Jahn-Teller effect and driven binary oscillators in $\mathcal{PT}$-symmetric potentials

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The dynamics of driven binary oscillators in the $\mathcal{PT}$-symmetric harmonic potential is investigated and the corresponding analytical solutions of the time-dependent wave function are obtained. Our numerical results exhibit that there is the population oscillation, which is different from the case of normal dissipation or gain systems. After studying the motion of driven harmonic oscillators which are coupled via the $E \bigotimes e$ Jahn-Teller effect in the Hermitian potential, we find that the dynamics behavior of these two systems is similar under certain conditions. As we know, the oscillation behavior is induced by the exchange interaction in the oscillator system with Jahn-Teller coupling. Our study shows that the binary quantum systems with $\mathcal{PT}$-symmetric Hamiltonians may be used to approximately describe the dynamics of coupling quantum systems and the quantum transport process in few-particle or other systems.

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I. INTRODUCTION

Since the pioneering work of Bender *et al.* [1], the theory of $\mathcal{PT}$-symmetric quantum mechanics has been extensively studied. In such quantum systems, the non-Hermitian Hamiltonians are assumed to be invariant under simultaneous parity transform $\mathcal{P}$ and time-reversal $\mathcal{T}$, which gives rise to real eigenvalues [2]. Some significant features of $\mathcal{PT}$-symmetric systems have been discovered [3–8]. Optical systems in which the Hermiticity can be broken by arranging the refractive index and gain or loss properly have important potential applications in constructing and simulating $\mathcal{PT}$-symmetric systems [3,4]. In addition to their value in theoretical research, such $\mathcal{PT}$-symmetric optical systems have important applications in chip-scale optical isolators [9]. We found that current studies are mainly concentrated on single-particle systems. Many-particle systems offer a number of new and challenging open problems. In our paper, we shall investigate, therefore, an elementary solvable model of the dynamics of two decoupled particles moving in two mutually conjugate $\mathcal{PT}$-symmetric potentials.

As we know, the model of parametrically driven harmonic oscillators provides an opportunity to study the dynamics of a freely oscillating system when driven by an external force (field). Parametrically driven harmonic oscillators have become one of the most important physical models [10]. This is because of the long list of applications in modeling many physical systems, whether in classical mechanics or in quantum mechanics, such as circuits and idealized spring systems, quantum Brownian motion, molecular or lattice vibrations, atomic systems interacting with a light field, the motion of ions in a Paul trap, and so on. Actually, large numbers of physical situations can be reduced to it either exactly or approximately. Such a model, although it is simple, can be used to interpret the fundamental features of the dynamical behavior. To study the property of a two-particle system with $\mathcal{PT}$-symmetric Hamiltonians, the model of parametrically driven oscillators in a harmonic potential is adopted as a working example. We analyze the dynamics behavior of the two oscillators driven by an external field in the $\mathcal{PT}$-symmetric harmonic potential. Nonconservation and oscillation behavior of the probability have been observed and relate with the form of applied fields. In the meantime, we investigate the motion of the harmonic oscillators coupled via the $E \bigotimes e$ Jahn-Teller effect and find that the population will be oscillating owing to the interaction of particle exchange. By comparing with the properties of these two systems, we find that the dynamics behavior of harmonic oscillators in the two different systems above is strikingly similar under some conditions, which means that the system with $\mathcal{PT}$-symmetric potential is in profound association with the one with Hermitian potential. Our results show that $\mathcal{PT}$-symmetric quantum theory has a potential application in the study of the dynamics of coupling quantum systems and the quantum transport process in few-particle or other systems.

The content of the paper is organized as follows: We first derive the analytic solutions of the driven oscillators in the $\mathcal{PT}$-symmetric potential in Sec. II. In Sec. III, the dynamics of harmonic oscillators coupled via the Jahn-Teller effect is researched and is compared with the results in the former model. We find that the dynamical behaviors of two different systems are similar under some conditions. The reason for the similar behaviors is explained in Sec IV. At the same time, we show how to realize our $\mathcal{PT}$-symmetric systems in this section. Finally, we present a brief summation. Some detailed derivations are appended.

II. DRIVEN OSCILLATORS IN $\mathcal{PT}$-SYMMETRIC POTENTIAL

In order to clarify the relationship of these two systems, we begin our analysis by considering the dynamics of the driven oscillators in the $\mathcal{PT}$-symmetric harmonic potential. For the system that we considered here, the equations of motion of wave functions Φ_a and Φ_b corresponding to harmonic oscillators a and b take the following forms [11] ($\hbar = 1$ and

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the masses of the oscillators are all unitary):

$$i\frac{\partial}{\partial t}\Phi_{a,b}=[\mathcal{H}_{a0,b0}-\hat{x}f(t)]\Phi_{a,b},$$
$$\mathcal{H}_{a0,b0}=\frac{\hat{p}^2}{2}+\frac{\omega^2}{2}\hat{x}^2\pm ig\hat{x}-\frac{g^2}{2\omega^2}. \quad (1)$$

Here, $\mathcal{H}_{a0,b0}$ denotes the Hamiltonians of the two oscillators a and b in the $\mathcal{PT}$-symmetric harmonic potential, which addresses the unperturbed part of the system Hamiltonian. $-\hat{x}f(t)$ represents the dipole interaction part. ω is the eigenfrequency of the harmonic oscillator, g is a factor that denotes the magnitude of the non-Hermitian term, and $f(t)$ is the time-dependent function of the external field. In writing the above $\mathcal{PT}$-symmetric Hamiltonians, we have assumed that the trapped potential of these two harmonic oscillators is chirally symmetric, namely, $\mathcal{H}_{b0}=\mathcal{P}\mathcal{H}_{a0}$. We can see that $\mathcal{H}_{a0,b0}$ is $\mathcal{PT}$ symmetric, which exhibits real spectrum. It is straightforwardly calculated that the spectrum is $E_n=(n+1/2)\omega$, in which the corresponding eigenfunction is $\tilde{\Phi}_{a,b}=H_n[\sqrt{\omega}(x\pm\frac{ig}{\omega^2})]\exp[-\omega(x\pm\frac{ig}{\omega^2})^2/2]=\exp(\mp\frac{g}{\omega^2}\hat{p})|n\rangle$. Here "plus" and "minus" are for the harmonic oscillators a and b, respectively. H_n is the nth-order Hermite polynomial and $|n\rangle=H_n[\sqrt{\omega}x]\exp[-\omega x^2/2]$ denotes the wave function of the number state of the Hermitian harmonic oscillator. The operator $S(\pm\frac{ig}{\omega^2})=\exp(\mp\frac{g}{\omega^2}\hat{p})$ is nonunitary, which displaces the wave packet in the position of the x direction by the amount $\pm\frac{ig}{\omega^2}$. It is interesting that eigenfunctions $\tilde{\Phi}_{a,b}$ with $n=0$ correspond to the wave functions of coherent states in "x" space.

These equations of motion in Eq. (1) can be solved exactly by the method introduced in Ref. [12]. If the harmonic oscillators are initially in the eigenstates with energy E_n, then the exact solutions for wave function Φ_a and Φ_b with respect to time can be worked out to be

$$\Phi^n_{a,b}(x,t)=C_n\exp\left[-\frac{\omega\left(x\pm\frac{ig}{\omega^2}-\beta\right)^2}{2}\right]\times\exp\left\{i\left[\frac{\partial\beta}{\partial t}\left(x\pm\frac{ig}{\omega^2}-\beta\right)+\int_0^t dt'\mathcal{L}'_{a,b}(t')\right]\right\},$$
$$\frac{\partial^2}{\partial t^2}\beta+\omega^2\beta=f(t),\quad C_n=C_0H_n\left[\sqrt{\omega}\left(x\pm\frac{ig}{\omega^2}-\beta\right)\right],$$
$$\mathcal{L}'_{a,b}=\frac{1}{2}\left(\frac{\partial\beta}{\partial t}\right)^2-\frac{\omega^2}{2}\beta^2+f(t)\left(\beta\mp\frac{ig}{\omega^2}\right), \quad (2)$$

where $\frac{\partial\beta}{\partial t}(0)=\beta(0)=f(0)=0$, and C_0 is a normalization constant at $t=0$. The detailed derivation is shown in the AppendixA. It should be noticed here that the initial wave function is normalized in the conventional Dirac way using the standard Hilbert-space inner product, $\langle\Phi_{a,b}(x,0)|\Phi_{a,b}(x,0)\rangle$, instead of the new inner product for the $\mathcal{PT}$-symmetric quantum theory [13]. Inspecting the solution given by expressions (2), we can see that the equation obeyed by β describes the motion of a classical harmonic oscillator driven by an external field $f(t)$. If g is equal to zero, then $\mathcal{L}'_{a,b}$ corresponds to the classical Lagrangian of a driven harmonic oscillator. By further

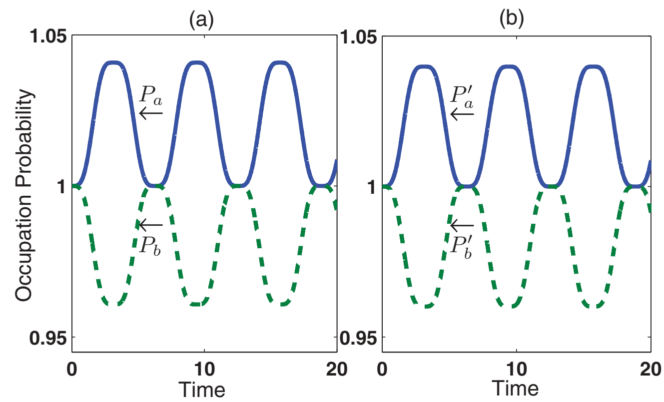

FIG. 1. (Color online) Occupation probability with respect to time. (a) Solutions of Eq. (1) with the initial state $\Phi_{a,b}(x,0)=(\frac{\omega}{\pi})^{1/4}\exp[-\frac{g^2}{2\omega^3}]\exp[-\omega(x\pm\frac{ig}{\omega^2})^2/2]$; (b) solutions of Eq. (5) with the initial state $\Psi_{a,b}(x,0)=(\frac{\omega}{\pi})^{1/4}\exp[-\omega x^2/2]$. The parameters for (a) and (b) are $\omega=1$, $f(t)=\sin(3t)$, and $g=g'=0.03$.

calculating, it is easy to achieve the occupation probability of these two harmonic oscillators, which is expressed as

$$P_{a,b}=\int_{-\infty}^{+\infty}|\Phi_{a,b}(x,t)|^2dx=\exp\left[\pm 2g\int_0^t\beta(t')dt'\right]. \quad (3)$$

Such a definition is convenient to experimentally verify if one uses an optical system to simulate the dynamics of our model in Eq. (1), which corresponds to the transverse intensity of the laser beam. The details of the optical simulation system will be discussed in Sec. IV. The above expression shows that the occupation probability $P_{a,b}$ becomes nonconservative if $f(t)\neq 0$. In such a case, the Hamiltonians are with broken $\mathcal{PT}$ symmetry. Recently, a similar feature has been reported in an optical structure with $\mathcal{PT}$-symmetry breaking [14]. The dissipation and gain behavior was shown in Figs. 1(a) and 2(a). The behavior can be distinguished as follows. Utilizing the displacement operator $S(\pm\frac{ig}{\omega^2})$, we get a transformed Hamiltonian $\bar{\mathcal{H}}_{a0,b0}=S^\dagger(\pm\frac{ig}{\omega^2})\mathcal{H}_{a0,b0}S(\pm\frac{ig}{\omega^2})=\frac{\hat{p}^2}{2}+\frac{\omega^2}{2}\hat{x}^2-\hat{x}f(t)\pm igf(t)=\bar{\mathcal{H}}_0(t)\pm igf(t)$ with a Hermitian part $\bar{\mathcal{H}}_0(t)$. The corresponding instantaneous evolution operator is thus $\exp[-i\bar{\mathcal{H}}_0(t)dt\pm gf(t)dt]$. It is easily seen that the real part $\pm g\int_0^t f(t')dt'$ in the instantaneous evolution operator directly brings on occupation probability with exponential increase or exponential decline. Thus, the oscillation

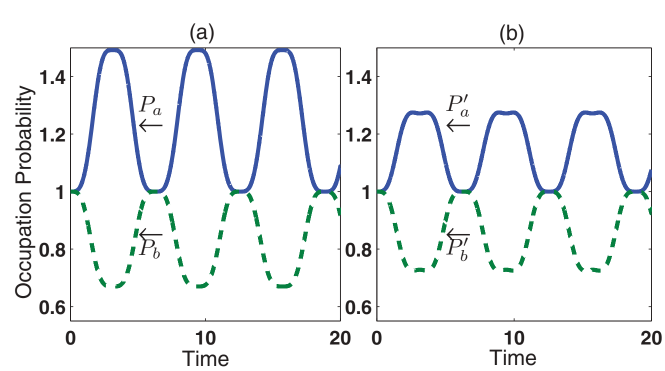

FIG. 2. (Color online) Occupation probability with respect to time. Same as Fig. 1, but for $g=g'=0.3$.

behavior of the occupation probability may be understood. In addition, the nonunitary operator $S(\pm\frac{ig}{\omega^2})$ is also a key ingredient of the oscillation behaviors of the occupation probability because the normalization constant in Eq. (2) is related to the displacement parameter g. This is also consistent with the fact that the occupation probability of the system with a non-Hermitian Hamiltonian is nonconservative. The increase and decline behavior depends on the form of $f(t)$, which differs from the normal case in amplification and loss media. Figures 1(a) and 2(a) depict the value of occupation probability $P_{a,b}$ at time t in different conditions. These figures show that the occupation probability is oscillating, which is greatly different from the ordinary decline (increase) behavior of dissipative (gain) systems. It is the same as the existence of the exchange of occupation probability between quantum oscillators and "surroundings" in our case. The $\mathcal{PT}$-symmetric system under perturbation should be treated as a pseudoclosed system. The more interesting thing is that the total occupation probability $P_a + P_b$ looks like conservation when g is small [shown in Fig. 1(a)]. In other words, such binary quantum systems with chirally symmetric Hamiltonians can be used to approximatively describe the system with a conservation particle in some conditions. A further discussion will be given in Sec. IV.

For optical and atomic physics, the dipole moment is an important parameter. Here the same concept is adopted. The dipole moment of the harmonic oscillator is defined as $D = \int_{-\infty}^{+\infty} x|\Phi(x,t)|^2 dx$, which expresses the location of the centroid of the harmonic oscillator. By substituting the expression for wave function $\Phi_{a,b}$ in Eq. (2) into that definition, the analytic expressions of the dipole moment for the harmonic oscillator a and b at time t are obtained:

$$D_{a,b} = \beta \exp\left[\pm 2g \int_0^t \beta(t')dt'\right]. \qquad (4)$$

As an illustration, the dipole moment with respect to time t is depicted in Figs. 3(a) and 4(a), which clearly indicates that the non-Hermitian term also trivially affects the instantaneous dipole moment if g is small enough. Hereto, we have studied the dynamics of driven harmonic oscillators in the $\mathcal{PT}$-symmetric potential. As a consequence, the non-Hermitian terms induce an exchange of quantized particles and impact the instantaneous location of the centroid of the harmonic oscillator. On the other hand, we see that the total of occupation probability $P_a + P_b$ can be considered approximatively as conservation under certain conditions.

III. HARMONIC OSCILLATORS COUPLED VIA JAHN-TELLER EFFECT

Subsequently, let us discuss the field-driven dynamics of two harmonic oscillators coupled via the Jahn-Teller effect in a Hermitian potential. The Jahn-Teller effect is usually used to describe the interaction of lattice vibrational modes with degenerate electronic states [15]. This effect is indispensable for a proper understanding of the physics of a variety of molecular systems, such as paramagnetic ions in nonmagnetic crystals [15], superconductivity in the fullerides [16], structural phase transitions [17], and so on. The motions of the harmonic oscillators coupled via the $E \otimes e$ Jahn-Teller effect and driven by an external field obey the following equations [15,18]:

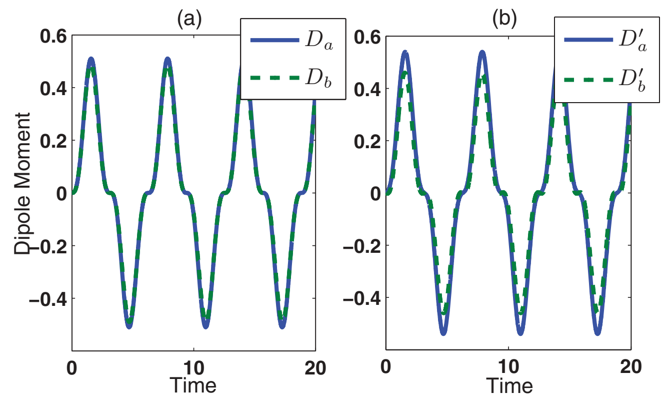

FIG. 3. (Color online) Dipole moment with respect to time. (a) and (b) are the solutions of Eqs. (1) and (5), respectively. The parameters are the same as Fig. 1.

$$\begin{aligned} i\frac{\partial}{\partial t}\Psi_a &= \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{2}x^2 - xf(t)\right]\Psi_a + ig'x\Psi_b,\\ i\frac{\partial}{\partial t}\Psi_b &= \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{2}x^2 - xf(t)\right]\Psi_b - ig'x\Psi_a. \end{aligned} \qquad (5)$$

The terms of spatial-dependent coupling in the right-hand side of the above equations are caused by the $E \otimes e$ Jahn-Teller effect. The factor g' has a clear physical meaning in Eq. (5), which describes the strength of coupling. As a matter of fact, the similar spatial-dependent coupling can exist in other systems, such as in a Bose-Einstein condensate [19] and in a superfluid [20]. It is clear that for this coupling system via the $E \otimes e$ Jahn-Teller effect, the total occupation probability $P'_a + P'_b$ is conservational [here, $P'_{a,b} = \int_{-\infty}^{+\infty} |\Psi_{a,b}(x,t)|^2 dx$]. Inspecting Eqs. (1) and (5), the non-Hermitian terms in Eq. (1) are replaced just by the terms of the Jahn-Teller coupling. The oscillatory behavior of the occupation probability is a prevalent speciality for coupled quantum systems, which is similar to the former system. The results of numerical investigation of this expected phenomenon are illustrated in Figs. 1(b) and 2(b). In addition, we can analogously define the dipole moment of the above system as $D'_{a,b} = \int_{-\infty}^{+\infty} x|\Psi_{a,b}(x,t)|^2 dx$, for which the

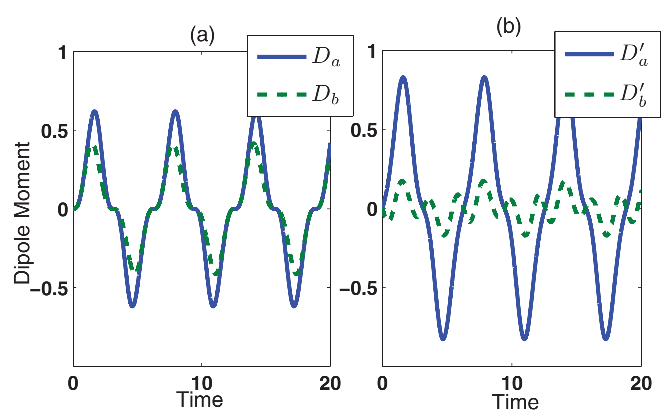

FIG. 4. (Color online) Dipole moment with respect to time. Same as Fig. 3, but for $g = g' = 0.3$.

numerical results are given in Figs. 3(b) and 4(b). Observing Figs. 1 and 3, we can conclude that the systems described by Eqs. (1) and (5) have strikingly similar dynamics behavior when the factors g and g' are small. Relying on these results, the factor g in Eq. (1) can be considered as the coupling strength of the harmonic-oscillator system and "surroundings" in a certain sense. In fact, expression (3) clearly indicates that g is related to the dissipation (gain) coefficient of the occupation probability of the harmonic oscillator in the $\mathcal{PT}$-symmetric potential.

IV. ANALYSIS

One naturally asks why the dynamics behavior of these two different systems is similar in some cases. In order to give a physical interpretation, we introduce the Lagrangian density of Eq. (5), which is given by

$$\mathcal{L} = \sum_{i=a,b}\left[i(\Psi_i\Psi_{i,t}^* - \Psi_i^*\Psi_{i,t}) + \frac{1}{2}|\Psi_{i,x}|^2\right] + \sum_{i=a,b}\left\{\left[\frac{\omega^2}{2}x^2 - xf(t)\right]|\Psi_i|^2\right\} + igx(\Psi_a^*\Psi_b - \Psi_a\Psi_b^*). \tag{6}$$

Here, $\Psi_{i,t} = \frac{\partial\Psi_i}{\partial t}$ and $\Psi_{i,x} = \frac{\partial\Psi_i}{\partial x}$. The crossover term $igx(\Psi_a^*\Psi_b - \Psi_a\Psi_b^*)$ is used to describe the particle exchange between waves Ψ_a and Ψ_b. At the same time, it is easy to find that this crossover term is Hermitian, which guarantees the conservation of the total probability. When a time-dependent driven field is applied, the balance of the population distributions between these two waves will be broken. It is the reason for the oscillation behavior. Considering this equation with the corresponding Lagrange equation,

$$\frac{\partial}{\partial t}\frac{\partial\mathcal{L}}{\partial\Psi_{i,t}^*} + \frac{\partial}{\partial x}\frac{\partial\mathcal{L}}{\partial\Psi_{i,x}^*} - \frac{\partial\mathcal{L}}{\partial\Psi_i^*} = 0, \tag{7}$$

one obtains Eq. (5). In the same way, the Lagrangian density, which corresponds to the $\mathcal{PT}$-symmetric systems that we considered in the former, is expressed as

$$\mathcal{L}' = \sum_{i=a,b}\left[i(\Phi_i\Psi_{i,t}^* - \Phi_i^*\Phi_{i,t}) + \frac{1}{2}|\Phi_{i,x}|^2\right] + \sum_{i=a,b}\left\{\left[\frac{\omega^2}{2}x^2 - xf(t) - \frac{g}{2\omega^2}\right]|\Phi_i|^2\right\} + igx(|\Phi_a|^2 - |\Phi_b|^2). \tag{8}$$

Inspecting Eqs. (1) and (5), there is a small difference between these two expressions. It is obvious that the Lagrange density (8) is non-Hermitian. In other words, the total probability of such a system is nonconservative. The term igx in Eq. (1) could be considered as a direct-current field with an imaginary amplitude. The wave functions then are approximated to be $\Phi_{a,b}(x,t+\delta t) = \exp\{[\pm gx + ixf(t)]\delta t\}\exp[-i(\frac{p^2}{2} + \frac{\omega^2}{2}x^2 - \frac{g}{2\omega^2})\delta t]\Phi_{a,b}(x,t)$ after a short-time evolution. By substituting the new evolute function $\Phi_{a,b}(t+\delta t)$ into the last term in Eq. (8), we can obtain the imaginary part of the Lagrange density, $gx[e^{2gx\delta t}|\Phi_a(t)|^2 - e^{-2gx\delta t}|\Phi_b(t)|^2]$. This term is equal to zero only at some special moments, such as at the initial time. It means the nonconservation of the total population with time evolution. When the non-Hermitian factor g and the driven field $f(t)$ are small enough, this imaginary part can be approximated to zero. This case corresponds to a system with particle conservation. In addition, the wave packets can be assumed to move as a whole and the position shift is set to be Δ when $f(t)$ is small. The absolute values of the wave functions then come to be $|\Phi_{a,b}(x,t+\delta t)| = \exp(\pm gx\delta t)|\Phi_{a,b}(x+\Delta,t)|$. Further, we get the population $P_{a,b}(t+\delta t) = \exp(\mp 2g\Delta\delta t)\int dx\exp(\pm 2gx\delta t)|\Phi_{a,b}(x,t)|^2$. From this expression, we can see that the signs of Δ and g determine the gain or loss property of the population. If the driven field has the form $\sin(at)$, then the shift Δ will oscillate around zero point. So the oscillation behavior of population $|\Phi_{a,b}|$ appears. As analyzed above, the oscillator models in the $\mathcal{PT}$-symmetric harmonic potential in our case may be looked upon as the approximate model of harmonic oscillators coupled via the Jahn-Teller effect in a Hermitian potential. Figures 1 and 3 indicate that such an approximation is quite effective for the case of weak coupling. However, with the increasing of g, the non-Hermitian term will become larger, which leads to the different dynamics behavior [see Figs. 2(b) and 4(b)]. To sum up, the system with the $\mathcal{PT}$-symmetric Hamiltonian is associated with the coupling system. The former can be considered as the approximate model of the latter under certain conditions, which provides a potential application of $\mathcal{PT}$-symmetric Hamiltonians.

In the following, we will demonstrate that the dynamics of quantum systems with $\mathcal{PT}$ symmetry may be simulated by optical beam propagation in complex optical potentials in which the special distributions of the refractive index and gain or loss are properly arranged [21]. It is helpful to carry out the corresponding experimental study. The complex refractive-index distribution always can be expressed as $n_0 + n_r(X,Z) + in_i(X,Z)$ with $n_0 \gg |n_{r,i}|$. Here, n_0 and n_r present the background refractive index and the perturbation of the refractive index, respectively, and n_i corresponds to the gain or loss phenomena. Assuming that the optical beam propagates along the Z axis, and X is the direction of transverse diffraction, as well as introducing the following scaled quantities: $x = X/x_0$, $z = Z/(k_0x_0^2)$ with wave number $k_0 = n_0\omega_0/c$, $U(x,z) = x_0^2k_0^2n_r(x,z)/n_0$, $V(x,z) = x_0^2k_0^2n_i(x,z)/n_0$, the resulting normalized equation of diffraction is $i\frac{\partial}{\partial z}\Phi = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Phi - U(x)\Phi - iV(x,z)\Phi$. Here, x_0 is an arbitrary scaling factor and Φ means the profile of the laser beam in the x-z (X-Z) plane. If $U(x,z) = \frac{\omega^2}{2}x^2 - \hat{x}f(z) - \frac{g^2}{2\omega^2}$ and $V(x,z) = \pm gx$, then the above equation is analogous to the time-dependent Schrödinger equation (1) in the former. So the variable novel behaviors discussed in the former can be verified with the help of such an optical structure.

V. CONCLUSIONS

In summary, we investigated the dynamics of driven binary oscillators in the $\mathcal{PT}$-symmetric harmonic potential. There is the oscillation behavior of the population, which is different from the dissipation or gain behavior in normal systems. At the same time, we studied the motion of the driven harmonic oscillators coupled via the $E\otimes e$ Jahn-Teller effect and observed that the population distribution was also oscillating owing to the interaction of particle exchange. By comparing the dynamics of these two systems, we found that the system with $\mathcal{PT}$-symmetric potential may be considered as the approximation of the coupled system with a Hermitian potential under certain conditions, and the corresponding explanation was given. In order to carry out the experimental work, we proposed an alternative experiment scheme to simulate such a $\mathcal{PT}$-symmetric quantum system.

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APPENDIX: DERIVATION OF THE EXPRESSIONS IN SEC. II

The Schrödinger equation which the harmonic oscillator a obeys is written as the following expression:

$$i\frac{\partial}{\partial t}\Phi_a = \left[\frac{\hat{p}^2}{2} + \frac{\omega^2}{2}\hat{x}^2 - (f(t) - ig)\hat{x} - \frac{g^2}{2\omega^2}\right]\Phi_a. \quad \text{(A1)}$$

By introducing the shifted coordinate $y = x - \xi(t)$ and substituting that into Eq. (A1), we can get

$$i\frac{\partial}{\partial t}\Phi_a(y,t) = \left[i\frac{\partial\xi}{\partial t}\frac{\partial}{\partial y} - \frac{1}{2}\frac{\partial^2}{\partial y^2}\right]\Phi_a(y,t) + \left[\frac{\omega^2}{2}(y+\xi)^2 - (y+\xi)(f(z) - ig)\right]\Phi_a(y,t). \quad \text{(A2)}$$

We then employ the transform $\Phi_a(y,t) = \exp[i\frac{\partial\xi}{\partial t}y]\Theta_a(y,z)$ with $\xi(t)$ obeying the equation of motion:

$$\frac{\partial^2}{\partial t^2}\xi + \omega^2\xi = f(t) - ig.$$

It yields

$$i\frac{\partial}{\partial t}\Theta(y,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial y^2} + \frac{\omega^2}{2}y^2 - \mathcal{L}'\right]\Theta(y,z).$$

Here, $\mathcal{L}' = \frac{1}{2}(\frac{\partial\xi}{\partial t})^2 - \frac{\omega^2}{2}\xi^2 + (f(t) - ig)\xi$.

In order to reduce the Schrödinger equation above, the transformation $\Theta_a(y,t) = \exp[i\int_0^t dt'\mathcal{L}'(t')]\Psi(y,t)$ is used. We then obtain the Schrödinger equation of the harmonic oscillator,

$$i\frac{\partial}{\partial t}\Psi_a(y,t) = \left[-\frac{1}{2}\frac{\partial^2}{\partial y^2} + \frac{\omega^2}{2}y^2\right]\Psi(y,t).$$

The eigenfunctions can be solved and written as

$$\Psi_n(y) = \sqrt{\frac{1}{2^n n!}\sqrt{\frac{\omega}{\pi}}}\exp\left(-\frac{\omega y^2}{2}\right)H_n(\sqrt{\omega}y).$$

The corresponding eigenvalue is $E_n = \omega(n + \frac{1}{2})$.

If the initial distribution of the wave function is equal to the eigenfunction $\Psi_n(y)$, then the solution for Eq. (A1) is of the form

$$\Phi_a(x,t) = \sqrt{\frac{1}{2^n n!}}\left(\frac{\omega}{\pi}\right)^{1/4} H_n[\sqrt{\omega}(x-\xi)] \times \exp\left[-\frac{\omega}{2}(x-\xi)^2 - \frac{\partial\zeta}{\partial t}(x-\xi)\right] \times \exp\left[-i\omega(n+1/2)t - i\int_0^t dt'\mathcal{L}'_a(t')\right].$$

By utilizing the shift transform $\xi = \beta - \frac{ig}{\omega^2}$, we will obtain the final solution of Eq. (1),

$$\Phi_a^n(x,t) = C_n \exp\left[-\frac{\omega\left(x + \frac{ig}{\omega^2} - \beta\right)^2}{2}\right] \times \exp\left\{i\left[\frac{\partial\beta}{\partial t}\left(x + \frac{ig}{\omega^2} - \beta\right) + \int_0^t dt'\mathcal{L}'_{a,b}(t')\right]\right\},$$

$$\frac{\partial^2}{\partial t^2}\beta + \omega^2\beta = f(t), \quad C_n = C_0 H_n\left[\sqrt{\omega}\left(x + \frac{ig}{\omega^2} - \beta\right)\right],$$

$$\mathcal{L}'_a = \frac{1}{2}\left(\frac{\partial\beta}{\partial t}\right)^2 - \frac{\omega^2}{2}\beta^2 + f(t)\left(\beta + \frac{ig}{\omega^2}\right), \quad \text{(A3)}$$

where C_0 is the normalized constant when $t = 0$. It should be noted that the value of C_0 depends on the label n. At the same time, the initial condition to solve the derivative equation obeyed by β is $\frac{\partial\beta}{\partial t}(0) = \beta(0) = f(0) = 0$.

When the initial wave function of harmonic oscillator a is $\Psi_0(x) = (\frac{\omega}{\pi})^{1/4}\exp(-\frac{\omega y^2}{2} - \frac{igx}{\omega})$ and the corresponding energy is $E_0 = \omega/2$, the solution has the form

$$\Phi_a^0(x,t) = C\exp\left[-\frac{\omega(x + \frac{ig}{\omega^2} - \beta)^2}{2}\right] \times \exp\left\{i\left[\frac{\partial\beta}{\partial t}\left(x + \frac{ig}{\omega^2} - \beta\right) + \int_0^t dt'\mathcal{L}'_a(t')\right]\right\}. \quad \text{(A4)}$$

Here, $C = (\frac{\omega}{\pi})^{1/4}\exp[-\frac{g^2}{2\omega^3}]$. It is straightforward to obtain the occupation probability P_a of harmonic oscillator a with respect to time with the help of Eq. (A4), which is

$$P_a = \int_{-\infty}^{+\infty}\Phi_a^0(x,t)\Phi_a^{0*}(x,t)dx = \exp\left\{\frac{2g}{\omega^2}\left[\int_0^t f(t')dt - \frac{\partial\beta}{\partial t}\right]\right\} = \exp\left[2g\int_0^t \beta(t')dt'\right]. \quad \text{(A5)}$$

In the derivation above, the equation of motion of β and the corresponding initial conditions are used. We then achieve the dipole moment $D_a = \beta \exp[2g \int_0^t \beta(t')dt']$ in the same way.

It is straightforward to get the wave function Φ_b, probability P_b, and dipole momentum D_b of the harmonic oscillator b by changing the sign of g.

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