind one that the polarization properties of the photon beam can be represented in terms of so-called Stokes parameters. While the parameter P_3 reflects the degree of circular polarization, which cannot be observed by available x-ray detectors, the two parameters P_1 and P_2 characterize the linear polarization of the light. These parameters are determined by the intensities of the light I_χ, which is linearly polarized under different angles χ with respect to the plane formed by the quantization axis and the wave vector of the photon $\mathbf{k}$ (see Fig. 1). Parameter P_1 characterizes the intensities of light, polarized in parallel and perpendicular to this plane:

$$P_1 = \frac{I_{0^\circ} - I_{90^\circ}}{I_{0^\circ} + I_{90^\circ}}, \qquad (10)$$

while the parameter P_2 is defined by the similar ratio taken at $\chi = 45^\circ$ and $\chi = 135^\circ$:

$$P_2 = \frac{I_{45^\circ} - I_{135^\circ}}{I_{45^\circ} + I_{135^\circ}}. \qquad (11)$$

Instead of Stokes parameters P_1 and P_2, the linear polarization of light can be represented in terms of the polarization ellipse. The degree of linear polarization, $P_L = \sqrt{P_1^2 + P_2^2}$, provides the relative length of the principal axis of such an ellipse. The orientation of the principal axis with respect to the reaction plane is determined by the angle χ_0, which is expressed in terms of Stokes parameters as follows [18]:

$$\cos 2\chi_0 = \frac{P_1}{P_L}, \quad \sin 2\chi_0 = \frac{P_2}{P_L}. \qquad (12)$$

The use of the polarization ellipse parameters for the description of the x-ray linear polarization is more convenient from a practical viewpoint since the application of the Compton x-ray polarimeters [9,27–36] allows direct determination of both P_L and χ_0.

The Stokes parameters $\{P_1, P_2, P_3\}$ are used to parametrize the density matrix of emitted photons as

$$\langle \mathbf{k}\epsilon_\lambda | \hat{\rho}_\gamma | \mathbf{k}\epsilon_{\lambda'} \rangle = \frac{1}{2} \begin{pmatrix} 1 + P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1 - P_3 \end{pmatrix}, \qquad (13)$$

where $\lambda = \pm 1$ is the helicity of the photon, i.e., its spin projection on the direction of propagation $\mathbf{k}$. The matrix on the left-hand side of Eq. (13) depends, of course, on the spin states of particles involved in the collision and on the setup under which the polarization of emitted light is observed. For the capture of unpolarized electrons into (initially) hydrogenlike ions, whose state is described by Eq. (9), the elements of the density matrix are

$$\langle \mathbf{k}\epsilon_\lambda | \hat{\rho}_\gamma | \mathbf{k}\epsilon_{\lambda'} \rangle = \frac{N_2}{2} \sum_{M_f \mu_s} \sum_{\mu_d \mu_d'} \langle J_f M_f, \mathbf{k}\epsilon_\lambda | \hat{\mathcal{R}}(1,2) | n_d\kappa_d\mu_d, p_i\mu_s \rangle \times \langle n_d\kappa_d\mu_d | \hat{\rho}_d | n_d\kappa_d\mu_d' \rangle \langle n_d\kappa_d\mu_d', p_i\mu_s | \hat{\mathcal{R}}^\dagger(1,2) | J_f M_f, \mathbf{k}\epsilon_{\lambda'} \rangle, \qquad (14)$$

where $|J_f M_f\rangle$ denotes final, heliumlike ionic state; the factor N_2 is introduced in order to the density matrix has a usual normalization (Tr $\hat{\rho}_\gamma = 1$). As seen from this expression, any further analysis of polarization Stokes parameters requires evaluation of the two-electron matrix elements of transition operator $\hat{\mathcal{R}}(1,2)$. In general, such an evaluation is not a simple task which requires application of sophisticated many-body theories. For heavy ions, however, the effects of electron-electron interaction are suppressed by a factor $1/Z$, compared to the interaction of the electrons with the Coulomb field of the nucleus. In this high-Z regime, the radiative recombination can be treated within the independent particle model, where the heliumlike wave function is constructed as

$$|J_f M_f\rangle = \Psi_f(\mathbf{x_1}, \mathbf{x_2}) = A_N \sum_{\mu_1\mu_2} \sum_{\mathcal{P}} (-1)^{\mathcal{P}} \mathcal{P} C^{J_f M_f}_{j_1\mu_1,\, j_2\mu_2} \times \psi_{n_1\kappa_1\mu_1}(\mathbf{x_1}) \psi_{n_2\kappa_2\mu_2}(\mathbf{x_2}), \qquad (15)$$

where $A_N = 1/2$ for equivalent electrons and $A_N = 1/\sqrt{2}$ for nonequivalent electrons, $(-1)^{\mathcal{P}}$ is the parity of the permutation, and $\mathcal{P}$ is the permutation operator. The representation (15) of the final-state wave function allows significant simplification of the amplitudes in Eq. (14) if the transition operator $\hat{\mathcal{R}}(1,2)$ is supposed to act locally on either of two electrons, leaving the other one unchanged:

$$\hat{\mathcal{R}}(1,2) = \hat{R}_1 \otimes \hat{I}_2 + \hat{R}_2 \otimes \hat{I}_1, \qquad (16)$$

where $\hat{R}_i = -e\boldsymbol{\alpha}_i \cdot \mathbf{A}_i$ is the one-electron interaction operator [see Eqs. (2) and (3)]. For the electron capture into the ground $(1s)^2$ state of a finally heliumlike ion we find, for example,

$$\langle \mathbf{k}\epsilon_\lambda | \hat{\rho}_\gamma | \mathbf{k}\epsilon_{\lambda'} \rangle = \frac{N_2}{2} \sum_{\mu_s\mu_{1s}\mu_{1s}'} \langle p_i\mu_s | R(\lambda') | 1s_{1/2} - \mu_{1s} \rangle \times \langle 1s_{1/2}\mu_{1s} | \hat{\rho}_d(\theta_1) | 1s_{1/2}\mu_{1s}' \rangle \times \langle 1s_{1/2} - \mu_{1s}' | R^\dagger(\lambda) | p_i\mu_s \rangle. \qquad (17)$$

By using this expression we can calculate the Stokes parameters of the second RR photons. Apart from the emission angles (θ_2, φ_2) of these photons, the Stokes parameters will also depend on the angle θ_1 under which the first x rays are observed. As seen from Eq. (17), this θ_1 dependence results from the density matrix of intermediate, hydrogenlike ions, providing thus a possibility to investigate the "angle-polarization" correlations in the two-step radiative recombination process.

IV. RESULTS AND DISCUSSION

The subsequent electron capture into the K shell of finally heliumlike heavy ions is likely to be studied at the GSI and future FAIR facilities. In these experiments, two subsequent

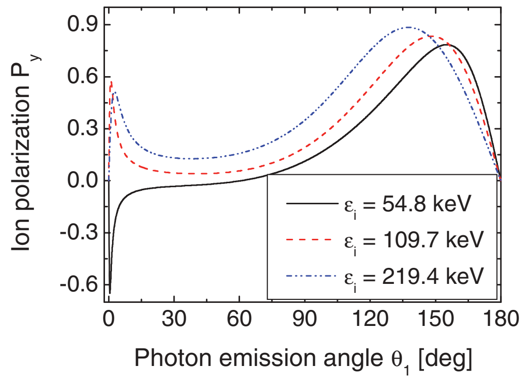

FIG. 2. (Color online) Component P_y of the polarization vector of hydrogenlike uranium ions ($Z = 92$) in the absence of magnetic field. The calculations are performed in the ion-rest frame for the kinetic energy $\varepsilon_i = 54.8$, 109.7, and 219.4 keV of the incoming electron.

recombinations into a single ion can be separated from the random coincidences by measuring the time difference between the emitted photons. In addition to that, the coincident registration of the doubly down-charged ion will help to suppress the unwanted background. In order to help plan and prepare such coincidence experiments, we have employed our theory in order to analyze the angle-polarization correlations between two photons emitted in the electron capture into a $1s$ state of hydrogenlike and, subsequently, into a $(1s)^2$ state of heliumlike uranium ions. Before we present predictions for the correlations, let us discuss the spin state of a (intermediate) hydrogenlike ion following the first electron capture. As mentioned above, the polarization of this ion is described by a single parameter P_y. In Fig. 2, the P_y is displayed as a function of the emission angle θ_1. The calculations have been performed in the ion-rest frame for the incident electron energies $\varepsilon_i =$ 54.8, 109.7, and 219.4 keV, which correspond to the projectile energy $T_p = 100$, 200, and 400 MeV/u in the laboratory frame, respectively. As seen from the figure, the polarization of the hydrogenlike ions following RR appears to be very sensitive to the geometry of the photon emission. For example, a very significant degree of polarization, $P_y \sim 80\%$–90%, can be achieved for those ions, whose production is accompanied by the photon emission under the angle $\theta \simeq 150^\circ$. In contrast, for the angles $15^\circ < \theta < 60^\circ$, the ion polarization does not exceed 20%. By choosing the geometry of the RR photon emission we can therefore "prepare" ions in a required polarization state.

Information about the polarization of the hydrogenlike ions can be obtained from the analysis of the linear polarization of the second recombination photons. In Fig. 3, for example, we display the Stokes parameter P_1 of these photons as a function of the angle θ_2. The calculations have been performed for the case of coplanar emission geometry, $\varphi_2 = 0^\circ$, and for four different angles $\theta_1 = 0^\circ$, 60°, 90°, and 120° of the first recombination photon with respect to the beam direction. Moreover, we assume for the moment no external magnetic field, $B = |\mathbf{B}| = 0$. Since for the $\varphi_2 = 0^\circ$, the second Stokes parameter vanishes identically, $P_2 = 0$, the photon polarization is completely defined by P_1. As seen from Fig. 3, this polarization is not very sensitive to the first photon angle θ_1 and, hence, to the ion spin states if both photons are emitted in the same plane. For such a coplanar geometry, variation of the P_1 does not exceed 2% over the entire angular range.

Since for the coplanar photon emission, the effect of correlation between the angle of the first photon and linear polarization of the second one is very small, we shall investigate other setup geometries. In Fig. 4, for example, we display the Stokes parameters P_1 (upper panel) and P_2 (bottom panel) of the second photons detected in the plane, perpendicular to the reaction one, $\varphi_2 = 90^\circ$. For such a perpendicular geometry, the P_1 is not affected by the θ_1, while the second Stokes parameter P_2 does strongly depend on this angle. As seen from the bottom panel of Fig. 4, this parameter vanishes completely for $\theta_1 = 0^\circ$, but may reach $P_2 \sim 20\%$–30% if the first photon is recorded under the angle $\theta_1 \simeq 120^\circ$. Such a behavior of the P_2 can be well understood from the fact that this parameter is directly proportional to the degree of the ion polarization, $P_{\rm ion}(B = 0) \equiv P_y$, and thus reflects the angular dependence of $P_y(\theta_1)$.

As one can see from Figs. 3 and 4, the Stokes parameters P_1 and P_2 of the (second) recombination photons behave in rather different ways with respect to the setup geometry. For example, for coplanar photon emission, $\varphi_2 = 0^\circ$, P_1 is only slightly affected by the ion polarization, and $P_2 = 0$. In contrast, if $\varphi_2 = 90^\circ$, P_2 is proportional to the degree of ion polarization, while P_1 is totally independent of this parameter. Such a behavior of polarization transfer between hydrogenlike ions and emitted light resembles the polarization

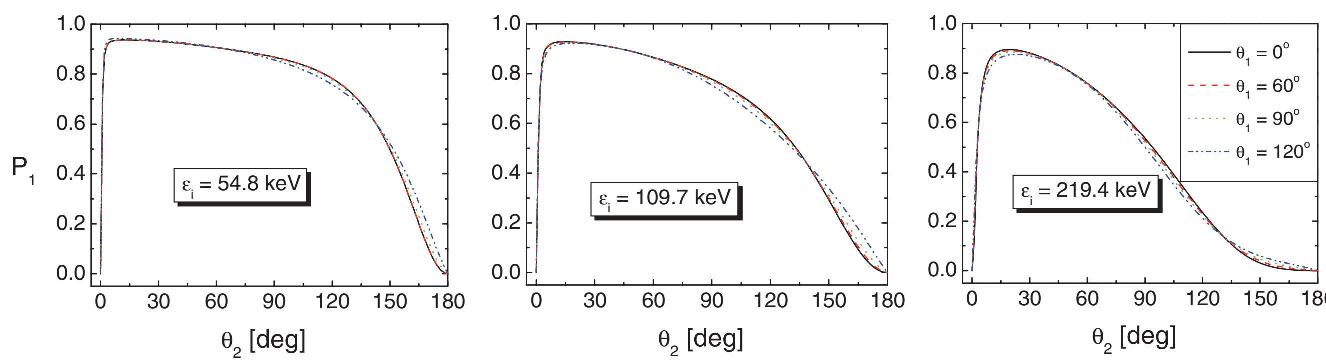

FIG. 3. (Color online) Stokes parameter P_1 for coplanar geometry ($\varphi_2 = 0^\circ$) as a function of the emission angle of the second photon θ_2. The calculations have been performed assuming no external magnetic field, $B = 0$, and for four angles of the first photon.

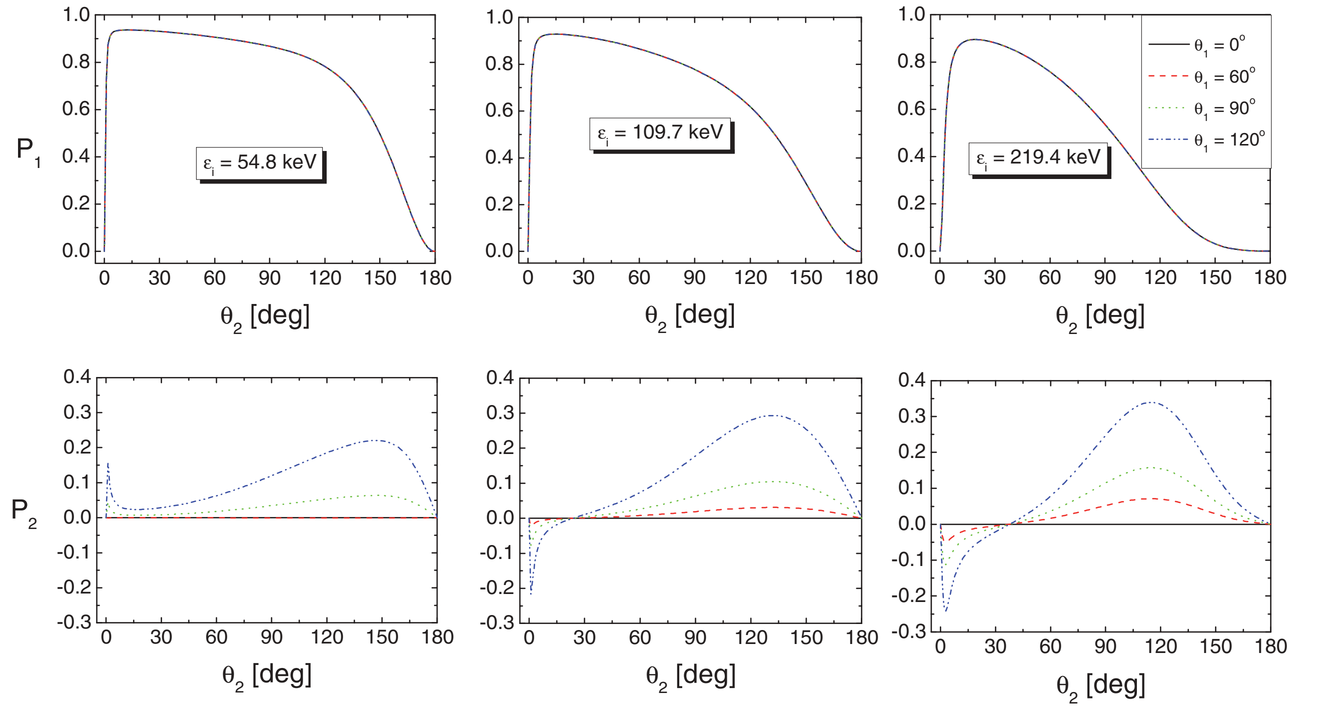

FIG. 4. (Color online) Stokes parameters P_1 (upper row) and P_2 (bottom row) as functions of the emission angle of the second photon θ_2 at $\varphi_2 = 90^\circ$. The calculations have been performed assuming no external magnetic field, $B = 0$, and for four angles of the first photon.

correlations described by Pratt and co-workers [37,38] for the recombination (and bremsstrahlung) of polarized electrons with bare unpolarized ions. Due to the fact that the electrons and ions appear rather symmetric in the recombination theory, we can use these correlations to understand the results presented in Figs. 3 and 4. In particular, it was shown by Pratt and co-workers, that RR of electrons, polarized perpendicular to the reaction plane, leads to emission of light, whose second Stokes parameter is always zero while P_1 is only slightly affected by the electron polarization. As seen from Fig. 4, this

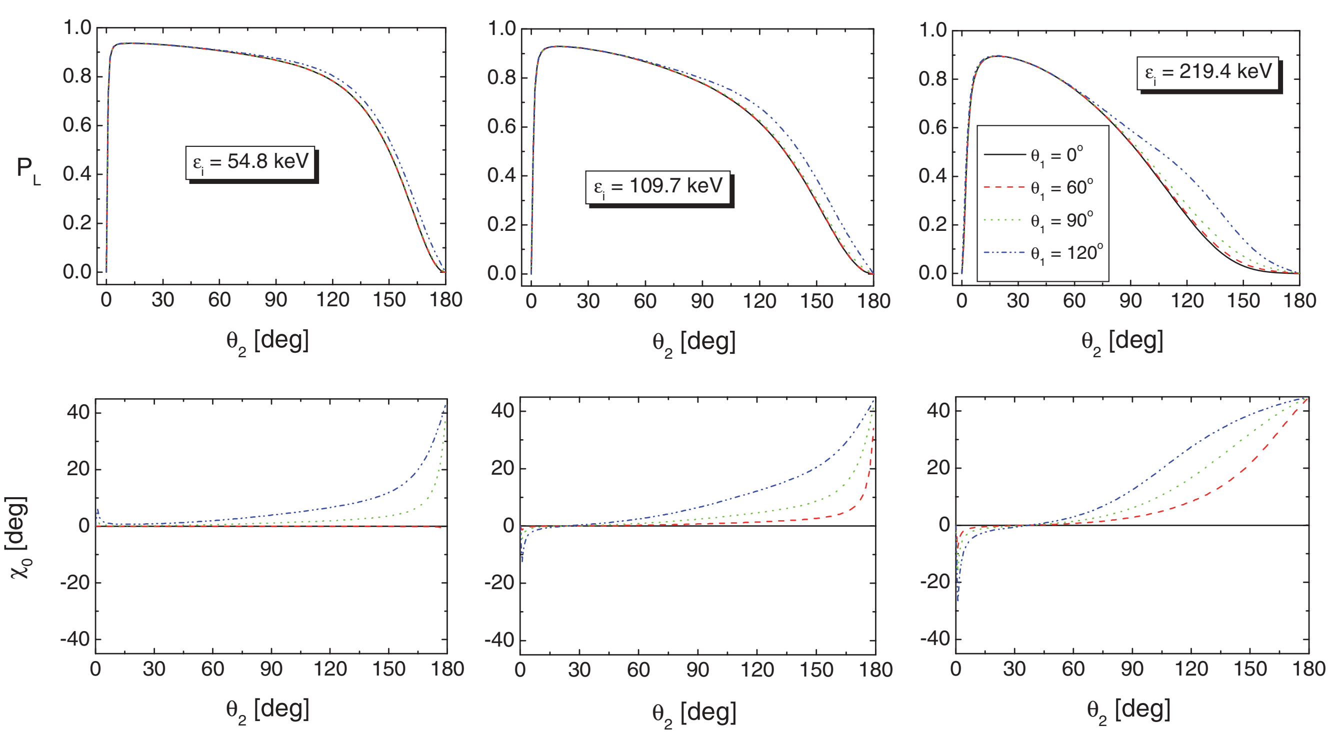

FIG. 5. (Color online) Parameter P_L (upper row) and angle χ_0 (bottom row) as functions of the emission angle of the second photon θ_2 at $\varphi_2 = 90^\circ$. The calculations have been performed assuming no external magnetic field, $B = 0$, and for four angles of the first photon.

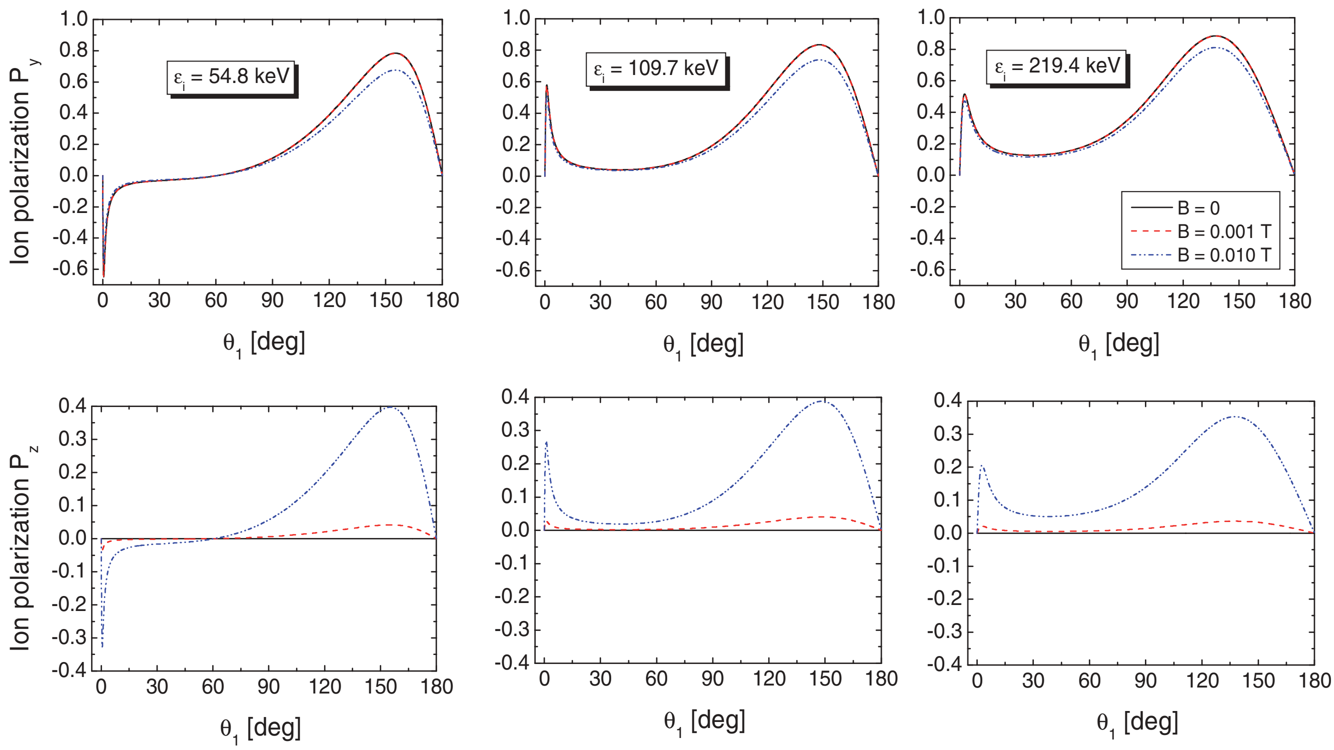

FIG. 6. (Color online) Components P_y (upper row) and P_z (bottom row) of the polarization vector of the hydrogenlike ion as a function of θ_1 in the presence of a magnetic field.

case corresponds to the coplanar geometry of the two-electron RR, where the ion spin is perpendicular to the plane of the photon emission. In contrast, for the orthogonal geometry, $\varphi_2 = 90^\circ$, the spin of the intermediate ion is within the (second) photon emission plane. From the symmetry viewpoint, such a setup corresponds to the polarization correlations between transversely polarized electrons, whose $\boldsymbol{\varepsilon}$ vector lays within the reaction plane, and linearly polarized photons [37]. As was shown by Pratt and co-workers, P_1 is independent of the electron polarization $\mathcal{P}_e$, while $P_2 \sim \mathcal{P}_e$ in such a case.

In order to better understand the influence of the intermediate ion spin state $P_y(\theta_1)$ on the polarization properties of the second RR x rays, we can express these properties in terms of polarization ellipse parameters P_L and χ_0 [see Eq. (12)]. In Fig. 5, the θ_2 dependence of the degree of polarization P_L and tilt angle χ_0 is shown again for the magnetic field-free case. As seen from the figure, χ_0 shows strong dependence on the θ_1 and, hence, on the degree of ion polarization. For example, while no rotation of the linear polarization of the second RR photons, $\chi_0 = 0^\circ$, can be observed for $\theta_1 = 0^\circ$, thus reflecting complete depolarization of the intermediate ion beam (see also Fig. 2), the tilt angle χ_0 may reach tens of degrees for $\theta_1 \simeq 120^\circ$ for which hydrogenlike projectiles appear to be significantly polarized. The strongest ion-polarization effect can be observed for the backward photon emission, $\theta_2 \gtrsim 120^\circ$, where χ_0 exceeds 20°–30°, which can be easily measured with rather high precision by means of x-ray polarimeters employing the Compton as well as Rayleigh scattering processes [9,27–36,39,40].

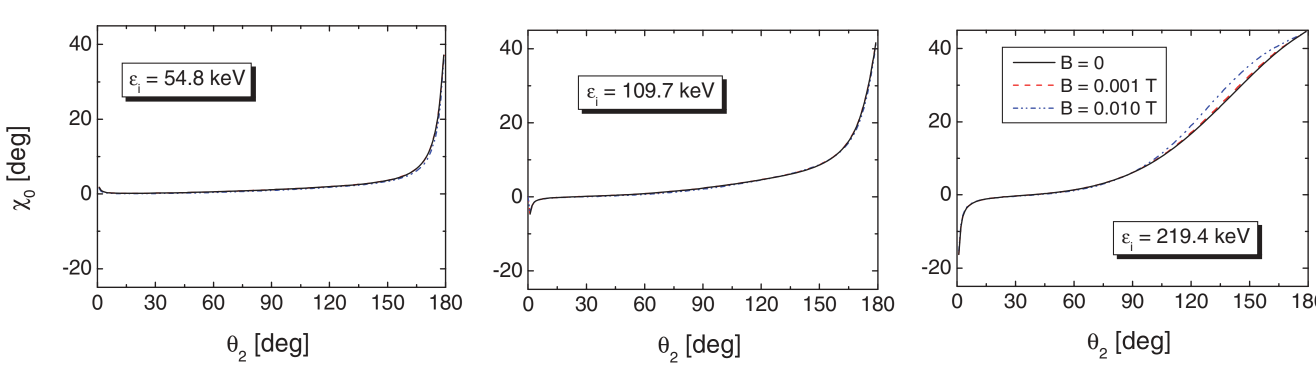

FIG. 7. (Color online) Angle χ_0 as a function of the emission angle of the second photon θ_2 at $\varphi_2 = 90^\circ$ and $\theta_1 = 90^\circ$ in the presence of a magnetic field.

Up to now we have discussed the process of two-step subsequent radiative recombination of initially bare ions in the absence of external fields. As already mentioned, if the ion between the first and the second capture processes is exposed to the external magnetic field, its polarization state will change, thus affecting the outcome of the γ-γ coincidence measurements. In order to analyze the influence of the magnetic field, we present in Fig. 6 the components P_y and P_z of polarization vector $\mathbf{P}$ of hydrogenlike ions following RR of the first electron and passage through the B-field region.

We estimate the time during which the ion is exposed to the magnetic field, as the time between the two electron bunches $\Delta t = \Delta l / v_i$, where we choose $\Delta l = 10$ cm. By using this penetration time, calculations have been performed for the magnetic field strengths $B = 0.001$ and 0.01 T and compared to the prediction for the field-free case. As seen from the figure, the magnetic field leads to a reduction of the P_y component by about 10% if first recombination photons are detected under large angles $\theta_1 \simeq 120^\circ$. In contrast, the parameter P_z increases from zero to almost 40% if B changes from $B = 0$ to $B = 0.01$.

In order to explore how the precession of the ion polarization in the external magnetic field affects the polarization properties of RR photons, emitted in the collisions with the second electron bunch, we display in Fig. 7 the tilt angle χ_0. Here, results are presented for the case of orthogonal photon detection geometry ($\theta_1 = 90^\circ$, $\varphi_2 = 90^\circ$) and, again, for three values of magnetic field strength: $B = 0$, 0.001, and 0.01 T. Based on our calculations, we argue that variation of the B field in this range does not strongly influence the tilt angle χ_0. As seen from Fig. 7, only for the largest collision energy $\varepsilon_i = 219.4$ keV, the external field effect rises to 10% (variation of the tilt angle $\triangle\chi_0 \sim 3^\circ$). This change in the tilt angle is hardly "seen" by the present-day polarization detectors. One can argue, therefore, that the presence of a weak magnetic field will not prevent observation of the angle-polarization correlations in the γ-γ coincidence experiments.

V. CONCLUSION

In the present paper we have investigated theoretically the subsequent two-electron recombination with initially bare ions having zero nuclear spin. We have shown that coincidence measurement of two RR photons emitted in such a two-step process allows one to study production and diagnostics of spin polarization of intermediate hydrogenlike ions. Most naturally, information about the ion-spin polarization can be obtained if linear polarization of the second recombination photon is observed in coincidence with the emission angle of the first one. In order to investigate in detail such angle-polarization correlations, we have employed the density matrix approach and relativistic Dirac's theory. Based on this theory, detailed calculations have been performed for the recombination of unpolarized electrons into the ground state of initially bare (finally heliumlike) uranium ions. Our calculations have demonstrated that angle-polarization correlations can be observed most easily if the second recombination photon is emitted in the plane that is perpendicular to the reaction plane, spanned by the directions of the incident electron and the first photon. For such an "orthogonal" geometry, we studied, moreover, the influence of an external magnetic field on the spin state of intermediate hydrogenlike ions. It was shown, that under the current experimental conditions this field will not significantly influence the outcome of angle-polarization correlation measurements.

The two-step recombination measurement, proposed in the present work, appears to be feasible with present-day available spectrometers at heavy-ion facilities. It may provide a first step towards experiments with spin-polarized heavy ions which are highly required today not only for the relativistic and QED-related studies but also for probing the parity-violation phenomena in a high-Z domain.

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